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ROBUST ADAPTIVE FUZZY SLIDING MODE CONTROL FOR A CLASS OF UNCERTAIN DISCRETE-TIME NONLINEAR SYSTEMS

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Abstract. *This paper presents an adaptive interval type-2 fuzzy sliding mode controller for a class of unknown nonlinear discrete-time systems with training data corrupted by noise or rule uncertainties involving external disturbances. Adaptive interval type-2 fuzzy control scheme and sliding mode control (SMC) approach are incorporated to implement the main objective of controlling the plant to track a reference trajectory and prevent big chattering of the control effort. The Laypunov stability theorem has been used to testify the asymptotic stability of the whole system and the free parameters of the adaptive fuzzy controller can be tuned on-line by an output feedback control law and adaptive laws. The overall adaptive scheme guarantees the global stability of the resulting closed-loop system in the sense that all signals involved are uniformly bounded. The simulation example is given to confirm validity and tracking performance of the advocated design methodology.*
Keywords: Sliding mode control, Type-2 fuzzy control, Discrete time system, Adaptive control, Lyapunov theorem

1. Introduction.

Nowadays, active research has been carried out in fuzzy-neural control. It has been proven that fuzzy logic system (FLS) can approximate any nonlinear function with any desired accuracy because of universal approximation theorem [1-10]. A great number of adaptive fuzzy neural control schemes have been proposed to get over the difficulty of extracting linguistic control rules from experts and deal with the system parameter uncertainties. Sliding mode control (SMC) as a general design approach for robust control systems is well established. There are many advantages of the sliding mode controller such as good transient, fast response, robustness of stability and insensitivity to the variation of plant parameters and external disturbances [5-9].

However, several adaptive fuzzy sliding mode control systems have been developed for continuous-time systems [11-15], but only a few of them are devoted to discrete-time systems [16-20]. Adaptive discrete-time fuzzy sliding mode control is proposed for anti-lock braking systems [27]. A discrete version of SMC is important when the implementation of the control is realized by computers with relative slow sampling period. It should be noticed that, theoretically, discrete-time SMC cannot be obtained from their continuous counterpart by means of simple equivalence.

Nevertheless, because of the environment changes and the associated uncertainties, linguistic uncertainty and noisy training data, the chosen type-1 sets might not be appropriate anymore. A type-2 fuzzy set is characterized by a fuzzy membership function

[21-26]. For each element of this set is a fuzzy set in $[0, 1]$, unlike a type-1 fuzzy set where the membership grade is a crisp value in $[0, 1]$. A type-2 FLS is characterized by IF-THEN rules, but its antecedent or consequent sets are type-2. Hence, type-2 FLSs can be used when the circumstances are too uncertain to determine exact membership grades such as when training data are corrupted by noise.

The type-2 FLS has been successfully applied to fuzzy neural network [28-33], VLSI testing and fuzzy controller designs [34]. The Lyapunov stability theorem has been used to testify the asymptotic stability of the whole system and the free parameters of the adaptive fuzzy controller can be tuned on-line by an output feedback control law and adaptive laws. It is proved that the overall control scheme can not only guarantee boundedness of the input and output of the closed-loop system but also make the tracking error converge to a small neighborhood of the origin.

The remaining of this paper is organized as follows. Problem formulation is given in Section 2. Adaptive fuzzy sliding mode control scheme for a class of nonlinear discrete-time system is presented in Section 3. The simulation example to demonstrate the performances of the proposed method is provided in Section 4. Section 5 gives the conclusions of the advocated design methodology.

2. **Problem Formulation.**

Consider the discrete-time single-input single-output (SISO) nonlinear systems having a state space representation in the form

$$\begin{cases} x_1(k+1) = x_2(k) \\ x_2(k+1) = x_3(k) \\ \quad\vdots \\ x_{n-1}(k+1) = x_n(k) \\ x_n(k+1) = f(x(k)) + u(k) + d(k) \\ y(k) = x_1(k) \end{cases} \tag{1}$$

where $x(k) = [x_1(k), x_2(k), \cdots, x_n(k)]$ is the state vector, $u(k) \in R$ and $y(k) \in R$ are the input and the output of the system, respectively. $f(x(k))$ is unknown but bounded function and $d(k)$ is bounded external disturbance. Let the tracking error be

$$\begin{aligned} e_1(k) &= x_1(k) - y_d(k) \\ e_2(k) &= x_2(k) - y_d(k+1) \\ &\vdots \\ e_n(k) &= x_n(k) - y_d(k+n-1) \end{aligned}$$

where $y_d(k)$ is the reference trajectory and the tracking error vector is denoted as $e(k) = [e_1(k), e_2(k), \cdots, e_n(k)]^T \in R^n$. Then, the tracking error dynamic equation can be expressed as

$$e(k+1) = Ae(k) + B\left[f(x(k)) + u(k) - y_d(k+n) + d(k)\right] \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{3}$$

The sliding surface in the space of the tracking-error vector can be defined as

$$s(k) = c_1e_1(k) + c_2e_2(k) + \cdots + c_{n-1}e_{n-1}(k) + e_n(k) = Ce(k) \tag{4}$$

where $C = [c_1, c_2, \cdots, c_{n-1}, 1]$ be selected so that $h(z) = z^n + c_{n-1}z^{n-1} + \cdots + c_2 z + c_1$ is stable. The process of sliding mode control can be divided into two phases, i.e., the approaching phase with $s(k) \neq 0$ and the sliding phase with $s(k) = 0$. A condition to guarantee that the trajectory of the error vector $e(k)$ will translate from the approaching phase to the sliding phase is select the control strategy such that [19]

$$s(k+1) - s(k) = -\varepsilon T s(k) - qT\text{sgn(s(k))} \tag{5}$$

where ε, $q > 0$ and $T > 0$ is the sampling period. In the sliding phase, if the function $f(x(k))$ is known and the system is free of external disturbance, $d(k) = 0$, then the equivalent control $u_{eq}(k)$ which will force the system dynamics to stay on the sliding surface can be derived as follows.

