

A CELLULAR AUTOMATA HYBRID QUASI-RANDOM MONTE CARLO SIMULATION FOR ESTIMATING THE ONE-TO-ALL RELIABILITY OF ACYCLIC MULTI-STATE INFORMATION NETWORKS

WEI-CHANG YEH^{1,2}, LONGBING CAO¹ AND JESSE S. JIN³

¹Advanced Analytics Institute
School of Software
University of Technology Sydney
P.O. Box 123, Broadway, NSW 2007, Australia
yeh@ieee.org

²Advanced Manufacturing and Service Management Center
Integration and Collaboration Laboratory
Department of Industrial Engineering and Management Engineering
National Tsing Hua University
No. 101, Sec. 2, Kuang-Fu Road, Hsinchu 30013, Taiwan

³School of Design Communication and IT
University of Newcastle
Cnr King and Auckland Streets, Newcastle, NSW 2300, Australia
Jesse.Jin@newcastle.edu.au

Received July 2010; revised January 2011

ABSTRACT. *Many real-world systems (such as cellular telephones and transportation) are acyclic multi-state information networks (AMIN). These networks are composed of multi-state nodes, with different states determined by a set of nodes that receive a signal directly from these multi-state nodes, without satisfying the conservation law. Evaluating the AMIN reliability arises at the design and exploitation stage of many types of technical systems. However, existing analytical methods fail to estimate AMIN reliability in a realistic time frame, even for smaller-sized AMINs. Hence, the main purpose of this article is to present a cellular automata hybrid quasi-Monte Carlo simulation (CA-HMC) by combining cellular automata (CA, to rapidly determine network states), pseudo-random sequences (PRS, to obtain the flexibility of the network) and quasi-random sequences (QRS, to improve the accuracy) to obtain a high-quality estimation of AMIN reliability in order to improve the calculation efficiency. We use one benchmark example from well-known algorithms in literature to show the utility and performance of the proposed CA-HMC simulation when evaluating the one-to-all AMIN reliability.*

Keywords: Network reliability, Multi-state node acyclic network (AMIN), Hybrid quasi monte carlo method, Minimal tree/cut

1. Introduction. Many real-world systems, such as computer and communication systems, power transmission and distribution systems, transportation systems and production systems [1-13], can be modeled as networks, such that each component in these networks refers to an object or operation. The recommended approach is to first measure and evaluate the performance of the systems that can be modeled as stochastic networks or into fault trees. Thus, system reliability plays an important role in modern society.

Depending on whether the flow (or information) transferred within systems conforms to the flow conservation law, there are two possible network structures [14,15]: the (multi/binary-state) flow network (MFN/BFN) [1-7], and the multi-state information network (MIN),

first investigated by Malinowski and Preuss [14]. The former obeys the flow conservation law, whereas the latter does not. In flow networks, the capacity of each element is a non-negative integer. However, in a MIN, each node has different states determined by a set of nodes that receive the signal directly from the multi-state nodes [14-19]. Both network structures have their own applications. However, MIN is more practical and reasonable than MFN/BFN in many real-life systems such as computer networks and cellular telephone networks [8-21]. Therefore, MIN analysis has become a new subject in system reliability.

The acyclic multi-state information network (AMIN) is an extension of MIN. It has a source node, which can only emit and send a signal to other nodes, a number of sink nodes, which can only receive information, and a number of intermediate nodes (neither source nor sink nodes) that retransmit the received information to other non-source nodes. The information is transmitted from a non-sink node to a number of non-source nodes along the edges between these nodes, in other words, multi-states. Information leaving a node cannot return to this node through any sequence of nodes, and therefore, the network is acyclic. The probability that a node (state) will transmit to the next nodes is assumed to be known and statistically independent for each non-sink node.

Most current methods for evaluating AMIN reliability are all derived from the universal generating function methods (UGFMs), which were proposed by Levitin [22], and have recently been improved by Yeh using some simple techniques [30,31]. Yeh used a special Branch-and-Bound algorithm to calculate the one-to-all reliability between the source node and the subset of sink nodes [19]. Yeh also proved that traditional algorithms (for example, the minimal cuts) for BFN reliability can easily be revised to solve the AMIN reliability problem [20]. However, these existing analytical methods [18-21] are still difficult to use to evaluate the exact AMIN reliability in a reasonable time, even for a smaller-sized AMIN. Therefore, there is a need for an efficient and intuitive method to evaluate the AMIN reliability.

To overcome the above obstacles completely and reduce the computational burdens, we introduce the Monte Carlo simulation (MCS) here to estimate AMIN reliability. MCS has now become one of the more efficient, effective and convenient approaches for estimating the reliability of MFN/BFN, under various kinds of network structures (e.g., series, parallel and complex networks) with numerous distribution components. However, as far as the author is aware, the development of MCS has received less attention in literature than other methods for evaluating AMIN/MIN reliability.

The main purpose of this study is to develop a cellular automata hybrid quasi-random Monte Carlo simulation (CA-HMC) by combining cellular automata (CA, to rapidly determine network states), pseudo-random sequences (PRS, to obtain the flexibility of the network) and quasi-random sequences (QRS, to improve the accuracy of the network) to obtain a high-quality estimation of the one-to-all AMIN reliability in order to improve the calculation efficiency. The paper is organized as follows. Section 2 describes the acronyms, notations and assumptions required. Section 3 describes the crude Monte Carlo method and its statistical properties. Section 4 discusses the proposed cellular automata. Section 5 gives an introduction to quasi-random sequences. Section 6 introduces the proposed CA-HMC, and computational experiments are provided in Section 7 using a benchmark AMIN to show the performance of the proposed CA-HMC. Section 8 contains concluding remarks.

2. Notation, Nomenclature and Assumption.

Acronym

MFN	Multi-state flow network
BFN	Binary-state flow network
MIN	Multi-state information network
AMIN	Acyclic MIN
MCS	Monte Carlo simulation
CMC	Crude MCS
CA	Cellular Automata
PRS	Pseudo-Random Sequence
QRS	Quasi-Random Sequences
pCMC	CMC based on PRS
qCMC	CMC based on QRS
CA-HMC	The proposed CA Hybrid Quasi-random CMC
UGFM	Universal Generating Function Method

Notation

$E[\bullet]$	the expected value of \bullet .
$Var[\bullet]$	the variance of \bullet .
$Pr\{\bullet\}$	the probability of event \bullet .
$G(V, E)$	an AMIN with the set of nodes $V = \{1, 2, \dots, n\}$ and the set of edges E , where node 1 is the source node, e.g., the network in Figure 1 is an AMIN. The nodes are numbered in such a way that for any edge $e_{uv} \in E$ with $u < v$ [19].

