International Journal of Innovative Computing, Information and Control
Volume **8**, Number **1(B)**, January **2012**
ICIC International ©2012 ISSN 1349-4198
pp. 677–689

BUMPLESS SWITCHING SCHEME DESIGN AND ITS APPLICATION TO HYPERSONIC VEHICLE MODEL

Wen Bao[1], Yiwen Qi[1], Daren Yu[1], Zhaohui Yao[1] and Jun Zhao[2]

[1]Hypersonic Technology Research Center
Harbin Institute of Technology
No. 92, West Da-Zhi Street, Harbin 150001, P. R. China
baowen@hit.edu.cn; qiyiwen@gmail.com

[2]College of Information Science and Engineering
Northeastern University
No. 11, Lane 3, Wenhua Road, Heping District, Shenyang 110004, P. R. China
zhaojun@ise.neu.edu.cn

Received October 2010; revised March 2011

Abstract. *This paper investigates the problem of switching performance optimal control for continuous-time controller switched systems. An extended bumpless switching technique is proposed via linear quadratic (LQ) optimal control and internal model principle. Then a sufficient condition for guaranteeing the switched closed-loop systems to be stable under bumpless switching is developed. Dwell-time and multiple Lyapunov functions methodologies are utilized for the stability analysis and switching law design. Simulations for the generic hypersonic vehicle model show the effectiveness of the proposed design method.*
Keywords: Controller switched systems, Bumpless switching, Dwell time, Hypersonic vehicle model

1. **Introduction.** In recent years, there has been increasing interest in the stability analysis and design methodology of switched systems due to their significance both in theory and applications [1]. Many researches have been presented on a wide range of topics, including the modeling, optimization, stability analysis and control, among which the stability issues have been a major focus [2-7]. In contrast, less attention has been paid to the issue of switching control performance, which is very important in practical applications and is also a very challenging problem. For instance, in flight control, multiple controllers are often used to control the same highly nonlinear flight vehicle, one for each operating point or task. Nevertheless, switching among controllers implies control discontinuities and undesired transients. Moreover, these discontinuities might directly lead to performance degradation, actuator saturation and even instability of the closed-loop system. The solution of this problem is called bumpless switching (transfer).

Generally, bumpless switching arises in several cases of practical interests. The first case is switching between manual and automatic mode [8]. Another case is switching among several linear controllers by gain scheduling [9, 10]. The third case is the improvement of system performance via switching between controllers with different properties [11]. Finally, switching can result from the need for online testing and evaluation of different control law designs. Bumpless switching is often performed in the steady state to meet safety requirements [12]. Approaches that focus on bumpless switching are mainly as follows: anti-windup bumpless transfer (AWBT) scheme [13], optimal linear quadratic control methods [12, 14], L_2 norm methods [8, 15], interpolation [9, 11], and observer-based technique [16], etc. In these mentioned works, the AWBT method [13] represents an AWBT operator forcing the output of the off-line controller to track that of the online one. Nevertheless, the input of off-line controller is conditioned by the operator and thus is not necessarily close to that of the online one at the switching instant [17]. The L_2 method [8, 15] is to design a compensator ensuring an L_2 bound on the mismatch between the actual plant output and that of the ideal closed-loop behavior. This method, unfortunately, is only applicable to linear plant with a sufficiently accurate model [10]. Moreover, as the authors themselves point out in [11], the interpolation algorithm relies only on stability considerations with the controller switched sufficiently slowly.

In fact, the LQ technique [14] stands out as a convenient tool for the steady-state bumpless switching synthesis. It has been applied to many physical systems such as vertical/short take-off land aircraft [10], coal-fired boiler/turbine [18] and magnetic bearings [19]. The central feature of LQ approach is the compensator F (in [14]), which is a static matrix and can drive the output of the off-line controller to track that of the online controller. However, because the matrix F is obtained by adjusting two weighting matrices and they have only limited amplitude of adjustment, the method does not produce an adequate solution. Nevertheless, the conceptual clarity and the computational simplicity of this method give a strong motivation to extend it.