From Equation (4), we have

$$s(k+1) = \sum_{i=1}^{n-1} c_i e_i(k+1) + f(x(k)) - y_d(k+n) + u(k) \tag{6}$$

and the incremental change in $s(k)$ can be expressed as

$$\Delta s(k+1) = \sum_{i=1}^{n-1} c_i e_i(k+1) + f(x(k)) - y_d(k+n) + u_{eq}(k) - \sum_{i=1}^{n-1} c_i e_i(k) - e_n(k) = 0 \tag{7}$$

then

$$u_{eq}(k) = -\sum_{i=1}^{n-1} c_i e_i(k+1) - f(x(k)) + y_d(k+n) + \sum_{i=1}^{n-1} c_i e_i(k) + e_n(k) \tag{8}$$

However, $f(x(k))$ is unknown and $d(k) \neq 0$, the ideal controller (7) cannot be implemented.

In the approaching phase $s(k) \neq 0$, in order to satisfy the sliding condition (5), a switching type control $u_{sw}(k)$ must be added

$$u_{sw}(k) = -\varepsilon T s(k) - qT\text{sgn(s(k))} \tag{9}$$

In Section 3, in order to handle training data corrupted by noise or rule uncertainties, we will use interval type-2 FLS described in [28-34] to approximate function $f(x(k))$ and adaptive laws can be derived.

3. **Robust Adaptive Fuzzy Sliding Mode Control Design.** An adaptive fuzzy system is a FLS equipped with a training algorithm to maintain a consistent performance under plant uncertainties. The most important advantage of the adaptive fuzzy control over conventional adaptive control is that adaptive fuzzy controllers are capable of incorporating linguistic fuzzy information from human operator, whereas conventional adaptive controller is not. In this section, we will develop the adaptive interval type-2 fuzzy controller that can incorporate with linguistic information to design an adaptive law for the adjustable parameters in the controller, such that the closed loop output trajectory $y(k)$ follows the reference trajectory $y_d(k)$.

To begin with, we replace $f(x(k))$ by interval type-2 FLS, $f(x(k)|\,\theta_f)$, described in [28-34] as

$$f(x(k)|\,\theta_f) = \theta_f^T(k)\xi(k) = \frac{\theta_{fr}^T(k)\xi_r(k) + \theta_{fl}^T(k)\xi_l(k)}{2} \tag{10}$$

where $\xi(k) = \frac{1}{2}[\xi_r(k)\xi_l(k)]^T$ is the fuzzy basis function vector, $\theta_f = [\theta_{fr}\ \theta_{fl}]^T$ is the corresponding adjustable parameter vector of each fuzzy logic system. The optimal parameter vector is defined as

$$\theta_f^* = \arg \min_{\theta_f \in \Omega_{\theta_f}} \left\{ \sup_{x \in \Omega_x} \left| f(x(k)) - f(x(k)|\, \theta_f) \right\} \right. \tag{11}$$

where Ω_{θ_f} and Ω_x are compact sets of suitable bounds on θ_f and x, respectively and they are defined as $\{\theta_f|\, |\theta_f| \leq M_f\}$ and $\Omega_x = \{x|\, |x| \leq M_x\}$ where M_f and M_x are positive constants. Define the minimum approximation error as

$$\omega_1(k) = f(x(k)) - f(x(k)|\, \theta_f^*) + d(k) \tag{12}$$

If the closed-loop system the fuzzy control $u(k)$ is chosen as

$$u(k) = -k_d s(k) + u_{eq}(k) + u_{sw}(k) + u_3(k) \tag{13}$$

then substituting (10)-(13) into (6), after simple manipulation, we have

$$\Delta s(k+1) = -k_d s(k) + u_{sw}(k) + u_3(k) + \frac{1}{2}\tilde{\theta}_{fr}^T(k)\xi_r(k) + \frac{1}{2}\tilde{\theta}_{fl}^T(k)\xi_l(k) + \omega_1(k) \tag{14}$$

where k_d is a small positive real number, $\tilde{\theta}_{fr}(k) = \theta_{fr}^* - \theta_{fr}(k)$, $\tilde{\theta}_{fl}(k) = \theta_{fl}^* - \theta_{fl}(k)$ and the robust controller $u_3(k)$ is employed to attenuate the external disturbance d given as

$$u_3(k) = -\mathrm{sgn}(\mathrm{s(k)}) \left[-\frac{1}{2}\mathrm{B_1(k)} + \frac{1}{2}\left(\mathrm{B_1^2(k)} - 4\mathrm{B_2(k)}\right)^{\frac{1}{2}} \right] \tag{15}$$

with

$$B_1(k) = 2\,|k_d s(k)| + 2A_0(k) - 2\,|s(k)| \tag{16}$$

$$B_2(k) = \left[|k_d s(k)| + A_0(k)\right]^2 \tag{17}$$

$$A_0(k) = s_{u2} + s_{fr}\,\|\xi_r(k)\| + s_{fl}\,\|\xi_l(k)\| + s_w \tag{18}$$

where s_w, s_{fr}, s_{fl} and s_{u2} are positive real constants and satisfy the following conditions.

$$|\omega_1(k)| \leq s_w, \quad \left|\tilde{\theta}_{fr}^T(k)\right| \leq s_{fr}, \quad \left|\tilde{\theta}_{fl}^T(k)\right| \leq s_{fl}, \quad |u_{sw}(k)| \leq s_{u2} \tag{19}$$

The main result of the robust adaptive interval type-2 fuzzy SMC scheme is summarized in the following theorem.

