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DYNAMIC INTEGRAL SLIDING MODE CONTROL FOR SISO UNCERTAIN NONLINEAR SYSTEMS

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Abstract. *In this work the authors present a dynamic integral sliding mode controller which is based on the existing dynamic sliding mode control and integral sliding mode control techniques. The proposed control law makes use of an integral manifold instead of the conventional sliding manifold which provides dynamic sliding mode without reaching phase. The robustness is inherited from dynamic sliding mode control and is enhanced by the elimination of reaching phase. In addition, this new designed control law reduces chattering with the incorporation of the dynamic sliding mode control concept. Furthermore, the performance is improved via the linear control law design. A comprehensive comparative analysis carried out with dynamic sliding mode control demonstrates the superiority of the proposed control law. A chatter free regulation control of a kinematic car model with improved performance in the presence of uncertainties certifies the robustness of the proposed dynamic integral sliding mode controller.*

Keywords: Dynamic integral sliding mode control, Nonlinear control, Chattering, Robustness

1. **Introduction.** Sliding mode control (SMC) plays a significant role in the theory of variable structure systems (VSSs) and it is suitable for the control of uncertain nonlinear systems. In SMC robustness is guaranteed against uncertainties, un-modeled dynamics, parametric uncertainties and external disturbances [1, 2]. However, it experiences chattering phenomenon which leads to the damage in actuators and the system itself. A brief overview about the contributions to ensure robustness and performance with reduced chattering is discussed below.

Robust stabilization of uncertain systems has been widely addressed in [1]. Zak et al. [3] developed a generic condition for the existence and stability of the reduced order sliding motion. Edwards [4] developed an algorithm to handle matched disturbances acting in the channel of input. Based on the work in [3], some dynamic output feedback control schemes have been proposed for robust control of uncertain systems [5, 6]. Silva et al. [7] have developed an algorithm in which the existence and the reachability problems have been formulated using a polytopic description in order to tackle mismatched uncertainties with reduced chattering. Bartolini et al. [8] have identified some conditions under which, even in the presence of uncertainty, the convergence to the sliding manifold is ensured via the application of a multi-input control. A nonlinear integral-type sliding surface is proposed in [9], for the system in the presence of both matched and unmatched uncertainties. The stability of the controlled system with unmatched uncertainties depends on the controlled nominal system, and the nature and size of the equivalent unmatched uncertainties.

The robustness of the SMC based control algorithms is directly related to chattering phenomena and vice versa. In order to avoid chattering and its adverse effects, in the last two decades many of the researchers devoted their efforts to handle the chattering reduction. The approach of Higher Order Sliding Mode Control (HOSM) generalizes the basic idea of SMC. The main idea of HOSM [10, 11] is to act on higher order time derivatives of sliding manifolds as compared with the 1st order derivative in standard sliding mode technique. With this action chattering is attenuated and high order precision is achieved but with a compromise on robustness [12]. The r^{th} order sliding mode is determined by $s = \dot{s} = \ddot{s} = \ldots = s^{(r-1)} = 0$, which form an r-dimensional condition on state of the dynamic system. The sliding order is a measure of the smoothness of the sliding variable in the vicinity of the sliding manifold. A number of such controllers were described in [12, 13, 14, 15]. Some realization problems of r-sliding mode are caused by the complicated structure of the transient process, which was difficult to monitor with $r > 2$ [17, 18]. Another Problem concerns the above mentioned procedure, when $u^{(1)}$ is treated as new control. Due to the intersection of u and its derivatives during the convergence of $(r+l)$-sliding mode $s = \dot{s} = \ldots = s^{(r+l-1)} = 0$, any $(r+l)$ sliding controller is only effective in some vicinity of the mode. Global convergence is only proved for $r = l = 1$ [15]. In [19], X. Liu et al. used higher order sliding mode for tracking control of piezoelectric systems to improve the chattering attenuation in the presence of some uncertainties. Furuta et al. [20] designed a variable structure (VS) controller with sliding sector that enables the system states to move from the outside to the inside of the sliding sector. The proposed VS controllers are quadratically stable and chatter free. In [21], Feng et al. have proposed a second order terminal sliding controller which utilizes a non-singular terminal sliding mode manifold for the input-output subsystem to realize fast convergence and better tracking precision. Meanwhile, a chattering-free second-order terminal sliding mode control law is presented. In [11] dynamic surfaces were under consideration with conventional sliding modes which provide a dynamic controller only if the system contains certain control derivatives $(u^{(\beta)}, \beta \geq 1)$ in the input output representation for chattering elimination.

The robust stabilization of uncertain systems attracted many researchers but performance was addressed by very few. J. Liu et al. [23] developed a dynamic terminal sliding mode based robust algorithm which provided chattering reduction and improved performance. Laghrouche et al. [18] proposed a HOSM algorithm which was based on the idea of integral sliding mode control. This algorithm improved the performance of the control law with reduced chattering along the switching manifold. In regards to applications, [24] used integral sliding mode for permanent-magnet synchronous motor (PMSM) speed regulation with improved performance and enhanced robustness. In [25], an indirect adaptive control scheme is derived in which the proposed scheme provides arbitrarily improved transient performance in the presence of disturbances. Dynamic Sliding Mode control (DSMC) is a robust output feedback control strategy which enforces additional dynamics which are termed as compensator dynamics. The dynamic sliding system becomes an augmented system with compensators, which is higher order system as compared with the original system. The compensator dynamics are designed to achieve and/or improve the stability of the sliding system, yielding chatter free control with desired performance. DSMC provides a dynamic control law [11, 21, 26, 27, 28] which robustly asymptotically stabilizes the nonlinear systems.

