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A HYBRID FUZZY AND NEURAL APPROACH WITH VIRTUAL EXPERTS AND PARTIAL CONSENSUS FOR DRAM PRICE FORECASTING

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Abstract. *To further enhance the accuracy and precision of DRAM price forecasting, a hybrid fuzzy and neural approach with virtual experts and partial consensus is proposed. In the proposed methodology, some virtual experts form a committee. These virtual experts construct their own fuzzy linear regression (FLR) equations to forecast the price of a DRAM product from various viewpoints. Each FLR equation can be transformed into two equivalent NP problems to be solved. Subsequently, partial-consensus fuzzy intersection is applied to aggregate fuzzy price forecasts into a polygon-shaped fuzzy number, in order to improve the precision. After that, a back propagation network is constructed to defuzzify the polygon-shaped fuzzy number and to generate a representative/crisp value, so as to enhance the accuracy. A practical case is used to evaluate the effectiveness of the proposed methodology. According to the experimental results, the proposed methodology improved both the precision and accuracy of DRAM price forecasting by 75% and 65%, respectively.*
Keywords: DRAM, Price, Forecasting, Fuzzy, Neural, Virtual expert, Partial consensus

1. Introduction.

Dynamic random access memory (DRAM) is a type of volatile memory products. It uses capacitors to store information, and can be encapsulated into memory working modules. DRAM has been widely used in computer related applications, communication systems and other electronic devices. DRAM price is therefore crucial for the electronics industry. Of course, DRAM price is determined by supply and demand sides in this industry and fluctuates over time, but the long-term trend does exist and can be roughly quantified [1].

There are two viewpoints when it comes to forecasting the price of a DRAM product [2]. The first viewpoint, the input-output relationship viewpoint, is to determine those factors (e.g., demand, supply, economic conditions and raw material costs) that are influential in the price, and then apply different approaches (e.g., multiple linear regression (MLR) and artificial neural network (ANN)) to modeling the relationship between the price and these factors in order to forecast the future price. The second viewpoint, the time-series viewpoint, is to treat the fluctuation in the price as a type of time series. Theoretically, there are many approaches, e.g., moving average (MA), weighted moving average (WMA), exponential smoothing (ES), MLR and ANN that can be applied to forecast the price. Generally speaking, an ANN is suitable for modeling a short-term nonlinear pattern of the price, while traditional approaches such as MA, WMA and ES have good performances when the trend in the price is stable. Sepselter and Sze [3] proposed the π rule, which describes the trend in the average price of packaged DRAM chips as a logarithmic function of time. The π rule states that in the face of rapid price declines, the peak volume of chips shipped corresponds to a per-chip price of π US dollars. The price continues to decline and eventually settles at about $\pi/2$ dollars per chip. This nicely described the trends in the prices of DRAMs up to 64K. Subsequently, Tarui and Tarui [1] modified the π rule and proposed the Bi-rule, taking into account that the bit cost will be reduced by half with each succeeding DRAM generation. The trends in the prices of DRAMs up to 16M did reflect this consideration.

Recently, some innovative attempts have been made. Chen and Wang [4] applied fuzzy interpolation to forecasting the price of a DRAM product. The accumulation in the fuzziness is a problem in their research. Recently, Ong et al. [5] combined genetic algorithm (GA) and auto-regressive integrated moving average (ARIMA) for the same purpose, which greatly improved the forecasting accuracy. ARIMA was first introduced by Box and Jenkins [6] to analyze stationary univariate time series data, and has been used in various fields so far. Tseng et al. [7] proposed the fuzzy ARIMA (FARIMA) approach to forecast the exchange rate. However, the precision of forecasting the exchange rate is an important issue, but has rarely been investigated in the past. That is, an interval that contains the actual value is to be generated, especially when the minimum and maximum possible profits of the factory needs to be estimated. Even with these attempts, it is still very difficult to accurately predict DRAM price. Cupertino [8] stressed the fact that it is only possible to quantitatively anticipate the turning points in this industry, by measuring independent economic factors affecting the purchase of DRAM products. In addition, it is relatively easy to estimate the demand/market for DRAM products than price.

Many studies have shown that fusion of soft computing technologies such as ANN, fuzzy logic and GA may significantly improve the performance of data analysis [9-11]. The reasons are explained below. First of all, these technologies are for the most part complementary and synergistic. ANNs are usually used for learning and curve fitting. Fuzzy logic is applied to deal with imprecision and uncertainty, while GA can be used for searching and optimization. Chen [2] proposed a fuzzy and neural approach in which some fuzzy linear regression (FLR) equations were applied to forecast the price of a DRAM product. Their reasons are described below. There is a considerable degree of volatility in the price of a DRAM product that is not easily to anticipate in advance. In addition, the price of a DRAM product is subject to two major stochastic factors – the supply and the demand in the industry. Fitting the demand or the supply within a future period with a distribution function is not easy, implying that a stochastic approach might not be applicable. For this reason, some studies [12-14] proposed a fuzzy approach to forecast the demand or the supply in an industry.

