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APPLYING SIGNED DISTANCE METHOD FOR FUZZY INVENTORY WITHOUT BACKORDER

Huey-Ming Lee[1] and Lily Lin[2]

[1]Department of Information Management
Chinese Culture University
55, Hwa-Kung Road, Yang-Ming-Shan, Taipei 11114, Taiwan
hmlee@faculty.pccu.edu.tw

[2]Department of International Business
China University of Technology
56, Sec.3, Hsing-Lung Road, Taipei 116, Taiwan
lily@cute.edu.tw

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Abstract. *For the total cost of the inventory without backorder model, if we fuzzify the order quantity, the total demand, the cost of storing and the cost of placing an order as fuzzy numbers then we can obtain the fuzzy total cost. In this paper, we apply the signed distance method to defuzzify the fuzzy total cost and then solve the optimal order quantity.*
Keywords: Fuzzy total cost, Signed distance

1. **Introduction.** There are several papers to treat the fuzzified problems of EOQ model. Vujosevic *et al.* [14] used trapezoidal fuzzy number to fuzzify the order cost in the total cost of the inventory model with backorder. Then, they got fuzzy total cost. They obtained the estimate of the total cost through centroid to defuzzify. Chen and Wang [3] used trapezoidal fuzzy number to fuzzify the order cost, inventory cost and backorder cost in the total cost of the inventory model without backorder. Then, they found the estimate of the total cost in the fuzzy sense by functional principle. Roy and Maiti [13] used nonlinear programming method to rewrite the classical economic order quantity problem. They fuzzified the objective function and storage area and solved this problem by fuzzy nonlinear and geometric programming. Ishii and Konno [6] fuzzified the shortage cost to L fuzzy numbers in the classical newsboy problem and got optimal ordering quantity by fuzzy ordering concept. In a series of papers, Yao *et al.* [2,4,9,15-18], considered the fuzzified problems for the inventory with or without backorder models. In [9,15,17], they considered the fuzzified problems for the inventory without backorder models. In [9], they fuzzified the order quantity q as the triangular fuzzy number, in [17], they fuzzified the order quantity q as the trapezoidal fuzzy number, and in [15], they fuzzified the order quantity q and the total demand quantity r as the triangular fuzzy numbers. In [9,15,17], they applied the extension principle to obtain the fuzzy total cost, and then, they defuzzified the fuzzy total cost by centroid. In [8], Lee and Chiang fuzzified the quantity produced per cycle, the holding cost, production cost, production quantity per day, the total demand quantity and the demand quantity per day, to triangular fuzzy numbers; and found the total costs in the fuzzy sense by signed distance and got the optimal solutions. In [2,4,16,17,18], they considered the fuzzified problems for the inventory with backorder models. In [2], they fuzzified the maximal stock quantity s as the triangular fuzzy number

and regarded the order quantity q as the crisp variable and obtained the fuzzy total cost by the extension principle, and then defuzzified by the centroid. In [4], they fuzzified the order quantity q, maximal stock quantity s, total demand r, storing cost a, backorder quantity b and order cost c as the triangular fuzzy number and obtained the fuzzy total cost by the decomposition theorem, then defuzzified by the signed distance to obtain the total cost in the fuzzy sense, and then obtain the optimal solution. In [16], they fuzzified the order quantity q as the triangular fuzzy number, in [17], they fuzzified the order quantity q as the trapezoidal fuzzy number. In [16,17], they applied the extension principle to obtain then fuzzy total cost, and then, they defuzzified by the centroid. In [18], they fuzzified the total demand quantity r by three methods, saying, triangular fuzzy number, the interval-valued fuzzy set based on two triangular fuzzy numbers, and interval-valued fuzzy set based on two trapezoid fuzzy numbers, and obtained the fuzzy total cost by the extension principle. The first one method, they defuzziifed by the centroid. The second and third method, let $f(x)$ = (the maximal membership degree at x)–(the minimal membership degree at x), for each x, and defuzzified the fuzzy total cost by the centroid of $f(x)$ and obtained the optimization. In papers [2,4,16,17,18], they applied the extension principle to find the membership functions of the fuzzy total cost. Then, they applied the centroid method to estimate the total cost in the fuzzy sense and obtain the optimization problems. But, it is very hard and complex to derive them. Chou [5] used the function principle to manipulate arithmetical operations, the graded mean integration representation method to defuzzify, and the Kuhn-Tucker conditions to find the optimal backorder quantity and shortage quantity for the fuzzy backorder quantity inventory model. Jia *et al.* [7] built a model to analyze the optimal problems of pricing competitions among multiple manufactures. Lin and Tsai [10] investigated solving the transportation problem with fuzzy demands and fuzzy supplies using a two-stage genetic algorithm.

