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PERFORMANCE ANALYSIS OF THE SLEEP/WAKEUP PROTOCOL IN A WIRELESS SENSOR NETWORK

Shunfu Jin[1], Wuyi Yue[2] and Qingtian Sun[1]

[1]College of Information Science and Engineering
Yanshan University
No. 438, Hebei Avenue, Qinhuangdao 066004, P. R. China
jsf@ysu.edu.cn

[2]Department of Intelligence and Informatics
Konan University
8-9-1 Okamoto 12, Higashinada-ku, Kobe 658-8501, Japan
yue@konan-u.ac.jp

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Abstract. *To conserve the energy of sensor nodes in Wireless Sensor Networks (WSNs), a sleep/wakeup protocol is introduced in IEEE 802.15.4. Performance analyses of such queueing systems have been done during the past years. However, most of these analyses are based on continuous-time models and the switching procedure from the sleep mode to the active mode is neglected in order to simplify the process of analysis. In this paper, we study a one-hop WSN system for digital transmission with a setup to capture the working principle of such a sleep/wakeup protocol as a discrete-time multiple vacation queueing model and also consider the WSN system by taking into account the switching procedure between the sleep mode and the active mode at sensor nodes. We present a new effective analysis method to give the formulas for performance measures in terms of the handover ratio, the average latency of data frames and the average energy consumption of the sensor node. We also present numerical results to discuss the relationship between the system performance and the modulation constellation size. In addition, we construct a cost function to give the optimal values of the modulation constellation size for minimizing the energy consumption of the sensor node.*
Keywords: Wireless sensor network, IEEE 802.15.4, Sleep/wakeup, Performance measures, Modulation constellation size, Queueing model, Setup

1. Introduction.

With the development of microsensor and microelectronic technology in parallel with wireless communication, the research into wireless sensor networks (WSNs) has attracted significant attention [1, 2]. For the characteristics of self-organization, microsizing, low-cost and flexibility, WSNs are being applied in many fields, such as military, environmental science, medical and health, space exploration, and commerce [3]. A WSN is, however, an energy-constrained system.

The sensor node in a WSN is a micro-embedded device, which typically carries only limited battery power [4]. Large numbers of sensor nodes are densely deployed over a wide region for monitoring complex environments. In some regions, replacement or recharging of the battery is impossible. This necessitates the designing of a low-powered mechanism which will minimize energy consumption. Therefore, for the design and tuning of an efficient modulation strategy, we must mathematically analyze and numerically evaluate the system performance. We also must investigate the effectiveness of the energy use and then optimize the system parameters to minimize the energy consumption.

On the other hand, data latency is also an important factor affecting user Quality of Service (QoS), and cannot be neglected when designing a practical system. With these in mind, some mathematical research models that faithfully reproduce the behavior of the WSNs and the system performance have been presented.

In [5], a data communication and aggregation framework is presented, in which the degree of data aggregation is manipulated to maintain specified acceptable latency bounds on data delivery while attempting to minimize energy consumption. An analytic model is constructed to describe the relationships between timeliness, energy consumption and the degree of aggregation. In [6], for satisfying a given throughput and delay requirement, considering the delay and peak-power constraints, a new modulation strategy is given to minimize the total energy consumption. In [7], to improve the performance of IEEE 802.15.4 beacon-enabled network, an improved mechanism called Scan First 3 Channels (SF3C) is developed to reduce the communicating frequency among devices and Personal Area Network (PAN) coordinator, and the other mechanism called Random Prime Double Hash (RPDH) is used to avoid the problem of channels collision. The performance of the proposed mechanisms is evaluated in the simple star topology based on the NS2 simulator.

In [8], the performance measures in terms of the average power consumption and the data delay are given based on a continuous-time queueing model. Unfortunately, the switching procedure from the sleep mode to the active mode is neglected in order to simplify the process of analysis.

However, most of these system models are based on continuous-time models. It is indicated that it would be more accurate and efficient to use discrete-time queueing models rather than their continuous counterparts when analyzing and designing digital transmitting systems [9].

In this paper, we consider a one-hop sensor network with digital transmission, where sensor nodes can transmit data frames with each other. Considering the digital nature and the switching procedure of the Phase Locked Loop from the sleep mode to the active mode at a sensor node in the network, we build a discrete-time multiple vacation queueing model with a setup to describe the working principle of the sleep/wakeup protocol for the sensor node. We analyze the relationships between the modulation constellation size and the performance measures in the sensor node. Moreover, numerical results are given to demonstrate the influence of the modulation constellation size on the system performance. Finally, we also determine the optimal value of the modulation constellation size for minimizing the energy consumption of the sensor node under various conditions.