Theorem 3.1. *Consider the uncertain nonlinear discrete time system (1) with the robust adaptive interval type-2 fuzzy controller given by (13). The parameter vectors $\theta_{fr}(k)$ and $\theta_{fl}(k)$ can be adjusted by the adaptive laws given by (20) and (21). It ensures that all the closed-loop signals are bounded and the tracking errors converge to zero.*

$$\Delta\theta_{fr}(k) = \alpha_r \xi_r(k) s(k) \tag{20}$$

$$\Delta\theta_{fl}(k) = \alpha_l \xi_l(k) s(k) \tag{21}$$

where α_r and α_l are positive constants which determine the rates of adaptation.

Proof: First, the Lyapunov function candidate is chosen as

$$V(k) = \frac{1}{2}s^2(k) + \frac{1}{2\alpha_r}\tilde{\theta}_{fr}^T(k-1)\tilde{\theta}_{fr}(k-1) + \frac{1}{2\alpha_l}\tilde{\theta}_{fl}^T(k-1)\tilde{\theta}_{fl}(k-1) \tag{22}$$

Then, the incremental change in $V(k)$ can be obtained as

$$\begin{aligned} \Delta V(k+1) &= V(k+1) - V(k) \\ &= \frac{1}{2}s^2(k+1) - \frac{1}{2}s^2(k) + \frac{1}{2\alpha_r}\tilde{\theta}_{fr}^T(k)\tilde{\theta}_{fr}(k) - \frac{1}{2\alpha_r}\tilde{\theta}_{fr}^T(k-1)\tilde{\theta}_{fr}(k-1) \\ &\quad + \frac{1}{2\alpha_l}\tilde{\theta}_{fl}^T(k)\tilde{\theta}_{fl}(k) - \frac{1}{2\alpha_l}\tilde{\theta}_{fl}^T(k-1)\tilde{\theta}_{fl}(k-1) \end{aligned} \tag{23}$$

Let

$$\Delta\theta_{tfr} = \frac{1}{2\alpha_r}\tilde{\theta}_{fr}^T(k)\tilde{\theta}_{fr}(k) - \frac{1}{2\alpha_r}\tilde{\theta}_{fr}^T(k-1)\tilde{\theta}_{fr}(k-1) \tag{24}$$

and

$$\Delta\theta_{tfl} = \frac{1}{2\alpha_l}\tilde{\theta}_{fl}^T(k)\tilde{\theta}_{fl}(k) - \frac{1}{2\alpha_l}\tilde{\theta}_{fl}^T(k-1)\tilde{\theta}_{fl}(k-1) \tag{25}$$

By using (24) and (25), (23) can be rewritten as

$$\begin{aligned}
\Delta V(k+1) &= \frac{1}{2}s^2(k+1) - \frac{1}{2}s^2(k) + \Delta\theta_{tfr} + \Delta\theta_{tfl} \\
&= \frac{1}{2}(\Delta s(k+1) + s(k))^2 - \frac{1}{2}s^2(k) + \Delta\theta_{tfr} + \Delta\theta_{tfl} \\
&= \frac{1}{2}\Delta s^2(k+1) + s(k)\Delta s(k+1) + \Delta\theta_{tfr} + \Delta\theta_{tfl} \\
&= \frac{1}{2}\Delta s^2(k+1) + s(k)\Big[-k_d s(k) + u_{sw}(k) + u_3(k) + \frac{1}{2}\tilde{\theta}_{fr}^T(k)\xi_r(k) \\
&\quad + \frac{1}{2}\tilde{\theta}_{fl}^T(k)\xi_l(k) + \omega_1(k)\Big] + \Delta\theta_{tfr} + \Delta\theta_{tfl} \\
&= \frac{1}{2}\Delta S^2(k+1) - k_d S^2(k) + u_{sw}(k)S(k) + u_3(k)S(k) + \omega_1(k)S(k) \\
&\quad + \frac{1}{2}\tilde{\theta}_{fr}^T(k)\xi_r(k)S(k) + \Delta\theta_{tfr} + \frac{1}{2}\tilde{\theta}_{fl}^T(k)\xi_l(k)S(k) + \Delta\theta_{tfl}
\end{aligned} \tag{26}$$

From (24) and (25), $\Delta\theta_{tfr}$ and $\Delta\theta_{tfl}$ can expressed by

$$\begin{aligned}
\Delta\theta_{tfr} &= \frac{1}{2\alpha_r}\left(\tilde{\theta}_{fr}^T(k)\tilde{\theta}_{fr}(k) - \left[\tilde{\theta}_{fr}(k) - \Delta\tilde{\theta}_{fr}(k)\right]^T\left[\tilde{\theta}_{fr}(k) - \Delta\tilde{\theta}_{fr}(k)\right]\right) \\
&= -\frac{1}{\alpha_r}\tilde{\theta}_{fr}^T(k)\Delta\tilde{\theta}_{fr}(k) - \frac{1}{2\alpha_r}\Delta\tilde{\theta}_{fr}^T(k)\Delta\tilde{\theta}_{fr}(k)
\end{aligned} \tag{27}$$

and

$$\begin{aligned}
\Delta\theta_{tfl} &= \frac{1}{2\alpha_l}\left(\tilde{\theta}_{fl}^T(k)\tilde{\theta}_{fl}(k) - \left[\tilde{\theta}_{fl}(k) - \Delta\tilde{\theta}_{fl}(k)\right]^T\left[\tilde{\theta}_{fl}(k) - \Delta\tilde{\theta}_{fl}(k)\right]\right) \\
&= -\frac{1}{\alpha_l}\tilde{\theta}_{fl}^T(k)\Delta\tilde{\theta}_{fl}(k) - \frac{1}{2\alpha_l}\Delta\tilde{\theta}_{fl}^T(k)\Delta\tilde{\theta}_{fl}(k)
\end{aligned} \tag{28}$$