In this paper, the authors propose a control design methodology which is based on the core ideas of dynamic sliding mode control and integral sliding mode control schemes.

This newly designed control law provides better results than that of DSMC and integral sliding mode. The sliding mode is established without reaching phase. The reaching phase is eliminated using an integral manifold and the chattering is reduced by using the concept of dynamic sliding mode control. The robustness is enhanced via the reaching phase elimination as well as via the robust nature of DSMC. In nutshell the proposed controller improved performance, robustness and chattering reduction. The rest of the paper is presented in the following sequence. In Section 2 the problem is converted into generalized controllable canonical form and in Section 3 the design method of the new control law, stability analysis and sliding mode existence is discussed. In Section 4, a standard car model is considered to clarify the design procedure. In Section 5, the comparative analysis of the simulation results of new control law with dynamic sliding mode is presented. Section 6 contains the comprehensive concluding remarks followed by references.

2. **Problem Formulation.** Consider a SISO nonlinear system described by the state equation

$$\dot{x} = f(x) + g(x)u + \zeta(x,t) \tag{1}$$

$$y = h(x) \tag{2}$$

where $x \in R^n$ is the measurable state vector, $u \in R$ is scalar control input, $f(x)$ and $g(x)$ are smooth vector fields, $\zeta(x,t)$ represents the uncertainties. These uncertainties occur due to unmodeled dynamics, parametric variations and external disturbances and $h(x)$ is a measurable scalar output function. The unknown function $\zeta(x,t)$ is some norm bounded scalar function which represents the uncertainties in the system, where $||\zeta(x,t)|| \leq \zeta_0$ represents some norm and ζ_0 is some positive constant. In order to present the new designed control law, a short discussion on the output feedback technique is presented in the forthcoming paragraphs which provide the standard form of the problem. The derivative of the output function $h(x)$ with respect to the function $f(x)$ is defined as [29].

$$Ł_f h(x) = \frac{\partial h(x)}{\partial x} f(x) = \nabla h(x) f(x)$$

Recursively it can be defined as

$$Ł_f^0 h(x) = h(x)$$

$$L_f^j h(x) = L_f(L_f^{(j-1)} h(x)) = \nabla(L_f^{(j-1)} h(x)) f(x), \quad j = 1, 2, \ldots$$

The r^{th} derivative of the output function in which the input u appears explicitly is called the relative degree. Mathematically, the r^{th} derivative of the output along the dynamics of system (1) becomes

$$y^{(r)} = L_f^r h(x) + L_g(L_f^{(r-1)} h(x))u + \zeta(x,t) \tag{3}$$

Subject to the following conditions

1. $L_g(L_f^{(i)} h(x)) = 0$ for all x in the neighborhood of x_0 for $i < r-1$.
2. $L_g(L_f^{(r-1)} h(x)) \neq 0$.

The n^{th} derivative of output function becomes,

$$\begin{aligned} y^{(n)} = {} & L_f^n h(x) + L_g(L_f^{(n-1)} h(x))u + \cdots + L_f L_g L_f^{(r-1)} h(x) u^{(k-1)} \\ & + L_g^2 L_f^{(r-1)} h(x) u u^{(k-1)} + L_g L_f^{(r-1)} h(x) u^{(k)} + \zeta^*(x, u, \dot{u}, \ldots, u^{(n-1)}, t) \end{aligned} \tag{4}$$

where $\zeta^*(x, u, \dot{u}, \ldots, u^{(n-1)}, t)$ is bounded function which represents the uncertainties and their time derivatives. The constant k shows the number of differentiation of (2). Following assumptions and transformations will be considered for the system defined in (3). Suppose that

$$\varphi(\hat{y}, \hat{u}) = L_f^n h(x) + L_g(L_f^{(n-1)} h(x))u + \cdots + L_f L_g L_f^{(r-1)} h(x) u^{(k-1)} + L_g^2 L_f^{(r-1)} h(x) u u^{(k-1)}$$

and

$$\gamma(\hat{y}) = L_g L_f^{(r-1)} h(x)$$

where $\hat{y} = (y, \dot{y}, \cdots, y^{(n-1)})$ and $\hat{u} = (u, \dot{u}, \cdots, u^{(n-1)})$. Now by defining the transformation $y^{(i-1)} = \xi_i$ for $i = 1, 2, \ldots, n$ and $\hat{y} = \hat{\zeta}$, the system (3) results in the form:

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \xi_3 \\ &\vdots \\ \dot{\xi}_n &= \varphi(\hat{\xi}, \hat{u}) + \gamma(\hat{\xi}) u^{(k)} + \zeta^*(\hat{\xi}, t) \\ &= \phi(\hat{\xi}, \hat{u}, u^{(k)}) + \zeta^*(\hat{\xi}, t) \end{aligned} \tag{5}$$

The representation in (4) is called Local Generalized Controllable Canonical Form(LGCCF) of Fliess [31].

Assumption 2.1. *Let $\zeta^*(\hat{y}, t)$ satisfy:*

$$||\zeta^*(\hat{\xi}, t)|| \leq K_1$$

where K_1 is the uncertainty bound in C. Edward, et al. [15].

