International Journal of Innovative
Computing, Information and Control
Volume 8, Number 1(B), January 2012

ICIC International ©2012 ISSN 1349-4198
pp. 539–552

TORQUE RIPPLE MINIMIZATION BY TRACKING NON-LINEAR CONTROL OF PM SYNCHRONOUS MOTORS

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Received August 2010; revised February 2011

Abstract. *Nowadays, Permanent Magnet Synchronous Motors have been considered as one of the best ones used in industry due to their high efficiency, low torque ripples and well known dynamical operation. Indeed, it is expected that this type of motors can be used more frequently. In this paper, a special type of these motors, the surface mounted PM motor, is considered. After introducing the mathematical model of the machine, a non-linear control approach according to Lyapunov control theory has been presented. According to this control strategy, all the states of the main system will track the states of the desired system, which are determined by our specifications. Due to the fact that all the states are not accessible in real plant, a nonlinear observer is designed to obtain motors states. Finally, simulation results show the effectiveness of the proposed methodology.*

Keywords: Permanent magnet synchronous motor, Lyapunov strategy, Tracking control, Nonlinear observer

1. **Introduction.** In permanent magnet synchronous motors (PMSMs) drive, the information about rotor position is vital for the inverter commutation to control the frequency and position of the stator current vectors. This information can be obtained by using sensors or alternatively by calculation in various sensorless schemes. According to the wide application of this type of motors, some handicaps, due to operating conditions, maintaining requirements and low system reliability, are appeared, which can be eliminated with sensorless schemes. These sensorless methods can be divided into two types: motional electromotive force and induction variation. In the first case, it is necessary to know that in the stop position, this method is not capable of detecting the initial rotor position. Typically, the changing rate of current is determined by the inductance as a function of rotor position and stator currents can be calculated either directly or indirectly. The problem is that the surface mounted PMSM (which is the main target in this paper), has not any significant saliency. This matter is a difficulty to detect the rotor position at zero or low speed under load, spontaneously. Lots of efforts have been made in this field by injecting signals to amplify the saturation saliency [1-7].

In [8], an initial rotor position estimation scheme using voltage pulses is presented, which offers a better solution rather than high frequency signal injection method.

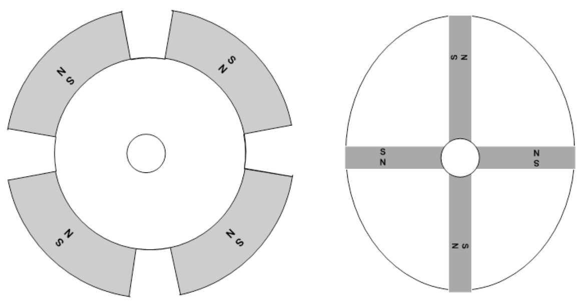
Sometimes, as the system parameter falling to a certain area, the permanent magnet synchronous motor is experiencing chaotic behavior which is the case of interest for a few numbers of researchers [9,10].

Focus has been made on torque ripple minimization especially in low speed conditions [11-14], and also, in some of the others, the speed ripple minimization is the main target [15]. Adaptive approach is one of various control strategies that have been applied to attain the mentioned targets [17-23].

According to [24,25], PM Synchronous Motors have multiple phases in the stator and the electrical frequency of the stator is proportional to the rotor velocity in the steady state. The permanent magnet motor results in better efficiency, decreased need of sleep rings on rotor, and will also suppress the electrical dynamics of the rotor which will simplify the control procedure. One of the necessities of control process of the PMSMs is the need for rotor position feedback.

According to the discrete magnetization of the stator, another problem occurs when it introduces a cogging torque in the output. It is noticeable that most frequent problems will emerge in low velocities. We can overcome these problems via a better design of machine or by selecting an appropriate control strategy to decrease or suppress such unfit torque. One of the best ways to control the torque of the machine, which is the main idea of this paper, is to control the stator current.

Due to different magnet positions in the rotor of the PMSMs, there exist different structures of these motors. In Figure 1, two types of them are shown. Our interest is the surface mounted type which has a multi-phase stator and a permanent magnet in the rotor.



(a) (b)

Figure 1. (a) Surface-mounted PM motor; (b) interior PM motor

These kinds of the PMSMs have a noticeable characteristic, which is the equivalence of inductance along the d and q axis so that there is no reluctance torque related to the interactions between the stator and the rotor.

This paper focuses on torque ripple minimization by designing an adaptive controller according to tracking of the desired trajectories based on Lyapunove stability method and also the controller is equipped with a nonlinear observer to get all the states accessible. Advantages of this method compared with others can be marked as: first, this method is applicable in all speed ranges and is not restricted to a specific rang of speed; secondly, the proposed control strategy is equipped with a nonlinear observer due to the fact that the whole parameters of the real system are not physically accessible.