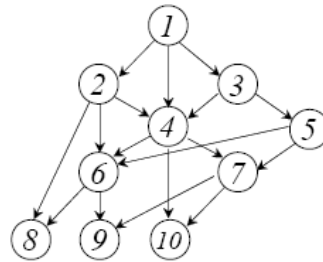


FIGURE 1. An example AMIN

$ \bullet $	the number of elements in \bullet .
T	the target node set ($\subseteq V - \{1\}$) of an AMIN.
s_i	the number of states of node i , where $i = 1, 2, \dots, V - T $.
s_{ij}	the j th state of node i , where $i = 1, 2, \dots, V - T $ and $j = 1, 2, \dots, s_i$.
S_{ij}	$\sum_{k=1}^j Pr\{s_{ik}\}$, where $i = 1, 2, \dots, V - T $ and $j = 1, 2, \dots, s_i$.
p_{ij}	the probability of all nodes in S_{ij} receive a signal directly from node i , where $\sum_j p_{ij}(t) = 1$ for $j = 1, 2, \dots, s_i$.
R_{1J}	the probability of all nodes in $J(\subseteq T)$ receiving a signal directly from node 1, i.e., the node 1 to target subset J reliability.
M	the total number of independent trials.

Nomenclature

- Acyclic network: a network containing no directed cycle.
- Target set/node: a non-empty subset of sink node set and its element is called the target node.

States:	it is a subset of nodes that one node can transmit signal to, e.g., \emptyset , $\{4\}$, $\{5\}$ and $\{4, 5\}$ are all states of node 3 in Figure 1.
AMIN Reliability:	the reliability of an AMIN at time t , $R(t)$, is defined as the probability that a signal can be transmitted from the source node to the target set which operates successfully throughout the time interval $(0, t]$. It can be denoted by $R(t) = P\{\text{there exists at least one path from node 1 to any target node in } G(V, E) \text{ during } (0, t)\} = P\{\phi(t) = 1\}$.
the flow conservation law:	the total flow (signal) through into and from a node (not a source node, target nodes or a target node) are all equal.
One-to-one:	between the source node and only one target subset.
One-to-all:	between the source node and all target subsets, i.e., all one-to-one.

Assumptions

The AMIN satisfies the following assumptions [19]:

1. All of nodes and arcs are perfectly reliable.
2. The signal can be retransmitted without following the flow conservation law.
3. All of the probabilities of states of each non-sink node are random variables according to a given distribution and assumed to be statistically independent.
4. No maintenance is considered.

3. The Crude Monte Carlo Method. MCS is a straightforward and efficient approach for obtaining a good quality estimation of the reliability of a relatively large and complex network, because the exact reliability is computationally intractable. The literature on MCS of BSN reliability is quite large. A brief survey of the previously published studies involving MCSs used to estimate network/system reliability are presented in chronological order as follows [6,7]. A more comprehensive review of recent literature of MCSs can be found in [7] and the references therein. It is worth noting that researchers have only focused on applying the MCS approach to estimating MFN/BFN reliability, without extending MCS to evaluate MIN/AMIN reliability.

As mentioned in [6], each proposed MCS should be compared with the most basic simulation, called the crude MCS (CMC), to determine its effectiveness (solution quality). In CMC, no enhancement is implemented when selecting and using random numbers [8-28]. The CMC for estimating the system reliability of a stochastic activity-of-arc BFN is as follows:

Procedure Crude-MCS()

Input: A BFN $G(V, E)$ with a source node 1, a sink node n , and the total number of replications equal to M .

Output: The estimator R^{CMC} of the system reliability.

STEP 0. Set the current replication number, i , to 1 and the current successful simulation number, $SUCCESS$, to 0.

STEP 1. Let $j = 1$ and $S_0 = \emptyset$.

STEP 2. If $\rho < (\text{the reliability of arc } j)$, then $S_0 \leftarrow S_0 \cup \{j\}$, where ρ is a random number generated from $U(0, 1)$.

STEP 3. If $j < |E|$ (the arc number), then let $j = j + 1$ and go to STEP 2.

STEP 4. If nodes 1 and n are connected in $G(V, E - S_0)$, then $SUCCESS = SUCCESS + 1$.

STEP 5. If $i < M$, then $i = i + 1$ and go to STEP 1. Otherwise, let $R^{\text{CMC}} = \frac{SUCCESS}{M}$ and halt.

The above CMC is suitable for BFNs, and can be extended easily to MFNs that satisfy the flow conservation law and MINs/AMINs that do not. Hence, all MCSs related to network reliability are based on the above CMC.

The fact that the CMC provides an unbiased estimator of the reliability of a system is well known to the technical community. Hence, we have the following statements, proofs of which can be found in any basic probability or simulation book:

Theorem 3.1. *The estimator R^{CMC} is an unbiased and consistency estimator with variance value $\frac{R(1-R)}{M}$, where R is the exact reliability and M is the replication number.*

Proof:

- a. Unbiased estimator of R : Let R_i^{CMC} be the the expected reliability of in $G(V, E - E_0)$, where $i = 1, 2, \dots, M$. Then,

$$E [R_i^{\text{CMC}}] = 1 \times Pr\{\text{nodes 1 and } n \text{ are connected}\} + 0 \times Pr\{\text{nodes 1 and } n \text{ are disconnected}\} = R. \quad (1)$$

Thus,

$$E \left[\sum_{i=1}^M \frac{R_i^{\text{CMC}}}{M} \right] = \sum_{i=1}^M \frac{E [R_i^{\text{CMC}}]}{M} = R. \quad (2)$$

- b. Variance of R^{CMC} : Since

$$Var [R_i^{\text{CMC}}] = E [(R_i^{\text{CMC}})^2] - E [R_i^{\text{CMC}}]^2 = R[1 - R], \text{ where } i = 1, 2, \dots, M. \quad (3)$$

Thus,

$$Var [R^{\text{CMC}}] = Var \left[\sum_{i=1}^M \frac{R_i^{\text{CMC}}}{M} \right] = \frac{R(1 - R)}{M}. \quad (4)$$

- c. Consistency estimator of R^{CMC} : From the above two parts, $E [R^{\text{CMC}}] = R$ and $\lim_{M \rightarrow \infty} Var [R^{\text{CMC}}] = 0$. This means that if any constant ε is given then $\lim_{M \rightarrow \infty} Pr\{|R^{\text{CMC}} - R| < \varepsilon\} = 1$. That is, if the replication number m is large enough, any precision requirement ε can be accomplished.