The method presented in this paper will be not only useful for the performance prediction and the optimum design of sensor nodes, but also applicable to cluster head nodes and relay nodes in the WSNs.

The structure of this paper is as follows. In Section 2, we describe the working mechanism for the sleep/wakeup protocol offered in IEEE 802.15.4, and model this sleep/wakeup protocol as a multiple vacation queueing model with a setup. The model is analyzed by using the method of an embedded-Markov chain in Section 3. As a result, the steady-state probability distribution of the system model is derived. In Section 4, we obtain the performance measures of the handover ratio, the average latency of data frames, and the average energy consumption of the sensor node. In Section 5, we give numerical results to illustrate the relationship between the system performance and the modulation constellation size, and then a cost function is constructed to investigate the optimal value of the modulation constellation size for minimizing the energy consumption of the sensor node. Conclusions are given in Section 6.

2. **System Model.** In order to minimize the energy consumption of sensor nodes, a sleep/wakeup protocol is offered in IEEE 802.15.4, in which each sensor node manages its state independently [10]. The communicated sensor nodes are required to be synchronous with the sleep/wakeup scheme so as to reduce the data latency. The implementation of such a sleep/wakeup scheme typically requires two different channels, namely, a data channel for normal data communication and a wakeup channel for awaking sensor nodes when needed.

In this system model, a sensor node (called "the sender") will communicate with a neighboring node (called "the receiver"). The sender can be a source sensor node or a relay sensor node and the receiver can be a relay sensor node or the base station (BS). We consider the transmission from the sender to the receiver as a research object.

Pulse Amplitude Modulation (PAM) is used and the channel between the sender and the receiver is supposed to be a Gaussian channel. In addition, we suppose that the error-correcting codes are not employed in the communication. A square wave pulse is applied in the system.

The sender in this system sends a series of signals on the wakeup channel. Once the receiver realizes that there is a pending signal, the receiver will send back a wakeup acknowledgement and turn on its data radio. The receiver wakes up periodically and listens to the channel for a short time in order to monitor whether there are data frames arriving or not. If a receiver does not detect any activity on the wakeup channel, the receiver will return to the sleep mode and turn off its data radio again.

Based on the sleep/wakeup protocol, the sender in this system works in two modes: the active mode and the sleep mode for all sensor nodes in a WSN. The active mode is also called working state, in which the data frames are transmitted normally.

Moreover, the sleep mode includes two stages: the sleep stage and the listening stage. In the sleep stage, the sender will turn off the wireless communication module, data frames will not be received or not be sent, and the energy will be saved; if some data frames are detected to be transmitted in the listening stage, the sender will enter into the active mode. Otherwise, the sender will return back to the sleep stage.

In this discrete-time system model, the time axis is segmented into a series of equal intervals, called slots. A late arrival system with immediate entrance is considered in this paper. Namely, we suppose that the departures occur at the moment immediately before the slot boundaries and the arrivals occur at the moment immediately after the slot boundaries.

Taking into account the memoryless nature of user-initiated data frame arrivals at the sender, we can suppose that the arrival process follows a Bernoulli distribution with arrival rate p ($0 < p < 1$). The probability that no arrival occurs is $\bar{p} = 1 - p$. The transmission time of a data frame is assumed to be an independent and identically distributed random variable denoted by S in slots.

We assume that the sleep stage of the sleep mode in the sender is regarded as a vacation period denoted by V_S. The listening stage of the sleep mode in the sender is regarded as another vacation period denoted by V_L. Obviously, the total vacation V in the sender is composed of V_S and V_L, namely, $V = V_S + V_L$. We assume that V_S and V_L have the fixed lengths of T_{V_S} and T_{V_L}, and V has the length of T_V, respectively, in slots. Then, we have that $T_V = T_{V_S} + T_{V_L}$, where $0 \le T_V \le \infty$.

The time period for transmitting data frames continuously is considered as a busy period denoted by Θ in slots. Moreover, the switching procedure of the Phase Lock Loop (PLL) in the frequency synthesizer when the sender transfers from the sleep mode to the active mode is regarded as a setup period denoted by U in slots.

It is assumed that data frames are transmitted according to a First-Come First-Served (FCFS) discipline at the sender. Moreover, the buffer capacity in the sender is assumed to be infinite.

Therefore, we can model the probability behavior of the sender as a multiple vacation Geom/G/1 queueing model with a setup. Below, we will call this queueing model the system model. The state transition of the system model is shown in Figure 1.

