International Journal of Innovative Computing, Information and Control
Volume **8**, Number **1(A)**, January **2012**
 ISSN 1349-4198
pp. 403-418

DESIGN OF STABLE AND QUADRATIC-OPTIMAL STATIC OUTPUT FEEDBACK CONTROLLERS FOR TS-FUZZY-MODEL-BASED CONTROL SYSTEMS: AN INTEGRATIVE COMPUTATIONAL APPROACH

Wen-Hsien Ho[1], Shinn-Horng Chen[2], I-Te Chen[3], Jyh-Horng Chou[2,4,*] and Chun-Chin Shu[4]

[1]Department of Medical Information Management
[3]Center for General Education
Kaohsiung Medical University
No. 100, Shi-Chuan 1st Road, Kaohsiung 807, Taiwan
{ whho; itchen }@kmu.edu.tw

[2]Department of Mechanical Engineering
National Kaohsiung University of Applied Sciences
No. 415, Chien-Kung Road, Kaohsiung 807, Taiwan
shchen@cc.kuas.edu.tw

[4]Institute of System Information and Control
National Kaohsiung First University of Science and Technology
No. 1, University Road, Yenchao, Kaohsiung 824, Taiwan
[*]Corresponding author: choujh@nkfust.edu.tw; u9516815@nkfust.edu.tw

Received September 2010; revised January 2011

Abstract. *By integrating the stabilizability condition, the orthogonal-functions approach (OFA) and the hybrid Taguchi-genetic algorithm (HTGA), an integrative computational method is presented in this paper to design the stable and quadratic-optimal static output feedback parallel-distributed-compensation (PDC) controller such that (i) the Takagi-Sugeno (TS) fuzzy-model-based control system can be stabilized, and (ii) a quadratic finite-horizon integral performance index for the TS-fuzzy-model-based control system can be minimized. In this paper, the stabilizability condition is proposed in terms of linear matrix inequalities (LMIs). By using the OFA and the LMI-based stabilizability condition, the stable and quadratic-finite-horizon-optimal static output feedback PDC control problem for the TS-fuzzy-model-based dynamic systems is transformed into a static constrained-optimization problem represented by the algebraic equations with constraint of LMI-based stabilizability condition, thus greatly simplifying the optimal static output feedback PDC control design problem. Then, for the static constrained-optimization problem, the HTGA is employed to find the stable and quadratic-optimal static output feedback PDC controllers of the TS-fuzzy-model-based control systems. A design example of stable and quadratic-optimal static output feedback PDC controller for a nonlinear inverted pendulum system controlled by a separately excited direct-current (DC) motor is given to demonstrate the applicability of the proposed integrative computational approach.*
Keywords: Quadratic optimal control, Static output feedback PDC controller, Takagi-Sugeno fuzzy model, Orthogonal-functions approach, Hybrid Taguchi-genetic algorithm, Linear matrix inequalities

1. Introduction.

Recently, it has been shown that the fuzzy-model-based representation proposed by Takagi and Sugeno [1], known as the TS fuzzy model, is a successful approach for dealing with the nonlinear control systems, and there are many successful applications of the TS-fuzzy-model-based approach to nonlinear control systems [2-15].

Unlike conventional modeling approaches where a single model is used to describe the global behavior of a nonlinear control system, the TS fuzzy modeling approach is essentially a multi-model approach in which the simple sub-models (typically linear models) are combined to describe the global behavior of the nonlinear control system. Each fuzzy rule for the TS fuzzy control system has a linear dynamic model as the consequent part that expresses the local dynamics of each fuzzy rule. Then, the overall fuzzy model is achieved by blending these rules. The advantage of controller synthesis for such a fuzzy model is that the linear control methods can be used.

Despite the success of applying the TS-fuzzy-model-based approach to nonlinear control systems, it has become evident that many research issues remain to be addressed. In fact, in many cases, it is very difficult, if not impossible, to obtain a full order output feedback controller of a nonlinear control system. This is due to inaccessible measurement or overly expensive measurement. Therefore, recently, some research studies [16-19] have proposed the linear-matrix-inequality-based (LMI-based) approach to design the static output feedback parallel-distributed-compensation (PDC) controllers of the TS-fuzzy-model-based control systems for the infinite-horizon (i.e., infinite-time) control problems. On the other hand, only robust stability and stabilization are often not enough in control design. In control systems design, it is often of interest to synthesize a quadratic-optimal controller such that the control objective of minimizing a quadratic integral performance criterion is achieved [20]. Hence, recently, some researchers [2,21,22] have proposed some LMI-based approaches to design the quadratic-optimal controllers of TS-fuzzy-model-based control systems. Tanaka and Wang [2], Zheng et al. [21] and Li [22] designed the quadratic-optimal parallel-distributed-compensation (PDC) controllers by minimizing the upper bound of a quadratic infinite-horizon integral performance index. However, under the design consideration of directly minimizing a quadratic infinite-horizon integral performance index, it is not easy for the LMI-based approaches presented by Tanaka and Wang [2], Zheng et al. [21] and Li [22] to solve the quadratic-infinite-horizon-optimal PDC control problem of such systems. For some practical problems, we need to deal with the finite-horizon (i.e., finite-time) optimal control problems [23]. However, it is also difficult to apply the LMI-based approaches proposed by Tanaka and Wang [2], Zheng et al. [21] and Li [22] to directly minimize the finite-horizon performance index for solving the quadratic-finite-horizon-optimal PDC control problem of these systems. Besides, for solving the optimal PDC control problems, there are some issues that need to be resolved, such as how to simplify the computation for the above control problem of such systems and also ensure some characteristics of closed-loop systems [24]. Therefore, one of the most important issues is to develop computational methods for designing the quadratic-finite-horizon-optimal PDC controllers where the performance index is directly minimized. Very recently, Ho and Chou [25] have proposed a computational optimization method, which integrates the orthogonal-functions approach (OFA) [26] and the genetic algorithm [27,28], to design quadratic-optimal PDC controllers for the finite-horizon optimal control problem of the TS-fuzzy-model-based control systems where the performance index is directly minimized. Since the method proposed by Ho and Chou [25] only involves algebraic computation and is straightforward and well-adapted to computer implementation, the design procedures of the controllers for these control systems may be either greatly reduced or much simplified accordingly. Ho and Chou [25] have also shown that the computational optimization method integrating the OFA and the genetic algorithm may obtain better results than the LMI-based approaches [2,21,22] for finding the quadratic-optimal PDC controllers of the TS-fuzzy-model-based control systems.

Summing up the above statements and reasons, although the LMI-based approach is successful in designing the static output feedback PDC controllers of the TS-fuzzy-model-based control systems for the infinite-horizon (i.e., infinite-time) control problems proposed by Fang et al. [16], Wu et al. [17], Chung et al. [18] and Huang and Nguang [19], to the authors' best knowledge, there are no studies investigating the issue of designing stable and quadratic-finite-horizon-optimal static output feedback PDC controllers for the TS-fuzzy-model-based control systems by directly minimizing the performance index subject to the constraint of stabilizability. On the other hand, in practice, in order to avoid high gains, the controller gains must be considered to satisfy the constraints. The LMI-based approach proposed by Fang et al. [16], Wu et al. [17], Chung et al. [18] and Huang and Nguang [19] cannot deal with the design problem of the static output feedback PDC controller gains having constraints. Therefore, we can see that it is worthwhile to present an efficiently numerical optimization approach accompanied with the stabilizability condition to design the stable and quadratic-finite-horizon-optimal static output feedback PDC controllers having constraints for the TS-fuzzy-model-based control systems, where the performance index subject to the constraint of stabilizability is considered to be directly minimized.

