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CONTROL DESIGN FOR TELE-OPERATION SYSTEM WITH TIME-VARYING AND STOCHASTIC COMMUNICATION DELAYS

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Abstract. *This paper investigates control design for the general bilateral tele-operation system with communication time delay. The time delay over the communication channel is assumed to be unknown and stochastically time varying. Then, an impedance controller is designed for the master side and an open-loop controller is designed for the slave side. In order to design the slave side controller, the control system is reformulated such that the slave side controller is converted to an equivalent dynamic output feedback controller in a standard control system representation. A Lyapunov-Krasovskii functional is defined for stability analysis. By choosing Lyapunov-Krasovskii functional. It is shown that the master-slave tele-operation system is stochastically stable under specific LMI conditions. Finally, simulations are performed to show the effectiveness of the proposed method.*

1. Introduction.

During the last two decades, kinds of tele-operation systems have been developed to allow human operators to execute tasks in remote or hazardous environments, with a variety of applications ranging from space to underwater, nuclear plants, and so on. A typical tele-operation system is composed of the human operator, the master robot, the communication networks, the slave robot and the environment. Tele-operation consists of unilateral and bilateral. The front only transmits the master motion and/or force to the slave site and the later includes the motion and/or force information transmissions from the slave site to the master site. Bilateral tele-operation is a challenging area of control technology with a number of traditional and potential applications ranging from space and undersea exploration to tele-surgery.

Due to information transmitted between master and slave via a communication channel, for example, Internet is of the most common communication channels used in this field. The induced time delay by Internet is stochastically time varying and it is well known that the time delays can destabilize the whole system if they are not well dealt with [3, 4]. Stability analysis and control design problems of time-delay systems have drawn increasing attention during the last few decades [1, 2].

Most of dynamic systems with time delays have been addressed in the context of functional differential equations, which leads to state space descriptions allowing recovering the time-domain stability analysis as introduced by Lyapunov approach [1, 28, 29]. A number of different control schemes have been proposed for tele-operation systems to provide a reliable and satisfactory control system, which includes passivity theory [5], wave variables [6], adaptive control [7], robust control [8], Smith predictor [10] and also non-time based reference for controller design [11].

Some work has considered time-delays to be random governed by a finite states homogeneous Markov process. For example, in [21], stochastic optimal control is investigated for a class of nonlinear systems governed by Markovian jump parameters. In [22], the control algorithm of an output feedback control scheme for robust optimal stabilizing control of a decentralized stochastic singularly-perturbed computer controlled system with multiple time-varying delays was designed. In [24], delay-dependent robust exponential stability was investigated for uncertain singular systems with state delay. In [23], the delay-dependent robust stochastic stabilization problem of stochastic TS-fuzzy delay systems with both norm bounded uncertainty and convex polyhedral uncertainty was considered.

Recently, the LMI-based approaches have been employed to deal with stability and stabilization problems [13, 14]. To the best of our knowledge, tele-operation control systems design has not been addressed by LMIs and strong features of this approach motivate the present study.

For most tele-operation approaches, time delay is assumed to be constant or varying with an upper boundary [25, 26]. Actually, the delay of Internet is characterized as random, unbounded and different for both branches in the control loop [27]. The Internet delay can be modeled as random delay with probability distributions governed by an underlying Markov chain [18], which is also called Markov jumping parameters. To the best of our knowledge, there is no such research about bilateral tele-operation systems with Markov jumping parameters. Therefore, in the paper, a new control strategy based on linear matrix inequalities and Markov jumping linear systems is designed for the tele-operation systems with Markov jumping parameters.

In this paper, stability analysis problem for a class of tele-operation systems is addresses. The forward and the backward transmission time delays are assumed to be both stochastic and time-varying, and then the tele-operation system is casted into the framework of Markov jumping systems. An impedance controller is designed for the master side and an open-loop controller is designed for the slave side. In order to design the slave side controller, the control system is reformulated such that the slave side controller is converted to an equivalent dynamic output feedback controller in a standard control system representation. A Lyapunov-Krasovskii functional is defined for stability analysis. By choosing Lyapunov-Krasovskii functional, we show that the master-slave tele-operation system is stochastically stable under specific LMI conditions. Finally, the simulations are performed to show the effectiveness of the proposed method.

2. **System Description.** A bilateral tele-operation system is presented in Figure 1. The overall master dynamic includes the master robot and its controller is denoted by P_M, while the slave side controller and the robot dynamics are represented by K_s and P_s respectively. The forwarding and returning communication channels time delays are labeled as $d_1(t)$ and $d_2(t)$ respectively. f_h and f_s are the force applied by the operator on the master and the force applied on the slave respectively. Besides, f'_s represents the slave force signal sent to the master through the communication channel. The dynamics of the master and slave are expressed as a $1DOF$ mass-damper system as follows:

$$m_m\dot{v}_m + \mu_m v_m = \tau_m + f_h \tag{1}$$

$$m_s\dot{v}_s + \mu_s v_s = f_s - f_e \tag{2}$$

where v_m, v_s and τ_m denote velocity and input torque, m_m and μ_m denote the mass and the viscous coefficients of the master respectively while m_s and μ_s denote the corresponding parameters of the slave respectively. Meanwhile, f_e represents the applied force by the

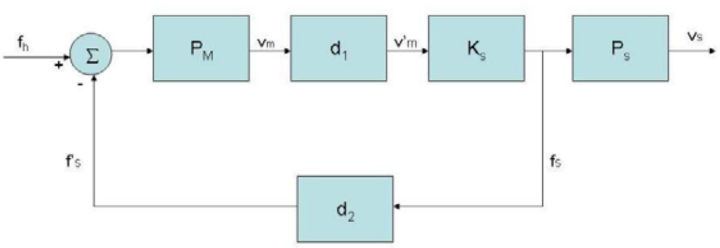

FIGURE 1. A bilateral teleoperation system

slave to the environment. Assume the desired impedance for the master is given by

$$M\ddot{x}_m + B\dot{x}_m + Kx_m = f_h - f'_s \tag{3}$$

where M, B and K are the inertia, damping and stiffness of the desired impedance respectively. Yet, as far as we are concerned, it is very hard to measure the acceleration $\ddot{x}_m$ in the presence of noise. Thus, by combining (3) and (1), we can obtain the control law of the master as follows:

$$\tau_m = \left(\mu_m - \frac{m_m}{M}B\right)v_m + \left(\frac{m_m}{M} - 1\right)f_h - \frac{m_m}{M}(f'_s + Kx_m) \tag{4}$$

which is for the slave manipulator to track the position of the master.