In Chen's approach, these FLR equations were fitted with some nonlinear programming (NP) models rather than the traditional linear programming (LP) models. After forecasting the price, fuzzy intersection (FI) and back propagation network (BPN) were applied to aggregate fuzzy price forecasts and to defuzzify the aggregation result, respectively. According to the experimental results, Chen's fuzzy and neural approach was superior to MA, ES, BPN, ARIMA and FARIMA, in particular the forecasting precision. However, Chen's fuzzy and neural approach has the following problems:

(1) Domain experts must give their opinions on Chen's fuzzy and neural approach, so that fuzzy forecasts meet the requirements of such experts. However, in case that there are no domain experts to consult, there is always the alternative of forming a committee of "virtual experts" for providing opinion sets for the proposed methodology.
(2) Sometimes, an expert's opinions may be too stringent to identify the corresponding FLR equation. In this case, there are no feasible solutions of the two NP problems, and the expert will be asked to modify his/her opinions.

(3) For the training data, fuzzy forecasts contained the actual values. However, it is not necessarily the case for the testing data.

For enhancing the performance of price forecasting, a hybrid fuzzy and neural approach with virtual experts and partial consensus is proposed in this study. The unique features of the proposed methodology include:

(1) Instead of gathering a small number of domain experts from different places, the proposed methodology uses virtual experts. It is possible to use any number of virtual experts, which leads to diverse views and is conducive to finding the global optimal solution.
(2) The concept of partial consensus is proposed to cope with the lack of consensus. In this way, the actual prices are more likely to be contained in the fuzzy forecasts.

In the proposed approach, some virtual experts form a committee and construct their own FLR equations to forecast the price of a DRAM product from various viewpoints. Each FLR equation is then converted into two equivalent NP problems to be solved. The future price forecasted with an FLR equation is a fuzzy value. The fuzzy price forecasts by different virtual experts might not be equal and therefore need to be aggregated, in order to optimize some criteria. Besides, these expert opinions can also be considered as unequally important. Therefore, a two-step aggregation mechanism is applied. At the first step, partial-consensus FI is applied to aggregate the fuzzy price forecasts into a polygon-shaped fuzzy number, in order to improve the precision. FI aims to identify the consensus among expert opinions. It is much easier for the past data than for the future data. For this reason, we define and derive the partial consensus FI among expert opinions. After taking into account the special shape of the polygon-shaped fuzzy number, a BPN is constructed to defuzzify the polygon-shaped fuzzy number and to generate a representative/crisp value, so as to enhance the accuracy.

The rest of this paper is organized as follows. Section 2 introduces the proposed methodology which is composed of four steps. A practical example is used to demonstrate the application of the proposed methodology. The performance of the proposed methodology is evaluated and compared with those of some existing approaches in Section 3. Based on the analysis results, some points are made. Finally, the concluding remarks and a view to the future are given in Section 4.

2. **Methodology.**

The proposed methodology is made of several steps (see Figure 1) which will be described in the following sections:

(1) Form a committee of virtual experts, and then generate the opinions of these virtual experts.
(2) Construct an FLR equation for each virtual expert to generate fuzzy price forecast.
(3) Apply partial-consensus FI to aggregate fuzzy DRAM price forecasts into a polygon-shaped fuzzy number.
(4) Defuzzify the aggregation result with a BPN.

2.1. **Step 1: Generate the opinions of virtual experts.**

Domain expert were asked to give their views on the following terms in Chen's fuzzy and neural approach, so that fuzzy forecasts meet the requirements of these experts:

(1) The sensitivity to the uncertainty (o_l): A large value of o_l means that the expert is very sensitive to the uncertainty in fuzzy forecasts which is usually expressed as the average range of the fuzzy forecasts.
(2) The desired range of every fuzzy foreign price forecast (d_l): The desired ranges by various experts usually depends on the purposes of application and might not be equal.

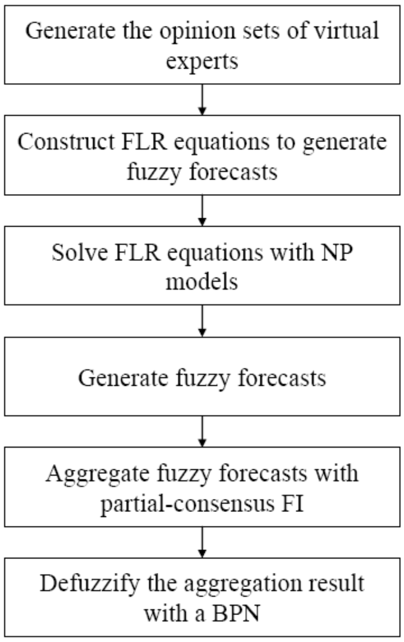

Figure 1. The steps of the proposed methodology

(3) The required satisfaction level (s_l): The required satisfaction levels by various experts are different in nature. A large value of s_l means that the core (i.e., values with memberships equal to 1) of a fuzzy forecast is very representative.
(4) The relative unimportance of the outliers of the sample data (m_l): A large value of m_l means that outliers are not important in fitting the FLR equation.

A set of real numbers, which contains four terms $OS_l = \{o_l, d_l, s_l, m_l\}$, is called an opinion set; $l = 1 \sim L$ (the number of experts). However, in case that there are no domain experts to consult, one alternative is to form a committee of "virtual experts" to provide opinion sets for implementing the fuzzy and neural approach. However, an opinion set cannot be randomly generated. It has to meet the following constraints:

(1) An opinion set (for example, with very small d_l or very large s_l) may result in no feasible solutions or increase the difficulty of solving the NP problems.
(2) Opinions that are close to each other may lead to the same or similar fuzzy forecasts. It is helpless to improve the precision of fuzzy forecasts [2].
(3) Increasing the difference between two opinion sets does not guarantee that the fuzzy forecasts generated by these two opinion sets will be more different.
(4) An opinion set is preferred if it generates a fuzzy forecast that is significantly different from the others, which is reflected in the contraction of the width of the aggregated result (see Figure 2).
(5) An opinion set is also preferred if it generates a fuzzy forecast that has more intersection points with the others, which is conducive to find out the actual value because of the increase in degrees of freedom (see Figure 3).