Thus, this theorem is true.

If the precision relative error, ε and the confidence interval $(1 - \alpha)\%$ are given for the n components, then the total number of trials (M) needed in the proposed method can be determined by the following theorem.

Theorem 3.2. *If the relative error, ε , and the confidence interval $(1 - \alpha)\%$ for the simulation are required, then the total number of trials for the simulation, M , must be at least $M \geq \frac{Z_\alpha^2}{4\varepsilon^2}$, where*

$$Z_\alpha = \frac{R^{\text{CMC}} - R}{\sqrt{Var[R^{\text{CMC}}]}}. \quad (5)$$

Proof: From the Central Limit Theorem, the limiting distribution of Z_α has a standard normal distribution. And by assumption:

$$P \{|R^{\text{CMC}} - R| < \varepsilon\} = (1 - \alpha)\% \quad (6)$$

we have

$$(1 - \alpha)\% = P \left\{ \frac{R^{\text{CMC}} - R}{\sqrt{Var[R^{\text{CMC}}]}} < \frac{\varepsilon}{\sqrt{Var[R^{\text{CMC}}]}} \right\} = P \left\{ |Z_\alpha| < \frac{\varepsilon}{\sqrt{R[1 - R]/M}} \right\} \quad (7)$$

Thus,

$$M = \frac{Z_\alpha^2 R [1 - R]}{\varepsilon^2}. \quad (8)$$

According to

$$\sqrt{R[1 - R]} \leq \frac{R + [1 - R]}{2} = \frac{1}{2}, \quad (9)$$

it follows that

$$M \geq \frac{Z_\alpha^2}{4\varepsilon^2}. \quad (10)$$

4. The Proposed Cellular Automata. Before introducing the proposed MCS, we give a brief introduction to the CA, including some of its useful properties. CA concepts were first devised by John von Neumann and Stanislaw Ulam in the 1940s while they studied self-replicating systems and lattice network models. Generally, the CA is a deterministic computation process that generates identical outputs for a given number of initial states. CA have been applied to studies of cryptography, complex system behaviors, and general phenomenological aspects.

Rocco and Moreno first introduced BFN reliability evaluation using a CA. Other CAs and MCSs can be found in [26-28]. Yeh also developed an algorithm similar to the CA in [21], but did not call it CA. Yeh et al. revised Yeh's algorithm in the CA format [31]. Although Yeh's CA is only based on the fundamental concept of graph scanning in graph theory, it is the more easily understood and implemented than existing CAs. Hence, Yeh's CA is revised and integrated into our proposed MCS to verify the connection between two given specified nodes in an AMIN, with the following three important elements [31]:

- 1) Cells and their states: A cell is a node, and its state is coded in integer digits: 0, 1, ..., (the number of states). If a cell's state is unknown, it is in a quiescent state. If a cell receives output from at least one of its connected nodes, it is in an activated state.
- 2) Neighborhoods: The neighborhood of node i is the set of all of its adjacent nodes.
- 3) Transition rules: In CA, a transition rule is the evolution strategy that delineates the next generation. A transition rule introduces dynamics between discrete time steps and the lattice.

Below is the proposed CA that we use to verify the state of the network.

Procedure Cellular-Automata()

Input: An AMIN $G(V_0, E_0)$ with the source node 1 and the target set T_0 .

Output: The connectedness between nodes 1 and all subsets of T .

STEP 0. Let $N = \{1\}$, $V^* = V_0 - \{1\}$, $T^* = T_0$ and $T^\# = \emptyset$.

STEP 1. Let node i be one of the element in N and $N^* = \{j \in V^* | e_{ij} \in E_0\}$.

STEP 2. If $(N^* \cap T^*) \neq \emptyset$, let $T^\# = T^\# \cup (N^* \cap T^*)$, $N^* = N^* - T^*$ and $T^* = T^* - N^*$.

STEP 3. Let $N = (N \cup N^*) - \{i\}$.

STEP 4. If $N \neq \emptyset$, let $V^* = V^* - N^*$ and go to STEP 1. Otherwise, nodes 1 and any proper subset of $T^\#$ are connected and halt.

As mentioned, Yeh's CA [31] is founded on scanning a graph, which is used to search for a path with a time complexity of $O(|V|)$. Hence, the running time of the above CA is $O(|V|)$ in the worst case, and its correctness can easily be assessed.

Lemma 4.1. *The above CA procedure is able to find a network state in $O(|V|)$.*

Figure 1, taken from Levitin [29], is a famous benchmark AMIN, and used here to show the procedure of the above CA. Figure 1 contains 10 perfectly reliable nodes and 9 unreliable arcs. The component (arc and node) label is listed either in a circle (node) or

link (arc). Suppose that the state reliability of each of the nodes, at some time (t), is as shown in Table 1. The values of $Pr\{s_{ij}\}$ are assumed by the user, and the accumulated final value of S_{ij} should be 1.