Substituting (27) and (28) into (26), we can obtain

$$\begin{aligned}
\Delta V(k+1) &= \frac{1}{2}\Delta s^2(k+1) - k_d s^2(k) + u_{sw}(k)s(k) + u_3(k)s(k) + \omega_1(k)s(k) \\
&\quad + \frac{1}{2}\tilde{\theta}_{fr}^T(k)\xi_r(k)s(k) + \Delta\theta_{tfr} + \frac{1}{2}\tilde{\theta}_{fl}^T(k)\xi_l(k)s(k) + \Delta\theta_{tfl} \\
&= \frac{1}{2}\Delta s^2(k+1) - k_d s^2(k) + u_{sw}(k)s(k) + u_3(k)s(k) + \omega_1(k)s(k) \\
&\quad + \tilde{\theta}_{fr}^T(k)\left[\frac{1}{2}\xi_r(k)s(k) - \frac{1}{\alpha_r}\Delta\tilde{\theta}_{fr}(k)\right] - \frac{1}{2\alpha_r}\Delta\tilde{\theta}_{fr}^T(k)\Delta\tilde{\theta}_{fr}(k) \\
&\quad + \tilde{\theta}_{fl}^T(k)\left[\frac{1}{2}\xi_l(k)s(k) - \frac{1}{\alpha_l}\Delta\tilde{\theta}_{fl}(k)\right] - \frac{1}{2\alpha_l}\Delta\tilde{\theta}_{fl}^T(k)\Delta\tilde{\theta}_{fl}(k)
\end{aligned} \tag{29}$$

From the adaptive laws (20) and (21), we get

$$\begin{aligned}\Delta V(k+1) = &- k_d s^2(k) + \frac{1}{2}\Delta s^2(k+1) + u_{sw}(k)s(k) + u_3(k)s(k) \\ &+ \omega_1(k)s(k) - \frac{1}{2\alpha_r}\Delta\tilde{\theta}_{fr}^T(k)\Delta\tilde{\theta}_{fr}(k) - \frac{1}{2\alpha_l}\Delta\tilde{\theta}_{fl}^T(k)\Delta\tilde{\theta}_{fl}(k)\end{aligned} \tag{30}$$

According to (14), we have

$$\begin{aligned}|\Delta s(k+1)| &\le |k_d s(k)| + |u_{sw}(k)| + |u_3(k)| + \left|\frac{1}{2}\tilde{\theta}_{fr}^T(k)\xi_r(k)\right| + \left|\frac{1}{2}\tilde{\theta}_{fl}^T(k)\xi_l(k)\right| + |\omega_1(k)| \\ &\le |k_d s(k)| + s_{u2} + s_{fr}\left\|\xi_r(k)\right\| + s_{fl}\left\|\xi_l(k)\right\| + s_w + |u_3(k)| \\ &\le |k_d s(k)| + A_0(k) + |u_3(k)|\end{aligned} \tag{31}$$

By taking square from both sides of (31), one can get

$$\begin{aligned}|\Delta s(k+1)|^2 &\le \big(|k_d s(k)|^2 + 2A_0(k)\,|k_d s(k)| + 2\,|k_d s(k)|\,|u_3(k)| + 2A_0(k)\,|u_3(k)| \\ &\quad + A_0^2(k) + |u_3(k)|^2\big) \\ &\le \big([|k_d s(k)| + A_0(k)]^2 + [2\,|k_d s(k)| + 2A_0(k) - 2\,|s(k)|]\,|u_3(k)| \\ &\quad + 2\,|s(k)|\,|u_3(k)| + |u_3(k)|^2\big) \\ &\le \left[-B_1(k) + \left(B_1^2(k) - 4B_2(k)\right)^{\frac{1}{2}}\right]|s(k)|\end{aligned} \tag{32}$$

Therefore, (30) can be expressed as

$$\begin{aligned}\Delta V(k+1) = &- k_d s^2(k) + \omega_1(k)s(k) + u_{sw}(k)s(k) - \frac{1}{2\alpha_r}\Delta\tilde{\theta}_{fr}^T(k)\Delta\tilde{\theta}_{fr}(k) \\ &- \frac{1}{2\alpha_l}\Delta\tilde{\theta}_{fl}^T(k)\Delta\tilde{\theta}_{fl}(k)\end{aligned} \tag{33}$$

Since $u_{sw}(k)s(k) < 0$, $\Delta\tilde{\theta}_{fr}^T(k)\Delta\tilde{\theta}_{fr}(k) > 0$, $k_d s^2(k) > 0$, $\Delta\tilde{\theta}_{fl}^T(k)\Delta\tilde{\theta}_{fl}(k) > 0$ and the term $\omega_1(k)s(k)$ should be very small if not equal to zero in the adaptive interval type-2 FLS. So, we have

$$\Delta V(k+1) \le 0 \tag{34}$$

By using the Barbalat's lemma [23] we can easily show that $S(k)$ will be decreased, and $s(k) \to 0$ as $k \to \infty$, i.e., $\lim_{k\to\infty} e(k) = 0$. The proof is completed.