The purpose of this paper is to propose a numerical optimization method accompanied with the stabilizability condition to design stable and quadratic-optimal static output feedback PDC controllers for the finite-horizon optimal control problem of the TS-fuzzy-model-based control systems by integrating the OFA, the hybrid Taguchi-genetic algorithm (HTGA) and the LMI technique, where the LMI technique is used to derive the stabilizability condition for ensuring that the closed-loop TS-fuzzy-model-based control systems can be stabilized. The proposed numerical optimization method can not only be applied to find the feedback gain matrices of the stable and quadratic-optimal static output feedback PDC controller for the TS-fuzzy-model-based control system under the minimization of a defined quadratic finite-horizon integral performance index, but also be applied to the case of the elements of the feedback gain matrices having constraints for practical consideration.

In this paper, by using the OFA and the LMI-based stabilizability condition, the stable and quadratic-finite-horizon-optimal static output feedback PDC control problem for the TS-fuzzy-model-based control systems is transformed into a static parameter constrained-optimization problem represented by algebraic equations with constraint of LMI-based stabilizability condition, thus greatly simplifying the optimal static output feedback PDC control design problem. The computational complexity for both differential and integral in the optimal static output feedback PDC control design of the original dynamic systems may therefore be reduced remarkably. Then, for the static constrained-optimization problem, the HTGA is employed to find the stable and quadratic-optimal static output feedback PDC controllers of the TS-fuzzy-model-based control systems. The proposed integrative computational method considers directly minimizing the quadratic finite-horizon integral performance index subject to the constraint of stabilizability in designing the stable and quadratic-optimal static output feedback PDC controllers. The reason why the HTGA is applied in this paper is that Tsai et al. [29,30] have shown that the HTGA may obtain better results than those existing improved genetic algorithms reported in the literature. An illustrative example is also given in this paper to demonstrate the applicability of the proposed integrative computational method.

2. **Problem Statement.** The TS-fuzzy-model-based control system for the nonlinear control system can be obtained as the following form:

$$\begin{aligned}&\tilde{R}^i: \text{ IF } z_1(t) \text{ is } M_{i1} \text{ and } \ldots \text{ and } z_g(t) \text{ is } M_{ig},\\ &\text{THEN } \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t), \\ y(t) = C_i x(t), \end{cases}\end{aligned} \tag{1}$$

with the initial state vector $x(0)$, where $\tilde{R}^i$ ($i = 1, 2, \ldots, N$) denotes the i-th implication, N is the number of fuzzy rules, $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}}$ denotes the n-dimensional state vector, $y(t) = [y_1(t), y_2(t), \ldots, y_r(t)]^{\mathrm{T}}$ denotes the r-dimensional output vector, $u(t) = [u_1(t), u_2(t), \ldots, u_p(t)]^{\mathrm{T}}$ denotes the p-dimensional input vector, $z_i(t)$ ($i = 1, 2, \ldots, g$) are the premise variables, A_i, B_i and C_i ($i = 1, 2, \ldots, N$) are, respectively, the $n \times n$, $n \times p$ and $r \times n$ consequent constant matrices, and M_{ij} ($i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, g$) are the fuzzy sets.

The resulting TS-fuzzy-model-based dynamic system inferred from (1) is represented as

$$\dot{x}(t) = \sum_{i=1}^{N} h_i(z(t))(A_i x(t) + B_i u(t)), \tag{2a}$$

$$y(t) = \sum_{i=1}^{N} h_i(z(t)) C_i x(t), \tag{2b}$$

in which $z(t) = [z_1(t), z_2(t), \ldots, z_g(t)]^{\mathrm{T}}$ denotes the g-dimensional premise vector, $h_i(z(t)) = w_i(z(t)) \Big/ \sum_{i=1}^{N} w_i(z(t))$, $w_i(z(t)) = \prod_{j=1}^{g} M_{ij}(z_j(t))$ and $M_{ij}(z_j(t))$ are the grades of membership of $z_j(t)$ in the fuzzy sets M_{ij} ($i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, g$). It can be seen that, for all t, $h_i(z(t)) \geq 0$ and $\sum_{i=1}^{N} h_i(z(t)) = 1$.

Before we are able to synthesize a static output feedback controller such that good control performance for a given dynamic system can be efficiently achieved, it is necessary that the given dynamic system can be stabilized by the following static output feedback PDC controller:

$$u(t) = -\sum_{i=1}^{N} h_i(z(t)) F_i y(t) = -\sum_{i=1}^{N} \sum_{j=1}^{N} h_i(z(t)) h_j(z(t)) F_i C_j x(t), \tag{3}$$

where F_i ($i = 1, 2, \ldots, N$) denote the $p \times r$ local static output feedback gain matrices.

By substituting (3) into (2a), we can get the closed-loop TS-fuzzy-model-based dynamic system as

$$\dot{x}(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} h_i(z(t)) h_j(z(t)) h_k(z(t))(A_i - B_i F_j C_k) x(t). \tag{4}$$

Before we investigate the stabilizability condition to design the stable and quadratic-finite-horizon-optimal static output feedback PDC controllers for the TS-fuzzy-model-based dynamic systems (4), the following lemmas need to be introduced first.

Lemma 2.1. [2] *If the number of rules that fire for all t is less than or equal to $\bar{s}$ where $1 < \bar{s} \leq N$, then*

$$\sum_{i=1}^{N} h_i^2(z(t)) - \frac{1}{\bar{s} - 1} \sum_{i<j}^{N} 2 h_i(z(t)) h_j(z(t)) \geq 0. \tag{5}$$

For the closed-loop TS-fuzzy-model-based dynamic system (4), the problem of stabilizability analysis is (under the condition that the local static output feedback gain matrices F_i ($i = 1, 2, \ldots, N$) of the static output feedback PDC controller in (3) have been specified in advance) to derive a stabilizability criterion for checking whether the closed-loop TS-fuzzy-model-based dynamic system (4) can be stabilized by the specified static output feedback PDC controller or not. Hence, in what follows, we present an LMI-based stabilizability criterion to analyze whether the closed-loop TS-fuzzy-model-based dynamic system (4) can be stabilized by the static output feedback PDC controller or not, where the local static output feedback gain matrices F_i have been specified in advance.

Theorem 2.1. *The closed-loop TS-fuzzy-model-based dynamic system (4) is stable, if, for the specified local static output feedback gains matrices F_j ($j = 1, 2, \ldots, N$) in (3), there exists a symmetric positive definite matrix P and the symmetric positive semi-definite matrices Q_k ($k = 1, 2, \ldots, N$) such that the following LMIs are simultaneously satisfied:*

$$G_{kii}^T P + P G_{kii} + (\bar{s} - 1) Q_k < 0, \tag{6a}$$

and

$$\left(\frac{G_{kij} + G_{kji}}{2}\right)^T P + P\left(\frac{G_{kij} + G_{kji}}{2}\right) - Q_k \leq 0, \tag{6b}$$

where $G_{ijk} = A_i - B_i F_j C_k$, $i < j$, $\bar{s} > 1$, and $i, j, k = 1, 2, \ldots, N$.

Proof: See Appendix A.