For these reasons, the following systematic procedure is established for generating the opinion sets of virtual experts:

Step 1. Determine the range of each parameter.
Step 2. Let $l = 0$.
Step 3. Generate a new opinion set randomly. Use it to generate fuzzy forecasts, and then re-aggregate the results.

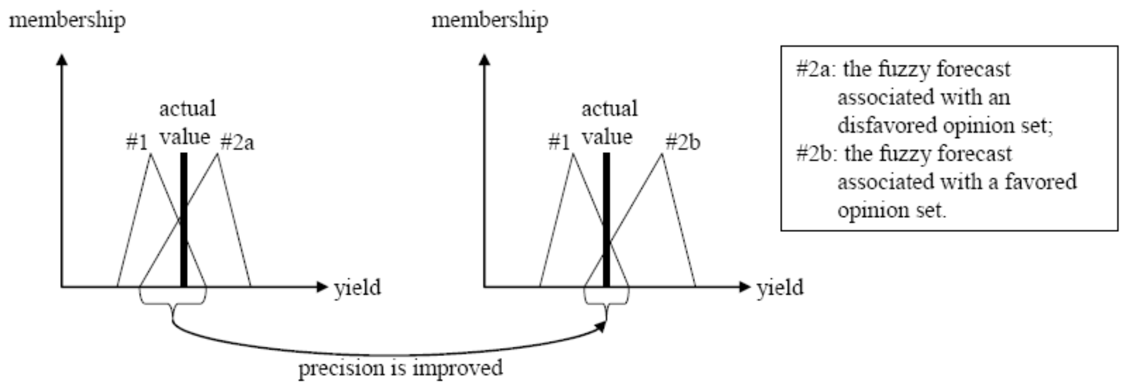

Figure 2. Comparison of two fuzzy forecasts associated with different opinion sets

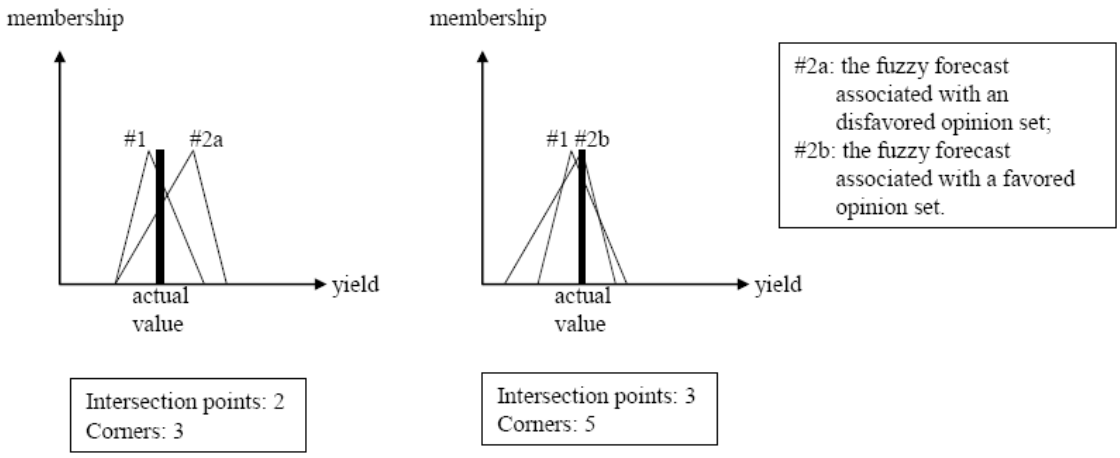

Figure 3. Comparison of two fuzzy forecasts associated with different opinion sets

Step 4. If the aggregation result has a narrower range or more corners than ever before, go to the next step; otherwise, return to Step 3.

Step 5. $l = l + 1$. If $l = L$ (the number of virtual experts), go to the next step; otherwise, return to Step 3.

Step 6. If the range of the aggregation result $\leq$ a threshold and the corners of the aggregation result $\geq$ another threshold, stop; otherwise, return to Step 3.

2.2. **Step 2: Constructing FLR equations.** In the proposed methodology, multiple virtual experts construct their own FLR equations to forecast the price of a DRAM product.

$$\tilde{P}_t = \tilde{w}_0(+)\sum_{k=1}^{K}\tilde{w}_k(\times)\tilde{P}_{n-k} \tag{1}$$

where $\tilde{P}_t$ is the fuzzy price forecast of month t; $\tilde{w}_k$ are constants or coefficients; $k = 0 \sim K$. $(+)$ and $(\times)$ denote fuzzy addition and multiplication, respectively. Assuming all variables are given in triangular fuzzy numbers (TFNs):

$$\tilde{P}_t = (P_{t1}, P_{t2}, P_{t3}) \tag{2}$$

$$\tilde{w}_k = (w_{k1}, w_{k2}, w_{k3}) \tag{3}$$

In the proposed methodology, the FLR equation is fitted by solving the following two NP problems [2,12]:
(Model NPI)

