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# LMI-BASED FAULT DETECTION FUZZY OBSERVER DESIGN WITH MULTIPLE PERFORMANCE CONSTRAINTS FOR A CLASS OF NON-LINEAR SYSTEMS: COMPARATIVE STUDY

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Abstract. *In view of the conservatism of the conventional linear matrix inequality (LMI) based fault detection observer design for Takagi-Sugeno fuzzy nonlinear systems with more If-Then rules, an improved fuzzy observer design is presented. The identical transformation of matrix inequalities is employed to reduce the conservatism and the number of LMI constraints, which can accommodate to the models with more rules. The multiobjective optimization strategy is also applied to dealing with the multiobjective constraints on the disc poles index, the quasi $L_2$-norm indices of the residual's robustness to disturbances and sensitivity to faults. The resulting observer not only is less conservative, but also meets the multiple performance requirements of fault detection. Meanwhile, two other methods are introduced for comparative study. Moreover, to enhance the effect of fault detection in residual evaluation, a weighted BIC criterion-based algorithm is introduced to determine the finite-time window for online evaluation. Simulative examples demonstrate the effectiveness of the proposed method.*

**Keywords:** Takagi-Sugeno fuzzy model, Fault detection, Fuzzy observer, Multiobjective optimization, Matrix identical transformation

1. **Introduction.** Over the last decades, many researchers have paid attention to the problem of observer-based fault detection and diagnosis (FDD) for dynamic systems subjected to various possible faults [1-3]. Most of the early studies are focused on linear systems; see [4] and the references therein. In more recent years, observer-based fault detection (FD) for nonlinear systems has received a great deal of attention [5]. Whereas, due to the complexity in modeling nonlinearities, observer-based FDD for nonlinear system is still an open challenge. Recently, Takagi-Sugeno (T-S) fuzzy model as a typical description of nonlinear systems, its FDD problem has been widely studied; see for example [6-15] and the references therein. Through observing those results, it is obvious that many existing results are mainly focused on the LMI-based observer design to ensure only the stability of the residual systems, according to the conventional common quadratic Lyapunov function. Consequently, one problem is that only a few results touched upon the FD performance requirements [9, 13, 15]. Although many intelligent algorithms on the multiobjective programming and optimization have been developed [16, 17], it is negative to solve the FDD problem by such intelligent algorithms [17]. Another problem is

that the conventional common quadratic Lyapunov function and the multiple parameter-dependent Lyapunov functions or fuzzy Lyapunov function techniques often lead to too many LMIs constraints, which is too fragile to be solvable for systems with more If-Then rules [18-21]. Hence, it is also important to reduce the number of LMIs with acceptable conservatism of conditions [21]. Unfortunately, few results have been reported for this problem in the FD fuzzy observer design.

Therefore, in this paper, we investigate the fault detection problem for a class of discrete-time T-S fuzzy systems. Attention is focused on the FD observer with multiple performance constraints and its less conservative design method. Compared with existing works, the main contribustions are in two aspects: i) Multiobjective optimization idea [22] is employed to cope with the performance constraints on the regional eigenvalues, the quasi $\mathrm{L}_2$-norm indices of robustness against external disturbances and the sensitivity to fault, so that the residual system is asymptotically stable with the prescribed transient behavior as well as robust margin to neglected modeling dynamics, the expected robustness against disturbances and the desired sensitivity performance to faults; ii) Identical transformation of matrix inequalities and the technique of slack variables in [19] are applied to the developing process, which renders the results to be with smaller number of LMI constraints and less conservative. Thus, an improved LMI-based FD fuzzy observer design is developed for fuzzy systems with more rules. For comparative study, two typical strategies, the method based on the conventional common quadratic Lyapunov function and the method combining the fuzzy Lyapunov function with the slack of variables, are also presented respectively. All the results are formulated in the form of LMIs. In addition, residual evaluation function and threshold setting are discussed to enhance and achieve fault detection function.

The remainder of the article is organized as follows. Section 2 formulates the problem under consideration and presents some related preliminaries. The FD fuzzy observer design and comparative study are introduced in Section 3, along with simulative examples. Residual evaluation and detection threshold determination are discussed in Section 4. The numerical example showed the validity of the proposed approach in the same section. The paper is concluded in Section 5.

Throughout the paper, $\mathcal{Z}_+$ is the set of positive integers. The 2-norm of vector $x$ is defined as $\|x\| := \sqrt{x^{\mathrm{T}}(t)x(t)}$. The $\mathrm{L}_2$-norm is defined as $\|x\|_2 := \left[\sum_{t=0}^{\infty} x^{\mathrm{T}}(t)x(t)\right]^{1/2}$. The quasi $\mathrm{L}_2$-norm is defined as $\|x\|_{t_{\mathrm{d}}} := \left[\sum_{t=0}^{t_{\mathrm{d}}} x^{\mathrm{T}}(t)x(t)\right]^{1/2}$ or $\|x\|_{t_{\mathrm{d}}}^2 := \sum_{t=0}^{t_d} \left[x^{\mathrm{T}}(t)x(t)\right]$ over a finite-time interval $[0, t_{\mathrm{d}}]$.

## 2. **Problem Statement and Preliminaries.**

### 2.1. **Problem statement.**

We consider a class of discrete-time nonlinear systems with T-S fuzzy model as follows:

$$\begin{cases} x(t+1) = \displaystyle\sum_{i=1}^{N} h_i(\theta(t)) \left[A_i x(t) + B_i \nu(t) + D_i f(t)\right] \\ \qquad\quad = \left[\displaystyle\sum_{i=1}^{N} h_i(\theta) A_i\right] x(t) + \left[\displaystyle\sum_{i=1}^{N} h_i(\theta) B_i\right] \nu(t) + \left[\displaystyle\sum_{i=1}^{N} h_i(\theta) D_i\right] f(t) \\ \qquad\quad := A_g(h)x(t) + B_g(h)\nu(t) + D_g f(t) \\ y(t) = \displaystyle\sum_{i=1}^{N} h_i(\theta) C_i x(t) := C_g(h)x(t), \end{cases} \tag{1}$$

where $t$ is the time step, $x(t) \in \mathcal{R}^n$ and $y(t) \in \mathcal{R}^m$ are respectively the state and measurable output. $\nu(t) \in \mathcal{R}^p$ is the stationary exogenous disturbances with norm-bounded, and its L$_2$-norm $0 < \|\nu(t)\|_2 \le \phi_\nu < \infty$. The vector $f(t) \in \mathcal{R}^s$ stands for possible actuator or component fault signals which is norm-bounded. $A_i$, $B_i$, $C_i$ and $D_i$ are known constant matrices with appropriate dimensions. Here vector $\theta(t) = [\theta_1(t), \cdots, \theta_q(t)]$ is the premise variable on the fuzzy set $M_{ij}$ ($j = 1, \cdots, q$), and $N$ is the number of If-Then rules or local linear models. The fuzzy weighting function $h_i(\theta) = \omega_i(\theta)/\sum_{i=1}^N \omega_i(\theta)$ and $\omega_i(\theta) = \prod_{j=1}^q M_{ij}(\theta_j)$ with $M_{ij}(\theta_j)$ representing the grade of membership of $\theta_j$ in $M_{ij}$. It is obvious that $h_i(\theta)$ satisfies $h_i(\theta) \ge 0$ and $\sum_{i=1}^N h_i(\theta) = 1$. For simplicity, we assume that the system is stable as well as the pair of $\{C_i, A_i\}$ observable, and $\theta_j(t)$ is also known.