However, only stabilizability is often not enough in control design. The control objective of minimizing a quadratic finite-horizon integral performance criterion for the dynamic systems is also considered in many practical control-engineering applications [20,23]. On the other hand, before we are able to synthesize a controller such that good control performance for a given dynamic system can be efficiently achieved, it is necessary that the given dynamic system can be stabilized by the controller [31,32]. In addition, both optimality and stability should be simultaneously considered in the optimal controllers design [33]. Therefore, the problem considered in this paper is how to specify the local static output feedback gain matrices F_i ($i = 1, 2, \ldots, N$) of the static output feedback PDC controller in (3) such that the constraint of the LMI-based stabilizability condition (6) for the closed-loop TS-fuzzy-model-based dynamic system (4) can be satisfied, and such that the optimal control performance for the TS-fuzzy-model-based dynamic system (2) can be achieved by minimizing the following H_2 quadratic finite-horizon integral performance index:

$$\begin{aligned} J &= \int_0^{qt_f} \left[y^{\mathrm{T}}(t) Q y(t) + u^{\mathrm{T}}(t) R u(t)\right] dt \\ &= \sum_{k=0}^{q-1} \int_{kt_f}^{(k+1)t_f} \left[y^{\mathrm{T}}(t) Q y(t) + u^{\mathrm{T}}(t) R u(t)\right] dt, \end{aligned} \tag{7}$$

where t_f denotes a small time interval which is chosen for the independent variable t, q is a positive integer specified by the designer, Q is a symmetric positive semi-definite matrix, and R is a symmetric positive-definite matrix. Here the time interval of interest is designated as being from $t = 0$ to $t = qt_f$, where $t = 0$ is the initial time and $t = qt_f$ is the final time of the control period. The problem to be studied in this paper can be named the mixed H_2/LMI static output feedback PDC controllers design problem of the TS-fuzzy-model-based control systems, and the design procedures for the static output feedback PDC controllers can be described as follows:

Step 1: Check the constraint of LMI-based robust stabilizability condition (6).

Step 2: Minimize the H_2 quadratic finite-horizon integral performance index (7) for the TS-fuzzy-model-based dynamic system (2).

That is, the design problem of the mixed H_2/LMI static output feedback PDC controllers for the TS-fuzzy-model-based control systems is a constrained optimization problem. In the next section, we will integrate the OFA, the HTGA and the presented LMI-based stabilizability condition to solve the mixed H_2/LMI static output feedback PDC controllers design problem of the TS-fuzzy-model-based control systems, where the performance index subject to the constraint of stabilizability condition is considered to be directly minimized.

3. **Stable and Quadratic-Optimal Static Output Feedback PDC Controllers Design.** Here, consider the time interval $kt_f \le t \le (k+1)t_f$, where t_f is chosen for the independent variable t, and let us define

$$t = kt_f + \eta, \tag{8}$$

and

$$x_k = x(kt_f), \tag{9}$$

in which $k = 0, 1, 2, \ldots, q-1$ and $0 \le \eta \le t_f$.

The state vector $x(t)$, within $kt_f \le t \le (k+1)t_f$, can be approximated by the truncated orthogonal-functions (OF) representation as

$$x(t) = \sum_{s=0}^{m-1} x_s^{(k)} T_s(t) = \tilde{x}^{(k)} T(t), \tag{10}$$

where m is the number of terms required for the OF, $T(t) = [T_0(t), T_1(t), \ldots, T_{m-1}(t)]^{\mathrm{T}}$ denotes the $m \times 1$ OF basis vector, $T_i(t)$ $(i = 0, 1, \ldots, m-1)$ denotes the OF, $x_s^{(k)}$ $(s = 0, 1, \ldots, m-1)$ are the $n \times 1$ coefficient vector, and $\tilde{x}^{(k)} = \left[x_0^{(k)}, x_1^{(k)}, \ldots, x_{m-1}^{(k)}\right]$ is the $n \times m$ coefficient matrix of $x(t)$.

Substituting (3) and the truncated OF representation of $x(t)$ in (10) into the quadratic integral performance index (7), the quadratic integral performance index J becomes the following algebraic form:

$$\begin{aligned} J = \sum_{k=0}^{q-1} \mathrm{trace}\Bigg[& W(\tilde{x}^{(k)})^{\mathrm{T}} \Bigg(\sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{N} \sum_{o=1}^{N} h_i(z_k) h_j(z_k) h_l(z_k) h_o(z_k) \\ & C_i^{\mathrm{T}} (Q + F_j^{\mathrm{T}} R F_l) C_o \Bigg) (\tilde{x}^{(k)}) \Bigg], \end{aligned} \tag{11}$$

where the constant matrix W is the product-integration-matrix of two OF basis vectors [25].

Since, before the consequent output can be inferred within the small time interval $kt_f \le t \le (k+1)t_f$, the degree of fulfillment of the antecedent must be computed in advance, so, as in the studies given by Ho and Chou [34,35], we can let the value of $h_i(z(t))$, within $kt_f \le t \le (k+1)t_f$, be $h_i(z(kt_f))$. Then, integrating (2a) from $t = kt_f$ to $t = t$ within $kt_f \le t \le (k+1)t_f$, we obtain

$$x(t) - x(kt_f) = \sum_{i=1}^{N} h_i(z_k) \left[A_i \int_{kt_f}^{t} x(t)dt + B_i \int_{kt_f}^{t} u(t)dt \right], \tag{12}$$

where $h_i(z_k) = h_i(z(kt_f))$ and $k = 0, 1, 2, \ldots, q-1$.

Using the following integral property of the OF:

$$\int_{kt_f}^{t} T(t)dt = HT(t), \tag{13}$$

and applying (3), (9) and (10), (12) can be cast into the form

$$\tilde{x}^{(k)} - [x_k, 0, 0, \ldots, 0] = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N} h_i(z_k)h_j(z_k)h_l(z_k)(A_i - B_iF_jC_l)\tilde{x}^{(k)}H, \tag{14}$$

in which H is the operational matrix of integration for the OF [34,35].

Equation (14) can be rewritten as

$$\tilde{x}^{(k)} - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N} h_i(z_k)h_j(z_k)h_l(z_k)(A_i - B_iF_jC_l)\tilde{x}^{(k)}H = \tilde{Q}^{(k)}, \tag{15}$$

where $\tilde{Q}^{(k)} = [x_k, 0, 0, \ldots, 0]$ is an $n \times m$ matrix.

Making use of the Kronecker product, the explicit form for the coefficient matrix $\tilde{x}^{(k)}$ comes directly from (15) as

$$\hat{x}^{(k)} = \left[I_{mn} - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{N} h_i(z_k)h_j(z_k)h_l(z_k)(H^{\mathrm{T}} \otimes (A_i - B_iF_jC_l))\right]^{-1} \hat{Q}^{(k)}, \tag{16}$$

where I_{mn} denotes the $mn \times mn$ identity matrix, $\hat{x}^{(k)} = \left[x_0^{(k)\mathrm{T}}, x_1^{(k)\mathrm{T}}, \ldots, x_{m-1}^{(k)\mathrm{T}}\right]^{\mathrm{T}}$, $\hat{Q}^{(k)} = \left[x_k^{\mathrm{T}}, 0^{\mathrm{T}}, 0^{\mathrm{T}}, \ldots, 0^{\mathrm{T}}\right]^{\mathrm{T}}$, and $\otimes$ denotes the Kronecker product [36]. This implies that $\tilde{x}^{(k)}$ can be obtained from (16).