In this paper, we fuzzify the order quantity q, the total demand r, the cost of storing c, and the cost of placing an order a as the triangular fuzzy numbers for the inventory without backorder and obtain the fuzzy total cost, and then we apply the signed distance method instead of the extension principle and centroid method to solve the estimated total cost in the fuzzy sense to obtain the optimal order quantity.

Section 2 is the preliminaries, in which we consider the definition of the signed distance. In Section 3, we use the signed distance method to estimate the total cost in the fuzzy sense. In Section 4, we apply the numerical analysis method to solve the optimal order quantity and compare the results with the paper shown in [9]. In Section 5, we make a conclusion.

2. **Preliminaries.**

For the proposed algorithm, all pertinent definitions of fuzzy sets are given below [8,11,19,20].

Definition 2.1. *Let $\tilde{b}$ be a fuzzy set on $R = (\infty,\ \infty)$. It is called a fuzzy point if its membership function is*

$$\mu_{\tilde{b}}(x) = \begin{cases} 1, & \text{if } x = b \\ 0, & \text{if } x \neq b \end{cases} \tag{1}$$

Definition 2.2. *Let $[a,\ b;\ \alpha]$ be a fuzzy set on R. It is called a level α fuzzy interval, $0 \le \alpha \le 1$, $a < b$, if its membership function is*

$$\mu_{[a,\,b;\,\alpha]}(x) = \begin{cases} \alpha, & \text{if } a \le x \le b \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

If $a = b$, we call $[a,\ b;\ \alpha]$ a level α fuzzy point at a.

Definition 2.3. *The* α*-level set of the triangular fuzzy number* $\tilde{A} = (p,\ q,\ r)$ *is*

$$A(\alpha) = \{x\,|\mu_{\tilde{A}}(x) \geq \alpha\} \equiv [A_L(\alpha),\ A_R(\alpha)] \tag{3}$$

where

$$\begin{aligned} A_L(\alpha) &= p + (q-p)\alpha \\ A_R(\alpha) &= r - (r-q)\alpha, \quad \alpha \in [0,1] \end{aligned} \tag{4}$$

We can represent $\tilde{A} = (p,\ q,\ r)$ as

$$\tilde{A} = \underset{0\leq\alpha\leq1}{\cup}[A_L\alpha),\ A_R(\alpha);\ \alpha] \tag{5}$$

From Yao and Wu [19], we may define the signed distance from $[A_L(\alpha),\ A_R(\alpha);\ \alpha]$ to $\tilde{0}$ as

$$d([A_L(\alpha),\ A_R(\alpha);\ \alpha],\ \tilde{0})\ = \frac{1}{2}[A_L(\alpha) + A_R(\alpha)] \tag{6}$$

Let $\tilde{D}$ be a fuzzy set on R. Denote $D(\alpha) = \{x\,|\mu_{\tilde{D}}(x) \geq \alpha\} = [D_L(\alpha),\ D_R(\alpha)]$ as the α-cut of $\tilde{D}$, where $0 \leq \alpha \leq 1$, $D_L(\alpha)$ and $D_R(\alpha)$ are the left and right hand side of $D(\alpha)$. For each $\alpha \in [0,1]$, $D_L(\alpha)$ and $D_R(\alpha)$ uniquely exist and are integrable. In addition, we let F be the family of all these fuzzy sets $\tilde{D}$ on R. we have the following definition.

Definition 2.4. *Let* $\tilde{D} \in F$*, we define the signed distance of* $\tilde{D}$ *measured from* $\tilde{0}$ *as*

$$d(\tilde{D},\tilde{0}) = \frac{1}{2}\int_0^1 [D_L(\alpha) + D_R(\alpha)]d\alpha \tag{7}$$