For the fault detection fuzzy observer design, a fuzzy observer associated with the same premise variable as the model (1) is constructed by

$$\begin{cases} \hat{x}(t+1) = \sum\limits_{i=1}^{N} h_i(\theta(t)) \left[A_i\hat{x}(t) + G_i(y(t) - \hat{y}(t))\right] \\ \qquad\qquad = A_g(h)\hat{x}(t) + G_g(h)(y(t) - \hat{y}(t)) \\ \quad \hat{y}(t) = \sum\limits_{i=1}^{N} h_i(\theta)C_i\hat{x}(t) = C_g(h)\hat{x}(t), \end{cases} \tag{2}$$

where $\hat{x}(t) \in \mathcal{R}^n$ and $\hat{y}(t) \in \mathcal{R}^m$ are the state and output vectors of the observer, $G_i$ is the gain matrix designed later, $G_g(h) = \sum_{i=1}^N h_i(\theta)G_i$. Then, the state error $e(t) = x(t) - \hat{x}(t)$ and the output residual $\varepsilon(t) = y(t) - \hat{y}(t)$ are obtained by

$$\begin{cases} e(t+1) = A_{gc}(h)e(t) + B_g(h)\nu(t) + D_g(h)f(t) \\ \quad\;\; \varepsilon(t) = C_g(h)e(t), \end{cases} \tag{3}$$

where $A_{gc}(h) = [A_g(h) - G_g(h)C_g(h)] = \sum_{i=1}^N \sum_{j=1}^N [h_i(\theta)h_j(\theta)(A_i - G_iC_j)]$.

To make sure the stability and the expected rapidity of fault detection in the presence of uncertainty, the desired regional poles assignment will be studied. Furthermore, the quasi L$_2$-norm of $\varepsilon(t)$ and $\nu(t)$ is applied to indicate the robustness of residual against disturbances in normal case, namely,

$$\sum_{i=0}^{t_{\mathrm{d}}} \varepsilon^{\mathrm{T}}(t)\varepsilon(t) < \eta^2 \sum_{i=0}^{t_{\mathrm{d}}} \nu^{\mathrm{T}}(t)\nu(t). \tag{4}$$

When the fault occurs, the follow inequality is introduced to describe the robustness against disturbances and sensitivity to faults:

$$\sum_{i=0}^{t_{\mathrm{d}}} \varepsilon^{\mathrm{T}}(t)\varepsilon(t) < \beta^2 \sum_{i=0}^{t_{\mathrm{d}}} \nu^{\mathrm{T}}(t)\nu(t) + \gamma^2 \sum_{i=0}^{t_{\mathrm{d}}} f^{\mathrm{T}}(t)f(t) \tag{5}$$

where $t_{\mathrm{d}} \in \mathcal{Z}_+$ and $0 < t_{\mathrm{d}} < \infty$, the interval $[0,\ t_{\mathrm{d}}]$ is a finite-time window. To this end, the problem of robust fault detection fuzzy observer design is addressed as follows.

**Proposition 2.1.** *For the fuzzy nonlinear system* (1)*, given a prespecified region* $\mathcal{S}(0, r)$ *and three scalars* $\gamma > 0$*,* $\beta > 0$*,* $\eta > 0$ *with* $\gamma > \beta$ *and* $\gamma > \eta$*, find a fuzzy observer* (2) *or gain matrix* $G_i$ ($i = 1, \cdots, N$) *such that the following three constraints hold.*

*C1) The residual system* (3) *is asymptotically stable and its poles are assigned within the disc region* $\mathcal{S}(0, r)$*, where* $0 < r < 1$ *is the radius of disc centred at* $(0, 0)$*, i.e., the eigenvalues of* $A_{gc}(h)$ *are within* $\mathcal{S}(0, r)$*.*

*C2) When the system* (3) *is stable and fault-free, the inequality constraint* (4) *holds with initial condition* $e(0) = 0$*.*

*C3) When the fault has occurred, the inequality constraint* (5) *holds for the system* (3) *with initial condition* $e(0) = 0$*.*

**Remark 2.1.** *It is noted that constraint C2) represents the worst-case criterion for the effect of disturbances on the residual* $\varepsilon(t)$*, which is a well-known description of robustness against the disturbances. Nevertheless, the criterion C3) does not stands for the worst-case criterion for the sensitivity to faults* [4]*. However, to a great extent it does not deteriorate the sensitivity to fault while ensures the worst-case attenuation to disturbances, and makes the observer design solvable. In addition, the conditions* $\gamma > \beta$ *and* $\gamma > \eta$ *for the given three indices are just to enhance the effect of fault detection.*

2.2. **Several lemmas.**

**Lemma 2.1.** *(Elimination Lemma,* [23]*) For the following matrix inequality*

$$G(s) + U(s)XV^T(s) + V(s)X^TU^T(s) > 0 \tag{6}$$

*with* $s$ *and* $X$ *two variables, where* $U$ *and* $V$ *do not depend on* $X$*, and* $X$ *is a free matrix. Then, the inequality* (6) *is equivalent to the two inequalities* $G(s) - \rho U(s)U^T(s) > 0$ *and* $G(s) - \rho V(s)V^T(s) > 0$ *with* $s$ *the first variable and* $\rho \in \mathcal{R}$*.*

**Lemma 2.2.** *(Inversion Matrix Lemma) Let* $A$*,* $B$*,* $C$ *and* $D$ *be matrices of appropriate dimension, then* $(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}$*.*

**Lemma 2.3.** *A fuzzy observer* (2) *has its eigenvalues in the region* $\mathcal{S}(0, r)$*, if there exists a matrix* $P = P^{\mathrm{T}} > 0$ *such that the inequality* $A_{gc}^{\mathrm{T}}(h)PA_{gc}(h) - r^2P < 0$ *holds.*

It is a deduced result to LMI region $\mathcal{S}(0, r)$ from Lemma 2 in [24].

## 3. Fault Detection Fuzzy Observer Design and Comparative Study.

3.1. **Three kinds of fault detection fuzzy observer design.** First of all, our proposed design method is presented in Theorem 3.1 by using the identical transformation of matrix inequality and technique introduced in [19]. Then, two other methods (the first in Theorem 3.2 is based on the conventional common quadratic Lyapunov function; the second in Theorem 3.3 is derived from the fuzzy Lyapunov function) with less conservatism strategies such as in [6, 7, 9, 13, 15, 20, 25] are introduced for comparative study.