The contribution of paper is as follows: in Section 2, the model of the system is presented. The control strategy is mentioned in Section 3. Section 4 is devoted to the design of nonlinear observer. Finally, Section 5 contains all the simulation results that are shown in different initial conditions of stats and torque.

2. **Problem Statement and Preliminaries (Machine Equations).** In three phase machines, there are 120° differences between the phases [24,25]. So, the whole equations related to both the stator and rotor are classified as the procedure given below where B is the intensity of magnetic field, B_m is the intensity of magnetic field due to the presence of permanent magnet in the rotor, r is radius, r_S is the radius of stator, r_R is the radius of rotor, g is the air gap between stator and rotor, N_S is the number of round of stator wiring, K is the leakage coefficient, T_e is the electrical torque, T_l is the load torque, θ is the electrical angle, θ_R is the angle of the rotor, R_S is the stator resistance, L_S is the self-inductance of stator, J is the moment of inertia of the rotor, ω is the angular velocity, l is the length of rotor and

$$K_m = \sqrt{\frac{3}{2}}\,\frac{K \cdot \pi \cdot l^2 \cdot B_m \cdot N_S}{4}$$

Definition 2.1. *The equations of each phase of the stator magnetic field will be:*

$$\begin{cases} \vec{B}_{S1}(i_{S1}, r, \theta) = \dfrac{\mu_0 N_S}{2g}\dfrac{r_R}{r} i_{S1}\cos(\theta) \cdot \hat{r} \\ \vec{B}_{S2}(i_{S2}, r, \theta) = \dfrac{\mu_0 N_S}{2g}\dfrac{r_R}{r} i_{S2}\cos\left(\theta - \dfrac{2\pi}{3}\right) \cdot \hat{r} \\ \vec{B}_{S3}(i_{S3}, r, \theta) = \dfrac{\mu_0 N_S}{2g}\dfrac{r_R}{r} i_{S3}\cos\left(\theta - \dfrac{2\pi}{3}\right) \cdot \hat{r} \end{cases} \tag{1}$$

so we have:

$$\vec{B}_S(i_{S1}, i_{S2}, i_{S3}, r, \theta) = \frac{\mu_0 N_S}{2g}\frac{r_R}{r}\left(i_{S1}\cos\theta + i_{S2}\cos\left(\theta - \frac{2\pi}{3}\right) + i_{S3}\cos\left(\theta - \frac{4\pi}{3}\right)\right) \tag{2}$$

on the other hand, the equation of the permanent magnetic field of the rotor will be:

$$\vec{B}_R(r_S, \theta - \theta_R) = K B_m \frac{r_R}{r_S}\cos(\theta - \theta_R)\hat{r} \tag{3}$$

with K being the leakage constant.

Lemma 2.1. *Because there is no wiring in the rotor, the torque of the rotor is calculated from the stator torque, and also we know that: $T_R = -T_S$ [24,25].*

Considering the equations of B_R and B_S, the whole radial magnetic field is derived as:

$$\vec{B}(i_{S1}, i_{S2}, i_{S3}, r, \theta, \theta_R) \triangleq \vec{B}_S(i_{S1}, i_{S2}, i_{S3}, r_S, \theta) + \vec{B}_R(r_S, \theta - \theta_R) \tag{4}$$

To achieve the linkage flux of stator we have:

$$C_1 = \frac{2}{3}L_S \begin{bmatrix} 1 & \cos\dfrac{2\pi}{3} & \cos\dfrac{4\pi}{3} \\ \cos\dfrac{2\pi}{3} & 1 & \cos\dfrac{2\pi}{3} \\ \cos\dfrac{4\pi}{3} & \cos\left(-\dfrac{2\pi}{3}\right) & 1 \end{bmatrix} \tag{5}$$

so:

$$\Rightarrow \begin{bmatrix} \phi_{S1}(t) \\ \phi_{S2}(t) \\ \phi_{S3}(t) \end{bmatrix} = C_1 \begin{bmatrix} i_{S1}(t) \\ i_{S2}(t) \\ i_{S3}(t) \end{bmatrix} + \sqrt{\frac{2}{3}}K_m \begin{bmatrix} \cos(\theta_R) \\ \cos\left(\theta_R - \dfrac{2\pi}{3}\right) \\ \cos\left(\theta_R - \dfrac{4\pi}{3}\right) \end{bmatrix} \tag{6}$$