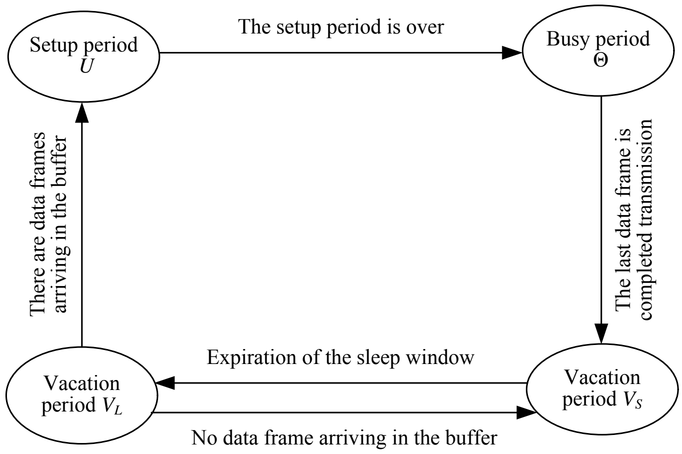

FIGURE 1. The state transition of the system model

From Figure 1, we know the following:

(1) If there is no data frame to be transmitted in the buffer of the sender, the busy period Θ will be over, and the vacation period V_S will begin.
(2) After the expiration of the vacation period V_S, the sender will begin another vacation period V_L.
(3) If there is no data frame arriving either within the vacation period V_S or within the vacation period V_L, the sender will return to the vacation period V_S after the vacation period V_L is over. Otherwise, the setup period will begin when the vacation period V_L expires.
(4) When the setup period U is over, the system will begin a new busy period Θ.

This process will be repeated.

3. **Performance Analysis.** In this section, we present a new effective analytical frame work based on the discrete-time multiple vacation Geom/G/1 queueing model with a setup.

An embedded Markov chain is constructed at the end of slots, where the data frame transmissions are completed. We define the system state by the number of data frames at the embedded Markov points.

Let B be the bandwidth and k ($k = 1, 2, \ldots$) be the modulation constellation size. So the numbers of bits transmitted per second are kB. We assume that the length of a data frame follows a general distribution with an average of H bits.

We can give the probability distribution s_j, the Probability Generating Function (PGF) $S(z)$ and the average $E[S]$ of the transmission time S of a data frame as follows:

$$P\{S = j\} = s_j, \quad j = 1, 2, \ldots, \quad S(z) = \sum_{j=1}^{\infty} z^j s_j, \quad E[S] = \frac{H}{kB}. \tag{1}$$

When the system load $\rho = pE[S] < 1$, the system will arrive at a state of equilibrium.

Letting A_V be the number of data frames arriving in a total vacation period V in the sender, the PGF $A_V(z)$ of A_V is given as follows:

$$A_V(z) = \sum_{j=0}^{T_V} P\{A_V = j\} z^j = \sum_{j=0}^{T_V} \binom{T_V}{j} p^j \bar{p}^{T_V - j} z^j = (\bar{p} + pz)^{T_V}, \quad T_V > 0. \tag{2}$$

Let the setup period U follow a general distribution. The probability distribution u_j, the PGF $U(z)$ and the average $E[U]$ of U are given as follows:

$$P\{U = j\} = u_j, \quad j = 1, 2, \ldots, \quad U(z) = \sum_{j=1}^{\infty} u_j z^j, \quad E[U] = \sum_{j=1}^{\infty} j z^j. \tag{3}$$

Let A_U and A_S be the number of data frames arriving during the setup period U and the transmission time S of a data frame, respectively. Considering that data frames arriving in the sender follow a Bernoulli arrival process, we can give the P.G.Fs. $A_U(z)$ and $A_S(z)$ of A_U and A_S as follows:

$$A_U(z) = \sum_{l=0}^{\infty} z^l \sum_{j=l}^{\infty} u_j \binom{j}{l} p^l \bar{p}^{j-l} = U(\bar{p} + pz),$$

$$A_S(z) = \sum_{l=0}^{\infty} z^l \sum_{j=l}^{\infty} s_j \binom{j}{l} p^l \bar{p}^{j-l} = S(\bar{p} + pz).$$

Let Q_B be the number of data frames at the beginning instant of a busy period Θ in the sender. Since there is at the most one data frame arriving in a slot, the maximal number of data frames arriving during a vacation period V with the time length T_V in slots will be the number of slots in T_V. Then we have the probability distribution b_j and the PGF $Q_B(z)$ of Q_B as follows:

$$b_j = P\{Q_B = j\} = \begin{cases} \dfrac{1}{1 - \bar{p}^{T_V}} \displaystyle\sum_{i=1}^{j} P\{A_V = i\} P\{A_U = j - i\}, & j = 1, 2, \ldots, T_V \\ \dfrac{1}{1 - \bar{p}^{T_V}} \displaystyle\sum_{i=1}^{T_V} P\{A_V = i\} P\{A_U = j - i\}, & j \geq T_V + 1, \end{cases} \tag{4}$$

$$Q_B(z) = \sum_{j=1}^{\infty} b_j z^j = \frac{1}{1 - \bar{p}^{T_V}} \left((\bar{p} + pz)^{T_V} - \bar{p}^{T_V} \right) U(\bar{p} + pz). \tag{5}$$