**Theorem 3.1.** *For the residual system* (3)*, given the performance constraints* $\mathcal{S}(0, r)$*,* $\eta > 0$*,* $\gamma > 0$ *and* $\beta > 0$ *with* $\gamma > \beta$ *and* $\gamma > \eta$*, if there exist matrices* $P = P^{\mathrm{T}} > 0$*,* $L_j$ $(j = 1, 2, \cdots, 9)$*,* $K_k$ *and* $\delta_k < 0$ $(k = 1, 2, 3)$ *such that for* $i = 1, 2, \cdots, N$*, the LMIs:*

$$\begin{bmatrix} P & * \\ C_i & I \end{bmatrix} > 0 \tag{7}$$

$$\begin{bmatrix} r^2P - L_1C_i - C_i^{\mathrm{T}}L_1^{\mathrm{T}} & * & * \\ -PA_i - L_2C_i & P & * \\ L_1^{\mathrm{T}} - K_1^{\mathrm{T}}C_i & L_2^{\mathrm{T}} & K_1 + K_1^{\mathrm{T}} - (r^2 + \delta_1)I \end{bmatrix} > 0 \tag{8}$$

$$\begin{bmatrix} P - L_3C_i - C_i^{\mathrm{T}}L_3^{\mathrm{T}} & * & * & * \\ -L_4C_i & \eta^2I & * & * \\ -PA_i - L_5C_i & -PB_i & P & * \\ L_3^{\mathrm{T}} - K_2^{\mathrm{T}}C_i & L_4^{\mathrm{T}} & L_5^{\mathrm{T}} & K_2 + K_2^{\mathrm{T}} - (1 + \delta_2)I \end{bmatrix} > 0 \tag{9}$$

$$\begin{bmatrix} P - L_6C_i - C_i^{\mathrm{T}}L_6^{\mathrm{T}} & * & * & * & * \\ -L_7C_i & \gamma^2I & * & * & * \\ -L_8C_i & 0 & \beta^2I & * & * \\ -PA_i - L_9C_i & -PD_i & -PB_i & P & * \\ L_6^{\mathrm{T}} - K_3^{\mathrm{T}}C_i & L_7^{\mathrm{T}} & L_8^{\mathrm{T}} & L_9^{\mathrm{T}} & K_3 + K_3^{\mathrm{T}} - (1 + \delta_3)I \end{bmatrix} > 0 \tag{10}$$

*have solution* $\{P, K_k, L_j, \delta_k\}$, *then the fuzzy observer* (2) *with gain matrix*

$$G_i = A_i(P - C_i^{\mathrm{T}}C_i)^{-1}C_i^{\mathrm{T}}\left[C_i(P - C_i^{\mathrm{T}}C_i)^{-1}C_i^{\mathrm{T}}\right]^{-1} \tag{11}$$

*drives the eigenvalues of the system* (3) *within the region* $\mathcal{S}(0,r)$ *and the performance constraints C2) and C3) are satisfied.*

**Proof:** With a quadratic Lyapunov's function $V(e(t)) = e^{\mathrm{T}}(t)Pe(t) \geq 0$, we first derive the result on the constraint C1). According to Lemma 2.3 and Schur Complement [23], the constraint C1) is met if the inequality

$$\begin{bmatrix} r^2(P - C_g^{\mathrm{T}}(h)C_g(h)) & * \\ PA_g(h) - N_g(h)C_g(h) & P \end{bmatrix} > 0, \tag{12}$$

where $N_g(h) = PG_g(h)$. By using Lemma 2.1, the inequality (12) is equivalent to the following two inequalities

$$P - C_g^{\mathrm{T}}(h)C_g(h) > 0, \tag{13}$$

$$\begin{bmatrix} r^2(P - C_g^{\mathrm{T}}(h)C_g(h)) - \delta_1 C_g^{\mathrm{T}}(h)C_g(h) & * \\ PA_g(h) & P \end{bmatrix} > 0 \tag{14}$$

for scalar $\delta_1 < 0$. By applying Lemma 3 in [26] and Schur Complement, the inequality (14) is equivalent to

$$\begin{bmatrix} r^2P - L_1C_g(h) - C_g^{\mathrm{T}}(h)L_1^{\mathrm{T}} & * & * \\ -PA_g(h) - L_2C_g(h) & P & * \\ L_1^{\mathrm{T}} - K_1^{\mathrm{T}}C_g(h) & L_2^{\mathrm{T}} & K_1 + K_1^{\mathrm{T}} - (\delta_1 + r^2)I \end{bmatrix} > 0 \tag{15}$$

with finding the matrices $P = P^{\mathrm{T}}$, $L_1$, $L_2$, $K_1$ and scalar $\delta_1 < 0$. Consequently, expanding the two inequalities (13) and (15) leads to the results (7) and (8) accordingly. To design the gain matrix $G_i$, applying Schur Complement to (12) and together with Lemma 2.2 to (14) gives, respectively,

$$P - (PA_g(h) - N_g(h)C_g(h))[r^2(P - C_g^{\mathrm{T}}(h)C_g(h))]^{-1}(PA_g(h) - N_g(h)C_g(h))^{\mathrm{T}} > 0 \tag{16}$$

and

$$P - PA_g(h)\left\{\begin{array}{l} (r^2[P - C_g^{\mathrm{T}}(h)C_g(h)])^{-1} - (r^2[P - C_g^{\mathrm{T}}(h)C_g(h)])^{-1}C_g^{\mathrm{T}}(h) \\ \times\left[(-\delta_1 I)^{-1} + C_g(h)(r^2[P - C_g^{\mathrm{T}}(h)C_g(h)])^{-1}C_g^{\mathrm{T}}(h)\right]^{-1} \\ \times C_g(h)(r^2[P - C_g^{\mathrm{T}}(h)C_g(h)])^{-1} \end{array}\right\}A_g^{\mathrm{T}}(h)P > 0. \tag{17}$$

Now, after some simplifications by Lemma 2.2, we get that the left part of (16) is always greater than the left part of (17) if the matrix $N_g(h)$ is set by

$$N_g(h) = PA_g(h)(P - C_g^{\mathrm{T}}(h)C_g(h))^{-1}C_g^{\mathrm{T}}(h)\left[C_g(h)(P - C_g^{\mathrm{T}}(h)C_g(h))^{-1}C_g^{\mathrm{T}}(h)\right]^{-1},$$

which means the gain matrix (11) making the inequality (12) holds.