Definition 2.2. *If we take R_S into account as the stator resistance in each phase and the voltages $u_{S1}(t)$, $u_{S2}(t)$ and $u_{S3}(t)$ as the stator voltage of each phase, according to Faraday's law we have:*

$$\begin{cases} u_{S1}(t) = R_S i_{S1} + \dfrac{d\phi_{S1}(t)}{dt} \\ u_{S2}(t) = R_S i_{S2} + \dfrac{d\phi_{S2}(t)}{dt} \\ u_{S3}(t) = R_S i_{S3} + \dfrac{d\phi_{S3}(t)}{dt} \end{cases} \tag{7}$$

knowing the fact [24,25]:

$$\begin{bmatrix} i_{Sa}(t) \\ i_{Sb}(t) \\ i_{So}(t) \end{bmatrix} \triangleq Q \cdot \begin{bmatrix} i_{S1}(t) \\ i_{S2}(t) \\ i_{S3}(t) \end{bmatrix} \qquad \begin{bmatrix} \lambda_{Sa}(t) \\ \lambda_{Sb}(t) \\ \lambda_{So}(t) \end{bmatrix} \triangleq Q \cdot \begin{bmatrix} \phi_{S1}(t) \\ \phi_{S2}(t) \\ \phi_{S3}(t) \end{bmatrix} \tag{8}$$

where:

$$Q = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \tag{9}$$

$$\begin{cases} \lambda_{So}(t) = \dfrac{1}{\sqrt{3}} \left(\phi_{S1}(t) + \phi_{S2}(t) + \phi_{S3}(t)\right) = 0 \\ i_{So}(t) = \dfrac{1}{\sqrt{3}} \left(i_{S1}(t) + i_{S2}(t) + i_{S3}(t)\right) = 0 \end{cases} \tag{10}$$

therefore, dynamical equation for the linkage flux of the stator will be:

$$\begin{cases} u_{Sa}(t) = R_S i_{Sa}(t) + \dfrac{d\lambda_{Sa}(t)}{dt} \\ u_{Sb}(t) = R_S i_{Sb}(t) + \dfrac{d\lambda_{Sb}(t)}{dt} \\ u_{So}(t) = R_S i_{So}(t) + \dfrac{d\lambda_{So}(t)}{dt} \end{cases} \tag{11}$$

according to Equations (8), (9) and (11) we have:

$$\begin{cases} u_{Sa} = L_S \dfrac{d}{dt} i_{Sa} + K_m \dfrac{d}{dt} \sin(\theta_R) + R_S i_{Sa} \\ u_{Sb} = L_S \dfrac{d}{dt} i_{Sb} + K_m \dfrac{d}{dt} \sin(\theta_R) + R_S i_{Sb} \\ u_{So} = 0 \end{cases} \tag{12}$$

with respect to:

$$\vec{F} = i\vec{l} \times \vec{B} \tag{13}$$

and

$$T = \vec{r} \times \vec{F} \tag{14}$$

for stator torque we have:

$$T_S = K_m (i_{Sa} \sin\theta_R - i_{Sb} \cos\theta_R) \tag{15}$$

so, as it mentioned before:

$$T_R = -T_S = -K_m(i_{Sa}\sin\theta_R - i_{Sb}\cos\theta_R) \tag{16}$$

Finally, the mathematical model of the performance synchronous motor based on star connection of the phases is achieved as below:

$$\begin{cases} u_{Sa} = L_S\dfrac{di_{Sa}}{dt} + K_m\dfrac{d}{dt}\cos\theta_R + R_S i_{Sa} \\ u_{Sb} = L_S\dfrac{di_{Sb}}{dt} + K_m\dfrac{d}{dt}\sin\theta_R + R_S i_{Sb} \\ J\dfrac{d\omega_R}{dt} = K_m(i_{Sb}\cos\theta_R - i_{Sa}\sin\theta_R) - T_L \\ \dfrac{d\theta_R}{dt} = \omega_R \end{cases} \tag{17}$$

Consequently, the model of a two phase permanent magnet machine with n_P poles and sinusoidal distributed wiring is considered as:

$$\begin{cases} L_S\dfrac{di_{Sa}}{dt} = -R_S i_{Sa} + K_m\sin(n_p\theta)\cdot\omega + u_{Sa} \\ L_S\dfrac{di_{Sb}}{dt} = -R_S i_{Sb} + K_m\cos(n_p\theta)\cdot\omega + u_{Sb} \\ J\dfrac{d\omega}{dt} = K_m(-i_{Sa}\cdot\sin(n_p\theta) + i_{Sb}\cdot\cos(n_p\theta)) - T_L \\ \dfrac{d\theta}{dt} = \omega \end{cases} \tag{18}$$

Definition 2.3. *We have a d-q transformer for the current and voltage as:*

$$\begin{aligned} \begin{bmatrix} u_d \\ u_q \end{bmatrix} &\triangleq \begin{bmatrix} \cos(n_p\theta) & \sin(n_p\theta) \\ \sin(n_p\theta) & \cos(n_p\theta) \end{bmatrix} \begin{bmatrix} u_{Sa} \\ u_{Sb} \end{bmatrix} \\ \begin{bmatrix} i_d \\ i_q \end{bmatrix} &\triangleq \begin{bmatrix} \cos(n_p\theta) & \sin(n_p\theta) \\ -\sin(n_p\theta) & \cos(n_p\theta) \end{bmatrix} \begin{bmatrix} i_{Sa} \\ i_{Sb} \end{bmatrix} \end{aligned} \tag{19}$$