In this paper, we present a new extended scheme for the bumpless switching of a controller switched system. Firstly, we will design a set of compensatory controllers (CCs) for the off-line controllers respectively. With the effect of CC, at the instant of switching, the output of online controller and that of the off-line controller are as close as possible so as to reduce the magnitude of the discontinuity (or error). We refer to [14], the LQ framework and the input weighting of off-line controller is used for design. However, the proposed method is mainly different from that in [14]. In our scheme, these CCs, designed based on internal model principle, can make the off-line controllers track the output of online controller with nearly zero steady state error such that the "continuity" of plant input becomes much stricter at switching instants. Furthermore, an integral action is introduced to eliminate the (tracking) error. By considering the integral of error as an extra set of state, the error minimization problem is transformed into an augmented system stability problem. It indicates that once the augmented system is stable, the tracking error will asymptotically converge to zero, or be reduced to an acceptable value as close to zero as possible in finite time. And based on this fact, a CC can be obtained.

In addition, based on the multiple Lyapunov functions combined with the dwell time technique, a switching law design and a condition rendering switched closed-loop systems exponentially stable are given. Note that this condition is developed in the switched systems framework, which only a few articles have addressed [20]. The paper is organized as follows. Section 2 gives the problem formulation and some preliminaries. In Section 3, we present the CC design. The switching law design and stability analysis are given in Section 4. An algorithm is then presented in Section 5. Simulations are shown in Section 6. Conclusions are given in Section 7.

2. **Problem Statement and Preliminaries.** Consider the controller switched system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \quad x(t_0) = x(0), \\ u(t) &= u_{\sigma(t)}(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $\sigma(t) : [0, +\infty) \to \mathcal{I}_N = \{1, 2, \ldots, N\}$ is the switching signal to be designed, $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $u_i(t) \in \mathbb{R}^m$ are the sub-controllers outputs, $y(t) \in \mathbb{R}^l$ is the vector of plant measurements. A, B and C are real constant matrices of appropriate dimensions. Corresponding to the switching signal σ, we have the switching sequence $\Lambda = \{x_0; (i_0, t_0), (i_1, t_1), \ldots, (i_k, t_k), \ldots, \ |i_k \in \mathcal{I}_N\}$. When $t \in [t_k, t_{k+1})$, $\sigma(t) = i_k$, that is, the i_k-th subsystem is activated.

Assumption 2.1. *The pair (A, B) is stabilizable.*

Remark 2.1. *It should be noticed that this assumption is necessary for stabilizability of a linear system. If a system is stabilizable, then its all unstable eigenvalues can be made stable by means of a feedback gain.*

A set of controllers can be designed in advance based on the method in [21] as follows:

$$\mathcal{C}_i : \begin{cases} \dot{x}_{c_i}(t) = A_{c_i} x_{c_i}(t) + B_{c_{i_1}} x(t) + B_{c_{i_2}} r_{c_i}(t), \\ u_i(t) = C_{c_i} x_{c_i}(t) + D_{c_i} x(t), \end{cases} \tag{2}$$

where $x_{c_i}(t) \in \mathbb{R}^{n_c}$ is the i-th controller state. For the online controller, $r_{c_i} = r(t) \in \mathbb{R}^l$ is the reference command signal. For the off-line one, $r_{c_i}(t) = \bar{r}_i(t)$ is the CC's output to be designed (see Figure 1). An off-line controller is closed by its CC to drive the control signal close to the online one. Since the purpose of this article is to extend the results of literature [14], the scope of this paper is to discuss the steady-state switching.