TABLE 1. Probability distributions of the states of nodes in Figure 1

i	j	State s_{ij}	$Pr\{s_{ij}\}$	$S_{ij} = \sum_{k=1}^j Pr\{s_{ik}\}$	i	j	State s_{ij}	$Pr\{s_{ij}\}$	$S_{ij} = \sum_{k=1}^j Pr\{s_{ik}\}$	
1	6	{2,3,4}	.75	1.00	4	8	{6,7,10}	.62	1.00	
	5	{2,3}	.10	.25		7	{6,7}	.08	.38	
	4	{3,4}	.08	.15		6	{6,10}	.06	.30	
	3	{2}	.02	.07		5	{7,10}	.02	.24	
	2	{3}	.01	.05		4	{6}	.05	.22	
	1	\emptyset	.04	.04		3	{7}	.05	.17	
2	8	{4,6,8}	.65	1.00	2	{10}	.07	.12		
	7	{4,6}	.08	.35		1	\emptyset	.05	.05	
	6	{4,8}	.05	.27		5	4	{6,7}	.83	1.00
	5	{6,8}	.08	.22		3	{6}	.04	.17	
	4	{4}	.05	.14		2	{7}	.07	.13	
	3	{6}	.02	.09		1	\emptyset	.06	.06	
	2	{8}	.05	.07		6	4	{8,9}	.80	1.00
	1	\emptyset	.02	.02		3	{8}	.06	.20	
3	4	{4,5}	.85	1.00	2	{9}	.10	.14		
	3	{4}	.06	.15		1	\emptyset	.04	.04	
	2	{5}	.04	.09		7	4	{9,10}	.60	1.00
	1	\emptyset	.05	.05		3	{9}	.35	.40	
						2	{10}	.02	.05	
				1	\emptyset	.03	.03			

TABLE 2. The random number, ρ_i , generated for node i for $i = 1, 2, \dots, 7$

i	ρ_i	Remark
1	.2073	$Pr\{S_{i4}\} < \rho_i < Pr\{S_{i5} = \{3, 4\}\}$
2	.7005	$Pr\{S_{i7}\} < \rho_i < Pr\{S_{i8} = \{4, 6, 8\}\}$
3	.7932	$Pr\{S_{i3}\} < \rho_i < Pr\{S_{i4} = \{4, 5\}\}$
4	.3140	$Pr\{S_{i6}\} < \rho_i < Pr\{S_{i7} = \{6, 7\}\}$
5	.0066	$Pr\{S_{i0}\} < \rho_i < Pr\{S_{i1} = \emptyset\}$
6	.1289	$Pr\{S_{i1}\} < \rho_i < Pr\{S_{i2} = \{9\}\}$
7	.9915	$Pr\{S_{i3}\} < \rho_i < Pr\{S_{i4} = \{9, 10\}\}$

Assume that a random number, ρ_i , is generated for each node, as shown in Table 2.

If $Pr\{S_{i,j-1}\} < \rho_i < Pr\{S_{ij}\}$, then e_{ik} needs to be removed for all $k \notin s_{ij}$ (i.e., e_{12} , e_{56} , e_{57} , $e_{4,10}$ and e_{68}), as shown by the dashed arcs in Figure 2. After that, there is a direct path from node 1 to each proper subset of $\{9, 10\}$ in the residual network by using the proposed CA to scan Figure 3.

The summarized result for each iteration using the above CA, referred to as Procedure Cellular-Automata(), is listed as shown in Figure 3.

5. Introduction to Quasi-Random Sequences. Most traditional CMCs are all based on pseudo-random sequences (PRS), or pCMC, which are generated by a uniform distribution. In other words, each pseudo-random number has the same probability of being

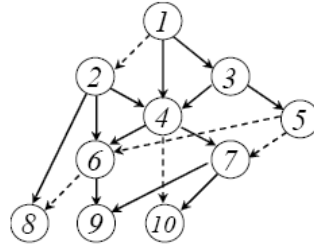


FIGURE 2. These dashed lines are the corresponding arcs in Table 2

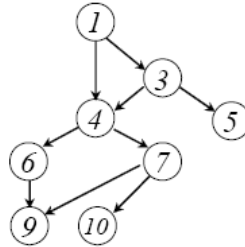


FIGURE 3. The residual connected network after removing $\{e_{12}, e_{56}, e_{57}, e_{4,10}, e_{68}\}$ and the corresponding nodes 2 and 8

TABLE 3. The summary of each iteration when applying the above CA to Figure 3

iteration	node i	N	N^* in STEP 1	N^* in STEP 2	V^*	T^*	$T^\#$
0		{1}			{3,4,5,6,7,9,10}	{9,10}	\emptyset
1	1	{3,4}	{3,4}	{3,4}	{5,6,7,9,10}	{9,10}	\emptyset
2	3	{4,5}	{5}	{5}	{6,7,9,10}	{9,10}	\emptyset
3	4	{5,6,7}	{6,7}	{6,7}	{9,10}	{9,10}	\emptyset
4	5	{6,7}	\emptyset	\emptyset	{9,10}	{9,10}	\emptyset
5	6	{7}	{9}	\emptyset	{10}	{10}	{9}
6	7	\emptyset	{10}	\emptyset	\emptyset	\emptyset	{9,10}

generated. Hence, pseudo-random numbers have the same relevant statistical properties [27], and may exhibit crowding in some ranges, and no pseudo-random numbers in other ranges [28]. Note also that, in many situations, pCMCs exhibit a rate of convergence that is too slow for the main problem of a CMC. Thus, various quasi-random sequences (QRS) have been constructed for the CMC [27,28], or qCMC.

QRSs are also called low discrepancy sequences, and the quasi-random numbers that are generated are totally deterministic. In most cases, QRSs can simulate more efficiently than pseudo-random sequences with a smaller estimated error by sacrificing some statistical properties such as independence [27,28]. Furthermore, a higher number of simulations always improves the accuracy of the simulation based on quasi-random sequences [27,28].

The hybrid QRS (HRS) integrated into the proposed CA-HMC is adapted from the van der Corput sequence. It combines the quasi-random numbers with the pseudo-random numbers. Consider the following radical inverse function:

$$\rho_b(k) = \sum_{j=0}^i a_j(k)b^{-j-1} \text{ when } k = \sum_{j=0}^i a_j(k)b^j, \tag{11}$$

where k is the corresponding van der Corput sequence (in base b for the required decimal number k), $a_j(k)$ is the sequence of digits (e.g., for base $b = 2$, a sequence of 0s and 1s; for base 3 a sequence of 0s, 1s and 2s, etc.), the natural number $b > 1$ is the base, and i is the lowest positive integer that makes $a_j(k) = 0$ for all $j > i$. The values of $\rho_b(k)$ are always in the unit interval $[0,1)$.