Remark 2.1. *If* $\tilde{C} = (u,\ v,\ w)$ *then the left endpoint and the right endpoint of the* α*-cut of* $\tilde{C}$ *are* $C_L(\alpha) = u + (v-u)\alpha$ *and* $C_R(\alpha) = w - (w-v)\alpha$*, respectively, where* $0 \leq \alpha \leq 1$*.* $C_L(\alpha)$ *and* $C_R(\alpha)$ *exist and are integrable for* $0 \leq \alpha \leq 1$*. Therefore,* $\tilde{C} \in F$*. By the Definition 2.4, we have*

$$d(\tilde{C},\tilde{0}) = \frac{1}{2}\int_0^1 [u + w + (2v - u - w)\alpha]d\alpha = \frac{1}{4}(2v + u + w) \tag{8}$$

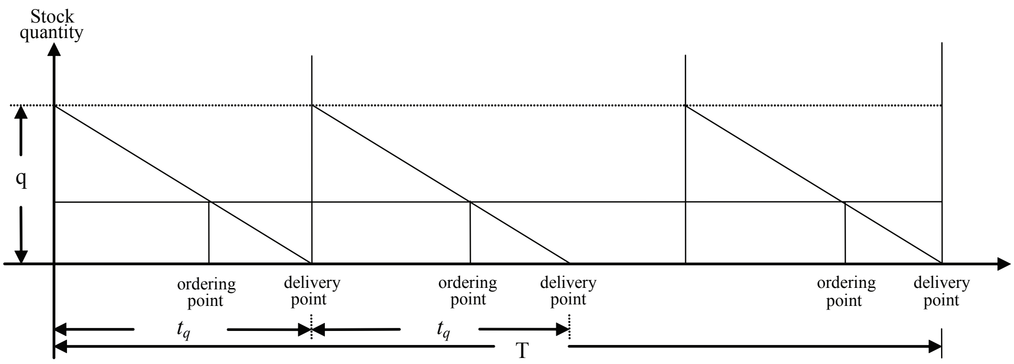

Figure 1. Inventory without backorder

3. **Fuzzy Inventory without Backorder Based on Signed Distance.** Figure 1 illustrates the role of all of the parameters, where

T: length of plan (in day)
c: the cost of storing one unit for one day
a: the cost of placing an order
r: the total demand over the planning time period $[0,T]$
t_q: length of a cycle

q: order quantity per cycle

We have $\dfrac{q}{t_q} = \dfrac{r}{T}$.

The crisp total cost function of the inventory without backorder on the planning time period $[0, T]$ is given by

$$F(q) = \frac{1}{2} c \cdot T \cdot q + \frac{a \cdot r}{q}, \ (q > 0) \tag{9}$$

In Equation (9), the crisp optimal solutions are as follows:

$$\text{The optimal order quantity } q_* = \sqrt{\frac{2a \cdot r}{c \cdot T}} \tag{10}$$

$$\text{The minimal total cost } F(q_*) = \sqrt{2a \cdot c \cdot r \cdot T} \tag{11}$$

Equations (10) and (11) are derived under the condition that the period from the ordering to delivery is fixed per cycle. But, in the real situation, it will fluctuate a little. The total demand over the period $[0, T]$ may not be equal to r, therefore, we set the total demand in the interval $[r - \Delta_1, \ r + \Delta_2]$, where $0 < \Delta_1 < r$, $0 < \Delta_2$. By the same way, the cost of storing one unit for one day is in the interval $[c - \Delta_3, \ c + \Delta_4]$, the cost of placing an order in the interval $[a - \Delta_5, \ a + \Delta_6]$, where $0 < \Delta_3 < c$, $0 < \Delta_4$, $0 < \Delta_5 < a$, $0 < \Delta_6$. In the total cost function $F(q)$, the total demand, cost of storing and cost of ordering are in the intervals, therefore, we should consider the ordering quantity in the interval $[q - w_1, \ q + w_2]$, too, where $0 < w_1 < q$ and $0 < w_2$. Referred to [4,8], corresponding to the above four intervals, we have the fuzzy total demand $\tilde{r} = (r - \Delta_1, \ r, \ r + \Delta_2)$, fuzzy cost of ordering $\tilde{c} = (c - \Delta_3, \ c, \ c + \Delta_4)$, fuzzy cost of ordering $\tilde{a} = (a - \Delta_5, \ a, \ a + \Delta_6)$, and the fuzzy order quantity $\tilde{q} = (q - w_1, \ q, \ q + w_2)$. The above all are triangular fuzzy numbers. We use these triangular fuzzy numbers to fuzzify the total cost in Equation (9), then, we have the following fuzzy total cost