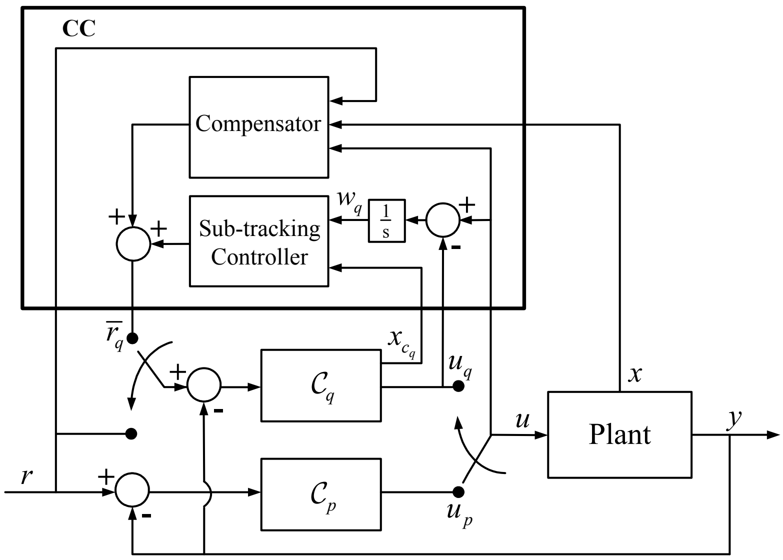

FIGURE 1. The proposed bumpless switching scheme with $\mathcal{I}_N = \{p, q\}$

Assumption 2.2. *Throughout the paper, we consider the situation that the system (1) is under steady-state switching.*

Remark 2.2. *The closed-loop system under the initial implemented controller performs a linear behavior before controller switching. Thus the steady-state switching here means to start controller switching after the closed-loop system under the initial implemented controller has reached its steady state. Here it is assumed that steady state is reached once the closed-loop system outputs remain within 99% and 101% of the steady state values.*

Integral action is used in classical control to eliminate steady state errors when tracking signals. It can be introduced into the LQ framework by considering the integral of the tracking error as an extra set of state variables. For any off-line controller $\mathcal{C}_i$, $i \in \mathcal{I}_N$, define the error state as

$$\dot{w}_i(t) = u(t) - u_i(t) = u(t) - C_{c_i} x_{c_i}(t) - D_{c_i} x(t). \tag{3}$$

Definition 2.1. *Given a positive scalar* $\varepsilon > 0$, *a switching (online) controller* $\mathcal{C}_p$ ($\forall p \in \mathcal{I}_N$) *is said to perform a* bumpless switching *if, whenever controller is switched, there exists a finite time* $T_\varepsilon > 0$ *such that the output of a controller* $\mathcal{C}_q$ ($\forall q \neq p \in \mathcal{I}_N$) *to be switched satisfies the condition* $\lim_{t\to T_\varepsilon} |u(t)-u_q(t)| \leq \varepsilon$, *where* $u(t) = u_p(t)$. *In particular,* $\mathcal{C}_p$ *is said to perform a* strictly bumpless switching *if* $\lim\limits_{t\to\infty} |u(t) - u_q(t)| = 0$.

Combining (2) and (3), we have the augmented system for the ith off-line controller:

$$\begin{bmatrix} \dot{x}_{c_i}(t) \\ \dot{w}_i(t) \end{bmatrix} = \begin{bmatrix} A_{c_i} & \mathbf{0} \\ -C_{c_i} & \mathbf{0} \end{bmatrix} \begin{bmatrix} x_{c_i}(t) \\ w_i(t) \end{bmatrix} + \begin{bmatrix} B_{c_{i_1}} \\ -D_{c_i} \end{bmatrix} x(t) + \begin{bmatrix} B_{c_{i_2}} \\ \mathbf{0} \end{bmatrix} \bar{r}_i(t) + \begin{bmatrix} \mathbf{0} \\ I \end{bmatrix} u(t). \tag{4}$$

For simplicity, the augmented system (4) is rewritten as

$$\dot{\bar{x}}_i(t) = \bar{A}_i\bar{x}_i(t) + \bar{B}_{i_1}x(t) + \bar{B}_{i_2}\bar{r}_i(t) + Fu(t), \tag{5}$$

where $\bar{x}_i = \begin{bmatrix} x_{c_i}^T & w_i^T \end{bmatrix}^T$ is the augmented system state vector. The cost performance index for the design of the ith bumpless switching CC is defined as

$$J_i = \frac{1}{2}\int_0^T \left(\bar{x}_i^T Q_i\bar{x}_i + \vartheta_i^T R_i\vartheta_i\right) dt, \tag{6}$$

where $Q_i \geq 0$, $R_i > 0$, and $\vartheta_i = \bar{r}_i - r$ is the difference between CC output and switched systems' reference signal. Q_i for state variables is standard in optimal control, while the R_i is used for scaling ϑ_i. The size of ϑ_i will seriously influence the error size (3).