HRSs try to bridge the gap between the flexibility of a PRS and the advantages of a QRS. The HRS (as given in Equation (1)) listed in the proposed simulation is a revised version of the pseudo-code presented in Wilmott [23], but uses a random integer, k , instead of a deterministic integer. The complete HRS is given as follows:

Procedure Hybrid-Quasi-Random()

Input: *A positive integer random number k .*

Output: *A sequence of hybrid quasi-random numbers within $[0, 1)$.*

STEP 0: *Let $b = 2$, k be a positive integer random number, $\Phi = 0$ and $B = 1$.*

STEP 1: *Let $B = \frac{B}{b}$, $q =$ the quotient of $\frac{k}{b}$, $r =$ the remainder of $\frac{k}{b}$, $\rho = \rho + rB$ and $k = q$.*

STEP 2: *If $k > 0$, then go to STEP 1. Otherwise, return ρ .*

6. The Proposed CA-HMC. The proposed CA-HMC is a straightforward approach especially for the large complex system. It is based on CA, CMC but using hybrid quasi-random sequences instead of PRS and QRS. The complete proposed CA-HMC which estimates AMIN reliability is given as follows:

Purpose: Estimate the one-to-all reliability of an AMIN.

Input: *AN AMIN $G(V, E)$ with the source node 1 and the target set $T(\subset V)$, and the total number of trials M .*

Output: *The estimator R^{CA-HMC} of the system reliability.*

STEP 0: *Let $i = 1$ and $R_S = 0$ for $S \subseteq T$.*

STEP 1: *Call Hybrid-Quasi-Random() to generate random numbers to simulate states of non-target nodes.*

STEP 2: *Call the proposed Cellular-Automata() to find the target subset, say $T^\#$, that connected to node 1 in the residual network based on STEP 2.*

STEP 3: *$R_S = R_S + 1$ for all $S \subseteq T^\#$.*

STEP 4: *If $i < M$, then $i = i + 1$ and go to STEP 1. Otherwise, $R_s^{CA-HMC} = \frac{R_s}{M}$.*

Even for a binary-state network with n nodes, it has 2^n possible system states. Therefore, M trials in the proposed CA-HMC can account at most for only a small portion of states even when n is moderate in size of the AMIN. This limitation introduces sampling error. Hence, the statistical methods are utilized here to analyze output during the simulation activities by the estimator of R . Hence, the following theorem is true for CA-HMC.

Theorem 6.1.

$$a. \quad E [R^{CA-HMC}] = R. \quad (12)$$

$$b. \quad Var [R^{CA-HMC}] = \frac{R(1 - R)}{2M}. \quad (13)$$

$$c. \quad R^{CA-HMC} \text{ is a consistency estimator.} \quad (14)$$

7. A Numerical Example. In order to introduce the advantages of the proposed CA-HMC over pCMC and qCMC for the one-to-all AMIN reliability, we implement all three of these methods. In other words, we implement $R_{1\{8\}}$, $R_{1\{9\}}$, $R_{1\{10\}}$, $R_{1\{8,9\}}$, $R_{1\{8,10\}}$, $R_{1\{9,10\}}$ and $R_{1\{8,9,10\}}$, as shown in Figure 1. All of the methods are run on the same

hardware, namely an Intel Pentium 900 MHz CPU and 2 GB memory. In addition, each method is coded using C++.

TABLE 4. The average absolute errors between the estimator and the exact reliability of the information in the target set in Figure 1

State	Exact Reliability	$M = 100 \sim 10000$			$M = 100 \sim 5000$			$M = 5100 \sim 10000$		
		pCMC	qCM	CA-HMC	pCMC	qCMC	CA-HMC	pCMC	qCMC	CA-HMC
$R_{1\{8\}}$.9202	.0009	.0272	.0003*	.0013	.0269	.0000*	.0005*	.0276	.0006
$R_{1\{9\}}$.9437	.0004	.0008	.0003*	.0011	.0009	.0002*	.0004	.0006	.0003*
$R_{1\{10\}}$.8588	.0008	.0359	.0000*	.0015	.0361	.0001*	.0001*	.0358	.0001
$R_{1\{8,9\}}$.9090	.0010*	.0160	.0021	.0014*	.0157	.0038	.0006	.0164	.0004*
$R_{1\{8,10\}}$.8253	.0010	.0179	.0001*	.0006	.0182	.0002*	.0014	.0176	.0001*
$R_{1\{9,10\}}$.8497	.0010	.0450	.0004*	.0010*	.0452	.0012	.0010	.0449	.0004*
$R_{1\{8,9,10\}}$.8185	.0008	.0247	.0005*	.0014	.0250	.0001*	.0002*	.0244	.0009

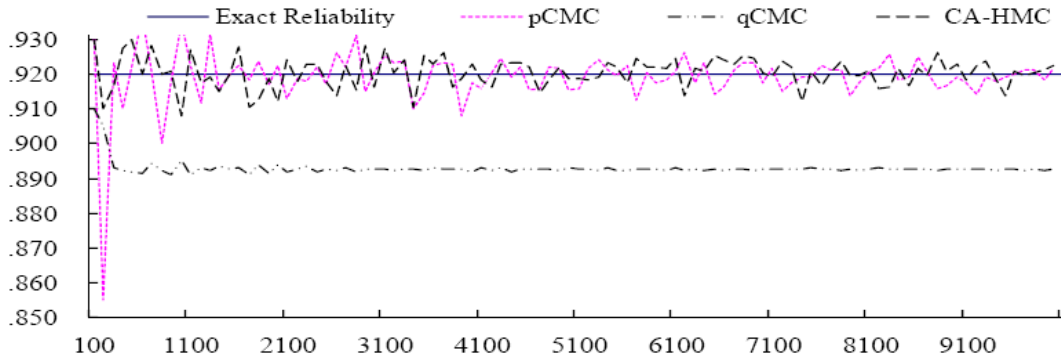
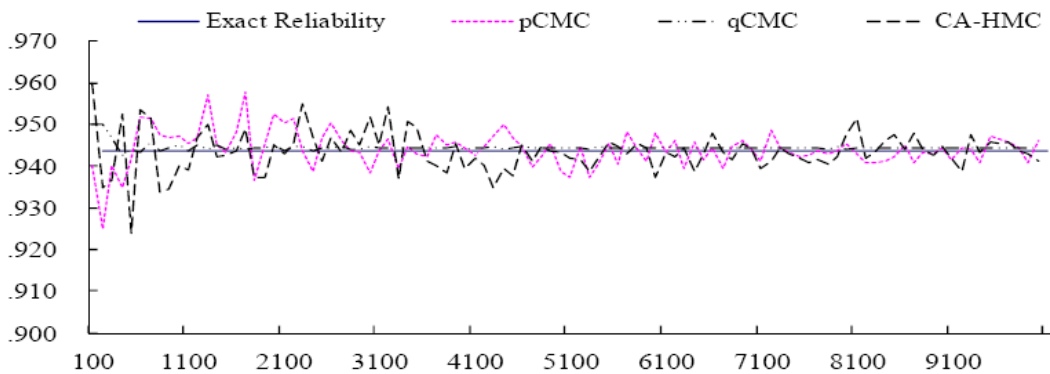
* the smallest absolute error among the corresponding items.