$$\text{Min} \quad Z_1 = \sum_{t=K+1}^{T} \left(w_{03} + \sum_{k=1}^{K} w_{k3} P_{t-k,3} - w_{01} - \sum_{k=1}^{K} w_{k1} P_{t-k,1} \right)^{o_l} \tag{4}$$

subject to

$$A_t \geq w_{01} + \sum_{k=1}^{K} w_{k1} P_{t-k,1} + s_l \left(w_{02} + \sum_{k=1}^{K} w_{k2} P_{t-k,2} - w_{01} - \sum_{k=1}^{K} w_{k1} P_{t-k,1} \right) \tag{5}$$

$$A_t \leq w_{03} + \sum_{k=1}^{K} w_{k1} P_{t-k,3} + s_l \left(w_{02} + \sum_{k=1}^{K} w_{k2} P_{t-k,2} - w_{03} - \sum_{k=1}^{K} w_{k3} P_{t-k,3} \right) \tag{6}$$

$$P_{t1} \geq 0 \tag{7}$$

$$t = K + 1 \sim T \tag{8}$$

$$w_{k1} \leq w_{k2} \leq w_{k3}; \quad k = 0 \sim K \tag{9}$$

(Model NPII)

$$\text{Max} \quad Z_2 = \bar{s} \tag{10}$$

subject to

$$Z_1 \leq (T - K) d_l^{o_l} \tag{11}$$

$$A_t \geq w_{01} + \sum_{k=1}^{K} w_{k1} P_{t-k,1} + s_t \left(w_{02} + \sum_{k=1}^{K} w_{k2} P_{t-k,2} - w_{01} - \sum_{k=1}^{K} w_{k1} P_{t-k,1} \right) \tag{12}$$

$$A_t \leq w_{03} + \sum_{k=1}^{K} w_{k1} P_{t-k,3} + s_t \left(w_{02} + \sum_{k=1}^{K} w_{k2} P_{t-k,2} - w_{03} - \sum_{k=1}^{K} w_{k3} P_{t-k,3} \right) \tag{13}$$

$$\bar{s} = \sqrt[m_l]{\frac{\sum_{t=K+1}^{T} s_t^{m_l}}{K - T}} \tag{14}$$

$$P_{t1} \geq 0 \tag{15}$$

$$0 \leq s_t \leq 1 \tag{16}$$

$$t = K + 1 \sim T \tag{17}$$

$$w_{k1} \leq w_{k2} \leq w_{k3}; \quad k = 0 \sim K \tag{18}$$

where A_t is the actual price of month t; $0 \leq s \leq 1$; T is the number of months; and K indicates the number of moving periods. The objective function Z_1 is to minimize the generalized sum of the ranges/supports of the fuzzy price forecasts, while the objective function Z_2 is to maximize the generalized average satisfaction level instead. Constraints (5), (6), (12) and (13) demand that the membership of the actual price in the fuzzy price forecast must be greater than or equal to the satisfaction level. Constraints (7) and (15) demand the price forecast to be non-negative. Constraints (9) and (18) show the sequence of the three corners of a TFN. o_l reflects the sensitivity of expert l to the uncertainty of the fuzzy price forecast; o_l ranges from 0 (not sensitive) to ∞ (extremely sensitive); s_l indicates the satisfaction level required by expert l; $0 \leq s_l \leq 1$; d_l is the desired range of every price forecast by expert l; $0 \leq d_l$; m_l represents the relative importance of the outliers in fitting the FLR equation to expert l; $m_l \in Z^+$. When $m_l = 1$, the relative importance of the outliers is the highest and is equal to that of the non-outliers; $l = 1 \sim L$ (the number of experts). o_l should be set within [0 1] if the variation in the price is less than 1. Otherwise, it should be greater than 1.

If there are L experts, then after incorporating experts' opinions into the two NP models there will be at most $2L$ NP problems to be solved. After solving each NP problem, the optimal solution is used to construct a corresponding FLR equation. Eventually, there will be at most $2L$ FLR equations, each of which generates a fuzzy price forecast. A mechanism is therefore required to aggregate these fuzzy price forecasts. There are many possible mechanisms applicable for this purpose, e.g., the fuzzy (weighted) average [15,16], a fuzzy back propagation network [17], the fuzzy intersection/AND operator [16] and the fuzzy union/OR operator [16]. Instead, in the proposed methodology, a two-step aggregation mechanism is applied for DRAM price forecasting.

2.3. **Step 3: Applying partial-consensus FI to aggregate fuzzy price forecasts into a polygon-shaped fuzzy number.** The aggregation mechanism is composed of two steps. At the first step, partial-consensus FI is applied to aggregate the fuzzy price forecasts into a polygon-shaped fuzzy number to improve the precision of price forecasting. The reasons are described below. For the training data, fuzzy forecasts obtained by the traditional FLR approaches (e.g., [15,16]) and the proposed methodology all contain the actual values. However, this situation might not be valid for the testing data. Because of the contraction of the average width, it becomes increasingly difficult for a fuzzy forecast to contain the actual value. Can fuzzy forecasts generated by these FLR approaches contain the actual values for the testing data? The answer is undoubtedly "yes". For example, let s_l be a value very close to 1, the spreads of fuzzy forecasts will be very large and it is very likely that all actual values will be contained in the fuzzy forecasts. Similarly, by setting d_l to a large value that may be greater than six times the standard deviation of the testing data, we can get the same result. However, we do not need a fuzzy forecast with such a wide range. For this purpose, the concept of partial consensus is defined as follows. The intersection of the opinions of some experts reflects the consensus of all of these experts. However, the future conditions might be completely different from when these experts were asked to give their opinions. Therefore, the opinions of some experts may be invalid for the testing data. To solve this problem, we try to rule out such opinions, and only the consensus of the remaining experts will be sought.