The overall Robust adaptive fuzzy sliding mode control scheme is as shown in Figure 1.

To summarize above analysis, the design algorithm for adaptive interval type-2 fuzzy sliding mode control is proposed as follows:

[Step 1]: Specify the desired coefficients c_i, such that $h(z)$ is a Hurwitz polynomial and obtain the sliding surface.

[Step 2]: Define the membership functions $\mu_{F_i^l}(x)$ for $i = 1, 2, \cdots, M$ and compute the fuzzy basis function vector $\underline{\xi}(x)$.

[Step 3]: Suitably select the adaptive parameters s_f, s_{fl}, s_{fr}, s_w, s_{u2}, ε, q.

[Step 4]: By using the adaptive laws (20) and (21) to adjust the parameter vectors $\underline{\theta}_{fr}\underline{\theta}_{fl}$, then control input $u(k)$ of the system can be constructed as (13).

4. **Simulation Example.** In this section, we will apply our adaptive interval type-2 fuzzy SMC to control a second-order discrete-time nonlinear system to track a desired trajectory.

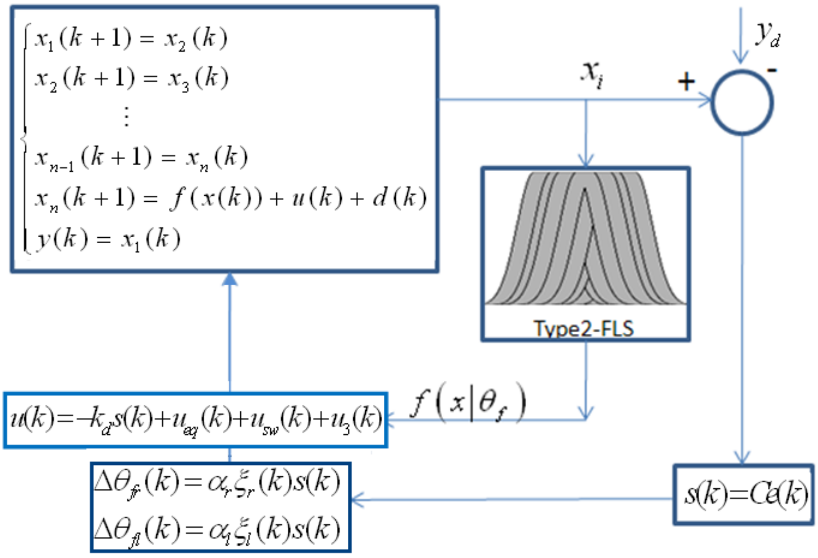

FIGURE 1. The overall robust adaptive fuzzy sliding mode control scheme

Example 4.1. *Consider the second-order discrete-time nonlinear system*

$$\begin{cases} x_1(k+1) = x_2(k) \\ x_2(k+1) = \dfrac{1-e^{-x_1(k)}}{1+e^{-x_1(k)}} - 0.1x_1(k)\sin(t) + u(k) + d(k) \\ y(k) = x_1(k) \end{cases} \tag{35}$$

where $d(t) = 0.1\cos(0.3t)$ *is the external disturbance.* $y_d(t) = \sin(0.1t) + \cos(0.3t)$ *is the reference trajectory.*

According to the design procedure, the design is given in the following steps:

[Step 1]: The feedback gain matrix is chosen as $C = [c_2\ c_1] = [1\ 0.1]$ and the sliding surface is obtained as $s = (c_1e_1 + c_2e_2)$.

[Step 2]: Specify the design parameters $s_f = s_{fr} = s_{fl} = 0.09$, $s_w = 0.05$, $s_{u2} = 0.12$, $\varepsilon = 1.666$, $q = 3.333$ in adaptive laws (20) and (21), simulation time $t_f = 150$ second and the step size $h = 0.06$.

[Step 3]: The following type-1 and interval type-2 fuzzy membership functions for x_i, $i = 1, 2$ are selected as F_i^j, $j = 1, \ldots, 7$ shown in Table 1. Also, the footprint of uncertainty of the type-2 membership function for x_i, $i = 1, 2$, is as shown in Figure 2.

$$\mu_{F_i^1}(x_i) = \frac{1}{1 + 0.1\exp(5(x_i + m_{i1}))}, \quad \mu_{F_i^2}(x_i) = \exp(-3(x_i + m_{i2})^2),$$
$$\mu_{F_i^3}(x_i) = \exp(-3(x_i + m_{i3})^2), \quad \mu_{F_i^4}(x_i) = \exp(-3(x_i + m_{i4})^2),$$
$$\mu_{F_i^5}(x_i) = \frac{1}{1 + 0.1\exp(5(x_i + m_{i5}))}.$$

[Step 4]: The control law is expressed as $u(k) = -k_d s(k) + u_{eq}(k) + u_{SW}(k) + u_3(k)$.

The simulation results are described as following two cases: free of internal noise case and training data corrupted by White Gaussian Noise (WGN) with signal-to-noise (SNR) = 20 dB case for both type-1 and interval type-2 adaptive fuzzy SMCs.