Next, we infer the result about the constraint C2) in the case of $f(t) = 0$ and $e(0) = 0$ for the system (3). According to the $\mathrm{H}_\infty$ optimization theory and the constraint C1), we consider the performance function $E(t) = \sum_{t=0}^{t_{\mathrm{d}}}[\varepsilon^{\mathrm{T}}(t)\varepsilon(t) - \eta^2\nu^{\mathrm{T}}(t)\nu(t)]$, and then get that the constraint C2) is met if the matrix

$$S = \begin{bmatrix} \begin{array}{l} C_g^{\mathrm{T}}(h)C_g(h) - P \\ +A_{gc}^{\mathrm{T}}(h)PA_{gc}(h) \end{array} & * \\ B_g^{\mathrm{T}}(h)PA_{gc}(h) & B_g^{\mathrm{T}}(h)PB_g(h) - \eta^2 I \end{bmatrix} < 0. \tag{18}$$

Transforming the inequality $S < 0$ by Schur Complement yields

$$\begin{bmatrix} P - C_g^{\mathrm{T}}(h)C_g(h) & * & * \\ 0 & \eta^2 I & * \\ N_g(h)C_g(h) - PA_g(h) & -PB_g(h) & P \end{bmatrix} > 0. \tag{19}$$

Then, with the similar manner for the constraint C1), we can obtain the inequality (9) and the gain matrices $G_i$ as (11).

Finally, as for the constraint C3) in the fault case $f(t) \neq 0$ of the system (3) with $e(0) = 0$, we consider the performance function $E_f(t) = \sum_{t=0}^{t_{\mathrm{d}}}[\varepsilon^{\mathrm{T}}(t)\varepsilon(t) - \beta^2\nu^{\mathrm{T}}(t)\nu(t) - \gamma^2 f^{\mathrm{T}}(t)f(t)]$. Then, similar to the strategy for the constraint C2), the inequality

$$\begin{bmatrix} P - C_g^{\mathrm{T}}(h)C_g(h) & * & * & * \\ 0 & \gamma^2 I & * & * \\ 0 & 0 & \beta^2 I & * \\ N_g(h)C_g(h) - PA_g(h) & -PD_g(h) & -PB_g(h) & P \end{bmatrix} > 0 \tag{20}$$

and the corresponding result (10) are obtained.

**Remark 3.1.** *In Theorem 3.1, the LMI conditions designing the gain matrices indirectly is potential less conservative. Moreover, the identical transformation were fully employed during the inferring procedure, which does not increase the number of LMIs. Thus, the conservatism is introduced as less as possible.*

Based on Theorem 3.1, we can achieve the object in Proposition 2.1 by minimizing the indices $\eta^2$ and $\beta^2$. As a result, the following corollary can be deduced.

**Corollary 3.1.** *For the residual system* (3)*, given the performance constraints $\mathcal{S}(0, r)$ and $\gamma > 0$, if there exist matrices $P = P^{\mathrm{T}} > 0$, $L_j$ $(j = 1, 2, \cdots, 9)$, $K_k$, $\delta_k < 0$ $(k = 1, 2, 3)$ and scalars $\beta > 0$ and $\eta > 0$ such that for $i = 1, 2, \cdots, N$, the optimization problem:*

$$\min_{L_j, P, \eta, \beta, \delta_k, K_k} (\eta^2 + \beta^2) \qquad \text{s.t.} \quad \mathit{LMIs}\ (7) \sim (10) \tag{21}$$

*has solution $\{P_0, K_{k,0}, L_{j,0}, \delta_{k,0}, \eta_0, \beta_0\}$, then the fuzzy observer* (2) *with gain matrices $G_i = A_i(P_0 - C_i^{\mathrm{T}}C_i)^{-1}C_i^{\mathrm{T}}\left[C_i(P_0 - C_i^{\mathrm{T}}C_i)^{-1}C_i^{\mathrm{T}}\right]^{-1}$ makes the poles of systems* (3) *within the region $\mathcal{S}(0, r)$ and the indices $\eta^2$ and $\beta^2$ of the constraints C2) and C3) minimized.*

**Theorem 3.2.** *For the residual system* (3)*, given the performance constraints $\mathcal{S}(0, r)$, $\eta > 0$, $\gamma > 0$ and $\beta > 0$ with $\gamma > \beta$ and $\gamma > \eta$, if there exist matrices $P = P^{\mathrm{T}} > 0$, $N_i$ such that for $i = 1, 2, \cdots, N$, the following LMIs:*

$$\begin{cases} U_{ii} > 0, & (i = 1, 2, \cdots, N); \\ U_{ij} + U_{ji} > 0, & (1 \le i < j \le N) \end{cases} \tag{22}$$

$$\begin{cases} Y_{ii} > 0, & (i = 1, 2, \cdots, N); \\ Y_{ij} + Y_{ji} > 0, & (1 \le i < j \le N) \end{cases} \tag{23}$$

$$\begin{cases} X_{ii} > 0, & (i = 1, 2, \cdots, N); \\ X_{ij} + X_{ji} > 0, & (1 \le i < j \le N) \end{cases} \tag{24}$$

*have solution $\{P, N_i\}$, then the fuzzy observer* (2) *with gain matrices $G_i = P^{-1}N_i$ drives the eigenvalues of the system* (3) *within the region $\mathcal{S}(0, r)$ and the constraints C2) and C3) are satisfied, where*

$$U_{ij} = \begin{bmatrix} r^2 P & * \\ PA_i - N_iC_j & P \end{bmatrix}, \quad Y_{ij} = \begin{bmatrix} P - C_i^{\mathrm{T}}C_j & * & * \\ 0 & \eta^2 I & * \\ N_iC_j - PA_i & -PB_i & P \end{bmatrix} \quad \textit{and}$$

$$X_{ij} = \begin{bmatrix} P - C_i^{\mathrm{T}} C_j & * & * & * \\ 0 & \gamma^2 I & * & * \\ 0 & 0 & \beta^2 I & * \\ N_i C_j - P A_i & -P D_i & -P B_i & P \end{bmatrix}.$$

**Proof:** Let a Lyapunov function $V(e(t)) = e^{\mathrm{T}}(t) P e(t)$. Then, according to Schur Complement, Lemma 2.3 and the similar technique in [9, 15], the results in the theorem can be yielded. Due to pages limitation, the detailed proof is omitted.

**Corollary 3.2.** *For the residual system* (3)*, given the performance constraints* $\mathcal{S}(0, r)$ *and* $\gamma > 0$*, if there exist matrices* $P = P^{\mathrm{T}} > 0$*,* $N_i$*, scalars* $\beta > 0$ *and* $\eta > 0$ *such that for* $i = 1, 2, \cdots, N$*, the following optimization problem*

$$\min_{P, \eta, \beta, N_i} (\eta^2 + \beta^2) \qquad \text{s.t.} \quad LMIs\ (22) \sim (24) \tag{25}$$

*has solution* $\{P_0, N_{i,0}, \eta_0, \beta_0\}$*, then the fuzzy observer* (2) *with gain matrices* $G_i = P_0^{-1} N_{i,0}$ *regulates the eigenvalues of the system* (3) *within the region* $\mathcal{S}(0, r)$ *and the indices* $\eta^2$ *and* $\beta^2$ *of the constraints C2) and C3) are minimized.*