Differentiating Equation (5) with respect to z at $z = 1$, we can obtain the average $E[Q_B]$ of Q_B as follows:

$$E[Q_B] = \frac{T_V p}{1 - \bar{p}^{T_V}} + pE[U]. \tag{6}$$

3.1. **Queueing length and waiting time.** Let L^+ be the queueing length at the transmission completion instant of a data frame. Obviously, we have $L^+ = L_0 + L_d$, where L_0 is the queueing length for the classical Geom/G/1 queueing model, and L_d is the additional queueing length introduced by the setup period and multiple vacations introduced in the system model presented in this paper.

Referencing to [11, 12], we can get the PGF $L_0(z)$ and the average $E[L_0]$ of L_0 as follows:

$$L_0(z) = \frac{(1-\rho)(1-z)S(\bar{p}+pz)}{S(\bar{p}+pz)-z}, \tag{7}$$

$$E[L_0] = \rho + \frac{p^2 E[S(S-1)]}{2(1-\rho)}. \tag{8}$$

Using the boundary state variable theory in [11], the PGF $L_d(z)$ and the average $E[L_d]$ of L_d are given as follows:

$$L_d(z) = \frac{1-Q_B(z)}{E[Q_B](1-z)} = \frac{1-\bar{p}^{T_V} - \left((\bar{p}+pz)^{T_V} - \bar{p}^{T_V}\right)U(\bar{p}+pz)}{(T_V p + (1-\bar{p}^{T_V})pE[U])(1-z)}, \tag{9}$$

$$E[L_d] = \frac{T_V(T_V-1)p + 2T_V pE[U] + p\left(1-\bar{p}^{T_V}\right)E[U(U-1)]}{2\left(T_V + (1-\bar{p}^{T_V})E[U]\right)}. \tag{10}$$

Combining Equations (7) and (9), we can obtain the PGF $L^+(z)$ of L^+ as follows:

$$\begin{aligned} L^+(z) &= L_0(z)L_d(z) \\ &= \frac{(1-\rho)(1-z)S(\bar{p}+pz)}{S(\bar{p}+pz)-z} \times \frac{1-\bar{p}^{T_V} - \left((\bar{p}+pz)^{T_V} - \bar{p}^{T_V}\right)U(\bar{p}+pz)}{(T_V p + (1-\bar{p}^{T_V})pE[U])(1-z)}. \end{aligned} \tag{11}$$

Combining Equations (8) and (10), we can get the average $E[L^+]$ of L^+ as follows:

$$\begin{aligned} E[L^+] &= E[L_0] + E[L_d] \\ &= \rho + \frac{p^2 E[S(S-1)]}{2(1-\rho)} + \frac{T_V\left(T_V-1\right)p + 2T_V pE[U] + p\left(1-\bar{p}^{T_V}\right)E[U(U-1)]}{2\left(T_V + \left(1-\bar{p}^{T_V}\right)E[U]\right)}. \end{aligned} \tag{12}$$

The stationary waiting time W of a data frame can be decomposed into the sum of two independent random variables, i.e., $W = W_0 + W_d$, where W_0 is the waiting time of a data frame for a classical Geom/G/1 queueing model, and W_d is the additional waiting time of a data frame due to the setup procedure and multiple vacations in this system model. Therefore, the PGF $W_0(z)$ of W_0 is given as follows:

$$W_0(z) = \frac{(1-\rho)(1-z)}{(1-z)-\rho(1-S(z))}. \tag{13}$$

Differentiating Equation (13) with respect to z at $z = 1$, the average $E[W_0]$ of W_0 is given as follows:

$$E[W_0] = \frac{pE[S(S-1)]}{2(1-\rho)}. \tag{14}$$

By applying a similar method used when deriving the queueing length, we can get the PGF $W_d(z)$ of the additional waiting time W_d in this system model as follows:

$$W_d(z) = \frac{1-\bar{p}^{T_V} - (z^{T_V} - \bar{p}^{T_V})U(z)}{(T_V + E[U](1-\bar{p}^{T_V}))(1-z)}. \tag{15}$$

Accordingly, the average $E[W_d]$ of the additional waiting time W_d can be given as follows:

$$E[W_d] = \frac{T_V(T_V-1) + 2T_V E[U] + (1-\bar{p}^{T_V})E[U(U-1)]}{2(T_V + (1-\bar{p}^{T_V})E[U])}. \tag{16}$$