The current i_d is related to the magnetic field of the stator which is along the vector of the field of rotor as a direct vector and current i_q is related to the vertical vector of the stator magnetic field. By using the transformation matrix d-q, we have:

$$\begin{cases} L_S\dfrac{di_d}{dt} = -R_S\cdot i_d + n_p\cdot\omega\cdot L_S\cdot i_q + u_d \\ L_S\dfrac{di_q}{dt} = -R_S\cdot i_q - n_p\cdot\omega\cdot L_S\cdot i_d - K_m\cdot\omega + u_q \\ J\dfrac{d\omega}{dt} = K_m\cdot i_q - T_L \\ \dfrac{d\theta}{dt} = \omega \end{cases} \tag{20}$$

where u_d is the voltage of direct axis and u_q is the voltage of vertical axis.

3. **Control Strategy.** The control aim is to achieve a desired performance for machine torque. To achieve this purpose we use the Lyapunove stability theory.

The electromagnetic torque of the PMSM is controlled by the amplitudes and phase angle of the stator currents with respect to the rotor magnet orientation. Instantaneous torque control is conveniently achieved by controlling the q-axis current and setting the d-axis current to zero [24]. Besides tracking the desired performance, the designed controller will lead to optimum torque control. In order to consider the desired currents i_d^* and i_q^* we should notice that the quadrature component i_q of the current produces torque while the direct component i_d dose not produce any torque. However, to attain higher speeds range, it is necessary to apply a negative direct current to cancel the effect of the back-emf of the motor. On the other hand, according to Equation (20) the back-emf term in the dq coordinates is "$K_m \cdot w$", so the decoupling control low u_q should cancel this back-emf term and there is no necessity to producing negative i_d. By this assumption, we can both deplete the effect of the back-emf and also achieve the optimum control method by considering $i_d^* = 0$. (Noting to the fact that i_d dose not produce any torque.) So, to achieve the desired torque, the desired d-axis current of machine should be considered as: $i_d^* = 0$. To have the minimized consumption of power, i_q^* will be obtained from Equation (20) as [24,25]:

$$i_q^* = \frac{2}{3 * K_m} \times T_e^*$$

Theorem 3.1. *By defining the tracking errors as:*

$$\begin{aligned} e_d &= i_d^* - i_d \\ e_q &= i_q^* - i_q \end{aligned}$$

and by designing the control signal laws as follows:

$$\text{if:} \begin{cases} V_d = K_d \cdot L_d \cdot e_d + K_1 \cdot L_d \cdot \chi_d \cdot e_d + R \cdot i_d - \omega \cdot L_q \cdot i_q \\ V_q = K_q \cdot L_q \cdot e_q + K_2 \cdot L_q \cdot \chi_q \cdot e_q + R \cdot i_q - \omega \cdot L_d \cdot i_d + \omega \cdot K_m \end{cases} \tag{21}$$

the error dynamics will converges asymptotically stable to zero.

Proof: By considering Equation (20) and according to the definition of V_d and V_q, the dynamic of tracking errors will be as following equation:

$$\begin{cases} \dot{e}_q = -\dfrac{V_d}{L_d} + \dfrac{R}{L_d} \cdot i_d - \omega \dfrac{L_q}{L_d} \cdot i_q \\ \dot{e}_d = -\dfrac{V_q}{L_q} + \dfrac{R}{L_q} \cdot i_q + \omega \dfrac{L_d}{L_q} \cdot i_d + \omega \dfrac{K_m}{L_q} \end{cases} \tag{22}$$

assuming the Lyapunove candidate as:

$$V = \frac{1}{2} K_1 \chi_d^2 + \frac{1}{2} e_d^2 + \frac{1}{2} K_2 \chi_q^2 + \frac{1}{2} e_q^2 \geq 0 \tag{23}$$

where:

$$\begin{aligned} \chi_d &= \int_0^t e_d^2(\tau) d\tau \\ \chi_q &= \int_0^t e_q^2(\tau) d\tau \end{aligned}$$

Equation (23) implies that the function V is positive definite and also by obtaining the first order derivation of V, we have:

$$\Rightarrow \ \dot{V} = -K_d \cdot \chi_d \cdot e_d^2 - K_q \cdot \chi_q \cdot e_q^2 \leq 0 \tag{24}$$

Consequently, the function V is positive definite and its derivation is negative definite so V can be considered as a Lyapunov candidate that meets all the necessary conditions. It also supports that the error dynamic will converges asymptotically stable to zero by this Lyapunov candidate.