A. Free of internal noise case:

(I). Adaptive type-1 FNN SMC:

TABLE 1. Type-1 and interval type-2 fuzzy membership functions for x_i, $i = 1, 2$

	Variance (σ)	Mean (m)		
		m_1	m_2	m (type-1)
$\mu_{F_i^1}(x_i)$	1	2.5	1.5	2
$\mu_{F_i^2}(x_i)$	1	1.5	0.5	1
$\mu_{F_i^3}(x_i)$	1	0.5	–0.5	0
$\mu_{F_i^4}(x_i)$	1	–0.5	–1.5	–1
$\mu_{F_i^5}(x_i)$	1	–1.5	–2.5	–2

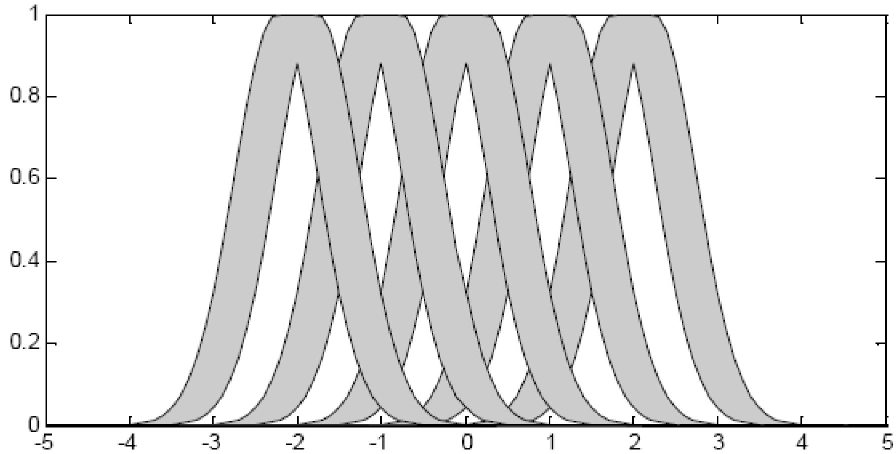

FIGURE 2. The FOU of the type-2 membership function for x_i, $i = 1, 2$

The tracking performance of the output trajectory $y(k)$ and the reference trajectory $y_d(k)$ is as shown in Figure 3 and Figure 4 shows control input $u(k)$. Trajectory of the sliding surface and mean square of the tracking error are shown in Figure 5 and Figure 6, respectively.

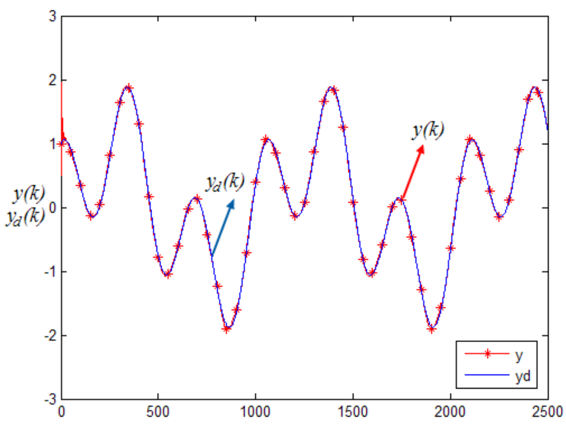

FIGURE 3. The output trajectories of $y(k)$ and $y_d(k)$

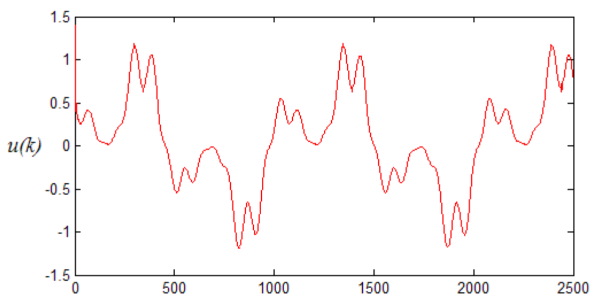

FIGURE 4. Control input $u(k)$

(II). <u>Adaptive interval type-2 FNN SMC:</u>
The tracking performance of the output trajectory $y(k)$ and the reference trajectory $y_d(k)$ is as shown in Figure 7 and Figure 8 shows control input $u(k)$. Trajectory of the

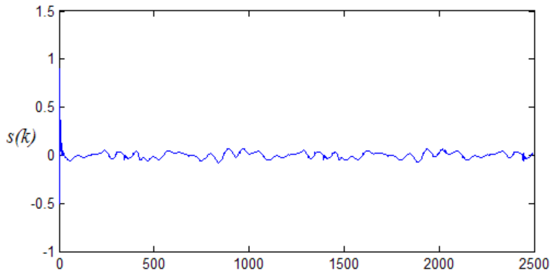

FIGURE 5. Trajectory of the sliding surface $s(k)$

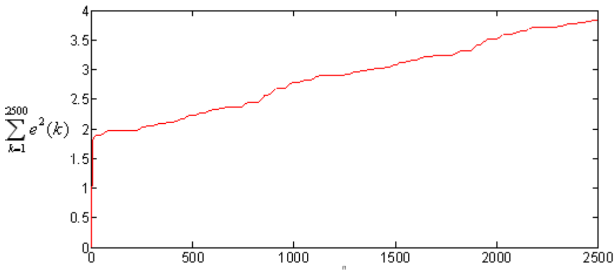

FIGURE 6. Mean square of the tracking error

sliding surface and mean square of the tracking error are shown in Figure 9 and Figure 10, respectively.