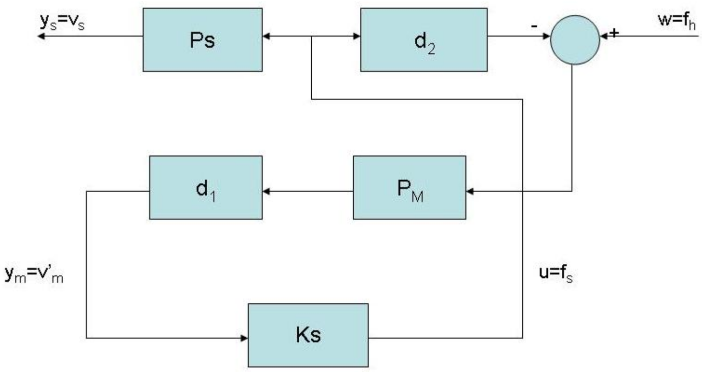

FIGURE 2. The standard representation of control system

In Figure 2, w denotes the exogenous input (human operator force) and y_s represents the controlled output which is the slave side velocity or displacement. Moreover, y_m and u are the equivalent measured output (delayed master side velocity command) and control inputs respectively. Assume $x_p = \left[x_{p1}^T, x_{p2}^T, x_{p3}^T, x_{p4}^T\right]^T = \left[x_m^T, \dot{x}_m^T, x_s^T, \dot{x}_s^T\right]^T$, then the overall state space representation of the equivalent plant P is as follows:

$$\dot{x}_p(t) = A_p x_p(t) + B_1 f_h(t - d_1(t)) + B_2 u(t) + B_3 u(t - d_1(t) - d_2(t)) \tag{5}$$

$$y_m(t) = C_{p1} x_p(t) \tag{6}$$

$$y_s(t) = C_{p2} x_p(t) \tag{7}$$

where

$$A_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K}{M} & -\frac{B}{M} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{\mu_s}{m_s} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_s} \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0 \\ -\frac{1}{M} \\ 0 \\ 0 \end{bmatrix}, \quad C_{p1} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}^T, \quad C_{p2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Consider a dynamic output controller in the following state space:

$$\begin{aligned} \dot{x}_k(t) &= A_k x_k(t) + B_k u_k(t), \quad u_k(t) \equiv y(t) \\ y_k(t) &= C_k x_k(t) + D_k u_k(t), \quad y_k(t) \equiv u(t) \end{aligned} \tag{8}$$

Define the tracking error e as:

$$e(t) = w(t - d_1(t)) - S_w z(t) \tag{9}$$

where S_w determines the output required to track. Now, adding the dynamic output feedback controller to this system model, we can obtain the following equations:

$$\dot{x}_a(t) = A_a x_a(t) + B_{a1} w(t - d_1(t)) + B_{a2} u(t) + B_{a3} u(t - d_1(t) - d_2(t))$$

$$y_{am}(t) = C_{am} x_a(t) \tag{10}$$

$$y_{as}(t) = C_{as} x_a(t) \tag{11}$$

where the augmented state vector is defined as $x_a^T(t) = [x_e^T, x_p^T]^T$ with $x_e(t) = \int_0^t e(\tau)d\tau$, $A_a = \begin{bmatrix} 0 & -S_w C_{p2} \\ 0 & A_p \end{bmatrix}$, $B_{a1} = \begin{bmatrix} I \\ B_1 \end{bmatrix}$, $B_{a2} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$, $B_{a3} = \begin{bmatrix} 0 \\ B_3 \end{bmatrix}$, $C_{as} = \begin{bmatrix} 0 \\ C_{p1}^T \end{bmatrix}^T$, $C_{am} = \begin{bmatrix} 0 \\ C_{p2}^T \end{bmatrix}^T$.

Let

$$x(t) = \begin{bmatrix} x_a(t) \\ x_k(t) \end{bmatrix} = \begin{bmatrix} x_e(t) \\ x_p(t) \\ x_k(t) \end{bmatrix} = \begin{bmatrix} x_e^T(t) & x_{p1}^T(t) & x_{p2}^T(t) & x_{p3}^T(t) & x_{p4}^T(t) & x_{k1}^T(t) & x_{k2}^T(t) \end{bmatrix}^T \tag{12}$$

Finally, the augmented system is given by the following state space equations:

$$\dot{x}(t) = A_1 x(t) + A_2 x(t - d_1(t) - d_2(t)) - E w(t - d_1(t)) \tag{13}$$

where $A_1 = \begin{bmatrix} A_a + B_{a2} D_k C_{am} & B_{a2} C_k \\ B_k C_{am} & A_k \end{bmatrix}$, $A_2 = \begin{bmatrix} B_3 D_k C_{p1} & 0 \\ 0 & B_3 C_k \end{bmatrix}$, $E = \begin{bmatrix} B_{a1} \\ 0 \end{bmatrix}$.

Thus, the control problem is changed into designing a robust stabilizer controller making the slave robot track the master side commands.