Combining Equations (13) and (15), we can give the PGF $W(z)$ of W as follows:

$$\begin{aligned} W(z) &= W_0(z)W_d(z) \\ &= \frac{(1-\rho)(1-z)}{(1-z)-p(1-S(z))} \times \frac{1-\bar{p}^{T_V} - (z^{T_V} - \bar{p}^{T_V})U(z)}{(T_V + E[U](1-\bar{p}^{T_V}))(1-z)}. \end{aligned} \tag{17}$$

Combining Equations (14) and (16), we can obtain the average $E[W]$ of W as follows:

$$\begin{aligned} E[W] &= E[W_0] + E[W_d] \\ &= \frac{pE[S(S-1)]}{2(1-\rho)} + \frac{T_V(T_V-1) + 2T_V E[U] + \left(1-\bar{p}^{T_V}\right) E[U(U-1)]}{2\left(T_V + (1-\bar{p}^{T_V})E[U]\right)}. \end{aligned} \tag{18}$$

3.2. **Busy cycle.** The busy cycle R in slots is defined as a time period from the instant in which one busy period finishes, to the instant in which the next busy period ends in the sender. The busy cycle R consists of one or more vacation periods V, a setup period U and a busy period Θ.

Each data frame at the beginning of a busy period Θ will introduce a sub-busy period θ. A sub-busy period θ of a data frame is composed of the transmission period S of this data frame and the sum of the sub-busy period θ incurred by all the data frames arriving during the transmission period S of this data frame.

All the sub-busy periods brought by the data frames at the beginning of the busy period combine to make a busy period Θ, so we have that

$$\theta = B + \underbrace{\theta + \theta + \cdots + \theta}_{A_S}, \quad \Theta = \underbrace{\theta + \theta + \cdots + \theta}_{Q_B}$$

where A_S is the number of data frames arriving in the sender during the transmission period S of a data frame, and Q_B is the number of data frames at the beginning instant of a busy period Θ.

Considering the Bernoulli arrival process in this system model, the PGF $\theta(z)$ and the average $E[\theta]$ of θ can be given as follows:

$$\theta(z) = S(z(\bar{p} + p\theta(z))), \quad E[\theta] = \frac{E[S]}{1-\rho}. \tag{19}$$

Then the average $E[\Theta]$ of Θ can be obtained as follows:

$$E[\Theta] = E[Q_B]E[\theta] = \frac{\rho}{1-\rho}\left(\frac{T_V}{1-\bar{p}^{T_V}} + E[U]\right). \tag{20}$$

Let N_V be the number of switches between the sleep mode and the active mode in a busy cycle. We can get the probability distribution and the PGF $N_V(z)$ of N_V as follows:

$$P\{N_V = r\} = \left(\bar{p}^{T_V}\right)^{r-1}\left(1-\bar{p}^{T_V}\right), \quad r \geq 1, \tag{21}$$

$$N_V(z) = \sum_{r=1}^{\infty} P\left\{N_V = r\right\} z^r = \frac{(1-\bar{p}^{T_V})z}{1-\bar{p}^{T_V}z}. \tag{22}$$

Differentiating Equation (22) with respect to z at $z = 1$, we can give the average $E[N_V]$ of N_V as follows:

$$E[N_V] = \frac{1}{1-\bar{p}^{T_V}}. \tag{23}$$

Consequently, we can give the average $E[R]$ of a busy cycle R as follows:

$$E[R] = E[\Theta] + E[U] + E[N_V]T_V = \frac{1}{1-\rho}\left(\frac{T_V}{1-\bar{p}^{T_V}} + E[U]\right). \tag{24}$$

Let p_b, p_v, p_u be the probabilities for the sender being in the busy period, the vacation period and the setup period. They then follow that

$$p_b = \frac{E[\Theta]}{E[R]} = \rho, \tag{25}$$

$$p_v = \frac{E[N_V]T_V}{E[R]} = \frac{T_V\,(1-\rho)}{T_V + (1-\bar{p}^{T_V})\,E[U]}, \tag{26}$$

$$p_u = \frac{E[U]}{E[R]} = \frac{(1-\rho)(1-\bar{p}^{T_V})E[U]}{T_V + (1-\bar{p}^{T_V})E[U]}. \tag{27}$$

4. **Performance Measures.** In this section, we present some important performance measures for the sleep/wakeup protocol concerned in this paper.