The nominal system corresponding to system (4) can be obtained by replacing $\zeta^*(\hat{\xi}, t) = 0$ and is termed as proper if *i*) It is single input single output; *ii*) $\phi(\hat{\xi}, \hat{u}, u^{(k)}) \in C^1$; *iii*) $\det\left[\frac{\partial \phi(\hat{\xi}, \hat{u}, u^{(k)})}{\partial u^{(k)}}\right] \neq 0$. A wide class of nonlinear systems can be put into I-O form with the addition of compensator term which appears as a chain of integrators [31].

Definition 2.1. *The zero dynamics of the nominal system in (5) are defined as* [28] $\phi(0, \hat{u}, u^{(k)}) = 0$. *That nominal system in (5) is called minimum phase if the aforementioned zero dynamics are uniformly asymptotically stable.*

3. **Control Law Design.**

The usual dynamic control law consists of a control law which is totally based on the sliding mode control theory. However, the proposed dynamic control law contains two dynamic terms which appear with the following mathematical expression:

$$u^{(k)} = u_0^{(k)} + u_1^{(k)} \tag{6}$$

The first part $u_0^{(k)} \in R$, is continuous which is used to stabilize the nominal system in finite time when sliding mode is being established from the beginning of the process. The second part $u_1^{(k)} \in R$ is discontinuous in nature called the dynamic integral control which efficiently rejects the uncertainties. These uncertainties may be due to external uncertainties and internal parametric uncertainties, etc. In the subsequent subsections, the design methodology is demonstrated.

3.1. **Design of $\boldsymbol{u_0^{(k)}}$.** To facilitate the design of the linear control law, the nominal system in (6) can be written in alternate form as follows:

$$\begin{aligned}\dot{\xi}_1 &= \xi_2\\ \dot{\xi}_2 &= \xi_3\\ &\vdots\\ \dot{\xi}_n &= \chi(\hat{\xi}, \hat{u}, u^{(k)}) + u^{(k)}\end{aligned} \tag{7}$$

where $\chi(\hat{\xi}, \hat{u}, u^{(k)}) = \varphi(\hat{\xi}, \hat{u}) + (\gamma(\hat{\xi}) - 1)u^{(k)}$. This linear control law is designed in ideal (nominal) case with the subsequent assumption:

Assumption 3.1. *The system in (7) is considered to be independent of nonlinearities, i.e.,* $\chi(\hat{\xi}, \hat{u}, u^{(k)}) = 0$ *in the very beginning and it is also supposed that the system operates under* $u_0^{(k)}$ *only from the beginning of the process.*

Consequently, the system in (7) takes the following form:

$$\dot{\xi} = A\xi + Bu_0^{(k)} \tag{8}$$

where $A = \begin{bmatrix} O_{(n-1)\times 1} & I_{(n-1)\times(n-1)} \\ O_{1\times 1} & O_{1\times(n-1)} \end{bmatrix}$ and $B = \begin{bmatrix} O_{(n-1)\times 1} \\ I_{1\times 1} \end{bmatrix}$. The control law $u_0^{(k)}$ is then designed via LQR method.

Remark 3.1. *The aforementioned control law performs best when it is designed by robust techniques such as LMIs. The proposed methodology assumes that system in (5) is either minimum phase in sense defined in Definition 2.1 or the zero dynamics are marginally stable.*

3.2. **Design of $\boldsymbol{u_1^{(k)}}$.** In the proposed design technique, the dynamic controller design uses an integral manifold instead of conventional sliding surface which is used in the existing dynamic sliding mode controller. In order to attain the desired performance and to robustly compensate the uncertainties with reduced chattering, the dynamic controllers $u_1^{(k)}$ is formulated by first defining the integral sliding surface. The integral sliding surface is designed in such a way that the reaching phase is eliminated. This elimination boosts the robustness against uncertainties from the very beginning. The integral manifold is defined as follows

$$\sigma(\xi) = \sigma_0(\xi) + z \tag{9}$$

where $\sigma_0(\xi)$ is the Hurwitz polynomial which is mathematically defined by $\sigma_0(\xi) = \sum_{i=1}^{n} c_i\xi_i$ with $c_n = 1$ and z is the integral term. The time derivative of (9) along (5) yields

$$\dot{\sigma}(\xi) = \sum_{i=1}^{n-1} c_i\xi_{i+1} + \chi(\hat{\xi}, \hat{u}, u^{(k)}) + (\gamma(\hat{\xi}) - 1)u^{(k)} + \zeta^*(\hat{\xi}, t) + u_0^{(k)} + u_1^{(k)} + \dot{z} \tag{10}$$

Inserting

$$\dot{z} = -\left(\sum_{i=1}^{n-1} c_i\xi_{i+1} + u_0^{(k)}\right) \tag{11}$$

with initial conditions $z(0) = -\sigma_0(\xi(0))$, one has

$$\dot{\sigma}(\xi) = \varphi(\hat{\xi}, \hat{u}) + (\gamma(\hat{\xi}) - 1)u_0^{(k)} + \gamma(\hat{\xi})u_1^{(k)} + \zeta^*(\hat{\xi}, t) \tag{12}$$

This initial condition, $z(0)$, of the integral terms is adjusted in such a way that the sliding surface starts at 0 at time $t = 0$. The design of the discontinuous control law can be facilitated with the use of Definition 2.1.