TABLE 5. ANOVA table of information from the target set in Figure 1 for $M = 100 \sim 10000$

State	Source	SS	DF	MS	F	P
$R_{1\{8\}}$	Between	0.04852	2	0.0243	781.7147	0.0000*
	Within	0.00922	297	0.0000		
	Total	0.05773	299			
$R_{1\{9\}}$	Between	0.00005	2	0.0000	1.8326	0.1618
	Within	0.00438	297	0.0000		
	Total	0.00443	299			
$R_{1\{10\}}$	Between	0.23803	2	0.1190	1941.3601	0.0000*
	Within	0.01821	297	0.0001		
	Total	0.25624	299			
$R_{1\{8,9\}}$	Between	0.01404	2	0.0070	164.3917	0.0000*
	Within	0.01269	297	0.0000		
	Total	0.02673	299			
$R_{1\{8,10\}}$	Between	0.02018	2	0.0101	225.8289	0.0000*
	Within	0.01327	297	0.0000		
	Total	0.03344	299			
$R_{1\{9,10\}}$	Between	0.13114	2	0.0656	1335.1661	0.0000*
	Within	0.01459	297	0.0000		
	Total	0.14573	299			
$R_{1\{8,9,10\}}$	Between	0.03868	2	0.0193	475.4754	0.0000*
	Within	0.01208	297	0.0000		
	Total	0.05076	299			

* the state with a statistically significant difference among the corresponding items.

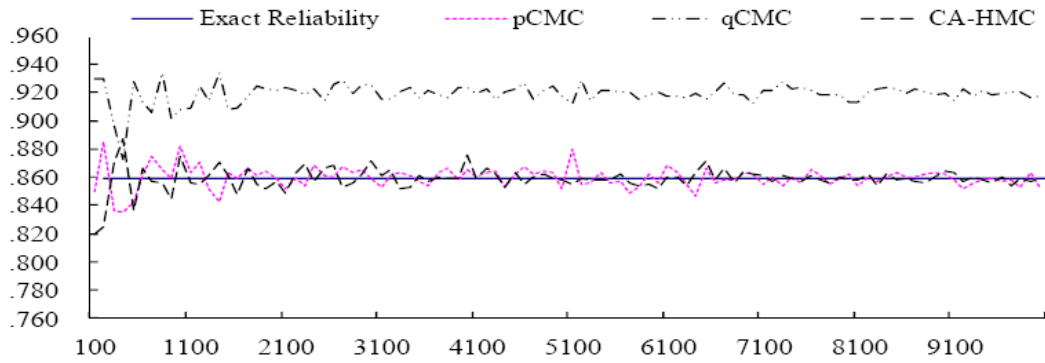
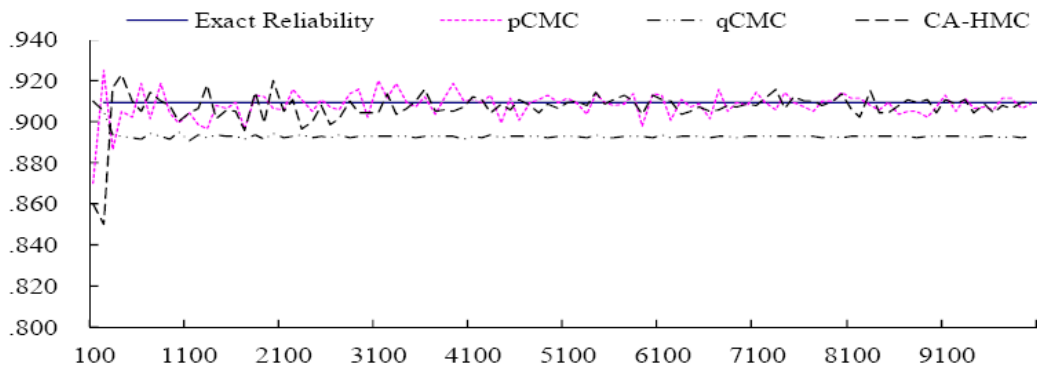
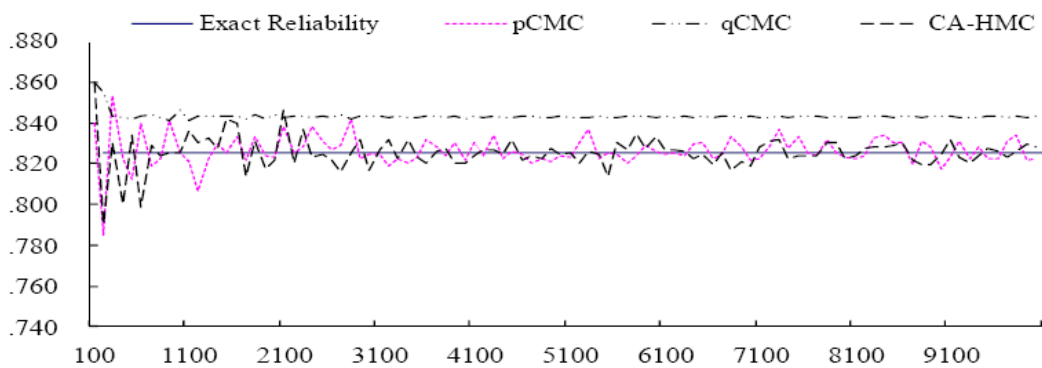
Each method is run with $M = 100, 200, \dots, 10000$. There are 100 test problems in each problem group. The exact reliabilities and the corresponding estimators obtained from pCMC, qCMC and CA-HMC for all the different combinations of sink nodes are displayed in Table 4 and Figures 4 to 10. In order to see the trend of the convergence of these methods, we not only list the average absolute errors between the exact reliability and the estimators, but also show the average absolute errors for $M = 100, 200$ to 5000,