**Theorem 3.3.** *For the residual system* (3)*, given the performance constraints* $\mathcal{S}(0, r)$*,* $\eta > 0$*,* $\gamma > 0$ *and* $\beta > 0$ *with* $\gamma > \beta$ *and* $\gamma > \eta$*, if there exist matrices* $R_i = R_i^{\mathrm{T}} > 0$*,* $N_i$ *and* $Z$ *such that for* $i = 1, 2, \cdots, N$*, the following LMIs:*

$$\begin{cases} \Xi_{ii} < 0, \quad (i = 1, 2, \cdots, N); \\ \frac{\Xi_{ii}}{N-1} + \frac{1}{2}(\Xi_{ij} + \Xi_{ji}) < 0, \quad (1 \le j \ne i \le N) \end{cases} \tag{26}$$

$$\begin{cases} \Psi_{ii} < 0, \quad (i = 1, 2, \cdots, N); \\ \frac{\Psi_{ii}}{N-1} + \frac{1}{2}(\Psi_{ij} + \Psi_{ji}) < 0, \quad (1 \le j \ne i \le N) \end{cases} \tag{27}$$

$$\begin{cases} \Phi_{ii} < 0, \quad (i = 1, 2, \cdots, N); \\ \frac{\Phi_{ii}}{N-1} + \frac{1}{2}(\Phi_{ij} + \Phi_{ji}) < 0, \quad (1 \le j \ne i \le N) \end{cases} \tag{28}$$

*have solution* $\{R_i, Z, N_i\}$*, then the observer* (2) *with gain matrices* $G_i = Z^{-\mathrm{T}} N_i^{\mathrm{T}}$ *regulates the poles of the system* (3) *within the region* $\mathcal{S}(0, r)$ *and the constraints C2) and C3) met, where*

$$\Xi_{ji} = \begin{bmatrix} -r(R_i - Z - Z^{\mathrm{T}}) & * \\ A_i^{\mathrm{T}} Z - C_i^{\mathrm{T}} N_j & -r R_i \end{bmatrix}, \quad \Psi_{ji} = \begin{bmatrix} -R_i + Z + Z^{\mathrm{T}} & * & * \\ A_i^{\mathrm{T}} Z - C_i^{\mathrm{T}} N_j & C_i^{\mathrm{T}} C_j - R_i & * \\ B_i^{\mathrm{T}} Z & 0 & -\eta^2 I \end{bmatrix}$$

$$\text{and} \quad \Phi_{ji} = \begin{bmatrix} -R_i + Z + Z^{\mathrm{T}} & * & * & * \\ A_i^{\mathrm{T}} Z - C_i^{\mathrm{T}} N_j & C_i^{\mathrm{T}} C_j - R_i & * & * \\ D_i^{\mathrm{T}} Z & 0 & -\gamma^2 I & * \\ B_i^{\mathrm{T}} Z & 0 & 0 & -\beta^2 I \end{bmatrix}.$$

**Proof:** The fuzzy Lyapunov function $V(e(t)) = \sum_{i=1}^{N} h_i(\theta) e^{\mathrm{T}}(t) P_i e(t) := e^{\mathrm{T}}(t) P(h) e(t)$ of the system (3) is defined, where $P_i = P_i^{\mathrm{T}} > 0$. Then, by introducing matrix $R(h) = \sum_{i=1}^{N} h_i(\theta) R_i > 0$, the slack matrix variable $R(h) - Z - Z^{\mathrm{T}}$ and matrix $N_i = G_i^{\mathrm{T}} Z$, the processing is similar to the procedure of Lemma 2 in [25], according to Lemma 2.3 and the inequalities (19) and (20) where the matrix $P$ is $P(h)$ instead. Thus, the corresponding results in the theorem can be obtained.

**Corollary 3.3.** *For the residual system* (3)*, given the performance constraints* $\mathcal{S}(0, r)$ *and* $\gamma > 0$*, if there exist matrices* $R_i = R_i^{\mathrm{T}} > 0$*,* $N_i$*,* $Z$ *and scalars* $\eta > 0$*,* $\beta > 0$ *such that for* $i = 1, 2, \cdots, N$*, the following optimization problem:*

$$\min_{R_i, \eta, \beta, N_i, Z} (\eta^2 + \beta^2) \qquad \text{s.t.} \quad LMIs\ (26) \sim (28) \tag{29}$$

*has solution* $\{R_{i,0}, \eta_0, \beta_0, N_{i,0}, Z_0\}$, *then the fuzzy observer* (2) *with gain matrix* $G_i = Z_0^{-\mathrm{T}} N_{i,0}^{\mathrm{T}}$ *makes the eigenvalues of the system* (3) *within the region* $\mathcal{S}(0, r)$ *and the indices* $\eta^2$ *and* $\beta^2$ *of the constraints C2) and C3) optimized.*

**Remark 3.2.** *As for the number of LMIs constraints of the above three design methods, it is obvious that for each performance requirement the method without any reducing technique is* $N^2$, *the one in Theorem 3.1 is* $N$, *in Theorem 3.2 is* $N(1+N)/2$ *and in Theorem 3.3 is* $N^2$. *Therefore, our proposed method in Theorem 3.1 is with the least number and is more suitable for systems with more rules. In this sense, the proposed design is said to be an improved one.*

3.2. **Numerical example.** For the sake of clarity, we named respectively the method in Theorem 3.1 and Corollary 3.1 as method I, in Theorem 3.2 and Corollary 3.2 as method II, in Theorem 3.3 and Corollary 3.3 as method III hereinafter. A three-dimension and four-rule fuzzy system (1) is considered. The parameters are as follows:

$$A_1 = \begin{bmatrix} 0 & -0.5 & 0.9 \\ 0.25 & 0.5 & 0.5 \\ 0.6 & 0.6 & 0.1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1.05 \\ 1.05 \\ 1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 & 0.1 \\ 0 & 0.5 \\ 0 & 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.7 & -0.2 & 0 \\ 0.2 & 0.9 & 0.0 \end{bmatrix};$$

$$A_2 = \begin{bmatrix} 0 & 0.5 & -1 \\ -0.5 & 0.5 & 0.5 \\ 0.6 & 0 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0.5 \\ 1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} -0.2 & 0.1 \\ 0 & -0.5 \\ 0 & -0.8 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.8 & -0.1 & 0 \\ 0.1 & 1.0 & 0.0 \end{bmatrix};$$

$$A_3 = \begin{bmatrix} 0 & -0.25 & 0.45 \\ 0.125 & 0.25 & 0.25 \\ 0.3 & 0.3 & 0.05 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.525 \\ 0.525 \\ 0.5 \end{bmatrix}, \quad D_3 = \begin{bmatrix} 0.5 & 0.05 \\ 0 & 0.25 \\ 0 & 0.5 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0.3 & 0 & 0.1 \\ 0.1 & 1.0 & 0.0 \end{bmatrix};$$

$$A_4 = \begin{bmatrix} 0 & -0.333 & 0.6 \\ 0.167 & 0.333 & 0.333 \\ 0.4 & 0.4 & 0.067 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 0.7 \\ 0.7 \\ 0.667 \end{bmatrix}, \quad D_4 = \begin{bmatrix} 0.667 & 0.067 \\ 0 & 0.333 \\ 0 & 0.667 \end{bmatrix},$$