Definition 3.1. *A general sliding convergence condition $\dot{\sigma}(\xi) = -\mu(k, \sigma)$ [28], is globally uniformly asymptotically stable if these conditions hold: i) $\mu(k, 0) = 0$, ii) $\mu(k, \sigma) \in C^1$ if $\sigma \neq 0$ and iii) $\dot{\sigma}(\xi) = -\mu(k, \sigma)$.*

For the proposed control law design this convergence condition is defined by the following mathematical form:

$$\dot{\sigma}(\xi) = -K_1(\sigma + W sign(\sigma)) \tag{13}$$

By comparing (12) and (13), the expression of dynamic controller $u_1^{(k)}$ becomes

$$u_1^{(k)} = -\frac{1}{\gamma(\hat{\xi})}\left(\varphi(\hat{\xi}, \hat{u}) + (\gamma(\hat{\xi}) - 1)u_0^{(k)} + K_1(\sigma + W sign(\sigma)\right) \tag{14}$$

This control law enforces sliding mode along the sliding manifold defined in (9). The constant K_1 is the control gain and can be selected according to uncertainty bounds [18], and the constant W can be defined according to application with value between $0 < W < 1$. Thus, the final control law can be obtained by replacing the optimal linear control law and the control law (14) in (6) and can be implemented by first integrating the derivative of the control $u^{(k)}$ k times.

Remark 3.2. *The coefficients of the conventional sliding surface are chosen by looking at the dynamic response of the system. However, in real application these constants can also be optimized using LMIs methods and PSO.*

Theorem 3.1. *Consider the nonlinear system in (4) subject to Assumptions 2.1 and 3.1. If the sliding surface is chosen according to (9), the discontinuous control law $u_1^{(k)}$ is selected according to (14) and the integral term is taken according to (11), then the asymptotic convergence condition is satisfied.*

Proof: Consider a Lyapunov function candidate as follows:

$$V = 1/2(\sigma)^2$$

Using (11), (14) and the mentioned assumptions, the time derivative of this Lyapunov function reduces to

$$\dot{V} \leq -\sigma\left(K_1(\sigma + W sign(\sigma))\right)$$

This expression shows that the gradient of the Lyanpunov function is negative which confirms that $\sigma = 0$ is stable equilibrium point and also guarantees the existence of dynamic integral sliding mode even in the presence of external disturbances and uncertainties.

Proposition 3.1. *The dynamics of the system (4), with control law (6) and integral manifold (11), in sliding mode is governed by the linear optimal control law.*

Proof: Consider (12), the expression of equivalent control law becomes

$$u_{eq}^{(k)} = -\frac{1}{\gamma(\hat{\xi})}\left(\varphi(\hat{\xi}, \hat{u}) - u_0^{(k)} + \zeta^*(\hat{\xi}, t)\right) \tag{15}$$

Now, using (15) in (5), ones has

$$\dot{\xi}_s = A\xi_s + Bu_0^{(k)} \tag{16}$$

Thus, it is proved that the system in sliding mode operates under the linear control law. The subscript s in (16) shows that the system is in sliding mode.

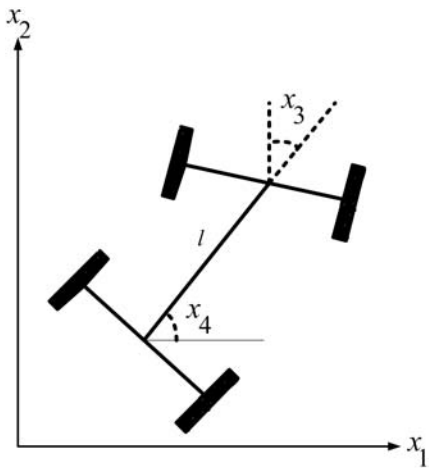

FIGURE 1. Kinematic car model

4. **Kinematic Car Model.** Consider a simple kinematic car model [33]

$$\begin{aligned} \dot{x}_1 &= w\cos(x_3) \\ \dot{x}_2 &= w\sin(x_3) \\ \dot{x}_3 &= w/l\tan(x_4) \\ \dot{x}_4 &= u + \zeta(x,t) \end{aligned} \tag{17}$$

where x_1 and x_2 are the Cartesian coordinates of the rear-axle middle point, x_3 the orientation angle and x_4 the steering angle, u the control input. w is the longitudinal velocity $w = 10\text{ms}^{-1}$, and l is the distance between the two axles ($l = 5$m). The term $\zeta(x,t) = 0.1\sin(x_3)^2 + 0.01x_2x_3$ in (17) represents some unknown bounded uncertainty. The objective is to regulate the output of the car from some initial position to the equilibrium point (origin). The output of interest is $y = h(x) = x_2$ and relative degree r of the system verses this output function is 3. The LGCCF form of the system (17) becomes

$$\begin{aligned} \dot{\xi}_i &= \xi_{i+1}, \quad i = 1,2,3 \\ \dot{\xi}_4 &= \varphi(\hat{\xi},\hat{u}) + \gamma(\hat{\xi})\dot{u} + \zeta^*(\hat{\xi},\hat{u},t) \end{aligned}$$

where $y = \xi_1$, $\gamma(\hat{\xi}) = \cos x_3 \sec^2 x_4$ and

$$\begin{aligned} \varphi(\hat{\xi},\hat{u}) = \frac{w^2}{l}\left(-\frac{w^2}{l^2}\cos x_3\tan^3 x_4 - \left(2\frac{w^2}{l^2} + \frac{w}{l}\right)\sin x_3\sec^2 x_4\tan x_4 u\right) \\ + \frac{w^2}{l}\left(2\cos x_3\sec^2 x_4\tan x_4 u\right) \end{aligned}$$

The control based on the methodology presented in the previous is given by

$$\dot{u} = -k_1\xi_1 - k_2\xi_2 - k_3\xi_3 - k_4\xi_4 - \frac{1}{\gamma(\hat{\xi})}\left(\varphi(\hat{\xi},\hat{u}) + (\gamma(\hat{\xi}) - 1)u_0^{(k)} + K_1(\sigma + W sign(\sigma)\right)$$

In the forthcoming section the simulation results are presented to look at the response of the system with the newly developed control law.