The handover ratio β is defined as the number of switches from the sleep mode to the active mode per slot. Note that there is only one switch procedure in a busy cycle, so the handover ratio β is given as follows:

$$\beta = \frac{1}{E[R]} = \frac{(1-\rho)(1-\bar{p}^{T_V})}{T_V + (1-\bar{p}^{T_V})E[U]}. \tag{28}$$

The average latency σ of data frames is the time period in slots elapsed from the arrival instant of a data frame, until the end instant of the transmission of that data frame. The average latency σ is the sum of the waiting time and the transmission time of a data frame, so we have that

$$\begin{aligned}\sigma &= E[W] + E[S] \\ &= \frac{pE[S(S-1)]}{2(1-\rho)} + \frac{T_V(T_V-1) + 2T_VE[U] + (1-\bar{p}^{T_V})E[U(U-1)]}{2\,(T_V + (1-\bar{p}^{T_V})E[U])} + E[S]. \end{aligned}\tag{29}$$

Let P_{CS} be the circuit power consumption in the sleep mode, P_{SA} be the power consumption when the sender switches from the sleep mode to the active mode and P_{CA} be the circuit power consumption in the active mode. P_{CS}, P_{SA} and P_{CA} can be considered as the system parameters. Let P_{amp} be the power consumption for the amplifier.

Let $E[P]$ be the average energy consumption of the sender per slot. Note that the energy is mainly consumed by a circuit and an amplifier. We can get $E[P]$ as follows:

$$E[P] = P_{CS}p_v + (P_{CA} + P_{amp})p_b + P_{SA}\frac{1}{E[R]}. \tag{30}$$

As set out in [8], the minimum power consumption P_{amp} of the amplifier is given by

$$P_{amp} = 8(M^2-1)\pi^2 d^2 B\mathrm{N}_0 \left(Q^{-1}(e_0)\right)^2 (3G\Gamma^2)^{-1} \tag{31}$$

where e_0 is the Bit Error Rate (BER), $M = 2^k$, k is the modulation constellation size and B is the bandwidth defined in Section 2. $\mathrm{N}_0/2$ is the variance of the Gaussian random variable, G is a constant defined by the antenna gain and other system parameters, Γ is the carrier wavelength, and

$$Q(e_0) = (2\pi)^{-1/2}\int_{e_0}^{\infty} e^{-x^2/2}dx,\ x \geq 0.$$

d is the distance between the sender and the receiver. The longer the distance between the sender and the receiver is, the greater the power consumption P_{amp} of the amplifier in the sender will be.

Substituting Equation (30) with Equation (31), then differentiating Equation (30) with respect to the modulation constellation size k, we can get the derivative function of $E[P]$ as follows:

$$\begin{aligned}\frac{\partial E[P]}{\partial k} &= \frac{pH}{k^2 B} \times (P_{CS} \times A_0 + P_{SA} \times C_0 - P_{CA}) \\ &+ \frac{pH}{k^2 B} \times 8\pi^2 d^2 B \mathrm{N}_0 \left(Q^{-1}(e_0)\right)^2 (3G\Gamma^2)^{-1} (4^k k \ln 4 - 4^k + 1) \end{aligned} \tag{32}$$

where H is the length of a data frame in bits defined in Section 3, and A_0 and C_0 can be given as follows:

$$A_0 = \frac{T_V}{T_V + (1 - \bar{p}^{T_V}) E[U]}, \quad C_0 = \frac{1 - \bar{p}^{T_V}}{T_V + (1 - \bar{p}^{T_V}) E[U]}.$$

5. **Numerical Results and Cost Function.** We numerically evaluate the system performance in this section. Referencing the calculations in [8], we set the system parameters as follows: a slot is 1 ms, $p = 0.05$, $d = 30$ m, $e_0 = 10^{-4}$, $G = 2$, $H = 16$ kb, $B = 10^6$ Hz, $P_{CS} = 10^{-4}$ mW, $P_{SA} = 5 \times 10^{-2}$ mW, $\mathrm{N}_0 = 2 \times 10^{-13}$ mW/Hz. Let the average setup period $E[U] = 3$ slots. The time length of the listening period $T_{V_L} = 3$ slots. The dependency relationships between the performance measures and the system parameters are shown in Figures 2-4.

Figure 2 shows the influence of the modulation constellation size k on the handover ratio β for different time lengths T_{V_S} of the sleep window.