$$C_4 = \begin{bmatrix} 0.5 & 0 & 0.1 \\ 0.05 & 1.0 & 0.0 \end{bmatrix}.$$

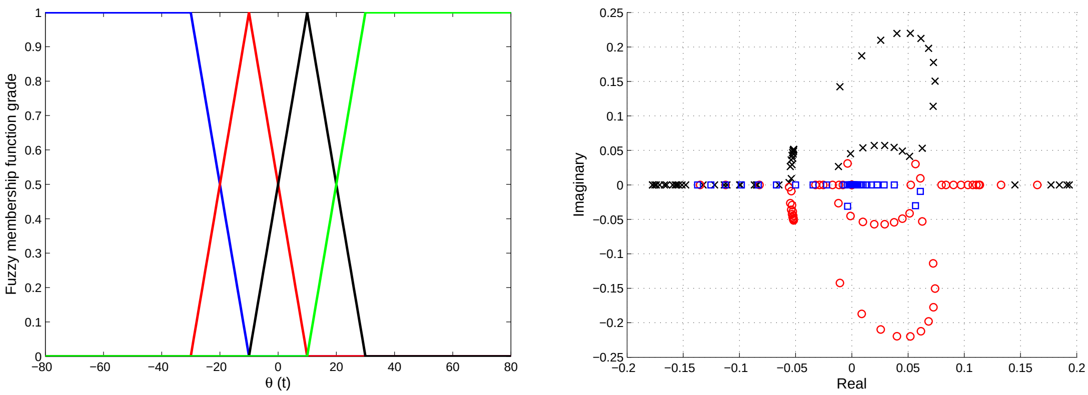
The fuzzy premise variable $\theta(t) = \theta_1(t)$ is assumed within the interval $[-80, 80]$. The membership grade functions are shown in Figure 1(a), and the corresponding coefficient $h_i(\theta)$ can be obtained. The initial state condition is $x(0) = [2.414, 1.413, 1.025]^{\mathrm{T}}$ and error $e(0) = 0$. The parameters of the performance constraints from C1) to C3) are, respectively, C1): $\mathcal{S}(0, r) = \mathcal{S}(0, 0.28)$, C2): $\eta^2 = 9.172$, C3): $\gamma^2 = 18.40$, $\beta^2 = 9.459$.

Through computing based on methods I – III, we found that only method I could obtain the desired results, denoted as fuzzy observer I:

$$G_1 = \begin{bmatrix} -0.65476 & 0.43777 \\ -0.26011 & 1.1304 \\ 0.53753 & 0.91265 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.58748 & -0.58008 \\ -1.0036 & 0.96905 \\ 0.091531 & 1.148 \end{bmatrix},$$

$$G_3 = \begin{bmatrix} -0.80586 & 0.35248 \\ -0.10288 & 0.54555 \\ 0.98053 & 0.24544 \end{bmatrix}, \quad G_4 = \begin{bmatrix} -0.56607 & 0.41388 \\ -0.0047241 & 0.7132 \\ 0.76938 & 0.38831 \end{bmatrix}.$$

The corresponding closed-loop poles of local observers are $z_{1,2,3} = \{-0.0986, 2.067\text{e-}015, 2.824\text{e-}016\}$ for $G_1$, $z_{1,2,3} = \{0.0186, 3.57\text{e-}015, 1.021\text{e-}011\}$ for $G_2$, $z_{1,2,3} = \{-0.1371, 7.65\text{e-}014, -5.908\text{e-}013\}$ for $G_3$ and $z_{1,2,3} = \{-0.1278, -1.756\text{e-}014, 5.957\text{e-}016\}$ for $G_4$, respectively. The corresponding global closed-loop poles distribution with $\theta(t) \in [-80,\ 80]$ is illustrated in Figure 1(b). The other two methods could not obtain their solutions. For the purpose of performance comparison, the optimal robustness indices $\eta^2$ and $\beta^2$ along with the parameter $r \in [0, 1]$ and the fixed index $\gamma^2 = 18.4$ were observed according to the corollaries 3.1 – 3.3. In this example, method III failed to get feasible solutions for given indices $r \in [0, 1]$. The results of the methods I and II are presented in Table 1. It is obvious that method I can get feasible solutions in a wider range of the parameter $r$. The above performance results indicated that our design is less conservative than the other two methods. It is more attractive for large-scale fuzzy systems with more rules.




(a) Fuzzy membership of premise variable $\theta(t)$

(b) Poles distribution of residual system by observer I

FIGURE 1. Fuzzy membership function and poles distribution

TABLE 1. Optimal robustness performance

| $r$ | | 0.15 | 0.2 | 0.328 | 0.35 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|
| $\eta^2$ | method I | 34.647 | 18.197 | 6.6513 | 5.8335 | 4.4568 | 2.3625 | 2.3554 | 2.3554 |
| $\eta^2$ | method II | – | – | 38.16 | 8.7401 | 4.4568 | 2.3625 | 2.3554 | 2.3554 |
| $\beta^2$ | method I | 327.54 | 19.358 | 6.8173 | 5.9663 | 4.5423 | 2.418 | 2.4149 | 2.4149 |
| $\beta^2$ | method II | – | – | 50.557 | 9.1753 | 4.5423 | 2.418 | 2.4149 | 2.4149 |

## 4. **Residual Evaluation and Fault Detection.**

4.1. **Residual evaluation and detection threshold.** Taking the on-line fault detection, time-finite evaluation and the performance requirements into accounts, here an evaluation function in terms of the 'quasi $\mathrm{L}_2$-norm' form of $\varepsilon(t)$ is introduced:

$$J(t) = \|\varepsilon(t)\|_{T_\mathrm{d}} = \left(\sum_{t-T_\mathrm{d}}^{t} \varepsilon^\mathrm{T}(t)\varepsilon(t)\right)^{1/2}, \tag{30}$$

where $T_\mathrm{d} \in \mathcal{Z}_+$ and $T_\mathrm{d} < \infty$ is an evaluation window. In the function (30), the parameter $T_\mathrm{d}$ is vital for FD. Thus, a weighted BIC (bayesian information criterion) information criterion [27] is introduced here to obtain the smaller value $T_0$ for parameter $T_\mathrm{d}$. The simple algorithm is listed as follows:

**Algorithm 1**

*Step 1:* For each disturbance signal $\nu_i(t)$ $(i = 1, \ldots, p)$ time-serial over the length $t_{\mathrm{d}}$, compute the parameter $L_{t_{\mathrm{d}},i} = \left[\sum_{t=0}^{t_{\mathrm{d}}} \nu_i^{\mathrm{T}}(t)\nu_i(t)\right]^{1/2}$. Set the loop variable $k = 1$, and begin the iterative process.

*Step 2:* Get the parameter $l = \mathrm{int}[t_{\mathrm{d}}/k]$, where $\mathrm{int}[t_{\mathrm{d}}/k]$ stands for the maximum integer less than $t_{\mathrm{d}}/k$. Then, calculate the parameter $\lambda_j = \left[\sum_{t=k(j-1)}^{j\cdot k} \nu_i^{\mathrm{T}}(t)\nu_i(t)\right]^{1/2}$, $(j = 1, 2, \cdots, l)$ in the interval $[0,\ t_{\mathrm{d}}]$.