$$G(\tilde{q}, \tilde{r}, \tilde{a}, \tilde{c}) = (\left(\frac{\tilde{T}}{2}\right) \otimes \tilde{c} \otimes \tilde{q}) \oplus (\tilde{a} \otimes \tilde{r} \ominus\!\!\!\!\!\div \tilde{q}) \tag{12}$$

where $\left(\frac{\tilde{T}}{2}\right) = \left(\frac{T}{2}, \frac{T}{2}, \frac{T}{2}\right)$ is the fuzzy point. The left and right hand side of the α-cut, $(0 \leq \alpha \leq 1)$, of the fuzzy sets

$$\tilde{P} \equiv (\left(\frac{\tilde{T}}{2}\right)) \otimes \tilde{c} \otimes \tilde{q}, \ \ \tilde{Q} \equiv \tilde{a} \otimes \tilde{r} \ominus\!\!\!\!\!\div \tilde{q} \tag{13}$$

are as follows

$$P_L(\alpha) = \frac{T}{2}[c - \Delta_3 + \Delta_3\alpha][q - w_1 + w_1\alpha]$$
$$P_R(\alpha) = \frac{T}{2}[c + \Delta_4 - \Delta_4\alpha][q + w_2 - w_2\alpha]$$
$$Q_L(\alpha) = [a - \Delta_5 + \Delta_5\alpha][r - \Delta_1 + \Delta_1\alpha]/[q + w_2 - w_2\alpha]$$
$$Q_R(\alpha) = [a + \Delta_6 - \Delta_6\alpha][r + \Delta_2 - \Delta_2\alpha]/[q - w_1 + w_1\alpha]$$

respectively. Therefore, the left and right hand side of the α-cut, $(0 \leq \alpha \leq 1)$, of the fuzzy set $G(\tilde{q}, \tilde{r}, \tilde{a}, \tilde{c})$ are

$$G(\tilde{q}, \tilde{r}, \tilde{a}, \tilde{c})_L(\alpha) = P_L(\alpha) + Q_L(\alpha)$$
$$G(\tilde{q}, \tilde{r}, \tilde{a}, \tilde{c})_R(\alpha) = P_R(\alpha) + Q_R(\alpha)$$

respectively.

Defuzzify the fuzzy total cost in Equation (12) by the signed distance, we have

$$\begin{aligned} &d(G(\tilde{q},\ \tilde{r},\ \tilde{a},\ \tilde{c}),\ \tilde{0}) \\ &= \tfrac{1}{2}\int_0^1 [G(\tilde{q},\tilde{R},\tilde{a},\tilde{c})_L(\alpha) + G(\tilde{q},\tilde{R},\tilde{a},\tilde{c})_R(\alpha)]d\alpha \\ &= \tfrac{T}{24}[(3c-2\Delta_3)q_1 + (6c-\Delta_3+\Delta_4)q + (3c+2\Delta_4)q_2] \\ &\quad +\tfrac{1}{2}H(\Delta_1\Delta_5,\ (a-\Delta_5)\Delta_1 + (r-\Delta_1)\Delta_5,\ (a-\Delta_5)(r-\Delta_1),\ -q_2+q,\ q_2) \\ &\quad +\tfrac{1}{2}H(\Delta_2\Delta_6,\ -(a+\Delta_6)\Delta_2 - (r+\Delta_2)\Delta_6,\ (a+\Delta_6)(r+\Delta_2),\ q-q_1,\ q_1) \end{aligned} \tag{14}$$

where

$$H(w,v,u,t,s) = \frac{w}{2t} + \frac{vt-sw}{t^2} + \frac{ut^2 - vts + s^2w}{t^3}\ln\left|\frac{t+s}{s}\right|$$

We let $FC(q_1,q,q_2;\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6) \equiv d(G(\tilde{q},\tilde{r},\tilde{a},\tilde{c}),\tilde{0})$. Then, we can obtain the estimated total cost $FC(q_1,q,q_2;\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6)$ in the fuzzy sense in Equation (14), where $q_1 = q - w_1$, $q_2 = q + w_2$.

In order to minimize $FC(q_1,q,q_2;\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6)$, where $\Delta_j \geq 0$, $j = 1,2,\ldots,6$, we apply the Nelder-Mead simplex algorithm [1,12]. The two transformations (15), (16) we used are shown in Figures 2 and 3 instead of the two transformations of Algorithm 6.5 of the Nelder-Mead method [12].