Remark 2.3. *In a viewpoint of an optimal control, the relative magnitudes of* Q_i *and* R_i *may be selected to trade off requirements on the smallness of the state* $\bar{x}_i$ *against that of* ϑ_i. *For instance, a larger* R_i *will make it necessary for* ϑ_i *to be smaller. On the other hand, to make* $\bar{x}_i$ *go to zero more quickly with time, we may select a larger* Q_i.

Assumption 2.3. $\begin{bmatrix} \bar{A}_i & \bar{B}_{i_2} \end{bmatrix}$ *is stabilizable,* $Q_i = \Pi^T\Pi \geq 0$ *and* $\begin{bmatrix} \bar{A}_i & \Pi \end{bmatrix}$ *is observable.*

Remark 2.4. *It is well known in the optimal control literature (for specific detail see* [22]*) that in order for the algebraic Riccati Equation (17) (in next Section), to have a positive definite stabilizing solution,* $(\bar{A}_i, \bar{B}_{i_2}, \Pi)$ *is required to be stabilizable and observable. In addition, it will be shown that the stability of the off-line closed-loop system composed of (5) and (20) is ensured as (17) has a constant, positive definite solution.*

In this paper, we are interested in the following two problems.

Problem 1 (Bumpless Switching CC Design Problem) Given a set of controllers (2), find a set of CCs respectively, each of which includes a compensator and a sub-tracking controller such that augmented system (5) is stabilized and cost functional J_i is minimized.

Problem 2 (Closed-loop System Stability Analysis Problem) Given a set of controllers (2) and their CCs, construct a switching law σ and derive a sufficient condition such that the closed-loop system composed of (1), (2) and CC is asymptotically stable and simultaneously guarantees the bumpless switching by switching controllers.

3. **Bumpless Switching CC Design.**

Now based on the above discussion, we give a solution to Problem 1. Suppose there exists a vector function $\lambda(t)$, such that the conditional extremum problem of (6) subject to constraints (5) can be transformed into a non-conditional extremum problem with the performance index

$$J_i = \frac{1}{2}\int_0^T \left(H(t) - \lambda^T\dot{\bar{x}}_i\right) dt, \tag{7}$$

where the Hamiltonian function is given by $H(t) = \frac{1}{2}\left[\bar{x}_i^T Q_i \bar{x}_i + (\bar{r}_i - r)^T R_i(\bar{r}_i - r)\right] + \lambda^T\left(\bar{A}_i\bar{x}_i + \bar{B}_{i_1}x + \bar{B}_{i_2}\bar{r}_i + Fu\right)$. A necessary condition for minimizing the cost index (7) is $\partial H/\partial\lambda = \dot{\bar{x}}_i$, $\partial H/\partial\bar{x}_i = -\dot{\lambda}$, $\partial H/\partial\bar{r}_i = 0$. Furthermore,

$$\partial H/\partial\bar{x}_i = Q_i\bar{x}_i + \bar{A}_i^T\lambda = -\dot{\lambda}, \tag{8}$$

$$\partial H/\partial\bar{r}_i = R_i(\bar{r}_i - r) + \bar{B}_{i_2}^T\lambda = 0. \tag{9}$$

From (9), it follows that

$$\bar{r}_i = r - R_i^{-1}\bar{B}_{i_2}^T\lambda. \tag{10}$$

Substituting (10) into (5) yields

$$\dot{\bar{x}}_i = \bar{A}_i\bar{x}_i + \bar{B}_{i_1}x + \bar{B}_{i_2}\left(r - R_i^{-1}\bar{B}_{i_2}^T\lambda\right) + Fu. \tag{11}$$

Defining

$$\lambda(t) = P_i(t)\bar{x}_i(t) - g(t), \tag{12}$$

then we can get

$$\dot{\lambda}(t) = \dot{P}_i(t)\bar{x}_i(t) + P_i(t)\dot{\bar{x}}_i(t) - \dot{g}(t). \tag{13}$$

Moreover, by (12), Equation (8) can be written as

$$\dot{\lambda}(t) = -Q_i\bar{x}_i(t) - \bar{A}_i^T\left(P_i(t)\bar{x}_i(t) - g(t)\right) = -\left(Q_i + \bar{A}_i^T P_i(t)\right)\bar{x}_i(t) + \bar{A}_i^T g(t). \tag{14}$$