5. **Controller Evaluation.** The proposed controller is evaluated for the predefined criterion that includes performance, chattering reduction and robustness. The regulation control of aforementioned academic car model is carried out with DSMC and DISMC and is analyzed in detail with low and high control gains. The controller gains with small values are defined in Table 1. Both DSMC and DISMC are evaluated with the same

design parameters. The assessment of the controllers is carried out on the basis of states convergence, sliding manifold convergence and controller effort under various types of uncertainties.

TABLE 1. Values of the controller gains used for both DISMC and DSMC controllers

Parameters	k_1	k_2	k_3	k_4	K_1	W	c_1	c_2	c_3	c_4
Small Gains	31.62	77.32	78.73	34.02	100	0.001	15	30	1	1
High Gains	31.62	77.32	78.73	34.02	650	0.001	30	25	10	1

5.1. **Input additive uncertainty.**

5.1.1. *Case 1:* $u = u + \zeta(x,t)$. This uncertainty $\zeta(x,t) = 0.1\sin x_3^2 + 0.01x_2x_3$ is of additive nature which is introduced at the input channel which may represent some noise in the input channel. The states convergence under the application of DSMC and the proposed DISMC with small control gains are shown in Figure 2. It can be observed that with DSMC the state x_2 settles to origin in 8 seconds with oscillatory response and with DISMC x_2 converges to origin in 5 seconds with slight overshoot is the response. Furthermore, a close view of convergence revealed that DSMC instead of converging to origin converged in the vicinity of origin, i.e., 0.01 and the proposed controller converged the state exactly to the origin. The convergence of sliding manifold makes it clear that DSMC sliding surface converges to -0.25 instead of origin and exhibits chattering in its response. However, DISMC sliding manifold converges exactly to the origin without any chattering. Similarly, the control effort of both the controllers are free of chattering but DSMC exhibits oscillations with considerable magnitude. These oscillations may not be too dangerous as chattering phenomena, but still a threat for system/actuator health. In comparison, the proposed controller effort is smooth, free of chattering and oscillations.

The comparison of both the control techniques is carried out with high control gains in order to make sure that the proposed control law performs better than DSMC in all case of control gains. This claim is verified in Figure 3. The output convergence under the action of the new control law is better than the DSMC controller. The control efforts of DSMC observe small chattering along with some oscillatory behavior in the very beginning of the process. However, the proposed controller is chatter free. The sliding surface convergence of DISMC is exactly to the origin while the convergence of DSMC is in the vicinity of the origin along with chattering. One more attribute that can be observed is that the proposed controller has to exert much lesser effort as compared with DSMC. Thus, the proposed controller evolves as cheaper controller as compared with DSMC.

5.1.2. *Case 2.* In this case, the uncertainty is introduced in the input channel. The control input is incremented by an additive term 3 when $t \in [11, 13]$. This disturbance is independent of the system parameters and is introduced after achieving steady state to evaluate the robustness of the proposed controller. The designed controllers (DSMC and DISMC) with same parameters as in Table 1 are chosen for evaluation. The results displayed in Figure 4 show that the output trajectory of the system under DSMC deviates from the origin and takes 10 seconds to achieve back the equilibrium position. However, the proposed controller efficiently tackles for the undesirable deviation and the trajectory stay at the origin even in the existence of the disturbance. The respective control efforts show that at the time of the introduction of the disturbance, the switching surface of DSMC oscillates with some undesirable peaks which degrades the robustness and performance of DSM control law. Conversely, the sliding manifold for DISMC remains at 0 with slight peaks, keeping the robustness intact.

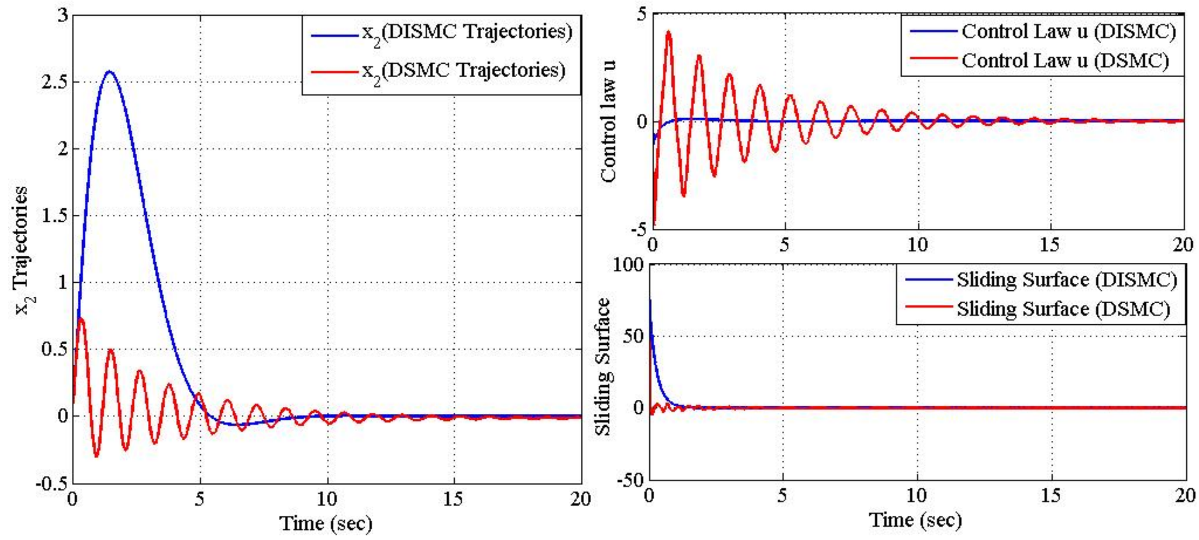

FIGURE 2. DISM and DSMC output regulation, control law efforts and sliding surface convergence