Remark 2.1. *As was mentioned above, we should introduced the stochastic model in analyzing the stability of the systems, because the communication delay over the Internet is both stochastic and time-varying.*

Remark 2.2. *For the imprecision of modeling, the introduced mathematical model should consider the uncertainty to ensure the performance of the real systems.*

3. **Problem Statement.** Since the system dynamics parameters are uncertain, and it is not easy to obtain the precise values beforehand. Moreover, for most tele-operation approaches, time delay is assumed constant or varying with an upper boundary in the previous works [20, 26]. Actually, the delay of Internet is characterized as random, unbounded and different for both branches in the control loop [27]. The Internet delay can be modeled as random delay with probability distributions governed by an underlying Markov chain [18], which is also called Markov jumping parameters. In this section, we introduce the following Markovian jump systems with two additive time-delays and the operator's force to the master.

$$\begin{cases} \dot{x}(t) = (A_1 + \Delta A_1)x(t) + (A_2 + \Delta A_2)x(t - d_{1r_t}(t) - d_{2r_t}(t)) + (E + \Delta E)(w(t - d_{1r_t}(t))) \\ x(s) = \phi(s), \quad r_s = r_0, \quad s \in \left[-2(\overline{d}_1 + \overline{d}_2), 0\right] \end{cases} \tag{14}$$

where $x(t) \in \mathbb{R}^n$ is the system state defined in (12), $\omega \in \mathbb{R}^m$ is the operator force, r_t is a right-continuous, discrete-state, homogeneous Markov process, taking values in the infinite set $S = \{1, 2, \ldots, N\}$, with the mode transition probability matrix

$$P(r_{t+\Delta t}{=}j|r_t = i) = \begin{cases} \pi_{ij}\Delta t + o(\Delta t) & i \neq j \\ 1 + \pi_{ii}\Delta t + o(\Delta t) & i = j \end{cases} \tag{15}$$

where $\Delta t > 0$, $\lim\limits_{\Delta t \to 0} o(\Delta t)/\Delta t = 0$, and $\pi_{ij} \geq 0$ $(i, j \in S, i \neq j)$ denotes the transition rate from mode i to j. $\pi_{ii} = -\sum\limits_{j=1, j\neq i}^{N} \pi_{ij}$, for all $i \in S$. A_1, A_2, B and E are all known constant matrices with appropriate dimensions; $d_{1r_t}(t)$ and $d_{2r_t}(t)$ are mode-dependent time-varying delay functionals satisfying

$$\begin{aligned} &0 \leq \underline{d}_1 \leq d_{1r_t}(t) \leq \overline{d}_1 |\dot{d}_{1r_t}(t)| \leq h_1 \\ &0 \leq \underline{d}_2 \leq d_{2r_t}(t) \leq \overline{d}_2 |\dot{d}_{2r_t}(t)| \leq h_2 \quad \forall r_t \in S \end{aligned} \tag{16}$$

where $\overline{d}_1$, $\underline{d}_1$, $\overline{d}_2$, $\underline{d}_2$, h_1, h_2 are all known positive scalars.

Assumption 3.1. *ΔA_1, ΔA_2, ΔE are all real-valued time-varying matrices representing the norm bounded parameters uncertainties satisfying*

$$\left[\Delta A_1 \ \Delta A_2 \ \Delta E\right] = HF(t)[D_1 \ D_2 \ D_3]$$

$F(t)$ is a real-valued matrix functional representing the uncertainty satisfying $F^T(t)F(t) \leq I$ and H, D_1 D_2, D_3 are all constant matrices known as a prior.

Definition 3.1. *(Boukas et al. [15]) System (14) is said to be stochastically stable if there exist a positive constant Γ such that*

$$E\left\{\int_0^\infty \|x(r_t, t)\|^2 dt | \phi(s), s \in [-2d, 0], r_0\right\} < \Gamma \tag{17}$$

Note that $\{\left(x(t, r_t), r_t\right), t \geq 0\}$ is non-Markovian because of the existence of $d_{1r_t}(t)$ and $d_{2r_t}(t)$. In order to cast the model into the framework of Markovian systems, we define a new process $\{\left(X_t, r_t\right), t \geq 0\}$ taking values in $\mathcal{C}_0$ as follows:

$$X_t \triangleq x(t + s), \quad s \in \left[-2(\overline{d}_1 + \overline{d}_2), 0\right]$$

where

$$\mathcal{C}_0 \triangleq \bigcup_{i \in S} \mathcal{C}[-2(\overline{d}_1 + \overline{d}_2), 0] \times \{i\}$$

and $\mathcal{C}[-2(\overline{d}_1+\overline{d}_2),0]$ denotes the continuous functions defined on $[-2(\overline{d}_1+\overline{d}_2),\ 0]$. Specifically, $\big((X_t,r_t),t\geq 0\big)$ can be verified as a strong Markov process with state space $\mathcal{C}_0$ similarly to [12].

Now, it is converted into a stochastic stability analysis problem which will be investigated in the following section.

4. **Main Results.** First, we will consider the nominal systems for (14), which implies that the uncertainty parameters $\Delta A_1=\Delta A_2=\Delta E=0$. The following theorem presents a sufficient condition to check the stochastic stability of the nominal systems for the investigated tele-operation systems (14).

Theorem 4.1. *Consider system (14) with $\Delta A_1=\Delta A_2=\Delta E=0$, then we conclude that the system is stochastically stable if there exist $n\times n$ matrices P_i, Q_k, Z_k, R_{il}, S_{il}, T_{il}, U_{il}, Y_{il}, $i\in S$, $k=1,2,3,4$, $l=1,2,\ldots,6$, such that (18) holds for all $i\in S$.*

$$M_i=\begin{pmatrix}\Psi_i & \overline{d}_1\widetilde{R}_i & \overline{d}_1\widetilde{U}_i & \overline{d}_2\widetilde{T}_i & \overline{d}_2\widetilde{S}_i\\ * & -\overline{d}_1Z_1 & 0 & 0 & 0\\ * & * & -\overline{d}_1Z_2 & 0 & 0\\ * & * & * & -\overline{d}_2Z_3 & 0\\ * & * & * & * & -\overline{d}_2Z_4\end{pmatrix}<0 \tag{18}$$

where $Q_m\leq Z_m$ $(m=1,2,3,4)$ and

$$\Psi_i=[\Psi^i_{jk}]_{6n\times 6n},\quad \widetilde{R}_i=\begin{pmatrix}R_{i1}\\ \vdots\\ R_{i6}\end{pmatrix},\quad \widetilde{U}_i=\begin{pmatrix}U_{i1}\\ \vdots\\ U_{i6}\end{pmatrix},\quad \widetilde{T}_i=\begin{pmatrix}T_{i1}\\ \vdots\\ T_{i6}\end{pmatrix},$$