Now, if one set of local static output feedback gain matrices $\{F_1, F_2, \ldots, F_N\}$ is given, then $\tilde{x}^{(k)}$ $(k = 0, 1, \ldots, q-1)$ can be calculated from the following algorithm only involving the algebraic computation.

Detailed Steps: Algebraic Algorithm

Step 1: Give a small time interval t_f, the specified positive integer q, and the initial state vector $x(0)$, and set $k = 0$.
Step 2: Calculate $h_i(z(k\,t_f))$ for $i = 1, 2, \ldots, N$.
Step 3: Calculate $\hat{x}^{(k)}$ from (16).
Step 4: Compute x_{k+1} by using $x_{k+1} = x((k+1)\,t_f) = \tilde{x}^{(k)}T((k+1)\,t_f)$.
Step 5: Set $k = k + 1$. If $k > q - 1$, then stop; otherwise go to Step 2.

From the above algorithm, it is obvious that if one set of local static output feedback gain matrices $\{F_1, F_2, \ldots, F_N\}$ is specified, then $\tilde{x}^{(k)}$ $(k = 0, 1, \ldots, q-1)$ can be determined, and thus the value of the performance index (11) corresponding to this set of $\{F_1, F_2, \ldots, F_N\}$ can be calculated. Given another set of local static output feedback gain matrices $\{F_1, F_2, \ldots, F_N\}$, there obtains another value of the performance index (11). That is, the value of the performance index of algebraic form (11) is actually dependent on the set of local static output feedback gain matrices $\{F_1, F_2, \ldots, F_N\}$, which means

$$J = G\left(f_{111}, f_{112}, \ldots, f_{Npr}\right) \tag{17}$$

where f_{ijk} $(i = 1, 2, \ldots, N,\ j = 1, 2, \ldots, p$ and $k = 1, 2, \ldots, r)$ denotes the elements of the local static output feedback gain matrices F_i. Hence, the design problem of the stable and quadratic-optimal static output feedback PDC controller for the TS-fuzzy-model-based control system is to search for the optimal f_{ijk} such that there exists a symmetric positive definite matrix P and the symmetric positive semi-definite matrices Q_k $(k = 1, 2, \ldots, N)$

to make the LMIs in (6) hold (i.e., such that the stabilizability condition in Theorem 2.1 is satisfied), and such that the performance index of algebraic form (11) for the TS-fuzzy-model-based dynamic system (2) is minimized. This is equivalent to the static constrained-optimization problem in the following:

$$\text{minimize } J = G\left(f_{111}, f_{112}, \ldots, f_{Npr}\right) \tag{18}$$

subject to $|f_{ijk}| \leq C_{ijk}$, and subject to the constraint that there exist a symmetric positive definite matrix P and the symmetric positive semi-definite matrices Q_k $(k = 1, 2, \ldots, N)$ to make the LMIs in (6) hold, for $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, p$ and $k = 1, 2, \ldots, r$, where C_{ijk} are the positive real numbers given from the practical consideration, respectively. This means that, by using the OFA and the LMI-based stabilizability condition, the stable and quadratic-finite-horizon-optimal static output feedback PDC control problem for the TS-fuzzy-model-based control systems can be replaced by a static constrained-optimization problem represented by the algebraic equations with constraints; thus greatly simplifying the stable and quadratic-finite-horizon-optimal static output feedback PDC control problem. Then, the HTGA described below can be employed to search for the optimal solution of the static constrained-optimization problem (18), where (18) is a nonlinear function with continuous variables.

The HTGA combines the traditional genetic algorithm (TGA) [37] with the Taguchi method [38-40]. In the HTGA, the Taguchi method is inserted between the crossover and mutation operations of a TGA. Then, by using two major tools (signal-to-noise ratio and orthogonal arrays) of the Taguchi method, the systematic reasoning ability of the Taguchi method is incorporated in the crossover operations to systematically select the better genes to achieve the crossover operations, and consequently enhance the genetic algorithms. The additional details regarding the HTGA can be found in the works proposed by Tsai et al. [29,30] and Ho and Chang [41].

4. **Illustrative Example.** A nonlinear inverted pendulum system controlled by a separately excited direct-current (DC) motor [42] is considered in this example as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ K_1 \sin x_1(t) + K_2 x_3(t) \\ K_3 x_2(t) + K_4 x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ K_5 \end{bmatrix} u\left(t\right), \tag{19}$$

where $x_1(t) \in [-\pi, \pi]$ is the angle of the pendulum, $x_2(t) = \dot{x}_1(t)$ is the angular velocity, $x_3(t)$ is the current of the motor, $u(t)$ denotes the control input voltage, $K_1 = 9.8$ ($1/\text{sec}^2$), $K_2 = 1$ $(1/\text{A} \cdot \text{sec}^2)$, $K_3 = -10$ $(\text{V} \cdot \text{sec/rad} \cdot \text{mH})$, $K_4 = -10$ (Ω/mH) and $K_5 = 10$ $(1/\text{mH})$. In this example, we also represent the nonlinear equation of motion of the system (19) into a TS-fuzzy-model-based dynamic system. The TS-fuzzy-model-based dynamic system for (19) can be obtained as following:

$$\tilde{R}^1 : \text{IF } z_1(t) \text{ is } M_{11}, \text{THEN } \left\{ \begin{array}{l} \dot{x}(t) = A_1 x(t) + B_1 u(t), \\ y(t) = C_1 x(t), \end{array} \right. \tag{20a}$$

$$\tilde{R}^2 : \text{IF } z_1(t) \text{ is } M_{21}, \text{THEN } \left\{ \begin{array}{l} \dot{x}(t) = A_2 x(t) + B_2 u(t), \\ y(t) = C_2 x(t), \end{array} \right. \tag{20b}$$

where $z_1(t) = \sin x_1(t)/x_1(t)$, $x(t) = [x_1(t), x_2(t), x_3(t)]^{\mathrm{T}}$, $x(0) = [30°, 0, 0]^{\mathrm{T}}$, $x_1(t) \in [0°, 360°]$, $A_1(t) = \begin{bmatrix} 0 & 1 & 0 \\ 9.8 & 0 & 1 \\ 0 & -10 & -10 \end{bmatrix}$, $A_2(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -10 & -10 \end{bmatrix}$, $B_1(t) = B_2(t) = [0, 0, 10]^{\mathrm{T}}$, $C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ denote the feedback of the angle and angular velocity of the pendulum, simultaneously, $M_{11}(z_1(t)) = \begin{cases} z_1(t), & x_1(t) \neq 0, \\ 1, & \text{otherwise}, \end{cases}$ and $M_{21}(z_1(t)) = 1 - M_{11}(z_1(t))$.

The quadratic finite-horizon integral performance index is

$$\begin{aligned} J &= \int_0^5 \left[y^{\mathrm{T}}(t) Q y(t) + u^{\mathrm{T}}(t) R u(t) \right] dt \\ &= \sum_{k=0}^{q-1} \int_{kt_f}^{(k+1)t_f} \left[y^{\mathrm{T}}(t) Q y(t) + u^{\mathrm{T}}(t) R u(t) \right] dt, \end{aligned} \tag{21}$$

where $q = 500$, $t_f = 0.01$, $Q = \operatorname{diag}\{1, 1\}$ and $R = 1$.