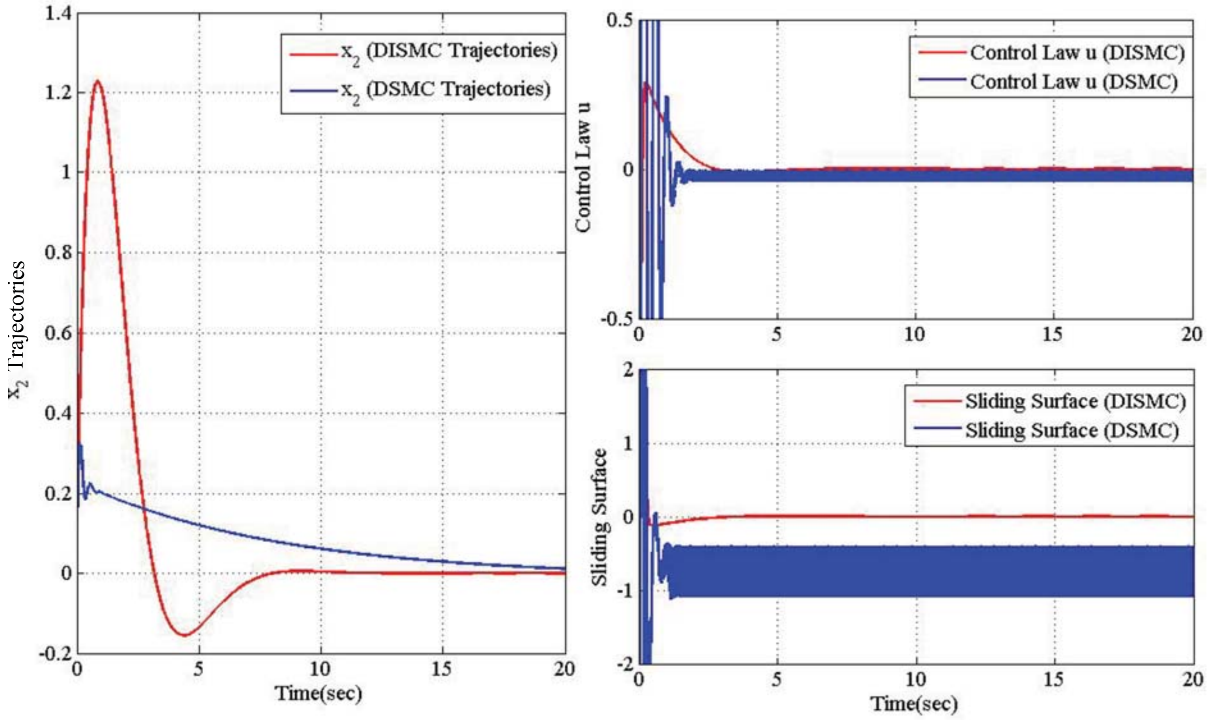

FIGURE 3. DISM and DSMC output regulation, control law efforts and sliding surface convergence

The proposed controller is tested with high control gains and their results are displayed in Figure 5. These results clarify that the proposed controller in output convergence, chattering reduction and performance improvement is better than DSMC.

5.2. **Parametric uncertainty.** This experiment involves the evaluation of the proposed controller under parametric variations in kinematic car model. The objective is to steer the output of the system to 0 in the presence of parametric variations. This system has two parameters w, and l with their nominal values 10m/s and 5m respectively. The

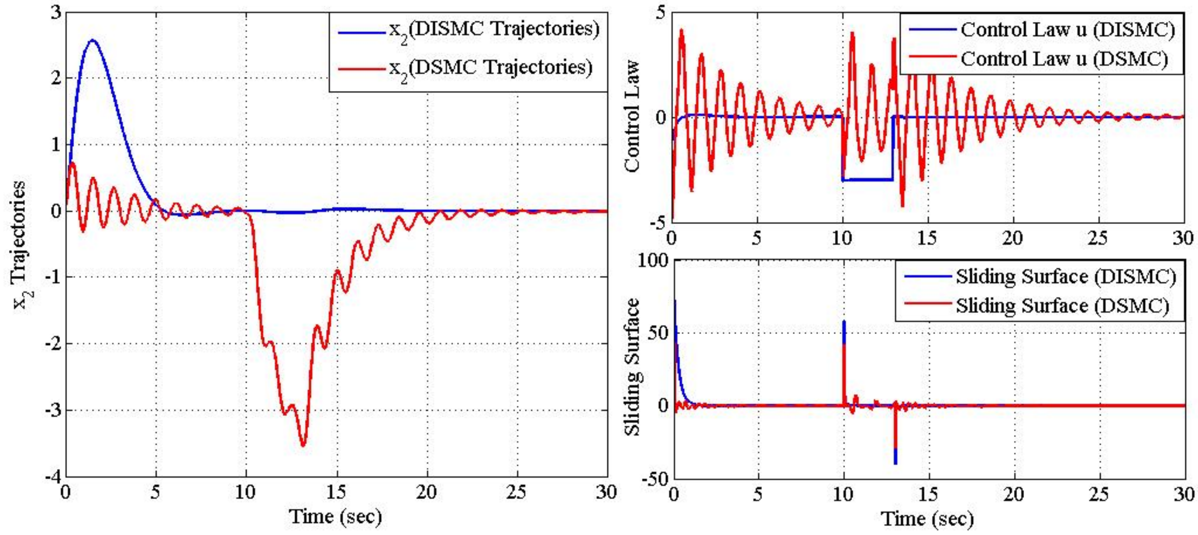

FIGURE 4. DISM and DSMC output regulation, control law efforts and sliding surface convergence