Example 3.1. *In this section, we propose a numerical example to demonstrate the effectiveness of the proposed control scheme. A permanent magnet synchronous motor with the following information is considered as a case study as below in which the parameters are chosen from [24]:*

TABLE 1. Parameters of the system

Parameters	***Value***
$L_d = L_q(H)$	7×10^{-3}
$R_s\,(Ohms)$	*3*
$K_m\,(N.m/A)$	*0.167*
n_p	*2*
$J\,(kg-m)^2$	0.134×10^{-3}

The behavior of system states without any control signal is shown in Figure 2. It is clear that i_q cannot produce any torque or i_d is not proper for our goals.

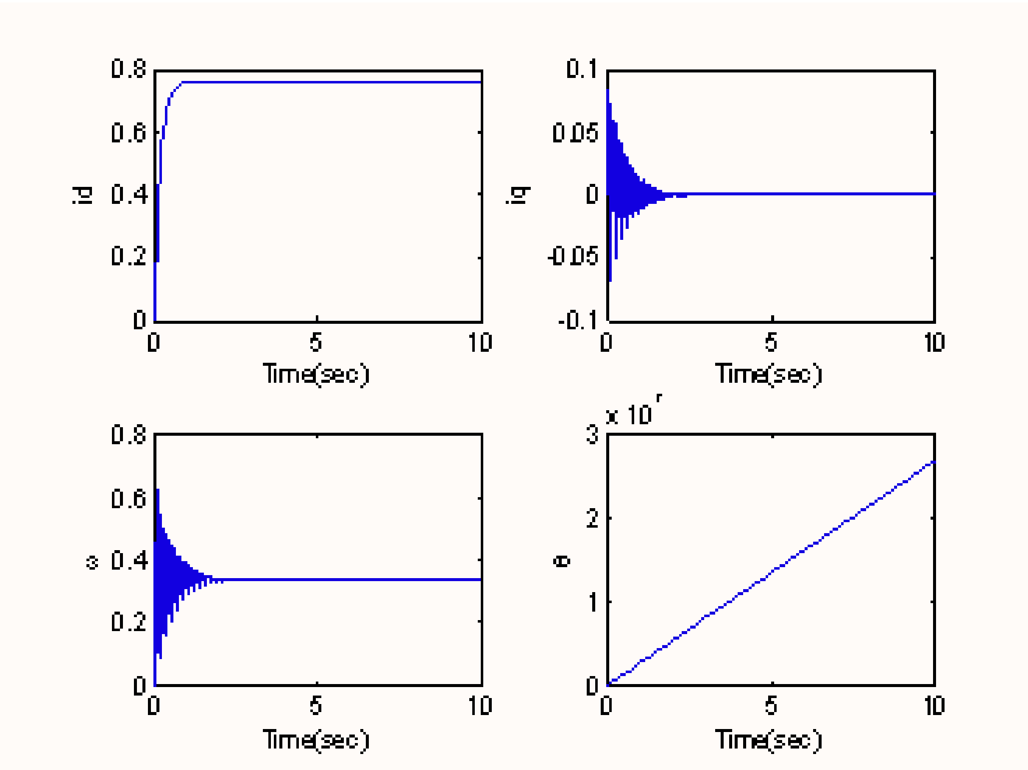


FIGURE 2. States of the model

Also, the torque is provided in Figure 3. According to this figure, motor cannot tolerate any load. Based on the figures above, it is obvious that non of the states of i_d and i_q converge to the desired and proper behavior.

Due to our desired control strategy (controlling the torque behavior according to controlling currents), we consider the desired torque T_e^* as a combination of deferent functions which varies by the time t.

$$T_e^* = \begin{cases} 5u(t) & 0 \le t < 1 \\ 4\sin(0.09t) & 1 \le t < 4 \\ 3u(t) & 4 \le t < 9 \\ \sin(0.05t) \times e^{-0.0099t} & 9 \le t \le 15 \end{cases} \tag{25}$$

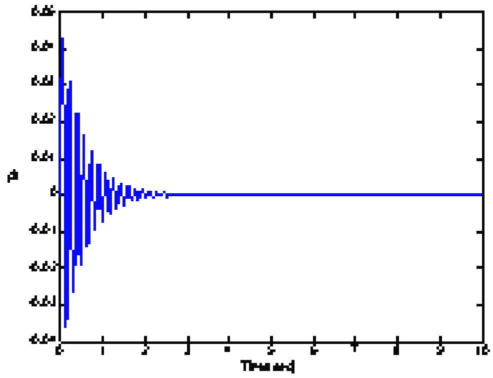


Figure 3. Torque response

And also, the load torque which is considered as a disturbance in different points of time (to show the effectiveness of control approach in presence of disturbances) will be shown as below:

$$T_l = \begin{cases} 3u(t) & 0 \leq t < 5 \\ 2u(t) & 5 \leq t < 11 \\ 1u(t) & 11 \leq t < 15 \end{cases} \tag{26}$$

the results of system behavior by applying the designed controller to the system is shown in Figure 4.

It can be observed from the figures that all the states track the desired trajectories and the torque T_e tracks T_e^*. As an instance, i_d is going to zero and also i_q has a proportional behavior like T_e^*. The tracking errors related to i_d and i_q are brought in Figure 5.

