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STABILITY OF ZEROS OF DISCRETE-TIME MULTIVARIABLE SYSTEMS WITH GSHF

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Abstract. *This paper is concerned with the stability of zeros of the discrete-time multivariable system composed of a generalized sample hold function (GSHF), a continuous-time plant with the degrees of infinite elementary divisors being two or three, i.e.,* $\mu_1 = \cdots = \mu_{m-k} = 2$ *and* $\mu_{m-k+1} = \cdots = \mu_m = 3$ $(k = 1, \ldots, m,\ 0 \leq k \leq m-1)$*, and a sampler in cascade. The properties of the limiting zeros are studied and conditions for ensuring the stable limiting zeros of the discrete-time systems for sufficiently small sampling periods are derived. It is a natural extension of Liang et al.'s result for single-input-single-output systems to multivariable systems.*

Keywords: Zeros, Stability, Generalized sampled hold function, MIMO, Discrete-time system

1. **Introduction.** It is well known that unstable zeros limit the performance that can be achieved. Some techniques for control system design are hard to be applied when a plant has unstable zeros [1, 2, 3]. Therefore, the properties of discrete-time zeros have received considerable attention from researchers [4, 5, 6, 7, 8, 9]. Some results therein showed the corresponding discrete-time systems arising from ZOH may have unstable discretization zeros when continuous-time systems with the relative degree $p \geq 2$. On the other hand, it was shown that the discretisation zeros can be placed inside the unit circle by the parameters of generalized sample hold function (GSHF) even if relative degree $p \geq 2$ [16, 17]. Moreover, the properties of zeros of the discrete-time systems with a piecewise constant GSHF, including the stability conditions, were presented. However, the properties of zeros of the discrete-time systems with GSHF are discussed mainly on SISO systems [17].

The properties of the zeros for multivariable systems are characterized by the degrees of the infinite elementary divisors $\mu_1, \ldots, \mu_m$ $(\mu_1 \leq \cdots \leq \mu_m)$ [10] of a system matrix. It was shown [11] that ÅSTRÖM *et al.*'s result [4] holds in the case of square multivariable systems when the difference between the largest and the smallest degrees of the infinite elementary divisors of the underlying continuous-time system is less than two, i.e., $\mu_m - \mu_1 < 2$. Weller [12] also demonstrated ÅSTRÖM *et al.*'s result [4] can be extended directly to decouplable MIMO systems. Hayakawa *et al.* [11] showed that if all the degrees of the infinite elementary divisors of a continuous-time system matrix are two, i.e., $\mu_1 = \cdots = \mu_m = 2$, and all zeros of the continuous-time system are stable, then