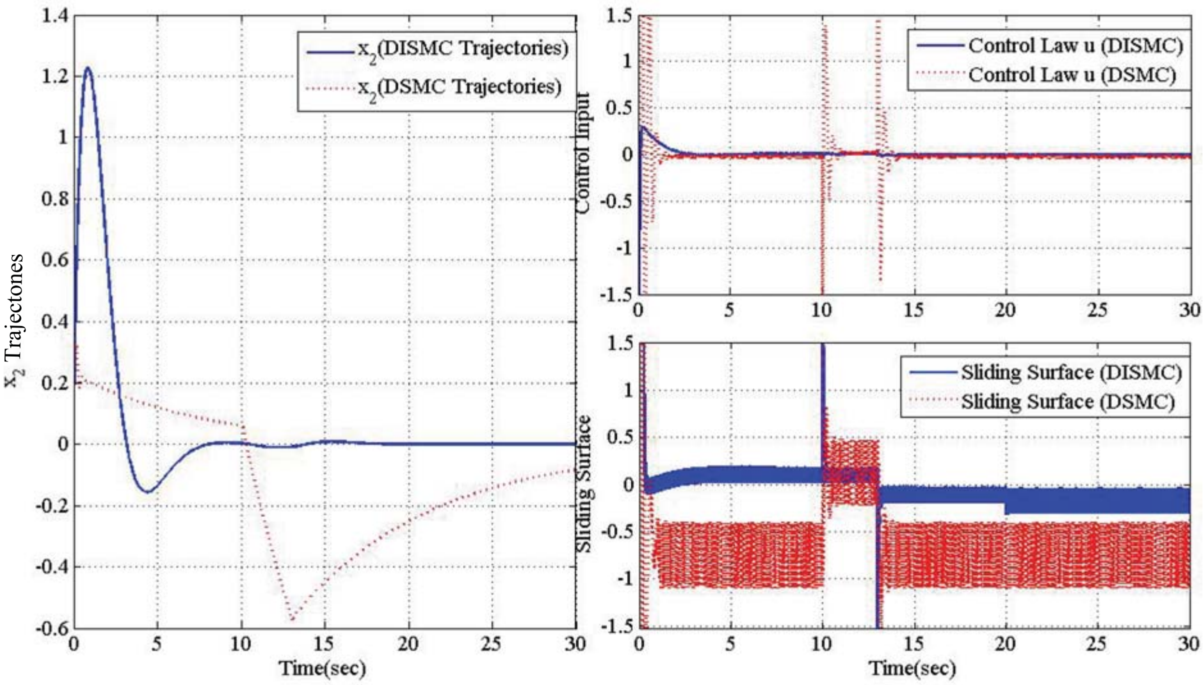

FIGURE 5. DISM and DSMC output regulation, control law efforts and sliding surface convergence

parametric uncertainty is introduced with a 40 percent variations in the nominal values of the actual parameters with the time frame $t \in [21, 25]$. These variations of the parameters were introduced after the achieving the steady state. The gains of the discontinuous controller are chosen to be $K_1 = 50$ and $W = 0.001$. The coefficients of the sliding surface are the same which were being used in Case 1 and Case 2. Figure 6 demonstrates the state convergence, control efforts and sliding surface convergence in the presence of these variations. It is clear that the steady state error of DSMC increases in the presence of

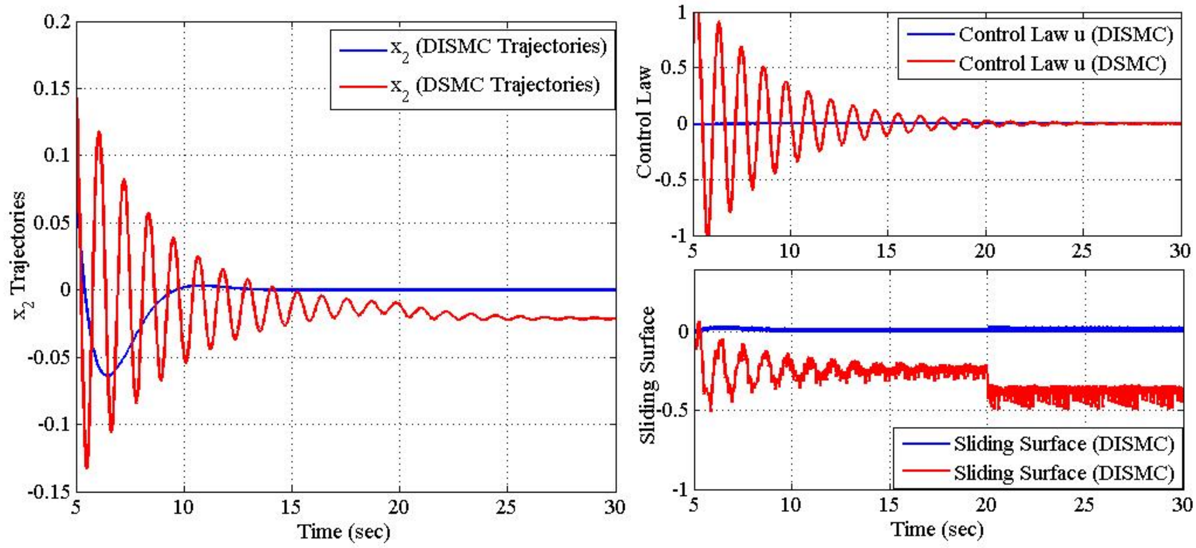

FIGURE 6. DISM and DSMC output regulation, control law efforts and sliding surface convergence