Definition 2.1 (Partial Consensus)**.** *The h/L partial consensus of fuzzy price forecasts $\tilde{P}_i(1), \ldots, \tilde{P}_i(L)$ is indicated with $I^{h/L}(\tilde{P}_i(1), \tilde{P}_i(2), \ldots, \tilde{P}_i(L))$ such that*

$$\mu_{I^{h/L}\left(\tilde{P}_i(1),\ldots,\,\tilde{P}_i(L)\right)}(x) = \max_{\text{all } g}\left(\min\left(\mu_{\tilde{P}_i(g(1))}(x), \ldots, \mu_{\tilde{P}_i(g(h))}(x)\right)\right) \tag{19}$$

where $g() \in Z^+$; $1 \le g() \le L$; $g(i) \cap g(j) = \varnothing$; $h \ge 2$.

For example, the 2/4 partial consensus of $\tilde{P}_i(1), \ldots, \tilde{P}_i(4)$ can be obtained as

$$\begin{aligned}\mu_{I^{2/4}(\tilde{P}_i(1),\ldots,\tilde{P}_i(4))}(x) = {} & \max(\min(\mu_{\tilde{P}_i(1)}(x), \mu_{\tilde{P}_i(2)}(x)), \min(\mu_{\tilde{P}_i(1)}(x), \mu_{\tilde{P}_i(3)}(x)), \\ & \min(\mu_{\tilde{P}_i(1)}(x), \mu_{\tilde{P}_i(4)}(x)), \min(\mu_{\tilde{P}_i(2)}(x), \mu_{\tilde{P}_i(3)}(x)), \\ & \min(\mu_{\tilde{P}_i(2)}(x), \mu_{\tilde{P}_i(4)}(x)), \min(\mu_{\tilde{P}_i(3)}(x), \mu_{\tilde{P}_i(4)}(x)))\end{aligned} \tag{20}$$

The 3/4 partial consensus of them can be obtained similarly:

$$\begin{aligned}\mu_{I^{3/4}(\tilde{P}_i(1),\ldots,\tilde{P}_i(4))}(x) = {} & \max(\min(\mu_{\tilde{P}_i(1)}(x), \mu_{\tilde{P}_i(2)}(x),\ \mu_{\tilde{P}_i(3)}(x)), \\ & \min(\mu_{\tilde{P}_i(1)}(x), \mu_{\tilde{P}_i(2)}(x), \mu_{\tilde{P}_i(4)}(x)), \\ & \min(\mu_{\tilde{P}_i(2)}(x), \mu_{\tilde{P}_i(3)}(x), \mu_{\tilde{P}_i(4)}(x)))\end{aligned} \tag{21}$$

The following properties hold for the partial consensus:

(1) The range of $I^{h_1/L}(\tilde{P}_i(1), \ldots, \tilde{P}_i(L))$ is wider than that of $I^{h_2/L}(\tilde{P}_i(1), \ldots, \tilde{P}_i(L))$ if $h_1 < h_2$.
(2) The range of any partial consensus is obviously wider than that of the overall consensus.
(3) For the training data, every partial consensus contains the actual values.
(4) For the testing data, the probability that actual values are contained is higher in a partial consensus than in the overall consensus.

The result of partial-consensus FI is a polygon-shaped fuzzy number (see Figure 4). Compared with the original TFNs, the partial-consensus FI result has a narrower range while still containing the actual value. Therefore, the precision of price forecasting will be improved after applying partial-consensus FI.

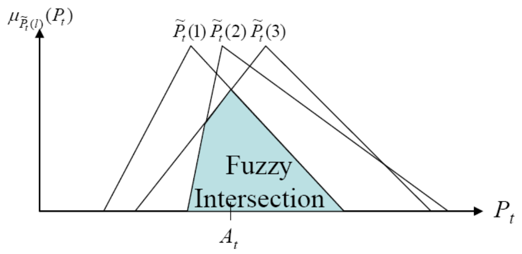

FIGURE 4. The fuzzy intersection of fuzzy price forecasts ($\tilde{P}_t(l)$: fuzzy price forecast by expert l)

The output of this step is a polygon-shaped fuzzy number that specifies the narrowest range of the price forecast. However, in practical applications, a crisp price forecast is usually required. Therefore, a crisp price forecast has to be generated from the polygon-shaped fuzzy number. For this purpose, many defuzzification methods are applicable [18]. After obtaining the defuzzified value, it is compared with the actual price to evaluate the accuracy. However, among the existing defuzzification methods, not one method is superior to all other methods in every case. Besides, the most suitable defuzzification method for a fuzzy variable is often chosen from the existing methods, which cannot guarantee the optimality of the chosen method. In addition, the shape of the polygon-shaped fuzzy number is special. These aforementioned phenomena provided the motive for proposing a tailored defuzzification method. To this purpose, a BPN is applied in this study because theoretically a well-trained BPN (without being stuck in a local minima) with a good selected topology can successfully map any complex distribution.