Then, for the TS-fuzzy-model-based control system (20), the proposed approach, which integrates the OFA, the HTGA, and the presented LMI-based stabilizability condition, is applied to design the stable and quadratic-optimal static output feedback PDC controller such that a symmetric positive definite matrix P and the symmetric positive semi-definite matrices Q_1 and Q_2 exist to make the LMIs in (6) hold, and such that the quadratic integral performance index (21) is minimized. Here, the type of OF considered in this example is also the shifted-Chebyshev functions. The evolutionary environments of the HTGA used in this paper are: the population size is 100, the crossover rate is 0.8, the mutation rate is 0.1, and the generation number is 50.

By using the proposed integrative computational approach and the LMI toolbox [43] with $m = 4$ and $|f_{ijk}| \leq 500$ in which f_{ijk} ($i = 1, 2$, $j = 1$ and $k = 1, 2$) are the elements of the local static output feedback gain matrices F_i ($i = 1, 2$) for $x(0) = [30°, 0, 0]^{\mathrm{T}}$, we can derive the performance index as $J = 62.3289$, and the stable and quadratic-optimal local static output feedback gain matrices are $F_1 = \begin{bmatrix} 32.953 & 16.816 \end{bmatrix}$ and $F_2 = \begin{bmatrix} 10.376 & 28.287 \end{bmatrix}$, and we can obtain a symmetric positive definite matrix P and the symmetric positive semi-definite matrix Q_1 and Q_2, respectively, as $P = \begin{bmatrix} 167.7466 & 7.3397 & 0.7956 \\ 7.3397 & 6.4890 & 0.1333 \\ 0.7956 & 0.1333 & 0.0294 \end{bmatrix}$, $Q_1 = \begin{bmatrix} 6.6895 & 0.5061 & 0.5553 \\ 0.5061 & 12.5640 & 0.2238 \\ 0.5553 & 0.2238 & 0.0690 \end{bmatrix}$ and $Q_2 = \begin{bmatrix} 59.2127 & 18.7103 & 0.5746 \\ 18.7103 & 13.1713 & 0.0558 \\ 0.5746 & 0.0558 & 0.0463 \end{bmatrix}$. Therefore, we can conclude that the obtained stable and quadratic-optimal static output feedback PDC controller can make the closed-loop TS-fuzzy-model-based dynamic system of this example stable, and may simultaneously minimize the quadratic finite-horizon integral performance index (21).

The responses of the angle of the pendulum $x_1(t)$, the angular velocity $x_2(t)$ and the current of the motor $x_3(t)$, and the control input voltage $u(t)$ for a nonlinear inverted pendulum system controlled by a separately excited DC motor system with the designed stable and quadratic-optimal static output feedback PDC controller obtained by using the proposed integrative computational approach are, respectively, shown in Figures 1-4.

Figures 1-3 illustrate the responses $x_1(t)$, $x_2(t)$ and $x_3(t)$ of the controlled and uncontrolled nonlinear inverted pendulum system. The simulation of the optimal control input voltage $u(t)$ is shown in Figure 4. From Figures 1-4, it can be seen that the responses $x_1(t)$, $x_2(t)$ and $x_3(t)$ are remarkably suppressed by employing a control signal $u(t)$. Therefore, we can conclude that the proposed approach, which integrates the OFA, the HTGA and the presented LMI-based stabilizability condition, can make the nonlinear inverted pendulum system controlled by a separately excited DC motor system have satisfactory control results.

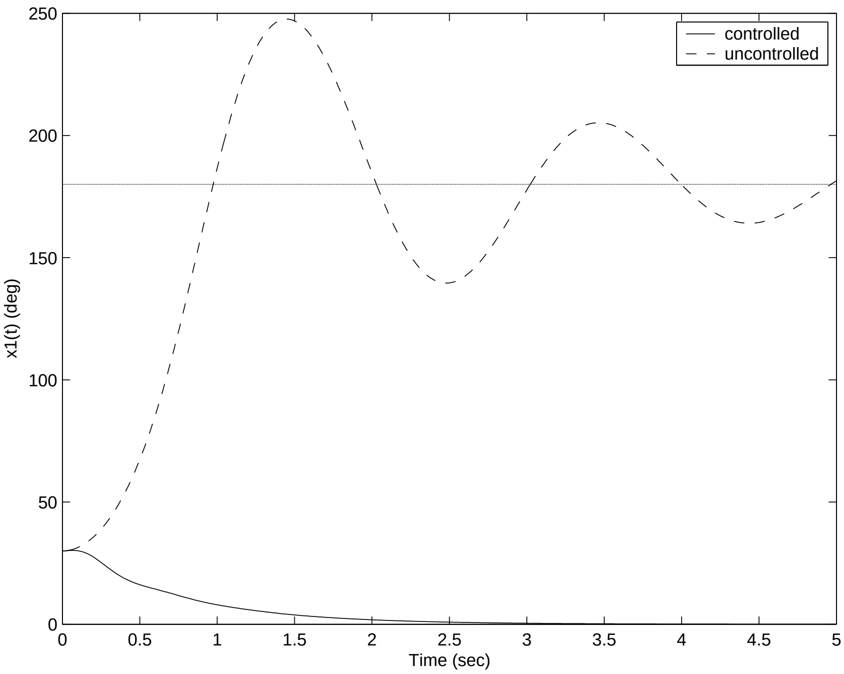

FIGURE 1. Response of the angle of the pendulum $x_1(t)$ for the nonlinear inverted pendulum system controlled by a separately excited DC motor with the designed stable and quadratic-optimal static output feedback PDC controller via the proposed integrative computational approach and without controller

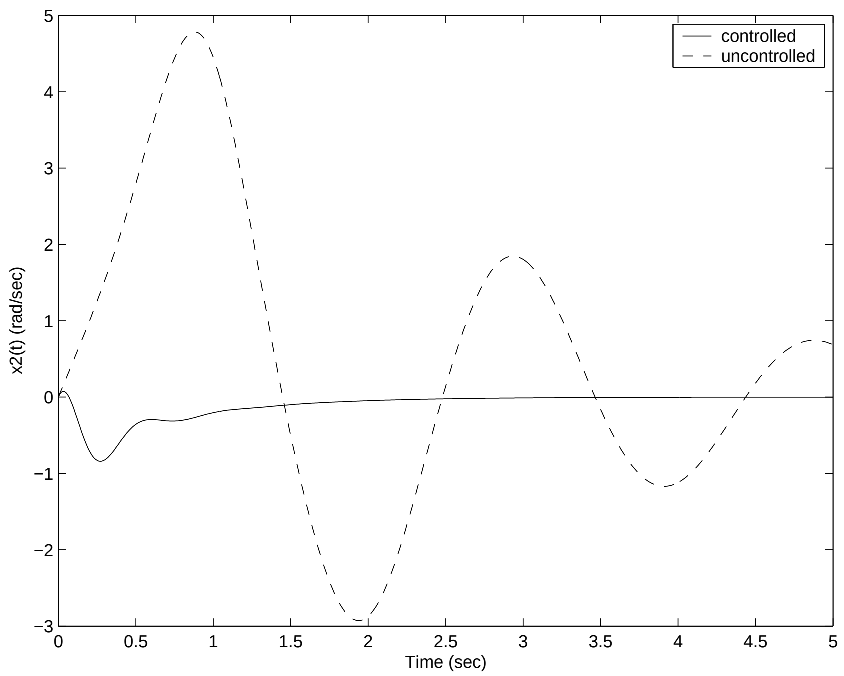

FIGURE 2. Response of the angular velocity $x_2(t)$ for the nonlinear inverted pendulum system controlled by a separately excited DC motor with the designed stable and quadratic-optimal static output feedback PDC controller via the proposed integrative computational approach and without controller