In addition, applying (11) and (12) to (13) results in

$$\begin{aligned}\dot{\lambda}(t) = &\dot{P}_i(t)\bar{x}_i(t) + P_i(t)\big[\bar{A}_i\bar{x}_i(t) + \bar{B}_{i_1}x(t) + \bar{B}_{i_2}r \\ &- \bar{B}_{i_2}R_i^{-1}\bar{B}_{i_2}^T(P_i(t)\bar{x}_i(t) - g(t)) + Fu\big] - \dot{g}(t).\end{aligned} \tag{15}$$

Then, from (14) and (15), it follows:

$$\begin{aligned}&\left[\dot{P}_i(t) + P_i(t)\bar{A}_i + \bar{A}_i^T P_i(t) - P_i(t)\bar{B}_{i_2}R_i^{-1}\bar{B}_{i_2}^T P_i(t) + Q_i\right]\bar{x}_i(t) \\ =&\dot{g}(t) + \left(\bar{A}_i^T - P_i(t)\bar{B}_{i_2}R_i^{-1}\bar{B}_{i_2}^T\right)g(t) - P_i(t)\bar{B}_{i_1}x(t) - P_i(t)\bar{B}_{i_2}r - P_i(t)Fu(t).\end{aligned} \tag{16}$$

Note that the left side of (16) is a product of a function of time and state variables $\bar{x}_i(t)$, while the right side is only a function of time. It means that for arbitrary t and $\bar{x}_i(t)$, the following two equations:

$$\dot{P}_i(t) + P_i(t)\bar{A}_i + \bar{A}_i^T P_i(t) - P_i(t)\bar{B}_{i_2}R_i^{-1}\bar{B}_{i_2}^T P_i(t) + Q_i = 0,$$

$$-\dot{g}(t) = (\bar{A}_i^T - P_i(t)\bar{B}_{i_2}R_i^{-1}\bar{B}_{i_2}^T)g(t) - P_i(t)\bar{B}_{i_1}x(t) - P_i(t)\bar{B}_{i_2}r - P_i(t)Fu(t).$$

must be satisfied. These two equations should be extended to infinite horizon (i.e., $T \to \infty$) for practical engineering consideration [22], which are

$$P_i\bar{A}_i + \bar{A}_i^T P_i - P_i\bar{B}_{i_2}R_i^{-1}\bar{B}_{i_2}^T P_i + Q_i = 0, \tag{17}$$

$$\left(\bar{A}_i^T - P_i\bar{B}_{i_2}R_i^{-1}\bar{B}_{i_2}^T\right)g - P_i\bar{B}_{i_1}x - P_i\bar{B}_{i_2}r - P_iFu = 0. \tag{18}$$

It follows from (17) and (18) that

$$g = \left(\bar{A}_i^T - P_i\bar{B}_{i_2}R_i^{-1}\bar{B}_{i_2}^T\right)^{-1}\left(P_i\bar{B}_{i_1}x + P_i\bar{B}_{i_2}r + P_iFu\right), \tag{19}$$

in which the symmetric positive definite matrix P_i is the solution of algebraic Riccati Equation (17). Thus using (10), (12) and (19), we have the CC of off-line controller $\mathcal{C}_i$, $i \in \mathcal{I}_N$ for bumpless switching as follows:

$$\bar{r}_i = K_{r_i}r + K_{c_i}x_{c_i} + K_{w_i}w_i + K_{x_i}x + K_{u_i}u, \tag{20}$$

where $K_{r_i} = I + R_i^{-1}\bar{B}_{i_2}^T\Sigma_1$, $K_{\bar{x}_i} = -R_i^{-1}\bar{B}_{i_2}^T P_i$, $K_{x_i} = R_i^{-1}\ \bar{B}_{i_2}^T\Sigma_2$, $K_{u_i} = R_i^{-1}\bar{B}_{i_2}^T\Sigma_3$, $\Sigma_1 = \Gamma\bar{B}_{i_2}$, $\Sigma_2 = \Gamma\bar{B}_{i_1}$, $\Sigma_3 = \Gamma F$, $K_{\bar{x}_i} = [K_{c_i}\ \ K_{w_i}]$ and $\Gamma = \left(\bar{A}_i^T - P_i\bar{B}_{i_2}R_i^{-1}\bar{B}_{i_2}^T\right)^{-1}P_i$. K_{c_i} and K_{w_i} are the sub-tracking controller gains, K_{r_i}, K_{x_i} and K_{u_i} are the compensator gains.