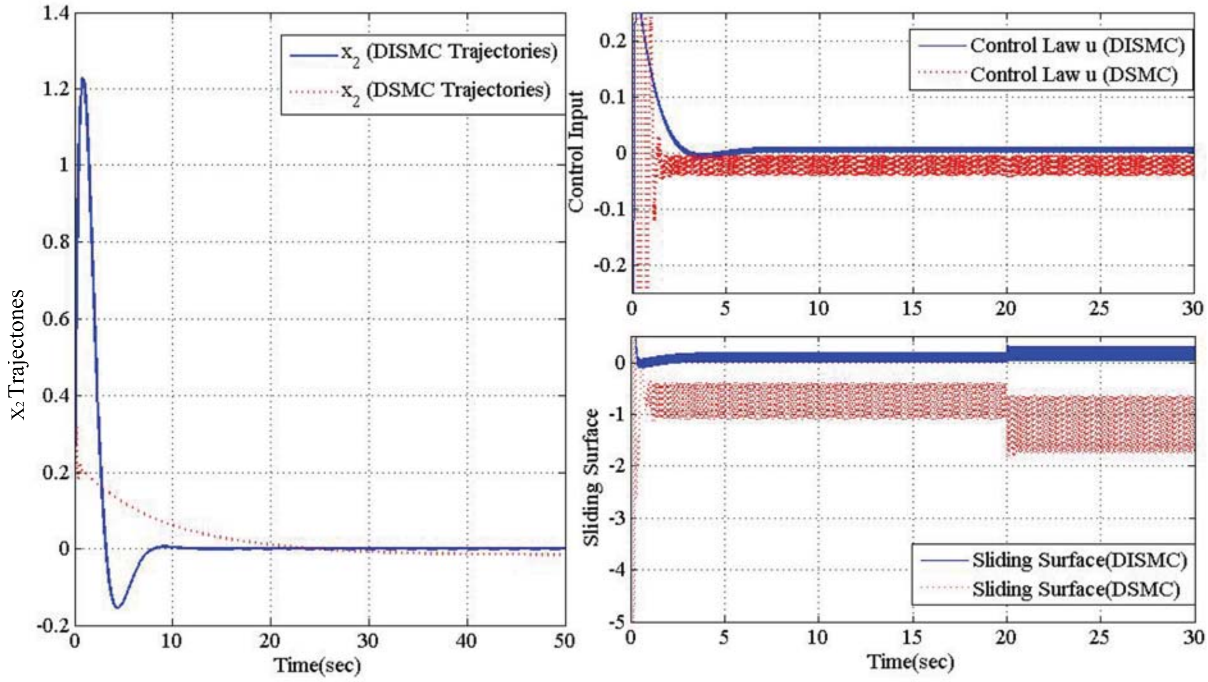

FIGURE 7. DISM and DSMC output regulation, control law efforts and sliding surface convergence

parametric variations whereas the new control law, DISMC, keeps the system at desired position. The sliding manifold is kept at zero by DISMC even in the presence of parametric deviation which certify the superiority of the proposed controller. Similar to the previous cases, the high gain performance of the controllers is displayed in Figure 7 which confirms the robustness of the proposed control law to parametric variations. The new controller evolves better than DSMC in output convergence, chattering reductions in the presence of parametric variations. The controller gains for these results are mentioned in Table 2.

Table 2. Comparative analysis of proposed controller with dynamic sliding mode controller

Attributes	DSMC	DISMC
Robustness	Rejects the disturbance but with deviation from the origin	Effectively rejects the disturbance
Settling Time	8 seconds	5 seconds
Oscillations	Oscillations	No Oscillations
Regulation	To the vicinity of the origin	Exactly to the origin
Overshoot	Exists with oscillations	No Over Shoot
Chattering Analysis	No chattering but oscillatory response	No Chattering
S. Surface Convergence	To the vicinity of origin with Chattering	To origin, No Chattering
Controller Gains	High Gains for desired performance	Small Gains (≈70 % small) for desired performance
Controller Effort	High Controller efforts	Low control efforts
Computational Complexities	Low computation complexities	High computation complexities

6. **Conclusions.** This paper has proposed a novel dynamic integral sliding mode (DISM) control design methodology for a class of SISO uncertain nonlinear systems. The control law is designed to ensure the asymptotic stabilization of the system in the presence of uncertainties. The control law incorporates an integral sliding manifold which guarantees the elimination of the reaching phase. Consequently, it enhances the robustness of the controller. The resulting controller considerably eliminates chattering at the system input. The performance of the controller has been proved far better than that of the dynamic sliding mode controller with low and high control gains. The better performance of the proposed control law depends on the integral sliding surface as well as on the continuous control law design. The simulation results confirm the applicability and efficient nature of this new control law.

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