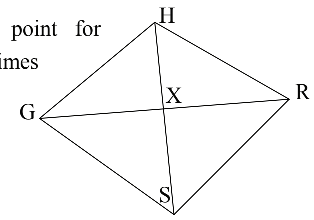

FIGURE 2. Contraction step

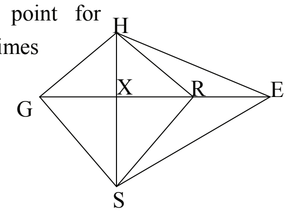

FIGURE 3. Expansion step

$$R = X + e(X-G) = (1+e)X - eG, \quad \text{where } 0 < e \leq 1 \tag{15}$$

$$E = X + d(R-X) = (1-d)X + dR, \quad \text{where } d > 1 \tag{16}$$

When we find the q_1^*, q^*, q_2^*, such that $FC(q_1^*,q^*,q_2^*;\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6)$ is the local minimal value, then by Equation (8), we get the best economic order quantity $q^{**} = \frac{1}{4}(q_1^* + 2q^* + q_2^*)$ and the minimal total cost $FC(q_1^*,q^*,q_2^*;\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6)$ in the fuzzy sense.

4. Example Implementation.

Since there are three variables in $FC(q_1,q,q_2;\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6)$ for $\Delta_j \geq 0$, $j = 1,2,\ldots,6$. By the algorithm discussed in Section 3, when we run the computer program to solve the optimal solution for $FC(q_1,q,q_2;\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6)$, we should assign a set of four initial points of q_1, q, q_2 which satisfies $0 < q_1 < q < q_2$.

$$\text{Let} \quad q_r^* = \frac{q^{**} - q_*}{q_*} \times 100\%, \quad F_r = \frac{FC(q_1^*,q^*,q_2^*,\Delta_1,\Delta_2,\cdots,\Delta_6) - F(q_*)}{F(q_*)} \times 100\%$$

Example 4.1. *Given $a = 8$, $c = 4$, $r = 18$, $T = 2$, we get the optimal order quantity q_*=6, the minimal total cost $F(q_*) = 48$ in the crisp. We have the following results for each case of given sets of initial points q_1, q, q_2.*

(Case 1)

q_1	q	q_2
4.5	5.5	7.0
3.5	5.5	6.5
5.5	6.5	7.0
4.0	5.0	7.0

then, we have the result as shown in Table 1.

Table 1. Optimal solution for Case 1

Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	q_1^*	q^*	q_2^*	q^{**}	FC	$q_r^*(\%)$	$F_r(\%)$
0.1	0.2	0.3	0.1	0.2	0.3	5.431	6.420	7.006	6.319	48.0715	5.31	0.149
0.2	0.2	0.2	0.2	0.2	0.2	5.377	6.358	7.009	6.276	48.2765	4.6	0.576
1.5	2.5	1.5	2.5	1.5	2.5	5.5	6.5	7.0	6.375	51.8816	6.25	8.087

(Case 2)

q_1	q	q_2
3.0	5.5	7.0
3.5	5.5	6.5
4.5	5.5	7.0
4.0	5.0	7.0

then, we have the result as shown in Table 2.

Table 2. Optimal solution for Case 2

Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	q_1^*	q^*	q_2^*	q^{**}	FC	$q_r^*(\%)$	$F_r(\%)$
0.1	0.2	0.3	0.1	0.2	0.3	4.5	5.5	7.0	5.625	48.5198	−6.25	1.083
0.2	0.2	0.2	0.2	0.2	0.2	4.5	5.5	7.0	5.625	48.6788	−6.25	1.414
1.5	2.5	1.5	2.5	1.5	2.5	4.5	5.5	7.0	5.625	52.9157	−6.25	10.241

(Case 3)

q_1	q	q_2
6.2	6.5	7.5
6.2	6.5	7.0
6.2	7.0	8.0
5.8	7.0	7.5

then, we have the result as shown in Table 3.