$$\widetilde{S}_i=\begin{pmatrix}S_{i1}\\ \vdots\\ S_{i6}\end{pmatrix},\quad \widetilde{Y}_i=\begin{pmatrix}Y_{i1}\\ \vdots\\ Y_{i6}\end{pmatrix}.$$

$$\begin{aligned}
\Psi^i_{11}&=A_1^TP_i+P_iA_1+\sum_{j=1}^N\pi_{ij}P_j+Q_1+Q_3+R_{i1}+R_{i1}^T+T_{i1}+T_{i1}^T-Y_{i1}A_1\\
&\quad-A_1^TY_{i1}^T\eta\overline{d}_1Z_1+\eta(\overline{d}_1+\overline{d}_2-\underline{d}_2)Z_2+\eta\overline{d}_2Z_3+\eta(\overline{d}_1+\overline{d}_2-\underline{d}_1)Z_4\\
\Psi^i_{12}&=-R_{i1}+R_{i2}^T+S_{i1}+T_{i2}^T-A_1^TY_{i2}^T,\quad \Psi^i_{13}=R_{i3}^T+T_{i3}^T-T_{i1}+U_{i1}-A_1^TY_{i3}^T\\
\Psi^i_{14}&=R_{i4}^T-S_{i1}+T_{i4}^T-U_{i1}-Y_{i1}A_2-A_1^TY_{i4}^T+P_iA_2\\
\Psi^i_{15}&=R_{i5}^T+T_{i5}^T+Y_{i1}-A_1^TY_{i5}^T,\quad \Psi^i_{16}=P_iE_i+R_{i6}^T+T_{i6}^T-A_1^TY_{i6}^T-Y_{i1}E\\
\Psi^i_{22}&=-(1-h_1)Q_1+(1+h_1)Q_4-R_{i2}-R_{i2}^T+S_{i2}+S_{i2}^T\\
\Psi^i_{23}&=-R_{i3}^T+S_{i3}^T-T_{i2}+U_{i2},\quad \Psi^i_{24}=-R_{i4}^T+S_{i4}^T-S_{i2}-U_{i2}-Y_{i2}A_2\\
\Psi^i_{25}&=-R_{i5}^T+S_{i5}^T+Y_{i2},\quad \Psi^i_{26}=-R_{i6}^T+S_{i6}^T-Y_{i2}E\\
\Psi^i_{33}&=-(1-h_2)Q_3+(1+h_2)Q_2-T_{i3}-T_{i3}^T+U_{i3}+U_{i3}^T\\
\Psi^i_{34}&=-S_{i3}-T_{i4}^T+U_{i4}^T-U_{i3}-Y_{i3}A_2,\quad \Psi^i_{35}=-T_{i5}^T+U_{i5}^T+Y_{i3}\\
\Psi^i_{36}&=-T_{i6}^T+U_{i6}^T-Y_{i3}E_i,\quad \Psi^i_{56}=Y_{i6}^T-Y_{i5}E,\quad \Psi^i_{66}=-Y_{i6}E-E^TY_{i6}^T\\
\Psi^i_{44}&=-(1-h_1-h_2)(Q_2+Q_4)-S_{i4}-S_{i4}^T-U_{i4}-U_{i4}^T-Y_{i4}A_2-A_2^TY_{i4}^T\\
\Psi^i_{45}&=-S_{i5}^T-U_{i5}^T-A_2^TY_{i5}^T+Y_{i4},\quad \Psi^i_{46}=-S_{i6}^T-U_{i6}^T-A_2^TY_{i6}^T-Y_{i4}E\\
\Psi^i_{55}&=\overline{d}_1Z_1+(\overline{d}_1+\overline{d}_2-\underline{d}_2)Z_2+\overline{d}_2Z_3+(\overline{d}_1+\overline{d}_2-\underline{d}_1)Z_4+Y_{i5}+Y_{i5}^T
\end{aligned}$$

Proof: Consider the following Lyapunov-Krasovskii functional:

$$V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7 \tag{19}$$

where

$$V_1 = x^T(t)P(r_t)x(t), \quad V_2 = \int_{t-d_{1r_t}(t)}^{t} x^T(s)Q_1x(s)ds + \int_{t-d_{1r_t}(t)-d_{2r_t}(t)}^{t-d_{1r_t}(t)} x^T(s)Q_4x(s)ds$$

$$V_3 = \int_{t-d_{2r_t}(t)}^{t} x^T(s)Q_3x(s)ds + \int_{t-d_{1r_t}(t)-d_{2r_t}(t)}^{t-d_{2r_t}(t)} x^T(s)Q_2x(s)ds$$

$$V_4 = \int_{-\overline{d}_1}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z_1\dot{x}(s)dsd\theta + \int_{-\overline{d}_1-\overline{d}_2}^{-\underline{d}_1}\int_{t+\theta}^{t} \dot{x}^T(s)Z_4\dot{x}(s)dsd\theta$$

$$V_5 = \int_{-\overline{d}_2}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z_3\dot{x}(s)dsd\theta + \int_{-\overline{d}_1-\overline{d}_2}^{-\underline{d}_2}\int_{t+\theta}^{t} \dot{x}^T(s)Z_2\dot{x}(s)dsd\theta$$

$$V_6 = \eta\int_{-\overline{d}_1}^{0}\int_{t+\theta}^{t} x^T(s)Z_1x(s)dsd\theta + \eta\int_{-\overline{d}_1-\overline{d}_2}^{-\underline{d}_1}\int_{t+\theta}^{t} x^T(s)Z_4x(s)dsd\theta$$

$$V_7 = \eta\int_{-\overline{d}_2}^{0}\int_{t+\theta}^{t} x^T(s)Z_2x(s)dsd\theta + \eta\int_{-\overline{d}_1-\overline{d}_2}^{-\underline{d}_2}\int_{t+\theta}^{t} x^T(s)Z_3x(s)dsd\theta$$

We define $\xi(t)$ as follows:

$$\xi(t) = \left[x^T(t), x^T(t-d_{1i}(t)), x^T(t-d_{2i}(t)), x^T(t-d_{1i}(t)-d_{2i}(t)), \dot{x}^T(t), w(t-d_{1i}(t))\right]^T$$