It can be seen that both tracking errors converge to zero so the tracking is perfectly achieved.

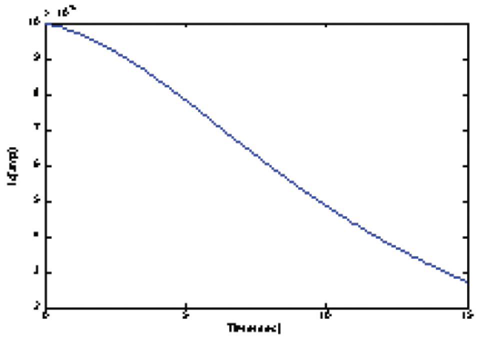
4. **Observer Design.** Because of the fact that, all of the machine parameters and variables are not physically accessible, we have to use a method to achieve them. Specially, the parameters θ and ω are the ones that need to use sensors, if not, we have to predict them. Use of sensors is accompanied with some restrictions and disadvantageous. For example, if a sensor is failed, the whole machine structure should be out of service to find the fault. The suitable place of the sensor is also a problem to be solved due to the placement restrictions. In this paper, a nonlinear observer design method is used to have an estimation of parameters. Actually, the observer is an estimator of system states to estimate the system behavior. The deference between observer predictions and real system states could be used as suitable bases for observer accuracy. The matter that is under study is that how we can estimate the unknown states of the model by using the inputs and outputs of the system. According to the fact that a linear observer has not enough complexity to determine the parameters and details, it is preferable to use a nonlinear observer design method [26,27].

Definition 4.1 (**Nonlinear Observer Design Method**). *Considering the whole nonlinear system by:*

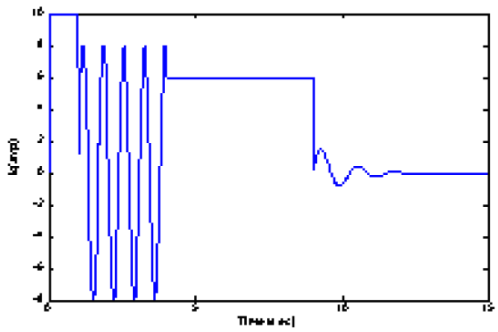
$$\dot{x}(t) = \phi(x(t), u(t), t) \tag{27}$$

And the nonlinear observer model as:

$$\begin{aligned} \dot{\widehat{x}}(t) &= \phi(\widehat{x}(t), u(t), t) + L(x(t) - \widehat{x}(t)) \\ \widehat{y}(t) &= \widehat{x}(t) \end{aligned} \tag{28}$$



(a)



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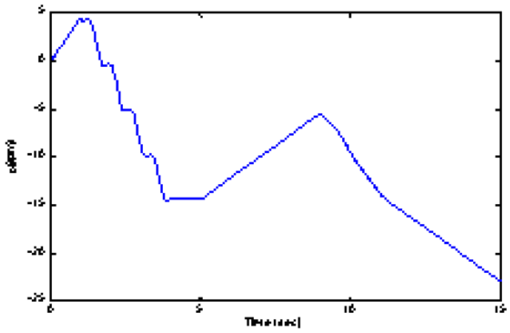
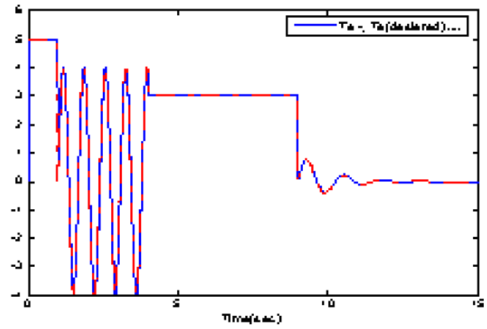


(c)



(d)

FIGURE 4. (a) I_d after control; (b) I_q after control; (c) ω after control; (d) T_e and T_e^* after control

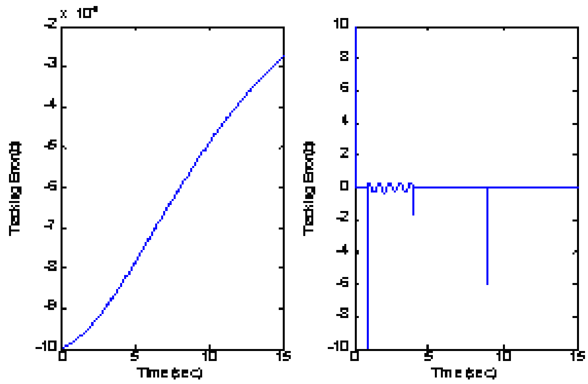


FIGURE 5. Tracking error of i_d and i_q

where $\widehat{x}(t)$ is the estimation of state $x(t)$. And by defining the estimation error as: $e(t) = x(t) - \widehat{x}(t)$, the model of nonlinear observer is rewritten as:

$$\begin{aligned} \dot{\widehat{x}}(t) &= \phi(\widehat{x}(t), u(t), t) + Le \\ \widehat{y}(t) &= \widehat{x}(t) \end{aligned} \tag{29}$$