2.4. **Step 4: Constructing a BPN to defuzzify the polygon-shaped fuzzy number.** The configuration of the BPN used in this study is established as follows:

(1) Inputs: $2v$ parameters corresponding to the v corners of the polygon-shaped fuzzy number and the membership function values of these corners. All input parameters have to be normalized before they are fed into the network.
(2) Single hidden layer: Generally speaking, one or two hidden layers are more beneficial for the convergence property of the BPN.
(3) Number of neurons in the hidden layer is chosen from $1 \sim 4v$ according to a preliminary analysis, considering both effectiveness (forecasting accuracy) and efficiency (execution time).
(4) Output: The crisp price forecast.
(5) Network learning rule: Delta rule.

(6) Transformation function: Sigmoid function,

$$f(x) = \frac{1}{1+e^{-x}} \tag{22}$$

(7) Learning rate (η): 0.01 $\sim$ 1.0.
(8) Batch learning.
(9) Number of epochs per replication: 60000.
(10) Number of initial conditions/replications: 1000. Among the results, the best one is chosen for the subsequent analyses.

The procedure for determining the parameter values refers to Chen [2].

3. **Practical Examples and Analyses.** To demonstrate the application of the proposed methodology, the practical data of a DRAM product (DDR2 1G) from www.insye.com were used. These data contained the price data of 256 days, including the highest, average and lowest values (see Figure 5). The average value is focused because it is more important than the other two values.

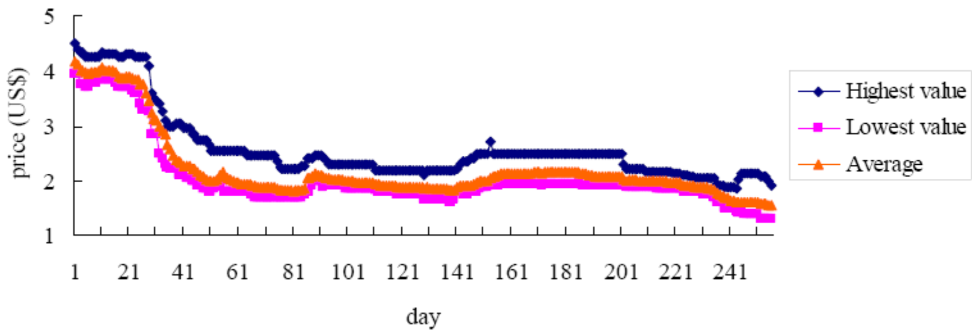

FIGURE 5. The collected price data

To compare with some existing approaches, MA, ES, BPN, and ARIMA, and Chen's fuzzy and neural approach [2], were also applied to the collected data. In MA, various numbers of moving periods (from 7 to 3 Step -1) were tried. Among them, the best one was chosen for the subsequent analyses. The number of inputs in BPN was determined in a similar way. In ES, the value of the smoothing constant changed from 0 to 1 with an interval of 0.1, and then the value contributing to the best performance was adopted. In ARIMA, there were three stages: model identification, model estimation and model checking. The minimum information criterion (MINIC) method [20] was used to identify the order in the ARIMA process. In addition, the augmented dickey fuller (ADF) unit root tests [21] were used to test the stationarity and seasonal stationarity in the price data.

In the BPN approach, the inputs were the normalized prices of the previous K periods, where K was equal to the optimal number of moving periods determined in the preliminary MA analysis for a fair comparison. Prices were normalized using the partial normalization approach [22-30]:

$$N(x) = N_L + \frac{x - x_{\min}}{x_{\max} - x_{\min}} \cdot (N_U - N_L) \tag{23}$$

where x indicates the original value; $N(x)$ is the normalized value of x; N_L and N_U indicate the lower and upper bounds of the range of the normalized value, respectively. $x_{\min}$ and $x_{\max}$ are the minimum and maximum of x, respectively. In this way, it is possible to deal with the condition in which the value of a new x has never happened before and falls outside the range containing all historical values. The output from the BPN was the normalized price forecast of the current period, which was compared with the target – the normalized actual price of the current period. There was one hidden layer with nodes that were twice as much as that of inputs. The number of epochs was set to be 60000 epochs. In addition, the initial values of all parameters in the BPN were randomly generated 1000 times. Among them, the best setting was kept for the subsequent analyses. Forecasts generated by the BPN approach were converted back to the un-normalized values as follows:

$$x = \frac{f(x) - N_L}{N_U - N_L} \cdot (x_{\max} - x_{\min}) + x_{\min} \tag{24}$$

A 4-fold evaluation process was applied to cross validate the data. Namely, if there are Rrecords then a 4-fold decomposition process makes each fold contain $R/4$ records. Each fold is used to be the testing data and the rest folds are merged as the training data. In this example, the testing data contained the prices during $256/4 = 64$ days.

In the proposed methodology, three virtual experts were used. To make a comparison, three real experts were asked to submit their opinions about the fuzzy price forecasts in Chen's approach, which were summarized in Table 1. As a result, there were six NP problems to be solved. From the optimization result of each NP problem, a corresponding FLR equation could be constructed. All the six FLR equations were applied to forecast the price of the product.

Table 1. Real experts' opinions in Chen's approach

l	o_l	d_l	s_l	m_l
1	1/4	0.75	0.7	3
2	1/5	0.7	0.7	5
3	1/2	0.65	0.4	4

Subsequently, the partial-consensus FI of the forecasting results was obtained, which was a polygon-shaped fuzzy number for each period. The data of the corners of all polygon-shaped fuzzy numbers, i.e., the aggregated fuzzy price forecasts of all periods, were used to train and/or test the BPN defuzzifier. The 4-fold cross-validation was adopted. Finally, the BPN was applied to defuzzify a polygon-shaped fuzzy number input to the network to generate the representative value, i.e., the crisp price forecast.