From the Newton-Leibniz formula and the system Equation (14), we have the following equalities:

$$2\xi^T(t)\widetilde{R}_i\left[x(t) - x(t-d_{1i}(t)) - \int_{t-d_{1i}(t)}^{t}\dot{x}(s)ds\right] = 0 \tag{20}$$

$$2\xi^T(t)\widetilde{S}_i\left[x(t-d_{1i}(t)) - x(t-d_{1i}(t)-d_{2i}(t)) - \int_{t-d_{1i}(t)-d_{2i}(t)}^{t-d_{1i}(t)}\dot{x}(s)ds\right] = 0 \tag{21}$$

$$2\xi^T(t)\widetilde{T}_i\left[x(t) - x(t-d_{2i}(t)) - \int_{t-d_{2i}(t)}^{t}\dot{x}(s)ds\right] = 0 \tag{22}$$

$$2\xi^T(t)\widetilde{U}_i\left[x(t-d_{2i}(t)) - x(t-d_{1i}(t)-d_{2i}(t)) - \int_{t-d_{1i}(t)-d_{2i}(t)}^{t-d_{2i}(t)}\dot{x}(s)ds\right] = 0 \tag{23}$$

$$2\xi^T(t)\widetilde{Y}_i\left[\dot{x} - A_1x(t) - A_2x(t-d_{1i}(t)-d_{2i}(t)) - Ew(t-d_{1i}(t))\right] = 0 \tag{24}$$

Noticing that

$$\sum_{j=1}^{N}\pi_{ij}\int_{t-d_{1j}(t)}^{t} x^T(s)Q_1x(s)ds \le \sum_{j\neq i}\pi_{ij}\int_{t-d_{1j}}^{t} x^T(s)Q_1x(s)ds \le \eta\int_{t-\overline{d}_1}^{t} x^T(s)Q_1x(s)ds \tag{25}$$

$$\sum_{j=1}^{N}\pi_{ij}\int_{t-d_{1j}(t)-d_{2j}(t)}^{t-d_{1j}(t)} x^T(s)Q_4x(s)ds \le \eta\int_{t-\overline{d}_1-\overline{d}_2}^{t-\underline{d}_1} x^T(s)Q_4x(s)ds \tag{26}$$

$$\sum_{j=1}^{N}\pi_{ij}\int_{t-d_{2j}(t)}^{t} x^T(s)Q_3x(s)ds \le \eta\int_{t-\overline{d}_2}^{t} x^T(s)Q_3x(s)ds \tag{27}$$

$$\sum_{j=1}^{N}\pi_{ij}\int_{t-d_{1j}(t)-d_{2j}(t)}^{t-d_{2j}(t)}x^T(s)Q_2x(s)ds \le \eta\int_{t-\overline{d}_1-\overline{d}_2}^{t-\underline{d}_2}x^T(s)Q_2x(s)ds \tag{28}$$

Then, applying the Markovian infinitesimal operator to $V_1 \sim V_7$, we have the following inequality

$$\begin{aligned}
\mathscr{L}V \le\; & x^T(t)\Big[A_1^TP_i + P_iA_1 + \sum_{j=1}^{N}\pi_{ij}P_j + Q_1 + Q_3 + R_{i1} + R_{i1}^T + T_{i1} + T_{i1}^T - Y_{i1}A_1 \\
& - A_1^TY_{i1}^T + \eta\overline{d}_1Z_1 + \eta(\overline{d}_1+\overline{d}_2-\underline{d}_2)Z_2 + \eta\overline{d}_2Z_3 + \eta(\overline{d}_1+\overline{d}_2-\underline{d}_1)Z_4\Big]x(t) \\
& + x^T(t)\left[-2R_{i1} + 2R_{i2}^T + 2S_{i1} + 2T_{i2}^T - 2A_1^TY_{i2}^T\right]x(t-d_{1i}(t)) \\
& + x^T(t)[2R_{i3}^T + 2T_{i3}^T - 2T_{i1} + 2U_{i1} - 2A_1^TY_{i3}^T]x(t-d_{2i}(t)) \\
& + x^T(t)\left[2R_{i4}^T - 2S_{i1} + 2T_{i4}^T - 2U_{i1} - 2Y_{i1}A_2 - 2A_1^TY_{i4}^T + 2P_iA_2\right]x(t-d_{1i}(t) \\
& - d_{2i}(t)) + x^T(t)\left[2R_{i5}^T + 2T_{i5}^T + 2Y_{i1} - 2A_1^TY_{i5}^T\right]\dot{x}(t) \\
& + x^T(t)\left[2P_iE + 2R_{i6}^T + 2T_{i6}^T - 2A_1^TY_{i6}^T - 2Y_{i1}E\right]w(t-d_{1i}(t)) \\
& + x^T(t-d_{1i}(t))\big[-(1-h_1)Q_1 + (1+h_1)Q_4 - R_{i2} - R_{i2}^T + S_{i2} + S_{i2}^T\big]x(t \\
& - d_{1i}(t)) + x^T(t-d_{1i}(t))[-2R_{i3}^T + 2S_{i3}^T - 2T_{i2} + 2U_{i2}]x(t-d_{2i}(t)) \\
& + x^T(t-d_{1i}(t))[-2R_{i4}^T + 2S_{i4}^T - 2S_{i2} - 2U_{i2} - 2Y_{i2}A_2]x(t-d_{1i}(t)-d_{2i}(t)) \\
& + x^T(t-d_{1i}(t))[-2R_{i5}^T + 2S_{i5}^T + 2Y_{i2}]\dot{x}(t) \\
& + x^T(t-d_{1i}(t))[-2R_{i6}^T + 2S_{i6}^T - 2Y_{i2}E]w(t-d_{1i}(t)) \qquad (29)\\
& + x^T(t-d_{2i}(t))\big[-(1-h_2)Q_3 + (1+h_2)Q_2 - T_{i3} - T_{i3}^T + U_{i3} + U_{i3}^T\big]x(t-d_{2i}(t)) \\
& + x^T(t-d_{2i}(t))[-2S_{i3} - 2T_{i4}^T + 2U_{i4}^T - 2U_{i3} - 2Y_{i3}A_2]x(t-d_{1i}(t)-d_{2i}(t)) \\
& + x^T(t-d_{2i}(t))[-2T_{i5}^T + 2U_{i5}^T + 2Y_{i3}]\dot{x}(t) \\
& + x^T(t-d_{2i}(t))[-2T_{i6}^T + 2U_{i6}^T - 2Y_{i3}E]w(t-d_{1i}(t)) \\
& + x^T(t-d_{1i}(t)-d_{2i}(t))[-(1-h_1-h_2)(Q_2+Q_4) - S_{i4} - S_{i4}^T - U_{i4} - U_{i4}^T \\
& - Y_{i4}A_2 - A_2^TY_{i4}^T]x(t-d_{1i}(t)-d_{2i}(t)) \\
& + x^T(t-d_{1i}(t)-d_{2i}(t))[-2S_{i5}^T - 2U_{i5}^T - 2A_2^TY_{i5}^T + 2Y_{i4}]\dot{x}(t) \\
& + x^T(t-d_{1i}(t)-d_{2i}(t))[-2S_{i6}^T - 2U_{i6}^T - 2A_2^TY_{i6}^T - 2Y_{i4}E]w(t-d_{1i}(t)) \\
& + \dot{x}^T(t)[\overline{d}_1Z_1 + (\overline{d}_1+\overline{d}_2-\underline{d}_2)Z_2 + \overline{d}_2Z_3 + (\overline{d}_1+\overline{d}_2-\underline{d}_1)Z_4 + Y_{i5} + Y_{i5}^T]\dot{x}(t) \\
& + \dot{x}^T(t)[2Y_{i6}^T - 2Y_{i5}E]w(t-d_{1i}(t)) \\
& + w^T(t-d_{1i}(t))[-Y_{i6}E - E^TY_{i6}^T]w(t-d_{1i}(t)) \\
& + \overline{d}_1\xi^T(t)\left[\widetilde{R}_iZ_1^{-1}\widetilde{R}_i^T + \widetilde{U}_iZ_2^{-1}\widetilde{U}_i^T\right]\xi(t) + \overline{d}_2\xi^T(t)\left[\widetilde{T}_iZ_3^{-1}\widetilde{T}_i^T + \widetilde{S}_iZ_4^{-1}\widetilde{S}_i^T\right]\xi(t)
\end{aligned}$$