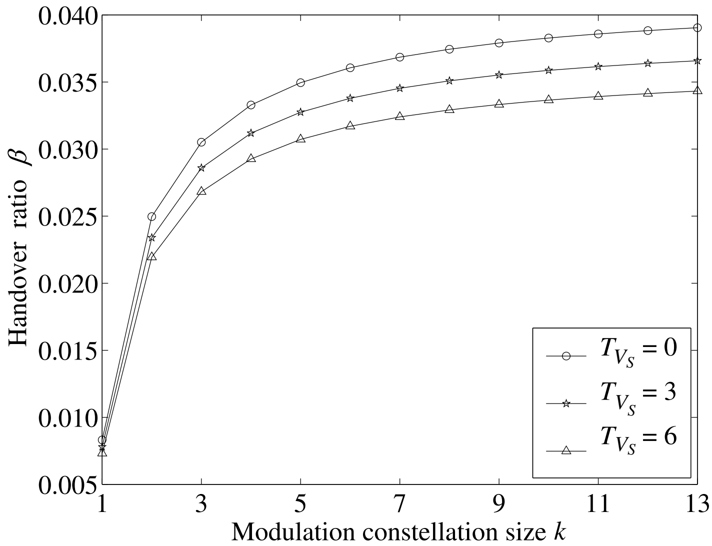

Figure 2. The handover ratio β vs. modulation constellation size k

It can be found that for the same modulation constellation size k, the handover ratio β will decrease as the time length T_{V_S} of the sleep window increases. This is because the larger the time length of the sleep window is, the longer the sender will be in the sleep mode, so the handover ratio will decrease. On the other hand, the handover ratio β will increase with the modulation constellation size k for all time lengths T_{V_S} of the sleep window. The reason is that the larger the modulation constellation size is, the shorter the transmission time of a data frame and the busy cycle are, so the greater the handover ratio will be.

Figure 3 shows how the average latency σ of data frames changes with the modulation constellation size k for different time lengths T_{V_S} of the sleep window.

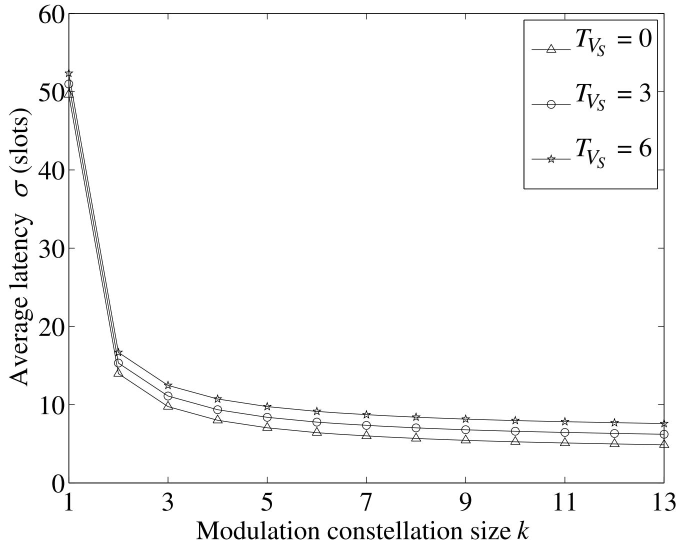

FIGURE 3. The average latency σ vs. modulation constellation size k

It is observed that for the same modulation constellation size k, the average latency σ of data frames will increase with the time length T_{V_S} of the sleep window. The reason is that the longer the time length of the sleep window is, the longer the waiting time of a data frame is, so the greater the average latency will be. On the other hand, the average latency σ of data frames will decrease as the modulation constellation size k increases for all time lengths T_{V_S} of the sleep window. This is because the larger the modulation constellation size is, the shorter the transmission time of a data frame is, so the less the average latency of data frames will be.

The influence of the modulation constellation size k on the average energy consumption $E[P]$ with $T_{V_S} = 6$ ms is plotted in Figure 4.

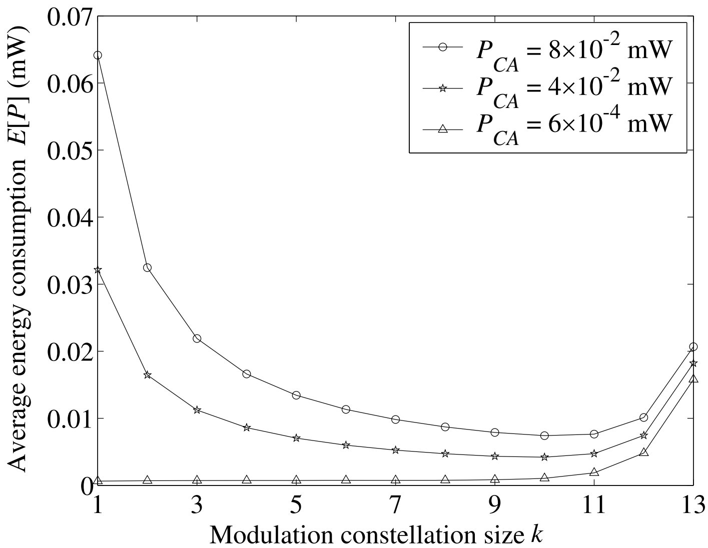

FIGURE 4. The average energy consumption $E[P]$ vs. modulation constellation size k

From Figure 4, we can conclude that the average energy consumption $E[P]$ will increase as the modulation constellation size k increases with $P_{CA} = 6\times10^{-4}$ mW. So we can obtain the minimum average energy consumption at $k = 1$ for this case. On the other hand, the average energy consumption $E[P]$ experiences two stages as the modulation constellation size k increases with $P_{CA} = 4 \times 10^{-2}$ mW and $P_{CA} = 8 \times 10^{-2}$ mW. When $1 \le k \le 10$, the average energy consumption $E[P]$ will decrease as the modulation constellation size k increases, when $k > 10$, the average energy consumption $E[P]$ will increase as the modulation constellation size k increases. Therefore, there is an optimal value k^* for a minimum energy consumption for this case.