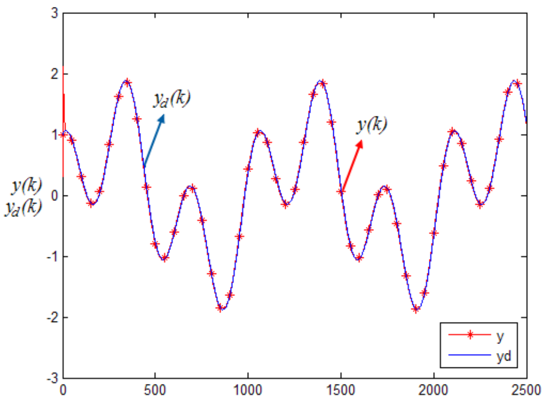

FIGURE 7. The output trajectories of $y(k)$ and $y_d(k)$

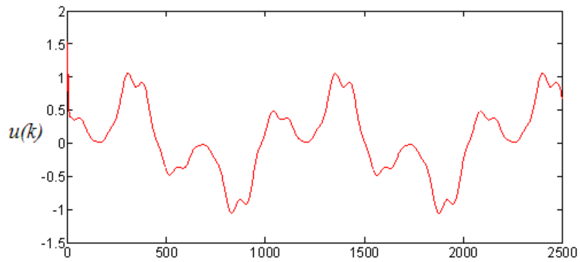

FIGURE 8. Control input $u(k)$

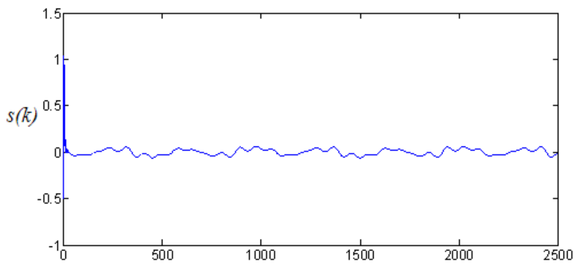

FIGURE 9. Trajectory of the sliding surface $s(k)$

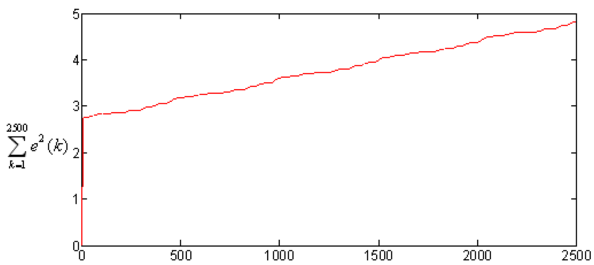

FIGURE 10. Mean square of the tracking error

B. Training data corrupted by WGN with SNR = 20 dB noise: In order to show that interval type-2 FNN SMC can handle numerical uncertainties associated with inputs and outputs of the FLC, the noisy training data is corrupted by WGN with SNR = 20 dB.

(I). Adaptive type-1 FNN SMC:

The tracking performance of the output trajectory $y(k)$ and the reference trajectory $y_d(k)$ is shown in Figure 11 and Figure 12 shows control input $u(k)$. Trajectory of the sliding surface and mean square of the tracking error are shown in Figure 13 and Figure 14, respectively.

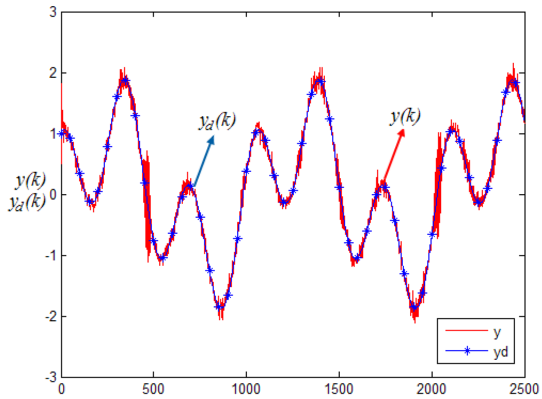

FIGURE 11. The output trajectories of $y(k)$ and $y_d(k)$

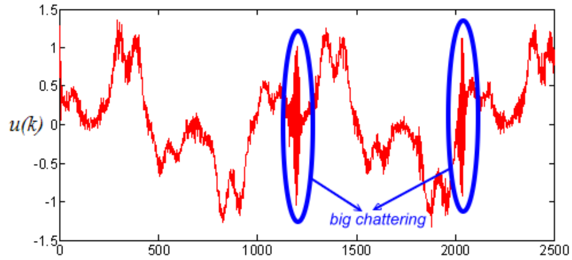

FIGURE 12. Control input $u(k)$

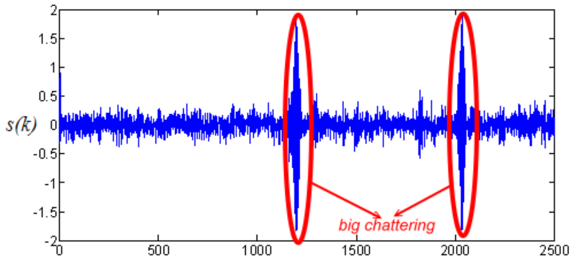

FIGURE 13. Trajectory of the sliding surface $s(k)$

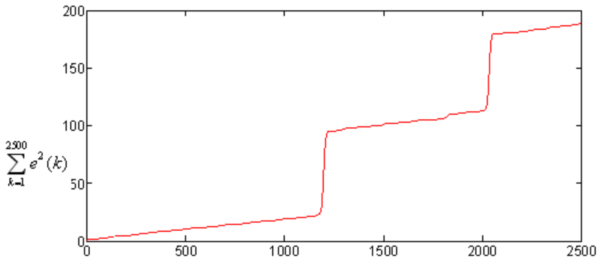

FIGURE 14. Mean square of the tracking error

(II). Adaptive interval type-2 FNN SMC:

The tracking performance of the output trajectory $y(k)$ and the reference trajectory $y_d(k)$ is as shown in Figure 15 and Figure 16 shows control input $u(k)$. Trajectory of the sliding surface and mean square of the tracking error are shown in Figure 17 and Figure 18, respectively.