The following Table 4 comes from the results of the Example 4.1 in [9], where

$$q_0^{**} = \frac{1}{3}(q_1^* + q_0^* + q_2^*),\ q_r = \frac{q_0^{**} - q_*}{q_*} \times 100\%,\ M \equiv M(q_1^*, q_0^*, q_2^*),\ M_r = \frac{M - F(q_*)}{F(q_*)} \times 100\%$$

Table 3. Optimal solution for Case 3

Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	q_1^*	q^*	q_2^*	q^{**}	FC	$q_r^*(\%)$	$F_r(\%)$
0.1	0.2	0.3	0.1	0.2	0.3	6.2	6.5	7.0	6.55	48.4110	9.167	0.856
0.2	0.2	0.2	0.2	0.2	0.2	6.2	6.5	7.0	6.55	48.2570	9.167	0.535
2.0	2.0	2.0	2.0	2.0	2.0	6.2	6.5	7.0	6.55	48.8429	9.167	1.756

Table 4. Results of the Example 4.1 shown in [9]

Case	q_1^*	q_0^*	q_2^*	q_0^{**}	M	q_r	M_r
1	5.22344	6.18254	7.02045	6.14214	48.18231	2.37	0.38
2	4.5	5.5	7.0	5.66667	48.27129	−5.56	0.565
3	6.2	6.5	7.0	6.56667	48.23762	9.4	0.5

Comparing the results of Tables 1 ∼ 3 with the Table 4, we have the following Tables 5 ∼ 7, where

$$q_r^{**} = \frac{q^{**} - q_0^{**}}{q_0^{**}} \times 100\%, \quad FC_r = \frac{FC - M}{M} \times 100\%$$

Table 5. Comparison the result of this paper with [9] for Case 1

This paper								Paper in [9]			
Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	q^{**}	FC	q_0^{**}	M	q_r^{**}	FC_r
0.1	0.2	0.3	0.1	0.2	0.3	6.319	48.0715	6.14214	48.18231	2.879	−0.230
0.2	0.2	0.2	0.2	0.2	0.2	6.276	48.2765	6.14214	48.18231	2.179	0.195
1.5	2.5	1.5	2.5	1.5	2.5	6.375	51.8816	6.14214	48.18231	3.791	7.678

Table 6. Comparison the result of this paper with [9] for Case 2

This paper								Paper in [9]			
Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	q^{**}	FC	q_0^{**}	M	q_r^{**}	FC_r
0.1	0.2	0.3	0.1	0.2	0.3	5.625	48.5198	5.66667	48.27129	0.735	0.515
0.2	0.2	0.2	0.2	0.2	0.2	5.625	48.6788	5.66667	48.27129	0.735	0.844
1.5	2.5	1.5	2.5	1.5	2.5	5.625	52.9157	5.66667	48.27129	0.735	9.621

Table 7. Comparison the result of this paper with [9] for Case 3

This paper								Paper in [9]			
Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	q^{**}	FC	q_0^{**}	M	q_r^{**}	FC_r
0.1	0.2	0.3	0.1	0.2	0.3	6.55	48.4110	6.56667	48.23762	0.254	0.359
0.2	0.2	0.2	0.2	0.2	0.2	6.55	48.2570	6.56667	48.23762	0.254	0.040
2.0	2.0	2.0	2.0	2.0	2.0	6.55	48.8429	6.56667	48.23762	−0.254	1.255

From these tables (Tables 5 ∼ 7), we find that if the $\Delta_j \geq 0$, $j = 1, 2, \ldots, 6$ are small then the computing results in this paper approach the results as in [9], otherwise, they have some differences.

5. Conclusions and Future Remarks. In this paper, we fuzzified the q, r, c, and a in Equation (9) to the triangular fuzzy numbers $\tilde{q} = (q_1, q, q_2)$, $\tilde{r} = (r - \Delta_1, r, r + \Delta_2)$, $\tilde{c} = (c - \Delta_3, c, c + \Delta_4)$, and $\tilde{a} = (a - \Delta_5, a, a + \Delta_6)$, respectively. From Equation (9), we have the fuzzy cost $G(\tilde{q}, \tilde{r}, \tilde{a}, \tilde{c})$ (in Equation (12)). Applying the signed distance method, we can easily obtain the estimate of the total cost in the fuzzy sense $FC(q_1, q, q_2; \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6)$ (in Equation (14)). In [9], Lee and Yao fuzzified the q in Equation (9) into the triangular fuzzy number $\tilde{q} = (q_1, q_0, q_2)$, where $0 < q_1 < q_0 < q_2$, obtained the fuzzy cost $F(\tilde{q})$, and applied the extension principle to find the membership function of $F(\tilde{q})$, then defuzzified by the centroid method to solve the estimate of the total cost in the fuzzy sense $M(q_1, q_0, q_2)$. This method is very hard and complex to solve the optimal solution. Based on the signed distance method, we easily derive the total cost in fuzzy sense and obtain the optimal order quantity.

As future studies, we'll treat the annual new demand based on the past n year intervals average by using both statistical and fuzzy methods.

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