FIGURE 4. $R_{1\{8\}}$ and its estimatorsFIGURE 5. $R_{1\{9\}}$ and its estimators

and $M = 5100, 5200$ to 10000 in Table 4. In addition, we provide a statistical study of the performance of these combinations of algorithms with more observations, and add the practical applications of the proposed algorithm. Also, understanding the effects of the various precedence relationship structures may benefit the contribution and improvement on the performance of proposed algorithm. Table 5 shows the ANOVA results among pCMC, qCMC, and CA-HMC for all the combinations of sink nodes. The formula for the absolute error is defined as $x_0 - x$, where x_0 is the measured value, and x is the actual value.

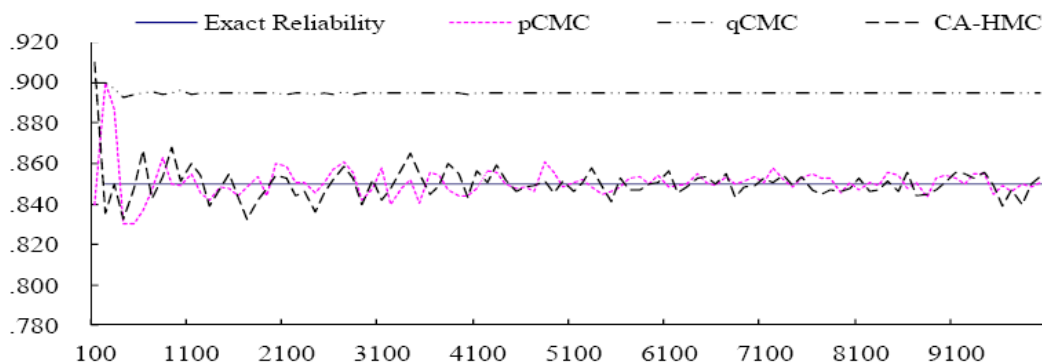
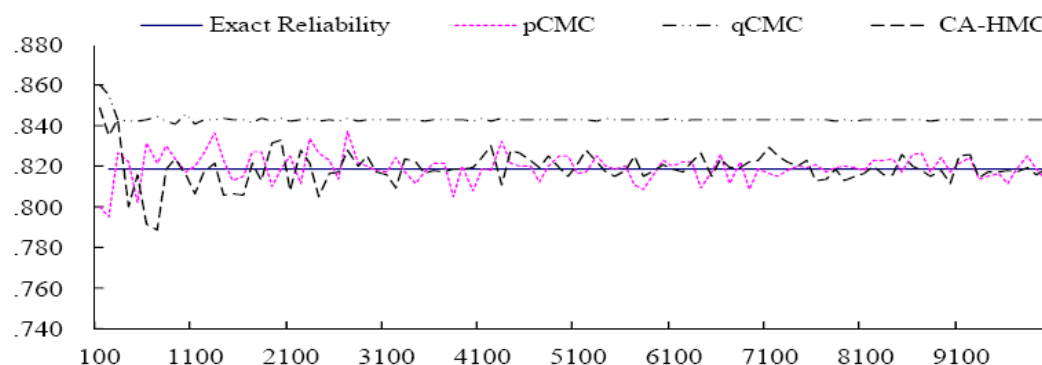
As can be seen from Figures 4 to 10, the estimators obtained from the qCMC clearly converge more rapidly than the estimators obtained from the pCMC and CA-HMC. However, there is a bias introduced in the estimate by using a qCMC chain. Hence, the simulation absolute error with qCMC is much higher than the absolute errors from using pCMC and CA-HMC, as shown in Table 4. In addition, qCMC seems to be either underestimating or overestimating the reliability in Figures 4 to 10. It is interesting and surprising to note that the estimators from qCMC for $R_{1\{8\}}$ and $R_{1\{8,9\}}$, for $R_{1\{10\}}$ and $R_{1\{9,10\}}$, and for $R_{1\{8,10\}}$ and $R_{1\{8,9,10\}}$ are all equal (see Figures 4 to 10).

From Table 4, it seems that CA-HMC clearly outperforms the classical PR for a number of Monte Carlo trials (M) up to 5000. When the number of trials (M) is increased in the range from 5100 to 10000, the two methods seem to be equal. This is because a possible bias effect is responsible for the lower performance of the method at higher number of Monte Carlo histories. In some cases, the absolute error with pCMC is lower than CA-HMC, but this occurs by chance in less frequent simulated cases, as shown in Table 4.

FIGURE 6. $R_{1\{10\}}$ and its estimatorsFIGURE 7. $R_{1\{8,9\}}$ and its estimatorsFIGURE 8. $R_{1\{8,10\}}$ and its estimators

From Table 5, the one-to-all AMIN reliability almost has a significant difference among pCMC, qCMC and CA-HMC in 100 test problems (except for $R_{1\{9\}}$). From our experiments, we showed that the proposed CA-HMC, which combines the CA, PRS and QRS, for evaluating the one-to-all AMIN reliability is slightly superior to pCMC, which uses the PRS and qCMC, which use the QRS.

8. Conclusions. AMINs have been applied extensively in many real-life situations and the analysis of AMINs has also become a new subject in system reliability. This work introduced a novel MCS, the CA-HMC, which integrates CA, pCMC and qCMC to provide a very useful means of evaluating the required one-to-all AMIN reliability. The proposed CA-HMC focuses on improving efficiency, which is a significant contribution not emphasized in literature related to network reliability. It improves the performance of the MCS

FIGURE 9. $R_{1\{9,10\}}$ and its estimatorsFIGURE 10. $R_{1\{8,9,10\}}$ and its estimators

in terms of solution quality when estimating the one-to-all AMIN reliability. Furthermore, the experimental results show that the proposed CA-HMC yields very good results when compared with the traditional Monte Carlo method by using pseudo-random numbers or quasi-random numbers.