It is observed that the tracking performance of the interval type-2 fuzzy SMC is much better than that of the type-1 fuzzy SMC. Meanwhile, in order to deal with noisy training data, type-1 fuzzy SMC must expend more control effort. For different internal noise levels, the mean square tracking errors of the type-1 and interval type-2 fuzzy SMCs are indicated in Figure 19. The comparison of tracking performance and control effort between type-1 and interval type-2 fuzzy SMCs are given in Table 2.

Remark 4.1. *From Figure 19 and Table 2, it is observed that for low internal noise levels (large SNR values), the tracking performance and control effort for both type-1 and interval type-2 fuzzy SMCs are almost the same. Nevertheless, for high internal noise levels (small SNR values), the tracking performance of the interval type-2 fuzzy SMC is much better than that of the type-1 fuzzy SMC and type-1 fuzzy SMC must expend more control effort.*

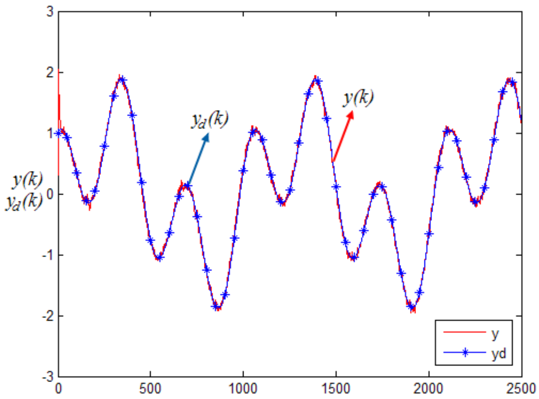

FIGURE 15. The output trajectories of $y(k)$ and $y_d(k)$

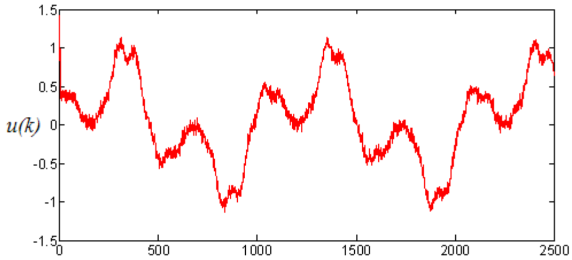

FIGURE 16. Control input $u(k)$

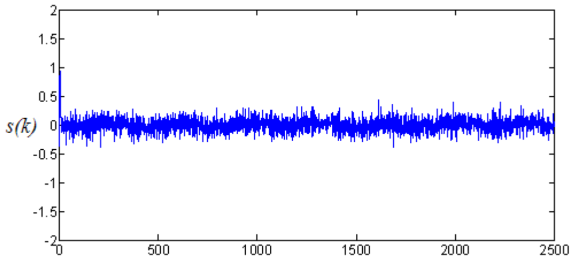

FIGURE 17. Trajectory of the sliding surface $s(k)$

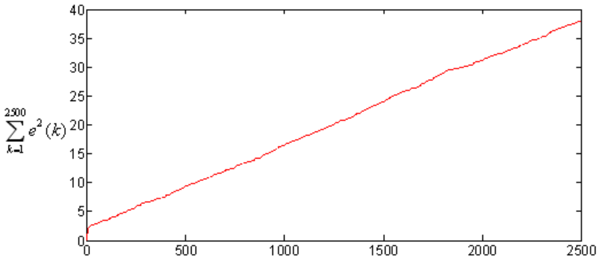

FIGURE 18. Mean square of the tracking error

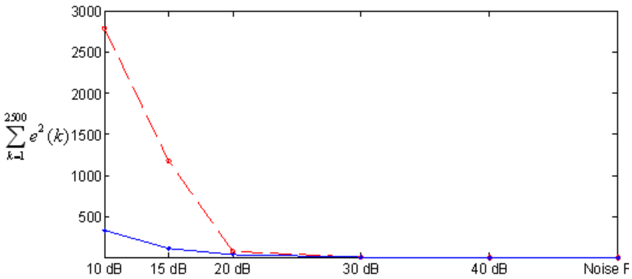

FIGURE 19. Mean square of the tracking errors for different noise levels of type-1 and interval type-2 fuzzy SMCs

TABLE 2. The comparison of tracking performance and control effort between type-1 and interval type-2 FNN SMCs

Name	Type-2		Type-1	
	Mean square tracking error	Square sum of control u	Mean square tracking error	Square sum of control u
Noise Free	4.81	731.43	3.83	761.64
Noise 40 dB	5.2	732.01	4.2	761.61
Noise 30 dB	7.8	731.66	7.44	763.53
Noise 20 dB	38.65	740.05	81.04	778.78
Noise 15 dB	118.9	746.92	1179.10	1090.30
Noise 10 dB	339.5	786.21	2787	1777.8

5. **Conclusions.** In this paper, we propose an adaptive interval type-2 fuzzy SMC for a class of unknown nonlinear discrete-time systems corrupted by internal noise and external disturbance. The simulation results show that the interval type-2 fuzzy SMC overcomes the limitations of type-1 fuzzy SMC. Especially, as in high level of uncertainties the type-2 fuzzy SMC has given better and smoother responses that have outperformed their type-1 fuzzy SMC counterparts.

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