Using the Schur-complement lemma, we could show that

$$\begin{aligned}
\mathscr{L}V \le\; & \xi^T(t)\Psi_i\xi(t) + \overline{d}_1\xi^T(t)\left[\widetilde{R}_iZ_1^{-1}\widetilde{R}_i^T + \widetilde{U}_iZ_2^{-1}\widetilde{U}_i^T\right]\xi(t) \\
& + \overline{d}_2\xi^T(t)\left[\widetilde{T}_iZ_3^{-1}\widetilde{T}_i^T + \widetilde{S}_iZ_4^{-1}\widetilde{S}_i^T\right]\xi(t) < 0
\end{aligned}$$

which is equivalent to inequality (18).

We choose

$$\beta = \max_{i \in S} \lambda_{\max} \left[\Psi_i + \overline{d}_1 \left(\widetilde{R}_i Z_1^{-1} \widetilde{R}_i^T + \widetilde{U}_i Z_2^{-1} \widetilde{U}_i^T \right) + \overline{d}_2 \left(\widetilde{T}_i Z_3^{-1} \widetilde{T}_i^T + \widetilde{S}_i Z_4^{-1} \widetilde{S}_i^T \right) \right]$$

Obviously, $\beta < 0$. Then, we have

$$\mathscr{L}V(X_t, r_t, t) \leq \beta \parallel \xi(t) \parallel^2 \leq \beta \parallel x(t) \parallel^2 \tag{30}$$

From the Dynkin's Formula [16],

$$E\{V(x(t), r_t, t)\} - V(x_0, r_0, 0) \leq \beta E \left\{ \int_0^t ||x(s)||^2 ds \right\}$$

Let $t \to +\infty$, then we have:

$$\lim_{t \to +\infty} E \left\{ \int_0^t ||x(s)||^2 ds \right\} \leq (-\beta)^{-1} V(x_0, r_0, 0)$$

It is noted that the stochastic stability is obtained from Definition 3.1. Therefore, the proof is completed.

Theorem 4.2. *Consider the investigated systems (14) with norm bounded uncertainties, then it follows that the systems will be stochastically stable irrespective of the modeling uncertainties if the following LMIS hold.*

$$\mathbb{M}_i = \begin{pmatrix} \Phi_i & \overline{d}_1 \widetilde{R}_i & \overline{d}_1 \widetilde{U}_i & \overline{d}_2 \widetilde{T}_i & \overline{d}_2 \widetilde{S}_i & N_i \\ * & -\overline{d}_1 Z_1 & 0 & 0 & 0 & 0 \\ * & * & -\overline{d}_1 Z_2 & 0 & 0 & 0 \\ * & * & * & -\overline{d}_2 Z_3 & 0 & 0 \\ * & * & * & * & -\overline{d}_2 Z_4 & 0 \\ * & * & * & * & * & -I \end{pmatrix} < 0 \tag{31}$$

where

$$\Phi_i = [\Phi^i_{jk}]_{6n \times 6n}, \quad \Phi^i_{11} = \Psi^i_{11} + E_1^T E_1, \quad \Phi^i_{14} = \Psi^i_{14} + E_1^T E_2, \quad \Phi^i_{16} = \Psi^i_{16} + E_1^T E_3,$$
$$\Phi^i_{44} = \Psi^i_{14} + E_2^T E_2, \quad \Phi^i_{46} = \Psi^i_{16} + E_2^T E_3, \quad \Phi^i_{66} = \Psi^i_{16} + E_3^T E_3.$$