So, for the estimation error we have:

$$\begin{aligned}&e(t) = x(t) - \widehat{x}(t) \Rightarrow \dot{e}(t) = \dot{x}(t) - \dot{\widehat{x}}(t) = \phi(x(t), u(t), t) - \phi(\widehat{x}(t), u(t), t) - Le(t) \Rightarrow \\ &\dot{e}(t) = \phi(x(t), u(t), t) - \phi((x(t) - e(t)), u(t), t) - Le(t)\end{aligned} \tag{30}$$

Lemma 4.1. *To have an accurate estimation, the estimation error $e(t)$ should converge to zero asymptotically. So, the error dynamic should be asymptotically stable. In the following statements, we will focus on the error dynamic stability [26,27].*

The gain L (of Liunberger estimation) is achieved by linearization at the operating point $Q_0 = x^*$. The Jacobian matrix related to $[\phi(x(t) - e(t), u(t), t)]$ for $e = 0$ and $x = x^*$ is:

$$A_{nl} = \left[\begin{array}{cc} \underbrace{\dfrac{\partial \phi}{\partial x}}_{A_{nl1}} & \underbrace{\dfrac{\partial \phi}{\partial e}}_{A_{nl2}} \end{array}\right] \Bigg| \begin{array}{l} x = x^* \\ e = 0 \end{array} \tag{31}$$

By considering Jacobian matrix for $[\phi(x(t), u(t), t)]$ at $x = x^*$, A_{nl1} would be as follow:

$$A_{nl1} = \left[\frac{\partial \phi}{\partial x}\right]\Bigg|_{x=x^*} \tag{32}$$

So, the error dynamic could be rewritten as:

$$\begin{aligned}[\dot{e}(t)] \approx\ &[\phi(x(t), u(t), t)] + [-\phi(x^*(t), u(t), t)] \\ &+[-A_{nl2}][e(t)] + [-A_{nl1}][x(t) - x^*(t)] - L[e(t)]\end{aligned} \tag{33}$$

According to Equation (31), Equation (33) can be obtained as:

$$\begin{aligned}[\dot{e}(t)] \approx\ &(A - L)[e(t)] \\ &+[\phi(x(t), u(t), t) - \phi(x^*(t), u(t), t)] + [-A_{nl1}][x(t) - x^*(t)]\end{aligned} \tag{34}$$

So it could be shown that $[\dot{e}(t)] = \zeta(x(t), e(t), t)$ and around the stable operating point $Q_0 = x^*$, the above equation could be considered as:

$$\zeta(x(t), u(t), t) = (A - L)e + \theta(x(t), e(t), u(t), t)$$

where:

$$A - L = \frac{\partial \zeta}{\partial e}\Bigg| \begin{array}{l} x = x^* \\ e = 0 \end{array} \quad \text{and} \quad \begin{aligned}\theta = &[\phi(x(t), u(t), t) - \phi(x^*(t), u(t), t)] \\ &+[-A_{nl1}][x(t) - x^*(t)]\end{aligned} \tag{35}$$

According to close loop system, matrix $(A-L)$ should be Hurwitz matrix, so the observer gain L must be chosen in order to the stability of the close loop matrix system $(A-L)$. In this way, we would have a matrix P for each positive definite matrix Q from the Lyapunove equation:

$$P(A - L) + (A - L)^T P = -Q \tag{36}$$

also P is a positive definite matrix.

By considering the Lyapunove candidate $V = e^T Pe > 0$, the first derivation function along the error dynamic trajectories would be given as:

$$\begin{aligned}\dot{V} &= e^T P\dot{e} + \dot{e}^T Pe = e^T P\zeta + \zeta^T Pe \\ &= e^T P[(A - L)e + \theta] + [(A - L)e + \theta]^T Pe \\ &= e^T[P(A - L) + (A - L)^T P]e + 2e^T P\theta \\ &= -e^T Qe + 2e^T P\theta\end{aligned} \tag{37}$$

It could be assumed that somewhere in state space Ω involving the operating point $Q_0 = x^*$, there exists conditions like: $\|\theta\|_2 \leq \gamma\ \|e\|_2$ (γ is a constant parameter). So, according to Equation (37), we will have:

$$\dot{V} < e^T Qe + 2\gamma\ \|P\|_2 \|e\|_2^2 \tag{38}$$

Mentioning that $e^T Qe \geq \lambda_{\min} Q \ \|e\|_2^2$ *with* $\lambda_{\min}$ *is the smallest eigenvalue of matrix* Q *where* $\lambda_{\min}$ *is real and positive,* Q *is a symmetric matrix and* p *is positive definite. In the operating point* $Q_0 \in \Omega$*, we have:*

$$\dot{V} < -[\lambda_{\min}(Q) - 2\gamma \ \|P\|_2] \ \|e\|_2^2 \tag{39}$$

By choosing $\gamma < \frac{1}{2}(\lambda_{\min}(Q)/ \ \|P\|_2)$*, it could be shown that* $\dot{V}$ *is negative and the error dynamic converge asymptotically to zero.*