*Step 3:* Compute the parameter: $Rss = \frac{1}{L_{t_{\mathrm{d}},i}} \mid L_{t_{\mathrm{d}},i} - \sup_j(\lambda_j) \mid$, and the BIC criterion: $BIC_k = \sigma \ln(Rss^2) + k[\ln(t_{\mathrm{d}})/t_{\mathrm{d}}]$, where $\sigma > 0$ is a specified weighting coefficient.

*Step 4:* If $k \geq t_{\mathrm{d}}$, then set the parameter $T_{i,0} = k$ when $BIC_k$ is the minimum one. The loop ends. Otherwise set $k = k + 1$ and return to Step 2 if $k < t_{\mathrm{d}}$.

*Step 5:* Set the parameter $T_0 = \max_i\{T_{i,0}\}$. The algorithm stops.

The resulted parameter $T_0$ is a lower-bound of $T_{\mathrm{d}}$ in the energy and entropy significance. The practical parameter $T_{\mathrm{d}}$ can be selected as severalfold $T_0$ in applications. Then as for the threshold setting, we select the maximum value of the above $J(t)$ over a reasonable time interval which is far more than $T_{\mathrm{d}}$ in fault-free case.

$$J_{\mathrm{th}} = \sup_{\nu \neq 0, f=0} \|\varepsilon(t)\|_{T_{\mathrm{d}}} \tag{31}$$

Thus, the logic for the fault detection is

$$J(t) = \left(\sum_{t-T_{\mathrm{d}}}^{t} \varepsilon^{\mathrm{T}}(t)\varepsilon(t)\right)^{1/2} \Rightarrow \begin{cases} \geq J_{\mathrm{th}} \Rightarrow \text{a fault is detectecd} \Rightarrow \text{alarm;} \\ < J_{\mathrm{th}} \Rightarrow \text{fault-free.} \end{cases}$$

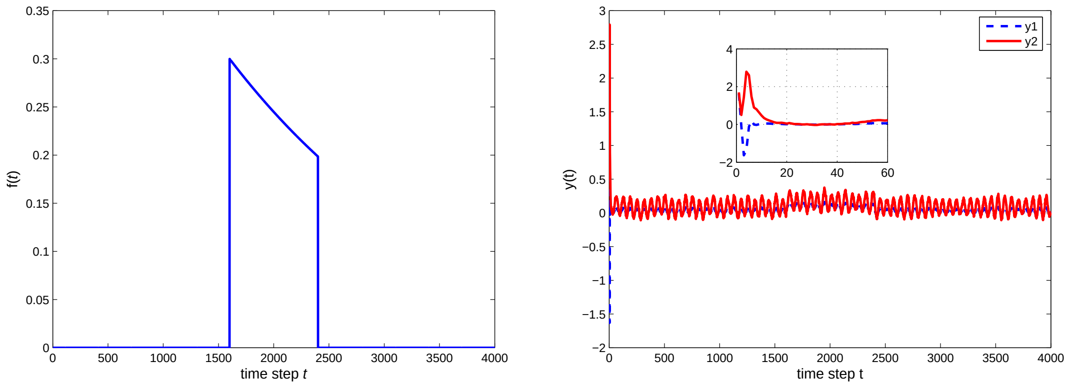
4.2. **Numerical example.** The FD effect of the designed observer I in Section 3.2 is demonstrated. We assumed that the nonlinear function of the premier variable $\theta(t)$ was

$$\theta(t) = \begin{cases} 18 - 40\exp(-0.01t) & t \leq 100, \\ 17.998 & t > 100. \end{cases} \tag{32}$$

Then, the fuzzy weighting coefficient $h_i(\theta(t))$ was obtained by fuzzy computation. To facilitate the simulation, we also supposed that the unknown disturbances were, respectively, a combined signal of a cosine wave $0.08\cos(0.1t)$, a step signal of amplitude 0.05, a noisy signal taking value randomly with normal distribution at zero mean and 0.02 intensity which is exponential attenuating, and a chirp signal with amplitude 0.02 and frequency varying linearly from 0.001Hz to 0.02Hz. Its upper bound of $\mathrm{L}_2$-norm over the whole simulation time was estimated as $\phi_\nu = 4.9837$. An abrupt fault occurred in the first channel between $t = 1600$ and $t = 2400$ (see Figure 2(a)). The output $y(t)$ of system (1) in the fault case is illustrated in Figure 2(b). Obviously, it is difficult to identify the fault from the output $y(t)$.

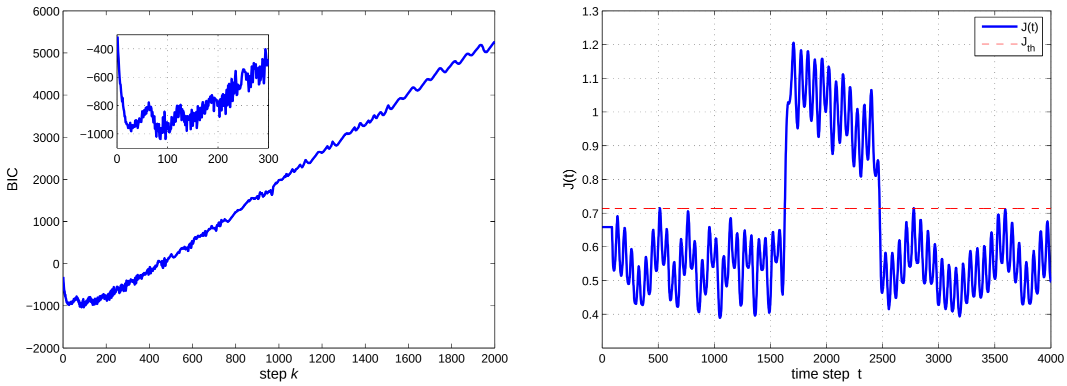
Using the FD observer I, the FD task can be accomplished. First of all, the detection iterative interval $T_{\mathrm{d}}$ was obtained as $T_{\mathrm{d}} = 86$ by the Algorithm 1 (see Figure 3(a), where weighting coefficient $\sigma = 2$). And so the detection result is illustrated in Figure 3(b). When we set the threshold $J_{\mathrm{th}} = \sup_{\nu \neq 0, f=0} \|\varepsilon(t)\|_{T_{\mathrm{d}}} = 0.7138$ by (31), the evaluation signal $J(t)$ was beyond the threshold at $t = 1627$ and then fell back at $t = 2474$ (see Figure 3(b)). To this end, it is evident that the FD observer design method I and the residual evaluation strategy are effective for fault detection.