For the other terms of $[\Phi^i_{jk}]_{6n \times 6n}$

$$\Phi^i_{jk} = \Psi^i_{jk}$$

and

$$N_i = \left(\begin{bmatrix} -Y_{i1}^T & -Y_{i2}^T & \cdots & -Y_{i6}^T \end{bmatrix}^T + \begin{bmatrix} P_i^T & 0 & \cdots & 0 \end{bmatrix}^T \right) H,$$
$$Q_m \leq Z_m \quad (m = 1, 2, 3, 4). \tag{32}$$

Proof: Replacing A_1, A_2, E in (18) with $A_1 + \Delta A_1$, $A_2 + \Delta A_2$, $E + \Delta E$, and considering Assumption 3.1, we have that inequality (18) gives

$$\begin{pmatrix} \Psi_i & \overline{d}_1 \widetilde{R}_i & \overline{d}_1 \widetilde{U}_i & \overline{d}_2 \widetilde{T}_i & \overline{d}_2 \widetilde{S}_i \\ * & -\overline{d}_1 Z_1 & 0 & 0 & 0 \\ * & * & -\overline{d}_1 Z_2 & 0 & 0 \\ * & * & * & -\overline{d}_2 Z_3 & 0 \\ * & * & * & * & -\overline{d}_2 Z_4 \end{pmatrix} + \begin{pmatrix} L \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} F^T \begin{pmatrix} N_i^T & 0 & 0 & 0 & 0 \end{pmatrix} < 0 \tag{33}$$

where $L_{6n \times 6n} = \begin{bmatrix} E_1 & 0 & 0 & E_2 & 0 & E_3 \end{bmatrix}^T$, and N_i is defined in (32).

Owing to Lemma 2.4 in [9], (33) holds if and only if there exist a positive constant λ, such that

$$\begin{pmatrix} \Psi_i & \overline{d}_1\widetilde{R}_i & \overline{d}_1\widetilde{U}_i & \overline{d}_2\widetilde{T}_i & \overline{d}_2\widetilde{S}_i \\ * & -\overline{d}_1Z_1 & 0 & 0 & 0 \\ * & * & -\overline{d}_1Z_2 & 0 & 0 \\ * & * & * & -\overline{d}_2Z_3 & 0 \\ * & * & * & * & -\overline{d}_2Z_4 \end{pmatrix} + \lambda^{-1}\begin{pmatrix} L \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} L^T & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$
$$+\lambda\begin{bmatrix} N_i^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T\begin{bmatrix} N_i^T & 0 & 0 & 0 & 0 \end{bmatrix} < 0 \quad (34)$$

Multiplying the both sides with λ, and letting $\lambda P_i = P_i$, $\lambda Q_k = Q_k$, $\lambda Z_k = Z_k$, $\lambda\widetilde{R}_i = \widetilde{R}_i$, $\lambda\widetilde{S}_i = \widetilde{S}_i$, $\lambda\widetilde{T}_i = \widetilde{T}_i$, $\lambda\widetilde{U}_i = \widetilde{U}_i$, $\lambda\widetilde{Y}_i = \widetilde{Y}_i$, $i \in S$ and $k = 1, 2, 3, 4$. we could obtain (31) without any difficulty owing to the Schur-complement lemma.

Remark 4.1. *From Theorems 4.1 and 4.2, the tele-operation system under consideration of human force and system uncertainty using the control (8), the forces exerted by human can be transferred to the environments by the slave robot, moreover, the positions of master robot can be tracked by the slave robot, even if there exists uncertain time delay through the network.*

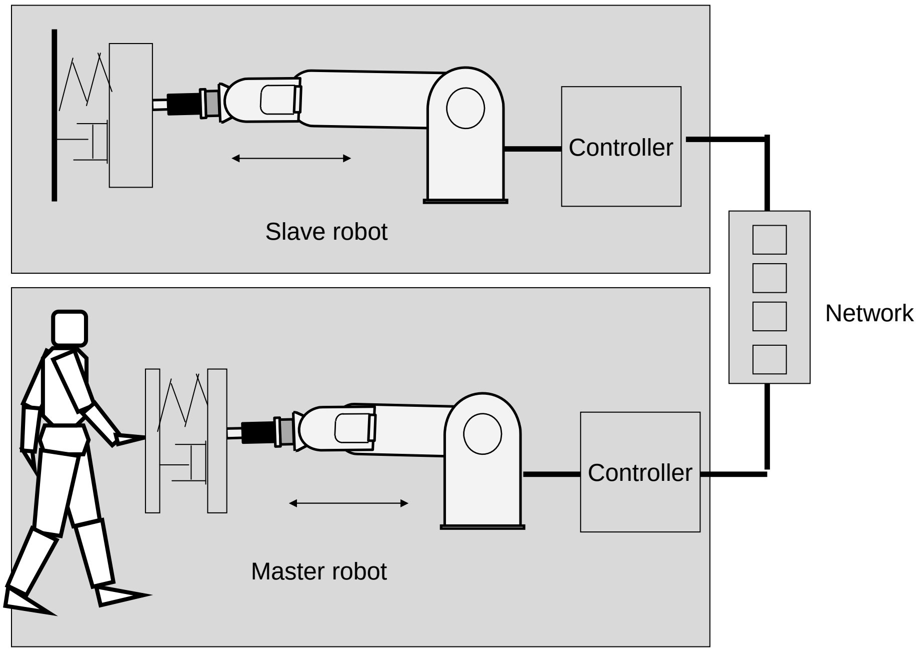

FIGURE 3. The tele-opeation robots

5. **Illustrative Example.** The tracking control performance is evaluated by applying a force exerted by human operator shown in Figure 3, where human interacts with the master robot by pushing, the force signal is transferred into the positions by the impedance approach, then the robots exchange the position information through the network. We consider equal randomly varying time delay in forwarding and returning communication path. Assume that the parameters of the bilateral system investigated in (1) $\sim$ (4) is given as follows. The master parameters: $M = 2.0 \pm 0.05Kg$, $B = 2.0 \pm 0.04Ns/m$, $K = 1.0 \pm 0.02N/m$. The slave parameters: $m_s = 1.0 \pm 0.04kg$, $\mu_s = 1 \pm 0.05N/m$, where the small uncertainty is introduced by modeling. The communication time-delay