Simulation results demonstrate that the proposed HMC is efficient and effective for simulating one-to-all AMIN reliability. However, the performance of the proposed CA-HMC is only slightly superior to pCMC. Therefore, future work will include applying variance reduction methods, such as stratified sampling and important sampling, to the proposed CA-HMC, and extending the proposed CA-HMC to general MINs.

Acknowledgment. This work is partially supported by the National Science Council Taiwan, under grant NSC-83-0415-E-035-005 and NSC-84-2213-E-035-015. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

- [1] Y. Lin, A simple algorithm for reliability evaluation of a stochastic-flow network with node failure, *Computers & Operations Research*, vol.28, pp.77-85, 2001.
- [2] Y. Lin, Study on the multicommodity reliability of a capacitated-flow network, *Computers & Mathematics with Applications*, vol.42, pp.255-264, 2002.
- [3] M. A. Samad, An efficient algorithm for simultaneously deducing MPs as well as cuts of a communication network, *Microelectronic Reliability*, vol.27, pp.437-441, 1987.
- [4] T. L. Landers, H. A. Taha and C. L. King, A reliability simulation approach for use in the design process, *IEEE Transactions on Reliability*, vol.40, pp.177-181, 1991.
- [5] W. C. Yeh, Search for all d -Mincuts of a limited-flow network, *Computers & Operations Research*, vol.29, no.13, pp.1843-1858, 2002.

- [6] W. C. Yeh, Search for minimal paths in modified networks, *Reliability Engineering & System Safety*, vol.75, no.3, pp.389-395, 2002.
- [7] W. C. Yeh, A simple method to verify all d -minimal path candidates of a limited-flow network and its reliability, *International Journal of Advanced Manufacturing Technology*, vol.20, no.1, pp.77-81, 2002.
- [8] T. Aven, Availability evaluation of oil/gas production and transportation systems, *Reliability Engineering*, vol.18, pp.35-44, 1987.
- [9] T. Aven, Some considerations on reliability theory and its applications, *Reliability Engineering and System Safety*, vol.21, pp.215-223, 1988.
- [10] J. Cha, J. Choi, D. Park, J. Yoon, S. Moon, J. Watada and R. Billinton, Reliability evaluation for interconnection planning in north east Asia, *International Journal of Innovative Computing, Information and Control*, vol.5, no.5, pp.1295-1312, 2009.
- [11] S. Wang and J. Watada, Reliability optimization of a series-parallel system with fuzzy random lifetimes, *International Journal of Innovative Computing, Information and Control*, vol.5, no.6, pp.1547-1558, 2009.
- [12] L. Hu, J. Li and W. Fang, Reliability analysis of an N-component series system with M failure modes and vacation, *ICIC Express Letters*, vol.2, no.1, pp.53-58, 2008.
- [13] L. Hu and J. Li, Reliability analysis of a three-unit system with vacation and priority, *ICIC Express Letters*, vol.3, no.2, pp.171-176, 2009.
- [14] J. Malinowski and W. Preuss, Reliability evaluation for tree-structured systems with multistate components, *Microelectronics Reliability*, vol.36, pp.9-17, 1996.
- [15] F. Hwang and Y. Yao, Multistate consecutively-connected systems, *IEEE Transactions on Reliability*, vol.38, pp.472-474, 1989.
- [16] A. Kossow and W. Preuss, Reliability of linear consecutively-connected systems with multistate components, *IEEE Transactions on Reliability*, vol.44, pp.518-522, 1995.
- [17] M. Zuo and M. Liang, Reliability of multistate consecutively-connected systems, *Reliability Engineering & System Safety*, vol.44, pp.173-176, 1994.
- [18] Y. Lin, Using minimal cuts to evaluate the system reliability of a stochastic-flow network with failures at nodes and arcs, *Reliability Engineering & System Safety*, vol.75, pp.1-6, 2002.
- [19] W. C. Yeh, Multistate-node acyclic network reliability evaluation, *Reliability Engineering & System Safety*, vol.78, no.2, pp.123-129, 2002.
- [20] W. C. Yeh, An evaluation of the multistate-node networks reliability using the traditional binary-state networks reliability algorithm, *Reliability Engineering & System Safety*, vol.81, no.1, pp.1-7, 2003.
- [21] W. C. Yeh, Multistate-node acyclic networks reliability evaluation based on MC, *Reliability Engineering & System Safety*, vol.81, no.2, pp.225-231, 2003.
- [22] G. Levitin, *The Universal Generating Function in Reliability Analysis and Optimization*, Springer-Verlag, London, 2005.
- [23] J. W. Barret, G. Moore and P. Wilmott, Inelegant efficiency, *Risk*, vol.5, no.9, pp.82-84, 1992.
- [24] H. Niederreiter, *Random Number Generation and Quasi-Monte Carlo Methods*, Capital Press, Montpelier, Vermont, 1992.
- [25] Y. Lin, Two-commodity algorithm for enumerating for a stochastic-flow network with node failure, *Computers & Operations Research*, vol.29, pp.1927-1939, 2002.
- [26] J. E. Ramirez-Marquez and D. W. Coit, A monte-carlo simulation approach for approximating multi-state two-terminal reliability, *Reliability Engineering & System Safety*, vol.87, pp.253-264, 2005.
- [27] B. D. Ripley, *Stochastic Simulation*, Wiley, 1987.
- [28] http://www.puc-rio.br/marco.ind/quasi_mc.html.
- [29] G. Levitin, Reliability evaluation for acyclic consecutively connected networks with multistate elements, *Reliability Engineering and System Safety*, vol.73, no.2, pp.137-143, 2001.
- [30] W.-C. Yeh and X. He, A new universal generating function method for estimating the novel multi-resource multistate information network reliability, *IEEE Transactions on Reliability*, vol.59, no.3, pp.528-538, 2010.
- [31] W.-C. Yeh, Y.-C. Lin and Y. Y. Chun, Performance analysis of cellular automata monte carlo simulation for estimating network reliability, *Expert Systems with Applications*, vol.37, no.5, pp.3537-3544, 2010.