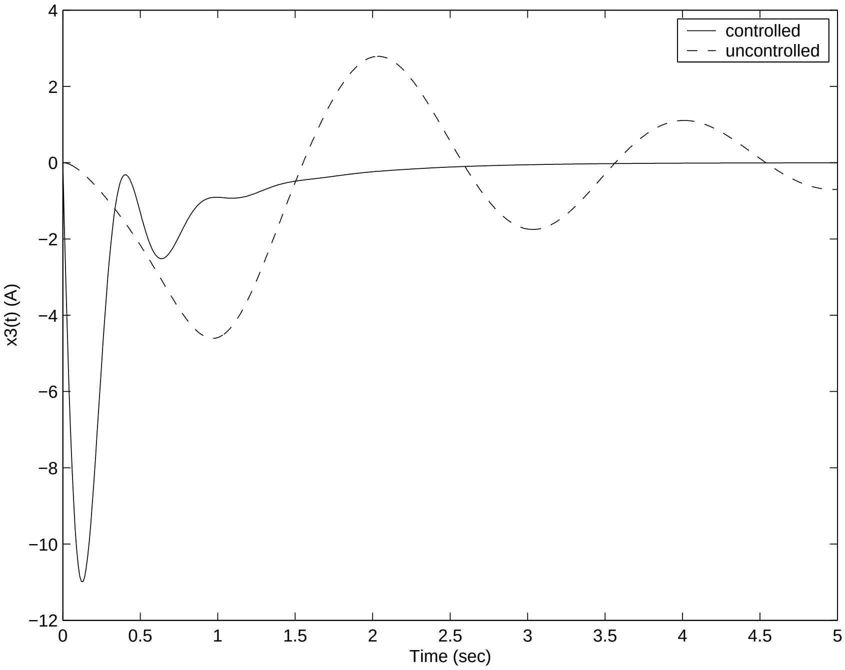

FIGURE 3. Response of the current of the motor $x_3(t)$ for the nonlinear inverted pendulum system controlled by a separately excited DC motor with the designed stable and quadratic-optimal static output feedback PDC controller via the proposed integrative computational approach and without controller

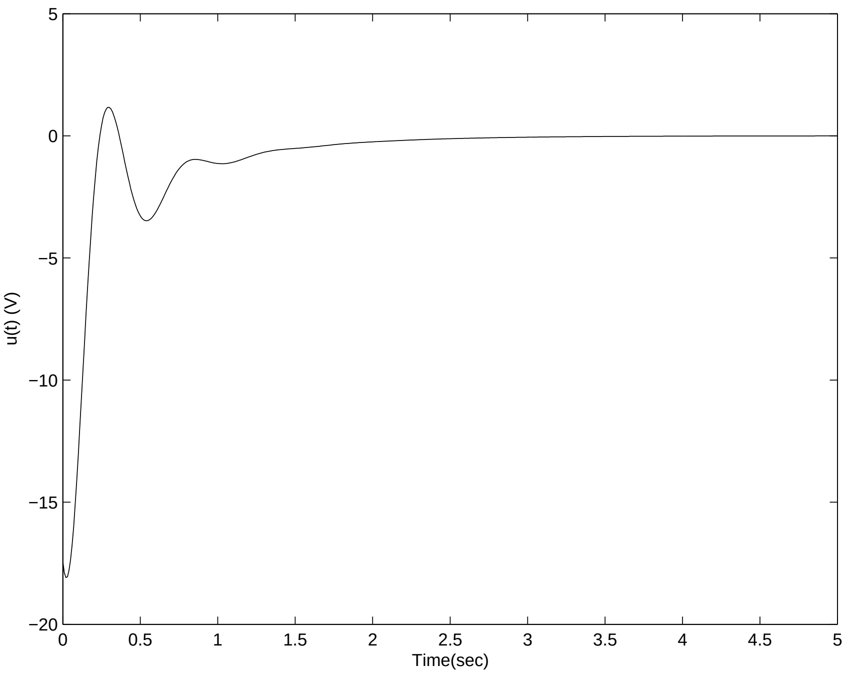

FIGURE 4. Control input voltage $u(t)$ of the nonlinear inverted pendulum system controlled by a separately excited DC motor with the designed stable and quadratic-optimal static output feedback PDC controller via the proposed integrative computational approach

5. **Conclusions.** Based on the OFA, an algorithm has been presented in this paper for solving the TS-fuzzy-model-based feedback dynamic equations. Then, the presented algebraic algorithm is integrated with the HTGA to design the stable and quadratic-optimal static output feedback PDC controllers of the TS-fuzzy-model-based control systems such that the control objective of directly minimizing a quadratic finite-horizon integral performance index subject to the constraint of LMI-based stabilizability condition can be achieved, where the quadratic integral performance index is also converted into the algebraic form by using the OFA. Since, by using the OFA and the LMI-based stabilizability condition, the stable and quadratic-finite-horizon-optimal static output feedback PDC control problem for the TS-fuzzy-model-based control systems can be replaced by a static parameter constrained-optimization problem represented by the algebraic equations with constraint of LMI-based stabilizability condition, and since the new proposed algorithm only involves the algebraic computation, the design procedures of the stable and quadratic-optimal static output feedback PDC controllers for the TS-fuzzy-model-based control systems may be either greatly reduced or much simplified accordingly. In addition, the presented integrative computational approach, which integrates the presented LMI-based stabilizability condition, the OFA and the HTGA, is non-differential, non-integral, straightforward, and well-adapted to computer implementation. Therefore, this proposed approach facilitates the design task of the stable and quadratic-optimal static output feedback PDC controllers for the TS-fuzzy-model-based control systems. On the other hand, the problem of determining the stabilizability has been turned into a LMI feasibility problem that can be easily solved by means of numerically efficient convex programming algorithms. The illustrative example regarding the nonlinear inverted pendulum system controlled by a separately excited DC motor has shown that the proposed approach is effective for designing stable and quadratic optimal static output feedback PDC controllers of the TS-fuzzy-model-based control systems. In future work, the LMI-based stabilizability criterion in (6) may be improved in order to make our results less conservative.

Acknowledgment. This work was in part supported by the National Science Council, Taiwan, under grant numbers NSC 98-2221-E-037-006 and NSC 99-2320-B-037-026-MY2.

REFERENCES

[1] T. Takagi and M. Sugeno, Fuzzy identification of systems and its applications to modeling and control, *IEEE Trans. on Systems, Man and Cybernetics*, vol.15, pp.116-132, 1985.

[2] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, John Wiley and Sons, New York, 2001.

[3] W. J. Chang, C. C. Ku, W. Chang and P. H. Huang, Passive fuzzy control with relaxed conditions for discrete affine T-S fuzzy systems, *International Journal of Innovative Computing, Information and Control*, vol.3, no.4, pp.853-871, 2007.

[4] W. H. Ho, J. T. Tsai, T. K. Liu and J. H. Chou, Optimal state-feedback control of time-varying TS-fuzzy-model-based systems by using an integrative computational approach, *International Journal of Innovative Computing, Information and Control*, vol.3, no.4, pp.873-885, 2007.

[5] H. Y. Chung and S. C. Chan, An alternate approach to the analysis of a T-S fuzzy model, *International Journal of Innovative Computing, Information and Control*, vol.4, no.11, pp.3067-3077, 2008.

[6] H. Han, Fuzzy controller design with input saturation, *International Journal of Innovative Computing, Information and Control*, vol.4, no.10, pp.2507-2521, 2008.