5. **Conclusions.** Aiming to the conservatism of existing methods for fuzzy system with more If-Then rules, an LMI-based fault detection fuzzy observer has been investigated




(a) Fault signal $f(t)$ (b) Output $y(t)$ in faulty case

FIGURE 2. Fault signal and system output




(a) Time window determination (b) Fault detection: time window $T_\mathrm{d} = 86$

FIGURE 3. Residual evaluation and fault detection

for a class of T-S fuzzy systems. The matrix identical transformation and relaxing techniques were utilized in our design. Meanwhile, the multiobjective optimization was also applied to meet the transient behavior, robustness against disturbances and satisfactory sensitivity to faults. Thus, the designed FD observer can guarantee the given multiple performance constraints and achieve the satisfied fault detection with the suggested residual evaluation in the paper. It is important to note that our proposed design is just sufficient condition based on the common Lyapunov function. Extending the strategy into the multiple Lyapunov functions may be a possible alternative for further improvement. Unfortunately, it is very tricky due to the complexity of identical transformation applied to the multiple Lyapunov functions. In addition, it is also interesting to extend the proposed strategy to the fuzzy systems with more complex structure of subsystems. They are the topics in our future research.

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## REFERENCES

[1] R. Isermann and P. Balle, Trends in the application of model-based fault detection and diagnosis of technical processes, *Control Engineering Practice*, vol.5, no.5, pp.709-719, 1997.

[2] P. M. Frank and S. X. Ding, Survey of robust residual generation and evaluation methods in observer-based fault detection systems, *Journal of Process Control*, vol.7, no.6, pp.403-424, 1997.

[3] R. J. Patton, F. J. Uppal, S. Simani and B. Polle, Robust FDI applied to thruster faults of a satellite system, *Control Engineering Practice*, vol.18, no.9, pp.1093-1109, 2010.

[4] S. X. Ding, *Model-based Fault Diagnosis Techniques: Design Schemes, Algorithms and Tools*, Springer-Verlag, Berlin, 2008.

[5] E. Sobhani-Tehrani and K. Khorasani, *Fault Diagnosis of Nonlinear Systems Using a Hybrid Approach*, Springer-Verlag, London, 2009.

[6] Y. Zhao, J. Lam and H. Gao, Fault detection for fuzzy systems with intermittent measurements, *IEEE Transactions on Fuzzy Systems*, vol.17, no.2, pp.398-410, 2009.

[7] S. K. Nguang, P. Shi and S. X. Ding, Delay-dependent fault estimation for uncertain time-delay nonlinear systems: An LMI approach, *Int. J. Robust Nonlinear Control*, vol.16, no.18, pp.913-933, 2006.

[8] M. Kowal and J. Korbicz, Fault detection under fuzzy model uncertainty, *International Journal of Automation and Computing*, vol.4, no.2, pp.117-124, 2007.

[9] S. He and F. Liu, Fuzzy model-based fault detection for markov jump systems, *Int. J. Robust and Nonlinear Control*, vol.19, no.11, pp.1248-1266, 2009.

[10] Z. Gao, B. Jiang, R. Qi, Y. Xu and Y. Cheng, Fuzzy observer design for near space vehicle with application to sensor fault estimation, *ICIC Express Letters*, vol.4, no.1, pp.177-182, 2010.

[11] Z. Gao, X. Shi and S. X. Ding, Fuzzy state/disturbance observer design for T-S fuzzy systems with application to sensor fault estimation, *IEEE Transactions on Systems, Man, and Cybernetics – Part B: Cybernetics*, vol.38, no.3, pp.875-880, 2008.

[12] Y. Zheng, H. Fang and H. O. Wang, Takagi-Sugeno fuzzy-model-based fault detection for networked control systems with Markov delays, *IEEE Transactions on Systems, Man, and Cybernetics – Part B: Cybernetics*, vol.36, no.4, pp.924-929, 2006.

[13] Y. Wang, W. Wang and D. Wang, LMI approach to design fault detection filter for discrete-time switched systems with state delays, *International Journal of Innovative Computing, Information and Control*, vol.6, no.1, pp.387-398, 2010.

[14] W. Chen and M. Saif, Fuzzy nonlinear unknown input observer design with fault diagnosis applications, *Journal of Vibration and Control*, vol.16, no.3, pp.377-401, 2010.

[15] H. Yang, Y. Xia and B. Liu, Fault detection for T-S fuzzy discrete systems in finite-frequency domain, *IEEE Transactions on Systems, Man, and Cybernetics – Part B: Cybernetics*, vol.41, no.99, pp.1-10, 2011.

[16] H. Yano, Interactive decision making for multiobjective programming problems with fuzzy domination structures, *International Journal of Innovative Computing, Information and Control*, vol.5, no.12(B), pp.4867-4876, 2009.

[17] C. Liu, New evolutionary algorithm for multi-objective constrained optimization, *ICIC Express Letters*, vol.2, no.4, pp.339-344, 2008.

[18] G. Feng, A survey on analysis and design of model-based fuzzy control systems, *IEEE Transactions on Fuzzy Systems*, vol.14, no.5, pp.676-697, 2006.

[19] F. Delmotte, T. M. Guerra and A. Kruszewski, Discrete Takagi-Sugeno's fuzzy models: Reduction of the number of LMI in fuzzy control techniques, *IEEE Transactions on Systems, Man, And Cybernetics – Part B: Cybernetics*, vol.38, no.5, pp.1423-1427, 2008.

[20] K. Tanaka, T. Hori and H. O. Wang, A multiple Lyapunov function approach to stabilization of fuzzy control systems, *IEEE Transactions on Fuzzy Systems*, vol.11, no.4, pp.582-589, 2003.

[21] A. Sala, T. M. Guerra and R. Babuska, Perspectives of fuzzy systems and control, *Fuzzy Sets and Systems*, vol.156, no.3, pp.432-444, 2005.

[22] C. Scherer, P. Gahinet and M. Chilali, Multiobjective output-feedback control via LMI optimization, *IEEE Transactions on Automatic Control*, vol.42, no.7, pp.896-911, 1997.

[23] S. Boyd, L. El Ghaoui, E. Feron and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory (Studies in Applied Mathematics)*, SIAM, Philadelphia, PA, USA, 1994.

[24] J. Chen, C. J. Lopez-Toribio and R. J. Patton, Non-linear dynamic systems fault detection and isolation using fuzzy observers, *Proc. of Instn. Mech. Engrs. Part I: Journal of Systems and Control Engineering*, vol.213, no.6, pp.467-476, 1999.

[25] H. N. Wu and H. Y. Zhang, Reliable $H_\infty$ fuzzy control for continuous-time nonlinear systems with actuator failures, *IEEE Transactions on Fuzzy Systems*, vol.14, no.5, pp.609-618, 2006.
[26] F. Delmotte, T. M. Guerra and M. Ksantini, Continuous Takagi-Sugeno's models: Reduction of the number of LMI conditions in various fuzzy control design technics, *IEEE Transactions on Fuzzy Systems*, vol.15, no.3, pp.426-438, 2007.
[27] G. Schwarz, Estimating the dimension of a model, *Annals of Statistics*, vol.6, no.2, pp.461-464, 1978.