The proposed methodology was implemented on a PC with an Intel Dual CPU E2200 2.2 GHz and 2.0G RAM. Lingo 8.0 was applied to solve the NP problems with the following conditions:

(1) The initial opinions of the three virtual experts are randomly generated using the linear congruential generator (LCG).
(2) Successive linear programming (SLP) was used to compute new search directions. Namely, a linear approximation was used in search computation to speed iteration times.
(3) The computation method used for computing derivatives was the numerical method. Numerical derivatives were computed using finite differences.

The BPN defuzzifier was implemented with the Neural Network Toolbox of MATLAB 2006a with the following conditions:

(1) Number of epochs per replication: 60000.
(2) Number of initial conditions/replications: 1000.
(3) Stop training if MSE $< 10^{-5}$ or 60000 epochs have been run.

The forecasting accuracy was measured with RMSE, MAE and MAPE. At the same time, the forecasting precision of a non-biased crisp approach can be measures with 6σ as follows:

$$6\sigma = 6\sqrt{\frac{\sum_{t=K+1}^{T} (F_t - A_t)^2}{T - K - K - 1}} \tag{25}$$

Theoretically, the probability that such an interval contains the actual value is about 99.7%, under the assumption that residuals follow a normal distribution. Comparatively, the precision of a fuzzy approach can be measured with the average spread (or range) of fuzzy price forecasts if all of them contain the actual value. The performances achieved by applying the five approaches were recorded and compared in Table 2.

Table 2. The forecasting performances of different approaches

		MA	ES	BPN	ARIMA	Chen's approach	The proposed methodology
accuracy	MAE	0.040	0.020	0.052	0.016	0.015	0.014
	MAPE	1.72%	0.87%	2.47%	0.69%	0.64%	0.61%
	RMSE	0.068	0.040	0.077	0.030	0.028	0.027
precision	6σ or average range	0.313	0.238	0.468	0.182	0.105	0.078

MA was adopted as the comparison basis, and the percentage of improvement on the performance measure by applying another approach is enclosed in parentheses following the performance measure. The effectiveness of the proposed methodology with respect to various performance measures is illustrated in Figures 6-9. According to the experimental results,

(1) The accuracy of DRAM price forecasting, measured in terms of MAPE of the proposed approach, was significantly better than those of the other approaches by achieving a 65% reduction in RMSE over the comparison basis – MA. The advantages over ES and BPN were 30% and 75%, respectively. The performance of the proposed methodology with respect to RMSE was also better than those of the other approaches. The proposed methodology provided an innovative way of hybridizing linear and nonlinear forecasting approaches (FLR and BPN).

(2) Compared with Chen's fuzzy and neural approach, the virtual expert and partial consensus adopted in the proposed methodology seemed to be a better way and contributed to the significant advantage of the proposed methodology.

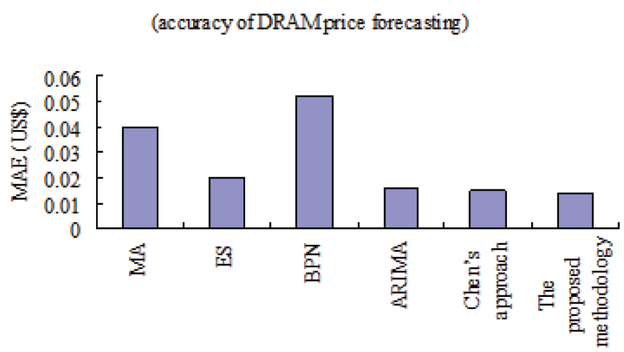

Figure 6. The accuracy of various approaches (MAE)

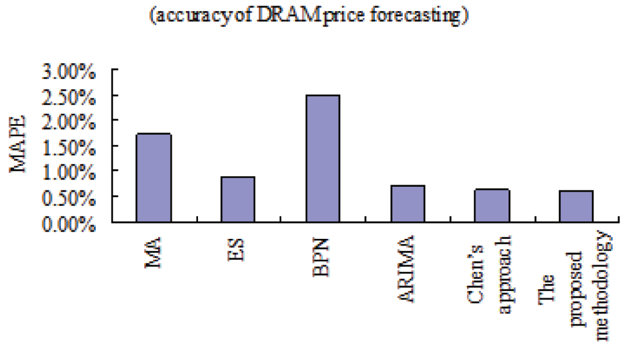

FIGURE 7. The accuracy of various approaches (MAPE)

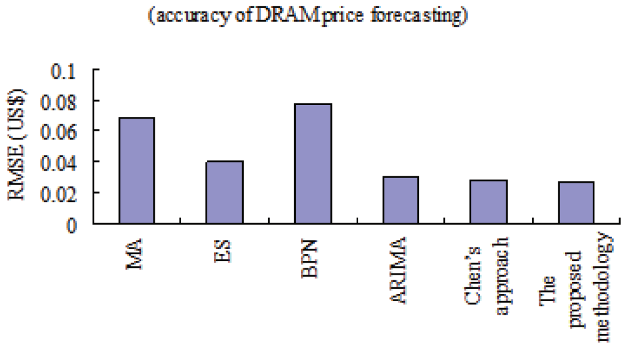

FIGURE 8. The accuracy of various approaches (RMSE)