[7] W. H. Ho, J. T. Tsai and J. H. Chou, Robust quadratic-optimal control of TS-fuzzy-model-based dynamic systems with both elemental parametric uncertainties and norm-bounded approximation error, *IEEE Trans. on Fuzzy Systems*, vol.17, pp.518-531, 2009.

[8] L. Luoh, Control design of T-S fuzzy large-scale systems, *International Journal of Innovative Computing, Information and Control*, vol.5, no.9, pp.2869-2880, 2009.

[9] D. Wang, W. Wang and X. Wang, Controller failure analysis for T-S fuzzy systems with time-varying delay via a switched approach, *International Journal of Innovative Computing, Information and Control*, vol.5, no.10(B), pp.3329-3340, 2009.
[10] Y. Yi and L. Guo, Constrained PI tracking control for the output PDFs based on T-S fuzzy model, *International Journal of Innovative Computing, Information and Control*, vol.5, no.2, pp.349-358, 2009.
[11] W. Zhang, T. Wang and S. Tong, Delay-dependent stabilization conditions and control of T-S fuzzy systems with time-delay, *ICIC Express Letters*, vol.3, no.4(A), pp.871-876, 2009.
[12] S. He and F. Liu, Robust finite-time stabilization of uncertain fuzzy jump systems, *International Journal of Innovative Computing, Information and Control*, vol.6, no.9, pp.3853-3862, 2010.
[13] X. Huang, K. Cao, X. Ban and X. Z. Gao, Some further results on the stability analysis of a type of T-S fuzzy control systems in the frequency domain, *ICIC Express Letters*, vol.4, no.3(A), pp.659-666, 2010.
[14] H. Wu and M. Bai, Stochastic stability analysis and synthesis for nonlinear fault tolerant control systems based on the T-S fuzzy model, *International Journal of Innovative Computing, Information and Control*, vol.6, no.9, pp.3989-4000, 2010.
[15] I. Zamani and M. H. Zarif, An approach for stability analysis of T-S fuzzy systems via piecewise quadratic stability, *International Journal of Innovative Computing, Information and Control*, vol.6, no.9, pp.4041-4054, 2010.
[16] C. H. Fang, Y. S. Liu, S. W. Kan, L. Hong and C. H. Lee, A new LMI-based approach to relaxed quadratic stabilization of T-S fuzzy control systems, *IEEE Trans. on Fuzzy Systems*, vol.14, pp.386-397, 2006.
[17] S. M. Wu, F. M. Yu, C. C. Sun and H. Y. Chung, Static output feedback fuzzy controller design via a mixed approach for region T-S fuzzy system, *Proc. of the 2006 IEEE International Conference on Systems, Man, and Cybernetics*, Taipei, Taiwan, pp.3074-3079, 2006.
[18] H. Y. Chung, S. M. Wu, F. M. Yu and W. J. Chang, Evolutionary design of static output feedback controller for Takagi-Sugeno fuzzy systems, *IET Control Theory and Applications*, vol.1, pp.1096-1103, 2007.
[19] D. Huang and S. K. Nguang, Static output feedback controller design for fuzzy systems: An ILMI approach, *Information Sciences*, vol.177, pp.3005-3015, 2007.
[20] G. C. Goodwin, S. F. Graebe and M. E. Salgado, *Control System Design*, Prentice-Hall, New Jersey, 2001.
[21] K. Zheng, J. M. Xu and L. Yu, Takagi-Sugeno model-based optimal guaranteed cost fuzzy control for inverted pendulums, *Control Theory and Applications*, vol.21, pp.703-708, 2004.
[22] D. Q. Li, Optimal fuzzy guaranteed cost control for delta-operator based nonlinear systems, *Systems Engineering and Electronics*, vol.27, pp.883-888, 2005.
[23] B. Friedland, *Control System Design: An Introduction to State-Space Methods*, McGraw-Hill, New York, 1986.
[24] C. Song and T. Chai, Comment on discrete-time optimal fuzzy controller design: Global concept approach, *IEEE Trans. on Fuzzy Systems*, vol.13, pp.285-286, 2005.
[25] W. H. Ho and J. H. Chou, Design of optimal controller for Takagi-Sugeno fuzzy model based systems, *IEEE Trans. on Systems, Man and Cybernetics, Part A*, vol.37, pp.329-339, 2007.
[26] W. H. Ho, C. H. Hsieh and J. H. Chou, Optimal course handling control for nonlinear ship maneuvering systems, *International Journal of Innovative Computing, Information and Control*, vol.6, no.10, pp.4379-4388, 2010.
[27] C. H. Hsieh, W. H. Ho and J. H. Chou, Optimal PID controllers design of PWM feedback time-varying systems using orthogonal-functions approach and genetic algorithm, *International Journal of Innovative Computing, Information and Control*, vol.5, no.2, pp.387-397, 2009.
[28] W. H. Ho, S. H. Chen, J. H. Chou and T. C. Wang, Design of robust optimal eigenvalue-assignable output feedback PID controllers for linear uncertain multivariable systems via orthogonal function approach and genetic algorithm, *International Journal of Innovative Computing, Information and Control*, vol.6, no.1, pp 409-422, 2010.
[29] J. T. Tsai, T. K. Liu and J. H. Chou, Hybrid Taguchi-genetic algorithm for global numerical optimization, *IEEE Trans. on Evolutionary Computation*, vol.8, pp.365-377, 2004.
[30] J. T. Tsai, J. H. Chou and T. K. Liu, Tuning the structure and parameters of a neural network by using hybrid Taguchi-genetic algorithm, *IEEE Trans. on Neural Networks*, vol.17, pp.69-80, 2006.
[31] N. S. Nise, *Control Systems Engineering*, John Wiley and Sons, New York, 2000.
[32] L. Yu, *Robust Control: Linear Matrix Inequality Approach*, Tsinghua University Press, Beijing, 2002.

[33] R. E. Skelton, *Dynamic Systems Control*, John Wiley and Sons, New York, 1988.
[34] W. H. Ho and J. H. Chou, Solutions of Takagi-Sugeno fuzzy-model-based dynamic equations via orthogonal functions, *IEICE Trans. on Fundamentals of Electronics, Communications and Computer Sciences*, vol.E87-A, pp.3439-3442, 2004.
[35] W. H. Ho and J. H. Chou, Solutions of time-varying TS-fuzzy-model-based dynamic equations using shifted Chebyshev series approach, *Int. J. of Systems Science*, vol.36, pp.767-776, 2005.
[36] S. Barnett, *Matrix Methods for Engineers and Scientists*, McGraw-Hill, New York, 1979.
[37] M. Gen and R. Cheng, *Genetic Algorithms and Engineering Design*, John Wiley and Sons, New York, 1997.
[38] G. Taguchi, S. Chowdhury and S. Taguchi, *Robust Engineering*, McGraw-Hill, New York, 2000.
[39] Y. Wu, *Taguchi Methods for Robust Design*, The American Society of Mechanical Engineers, New York, 2000.
[40] W. H. Ho, J. H. Chou and C. Y. Guo, Parameter identification of chaotic systems using improved differential evolution algorithm, *Nonlinear Dynamics*, vol.61, pp.29-41, 2010.
[41] W. H. Ho and C. S. Chang, Genetic-algorithm-based artificial neural network modeling for platelet transfusion requirements on acute myeloblastic leukemia patients, *Expert Systems With Applications*, vol.38, pp.6319-6323, 2011.
[42] K. Tanaka, T. Ikeda and H. O. Wang, Fuzzy regulators and fuzzy observers: Relaxed stability conditions and LMI-based designs, *IEEE Trans. on Fuzzy Systems*, vol.6, pp.250-265, 1998.
[43] P. Gahinet, A. Nemirovski, A. J. Laub and M. Chilali, *LMI Control Toolbox*, The Math Works Inc., Massachusetts, 1995.