bounds in (16) are set as: $d_1 = 0.5$, $h_1 = 1.5$; $d_2 = 0.8$, $h_2 = 1.2$, and the compatible time-varying delays are assumed to be $d_{11} = 0.3\sin^2 4t$, $d_{12} = 0.5\cos^2 t$; $d_{21} = 0.4\sin^2 3t$, $d_{22} = 0.8\cos^2 0.5t$. We assume that the output feedback in (8) controller is given as the following dynamics:

$$A_k = \begin{bmatrix} -27 & 12 \\ -3 & -66 \end{bmatrix},\ B_k = \begin{bmatrix} -19 \\ -15 \end{bmatrix},\ C_k = \begin{bmatrix} 23 & 3 \end{bmatrix},\ D_k = -0.6,\ \Pi = \begin{bmatrix} -50 & 50 \\ 10 & -10 \end{bmatrix}$$

Correspondingly, the time-varying delays governed by the Markov process are shown in Figure 4 and Figure 5. Using the LMI toolbox in Matlab, we could solve Theorem

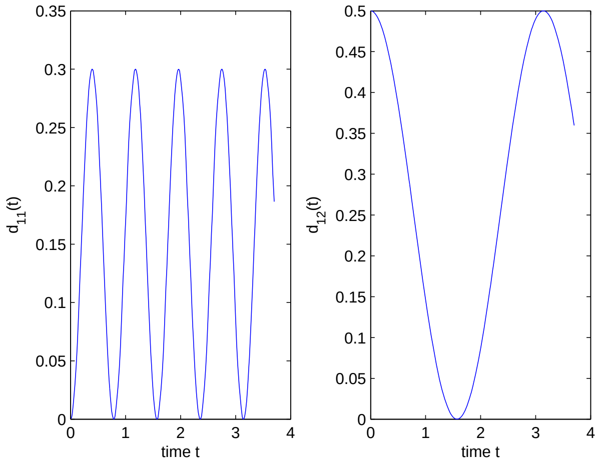

FIGURE 4. The time-varying delay $d_{1r_t}(t)$, at mode 1, $d_{1r_t}(t) = d_{11}(t)$; and at mode 2, $d_{1r_t}(t) = d_{12}(t)$

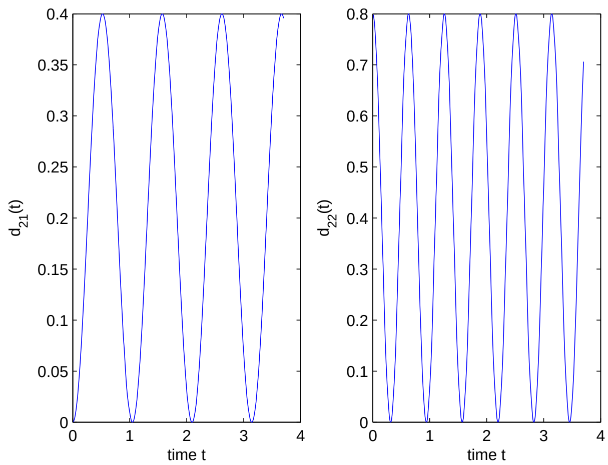

FIGURE 5. The time-varying delay $d_{2r_t}(t)$, at mode 1, $d_{2r_t}(t) = d_{21}(t)$; and at mode 2, $d_{2r_t}(t) = d_{22}(t)$

4.1, and get a group of solutions for P_i, Q_K, Z_K, etc. (See the appendix). Without loss of generality, we choose the initial states

$$\begin{aligned} x(t) = \phi(t) &= [x_e(t), x_{p1}(t), x_{p2}(t), x_{p3}(t), x_{p4}(t), x_{k1}(t), x_{k2}(t)] \\ &= [0.1\cos 2t, 0.4\cos t, -0.4\cos 2t, 0.3\cos 2t - 0.2\cos t, 0.2\cos 2t, -0.15\cos t]^T \end{aligned}$$

Then, we obtain the trajectories of the system states in Figure 6, which obviously shows the convergence of the states, and the trajectories of master and slave robot converge to the desired positions. The figure shows the excellent tracking performance.

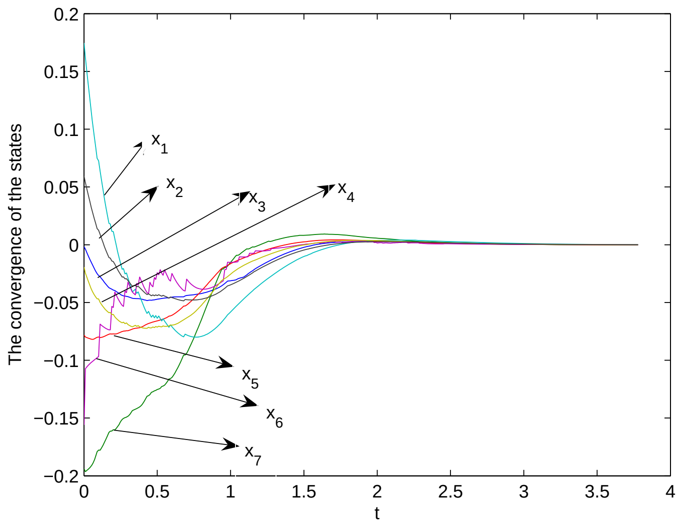

FIGURE 6. The trajectory of the states

And we could easily see the satisfactory performance of the system under the proposed control with time-varying and stochastic communication delays.

6. **Conclusions.** In this paper, stability analysis problem for a class of tele-operation systems with time-varying and stochastic communication delays is addresses. The forward and the backward transmission time delays are assumed to be time-varying and the tele-operation system is reconstructed by a Markov jumping system. An impedance controller is designed for the master side and an open-loop controller is designed for the slave side. In order to design the slave side controller, the control system is reformulated such that the slave side controller is converted to an equivalent dynamic output feedback controller in a standard control system representation. By choosing Lyapunov-Krasovskii functional, we show that the master-slave tele-operation system is stochastically stable under specific LMI conditions.

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