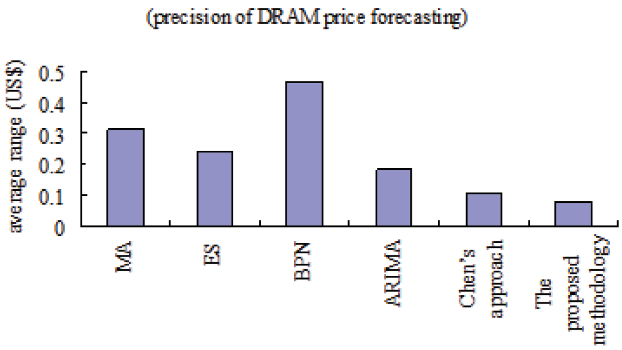

FIGURE 9. The precision of various approaches (average range)

(3) The two traditional linear forecasting approaches, MA and ES, achieved fair performances, measured in terms of MAE or MAPE, because of the stability in the collected data. Their performances in terms of RMSE were not satisfactory because there were sudden changes in the collected data. In the experiment, such a condition happened when the price was high, which enlarged the forecasting error. The forecasting error was further magnified in calculating the RMSE.
(4) The accuracy of the nonlinear approach BPN was poor because it could over-react when there was a linear trend in the data [31].
(5) As expected, the forecasting accuracy of ARIMA was very good and quite close to that of the proposed methodology.
(6) On the other hand, the precision of the proposed methodology was significantly better than those of the other approaches. The advantage over the baseline approach (MA) was up to 75%. In other words, with the proposed methodology, it is possible to

come up with a very small range for the future price, which was important to many planning purposes. The advantage of the proposed methodology came from the use of optimized virtual expert opinions.

(7) The effects of partial-consensus fuzzy intersection were illustrated in Figure 10. For the testing data, the probability that all actual values were contained in fuzzy forecasts was 88% with the overall consensus ($I^{3/3}$). With little expansion in the spreads of fuzzy forecasts, all actual values could be contained in fuzzy forecasts with the partial consensus ($I^{2/3}$).

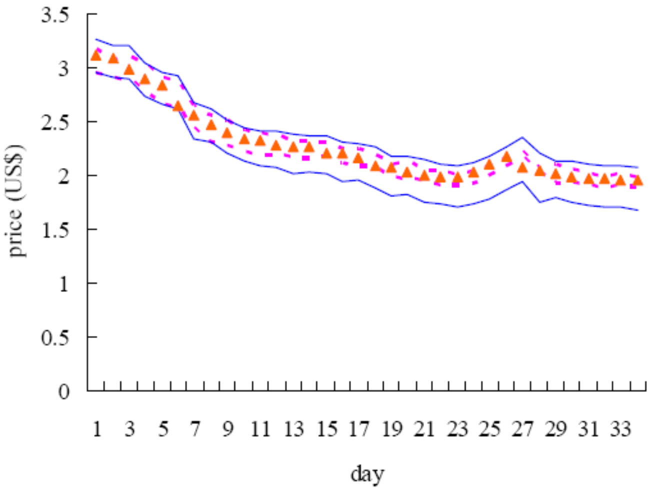

FIGURE 10. The effects of the partial-consensus FI

4. **Conclusion and Directions for Future Research.** For enhancing the performance of price forecasting, a hybrid fuzzy and neural approach with virtual experts and partial consensus is proposed in this study. In the proposed approach, some virtual experts form a committee and construct their own FLR equations to forecast the price of a DRAM product from various viewpoints. Subsequently, partial-consensus FI is then applied to aggregate the fuzzy price forecasts into a polygon-shaped fuzzy number, in order to improve the precision. After taking into account the special shape of the polygon-shaped fuzzy number, a BPN is constructed to defuzzify the polygon-shaped fuzzy number and to generate a representative/crisp value, so as to enhance the accuracy.

A practical case was used to demonstrate the application of the proposed methodology. According to experimental results, we found that

(1) The forecasting accuracy (measured in terms of MSE, MAPE and RMSE) of the proposed methodology was significantly better than that of some existing approaches.
(2) At the same time, the proposed methodology also outperformed these existing approaches in forecasting precision.
(3) The advantage of the proposed methodology was due to two sources. First, the use of virtual experts leads to diverse views and is conducive to find the global optimal solution. Second, to cope with the lack of consensus with partial-consensus fuzzy intersection, the actual prices are more likely to be contained in the fuzzy forecasts.

The advantages of the proposed methodology over the existing approaches include:

(1) Unlike the existing approaches, the proposed methodology forecasts the price of a DRAM product in a collaborative way, which conforms to the practical operations in a DRAM manufacturing factory.

(2) A sufficient number of domain experts are no longer needed for group forecasting. In this way, there can be significant cost savings and convenience.
(3) Compared with approaches that also use domain experts, the proposed methodology improves the precision further, and reduces the risk of financial planning.

Conversely, the possible deficiencies of the proposed methodology include:

(1) Long time is required for model learning and optimization in the proposed methodology. To tackle this problem, a dedicated software package can be developed in the future for implementing the proposed methodology.
(2) Even the virtual experts should be selected carefully. Lack of a better way to pick up virtual experts may harm the performance of forecasting.

More sophisticated price forecasting approaches can be developed in similar ways in future studies.

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