Appendix A. **Proof of Theorem 2.1.** Let $V(x(t)) = x^{\mathrm{T}}(t)\,Px(t)$ be a quadratic Lyapunov function candidate for the system (4), then we have

$$\begin{aligned}
\dot{V}(x(t)) =& x^{\mathrm{T}}(t)\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} h_i(z(t))h_j(z(t))h_k(z(t))\left(A_i - B_iF_jC_k\right)^{\mathrm{T}}\right)Px(t) \\
&+ x^{\mathrm{T}}(t)P\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} h_i(z(t))h_j(z(t))h_k(z(t))\left(A_i - B_iF_jC_k\right)\right)x(t) \\
=& \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} h_i(z(t))h_j(z(t))h_k(z(t))x^{\mathrm{T}}(t) \\
&\left(\left(A_i - B_iF_jC_k\right)^{\mathrm{T}}P + P\left(A_i - B_iF_jC_k\right)\right)x(t) \\
=& \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} h_i(z(t))h_j(z(t))h_k(z(t))x^{\mathrm{T}}(t)\left(G_{ijk}^{\mathrm{T}}P + PG_{ijk}\right)x(t) \\
=& h_1(z(t))\sum_{i=1}^{N}\sum_{j=1}^{N} h_i(z(t))h_j(z(t))x^{\mathrm{T}}(t)\left(G_{1ij}^{\mathrm{T}}P + PG_{1ij}\right)x(t) \\
&+ \ldots + h_N(z(t))\sum_{i=1}^{N}\sum_{j=1}^{N} h_i(z(t))h_j(z(t))x^{\mathrm{T}}(t)\left(G_{Nij}^{\mathrm{T}}P + PG_{Nij}\right)x(t) \\
=& h_1(z(t))\Bigg[\sum_{i=1}^{N} h_i^2(z(t))x^{\mathrm{T}}(t)\left(G_{1ii}^{\mathrm{T}}P + PG_{1ii}\right)x(t) \\
&+ \sum_{i<j}^{N} 2h_i(z(t))h_j(z(t))x^{\mathrm{T}}(t)\left(\left(\frac{G_{1ij}+G_{1ji}}{2}\right)^{\mathrm{T}}P + P\left(\frac{G_{1ij}+G_{1ji}}{2}\right)\right)x(t)\Bigg]
\end{aligned} \tag{A.1}$$

$$+\ldots+h_N(z(t))\Bigg[\sum_{i=1}^{N} h_i^2(z(t))x^{\mathrm{T}}(t)\Big(G_{Nii}^{\mathrm{T}}P+PG_{Nii}\Big)x(t)$$
$$+\sum_{i<j}^{N} 2h_i(z(t))h_j(z(t))x^{\mathrm{T}}(t)\Bigg(\bigg(\frac{G_{Nij}+G_{Nji}}{2}\bigg)^{\mathrm{T}}P+P\bigg(\frac{G_{Nij}+G_{Nji}}{2}\bigg)\Bigg)x(t)\Bigg],$$

where $G_{ijk}=A_i-B_iF_jC_k$.

From the condition (6a) and Lemma 2.1, we have

$$\begin{aligned}
\dot{V}(x(t)) \leq & h_1(z(t))\left[\sum_{i=1}^{N} h_i^2(z(t))\,x^{\mathrm{T}}(t)\left(G_{1ii}^{\mathrm{T}}P+PG_{1ii}\right)x(t)\right.\\
&\left.+\sum_{i<j}^{N} 2h_i(z(t))\,h_j(z(t))\,x^{\mathrm{T}}(t)\,Q_1x(t)\right]+\ldots\\
&+h_N(z(t))\left[\sum_{i=1}^{N} h_i^2(z(t))\;x^{\mathrm{T}}(t)\;\left(G_{Nii}^{\mathrm{T}}P+PG_{Nii}\right)x(t)\right.\\
&\left.+\sum_{i<j}^{N} 2h_i(z(t))\,h_j(z(t))\,x^{\mathrm{T}}(t)\,Q_Nx(t)\right]\\
\leq & h_1(z(t))\left[\sum_{i=1}^{N} h_i^2(z(t))\,x^{\mathrm{T}}(t)\left(G_{1ii}^{\mathrm{T}}P+PG_{1ii}\right)x(t)\right.\\
&\left.+(\bar{s}-1)\sum_{i=1}^{N} h_i^2(z(t))\,x^{\mathrm{T}}(t)\,Q_1x(t)\right]+\ldots\\
&+h_N(z(t))\left[\sum_{i=1}^{N} h_i^2(z(t))\,x^{\mathrm{T}}(t)\left(G_{Nii}^{\mathrm{T}}P+PG_{Nii}\right)x(t)\right.\\
&\left.+(\bar{s}-1)\sum_{i=1}^{N} h_i^2(z(t))\,x^{\mathrm{T}}(t)\,Q_Nx(t)\right]\\
= & h_1(z(t))\left[\sum_{i=1}^{N} h_i^2(z(t))\,x^{\mathrm{T}}(t)\left(G_{1ii}^{\mathrm{T}}P+PG_{1ii}+(\bar{s}-1)\,Q_1\right)x(t)\right]+\ldots\\
&+h_N(z(t))\left[\sum_{i=1}^{N} h_i^2(z(t))\,x^{\mathrm{T}}(t)\left(G_{Nii}^{\mathrm{T}}P+PG_{Nii}+(\bar{s}-1)\,Q_N\right)x(t)\right]\\
= & \sum_{i=1}^{N}\sum_{k=1}^{N} h_i^2(z(t))\,h_k(z(t))\,x^{\mathrm{T}}(t)\left(G_{kii}^{\mathrm{T}}P+PG_{kii}+(\bar{s}-1)\,Q_k\right)x(t).
\end{aligned} \tag{A.2}$$

It is obvious that $\dot{V}(x(t))<0$, $\forall x(t)\neq 0$, if, for the specified static output feedback gain matrices F_j $(j=1,2,\ldots,N)$ in (3), there exists a symmetric positive definite matrix P and the symmetric positive semi-definite matrices Q_k $(k=1,2,\ldots,N)$ such that

$$G_{kii}^{\mathrm{T}}P+PG_{kii}+(\bar{s}-1)\,Q_k<0, \tag{A.3a}$$

and

$$\left(\frac{G_{kij}+G_{kji}}{2}\right)^{\mathrm{T}}P+P\left(\frac{G_{kij}+G_{kji}}{2}\right)-Q_k\leq 0, \tag{A.3b}$$

where $G_{ijk}=A_i-B_iF_jC_k$, $i<j$, $\bar{s}>1$, and $i,j,k=1,2,\ldots,N$.

So, from the result mentioned above, we can derive the closed-loop TS-fuzzy-model-based dynamic system (4) is stable if, for the specified static output feedback gains matrices F_j ($j = 1, 2, \cdots, N$) in (3), a symmetric positive definite matrix P and the symmetric positive semi-definite matrices Q_k ($k = 1, 2, \ldots, N$) exist such that the LMIs in (6) are simultaneously satisfied. Thus, the proof is completed.