Example 4.1. *In this example, we will mention a PMSM model with characteristics which are shown in Table 1. Also, the torque parameters are selected as:* $T_l = 5u(t)\, N.M$ *and* $T_e = 6u(t)\, N.M$ *randomly. By considering the control strategy which is accompanied with the observer method, the result are shown as below:*

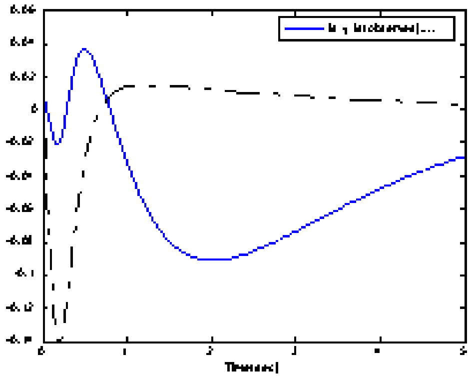


FIGURE 6. Stats i_d – (without observer) and i_{do} – (which is obtain from the observer) after applying the controller

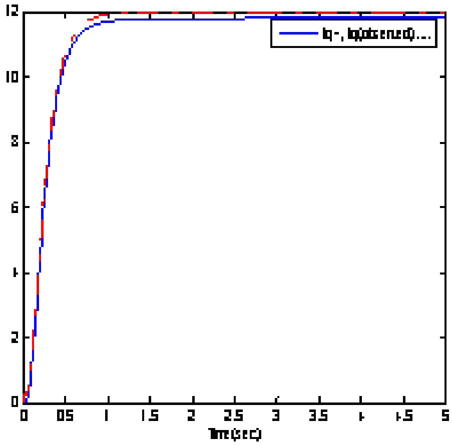


FIGURE 7. Currents i_q – (without observer) and i_{qo} – (which is obtain from the observer) after applying the controller

All the figures illustrate that the current vectors of the main system converge to the desired trajectories.

It is clear from the figures that the states ω *and* T *of the main system converge to their desired trajectories, too.*

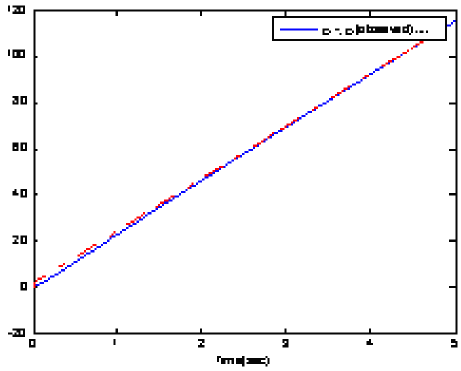


FIGURE 8. Angular speeds ω – (without observer) and ω_0 – (which is obtain from the observer) after applying the controller

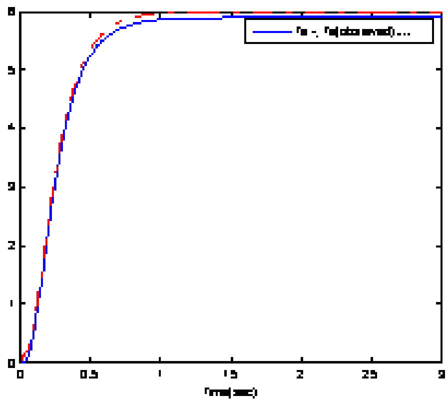


FIGURE 9. Torques T – (without observer) and ω_0 – (which is obtain from the observer) after applying the controller

5. **Conclusions.** Due to the existence of a unique model for a machine, there are many control methods to control the machine according to its velocity or torque. However, these methods have some restrictions such as speed ranges. Our proposed methodology not only does not consider any restrictions on the velocity but also could be used in general cases. As mentioned before, most of the problems of torque in these machines are related to low velocity ranges where there exist many torque ripples. However, with the Lyapunove stability theory the controller could lead to a good tracking torque with minimum ripples.

On the other hand, in the real case, we do not access to all of the machine states, so it seems that designing of an appropriate nonlinear observer is necessary. Therefore, by designing the nonlinear controller which is accompanied with observer dynamics, all the machine states converge to the desired behaviors in an asymptotically way. At the end, the simulation results are given to show the effectiveness of the proposed method.

As a future attempt this research could be improved by designing an integrated dynamical controlled system which consist both controller and observer in a unique dynamic. So, by this approach implementation of new controller would be more practical and comfortable.

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