Moreover, it can be found that $\partial E[P]/\partial k$ obtained by Equation (32) is always greater than zero under the condition that $P_{CA} \le (P_{CS} \times A_0 + P_{SA} \times C_0)$, so $E[P]$ is a monotone increasing function in this case. Therefore, we can derive the minimum value of $E[P]$ at $k = 1$. But, when $P_{CA} > (P_{CS} \times A_0 + P_{SA} \times C_0)$, there is a minimum value of $E[P]$ when the modulation constellation size k is set to an optimal value k^*.

Considering the trade-off between data latency and energy consumption requirements under a certain condition, we develop a cost function as follows:

$$F(k) = C_1\sigma + C_2E[P]$$

where C_1 = Cost of unit average latency of data frames, C_2 = Cost of unit average energy consumption.

By letting $T_{V_S} = 6$ ms, it can be shown how the cost function $F(k)$ changes with the modulation constellation size k in Figure 5.

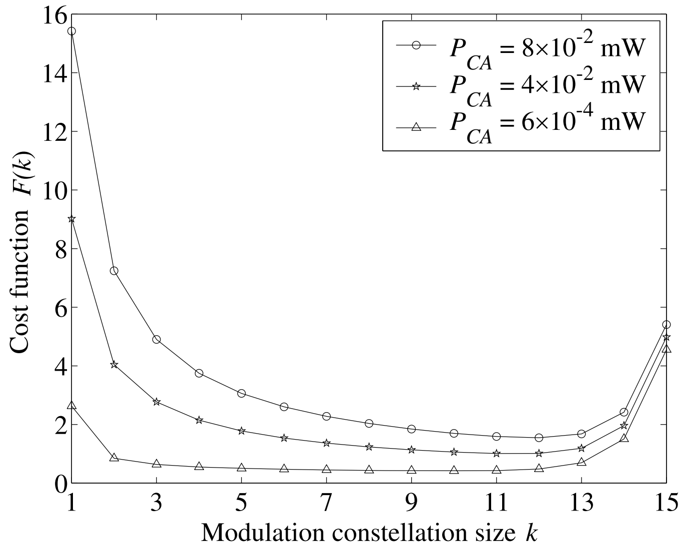

FIGURE 5. The cost function $F(k)$ vs. modulation constellation size k

From Figure 5, we can find that the cost function $F(k)$ experiences two stages for all the circuit power consumption P_{CA}. In the first stage, the cost function $F(k)$ will decrease along with the increase in the modulation constellation size k. During this stage, the larger the modulation constellation size is, the shorter the transmission period is, the less the average latency of data frames is, so the lower the cost will be.

In the second stage, the cost function $F(k)$ will increase with the modulation constellation size k. During this period, the larger the modulation constellation size is, the greater the power consumption of the amplifier is, consequently progressively increasing the average energy consumption, and therefore also increasing the cost.

Consequently, there is a minimal cost when the modulation constellation size is set to an optimal value. The optimal modulation constellation sizes for the different circuit power consumptions at the active mode of the sender are shown in Table 1.

Table 1. Optimal modulation constellation size k^*

Circuit power P_{CA} (mW)	Modulation constellation size k^*	Minimal cost $F(k^*)$
8×10^{-2}	12	1.5467
11×10^{-2}	4	1.0689
6×10^{-4}	10	0.4195

6. **Conclusions.** In a wireless sensor network (WSN), data latency and energy consumption are both important performance measures need to be considered. However, there is a trade-off between these two performance measures. We built a discrete-time multiple vacation queueing model with a setup to capture the working principle of the sleep/wakeup protocol offered in IEEE 802.15.4 for the sensor nodes in the WSNs. We analyzed the system model in the steady state, and proposed the formulas for the system performance measures in terms of the handover ratio, the average latency of data frames and the average energy consumption. Moreover, we presented numerical results to investigate the nature of the dependency relationships between the system parameters and the performance measures. Finally, we developed a cost function to optimize the modulation constellation size under certain conditions.

This paper provides a theoretical basis for the optimal setting of the system parameters in a wireless sensor network, and has potential applications in solving other energy conservation related problems.

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