Remark 3.1. *If Assumption 2.3 holds, then (17) has a unique solution, and the augmented closed-loop system composed of (5) and (20) is stable* [22] *which guarantees the bumpless switching of online controller. This will be discussed in the next section.*

Remark 3.2. *The derivation of compensator F in* [14] *is carried out through the minimization of a functional that includes two differences. One is the difference between the input signals of the controllers (both online and off-line), the other is the difference between the output signals. The functional is given by $J_i = \frac{1}{2}\int_0^T (z_u(t)^T W_u z_u(t) + z_e(t)^T W_e z_e(t))dt$, where $z_u(t) = u(t) - u_i(t)$, $z_e(t) = \bar{r}_i - r$, and W_u, W_e are weighting matrices. Especially, $z_u(t)$ represents the error between the off-line controller output and the online one, thus the smaller $z_u(t)$ is, the better the bumpless performance will be. Then, the key point of* [14] *is to choose appropriate weighting matrices W_u and W_e such that the J_i can be minimized under the effect of F. However, in Section 6, this technique will be found to provide an incomplete convergence of the $z_u(t)$ and produce a pronounced non-vanishing error of $z_u(t)$. In other words, the weighting matrices W_u and W_e have limited regulation range or ability, they can not be regulated arbitrarily to further reduce the index J_i value (i.e., J_i is bounded). That is the drawback or limitations of the LQ bumpless switching design.*

For this reason, the focus of our work is on "minimizing" the error to the maximum extent, which is different from [14], *and this is also the major contribution of our work. From (3) and (4), the proposed approach reduces the realization of error minimizing to the solving of the augmented system stability problem based on the internal model principle. By introducing an integral action, the method can make the off-line controller achieve zero-error tracking or even completely eliminate the error in theory (if $t \to \infty$).*

4. Switching Law Design and Stability Analysis.

The objective of this section is to derive a global exponential stability condition for the switched closed-loop systems under bumpless switching and to design a dwell time dependent switching law.

Without loss of generality, we assume that two controllers $\mathcal{C}_p$ and $\mathcal{C}_q$ ($\forall q \neq p \in \mathcal{I}_N$) are used for bidirectional switching. Substituting (20) into (2), we have the description of an off-line controller $\mathcal{C}_i$, $i \in \mathcal{I}_2 = \{p, q\}$ with compensatory effect. That is

$$\begin{aligned} \dot{x}_{c_i}(t) &= A_i x_{c_i}(t) + B_i v(t), \\ u_i(t) &= C_i x_{c_i}(t) + D_i v(t), \end{aligned} \tag{21}$$

where $A_i = A_{c_i} + B_{c_{i_2}} K_{c_i}$, $B_i = \begin{bmatrix} \Xi_1 & \Xi_2 & \Xi_3 & \Xi_4 \end{bmatrix}$, $C_i = C_{c_i}$, $D_i = \begin{bmatrix} D_{c_i} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$, $\Xi_1 = B_{c_{i_1}} + B_{c_{i_2}} K_{x_i}$, $\Xi_2 = B_{c_{i_2}} K_{r_i}$, $\Xi_3 = B_{c_{i_2}} K_{w_i}$, $\Xi_4 = B_{c_{i_2}} K_{u_i}$ and $v = \begin{bmatrix} x^T & r^T & w_i^T & u^T \end{bmatrix}^T$. Then, from (5) and (21), the augmented system for the off-line controller $\mathcal{C}_i$ is given by

$$\dot{\bar{x}}_{\mathrm{off}_i}(t) = \tilde{A}_i \bar{x}_{\mathrm{off}_i}(t) + \tilde{B}_{x_i} x(t) + \tilde{B}_{u_i} u(t) + \tilde{B}_{r_i} r(t), \tag{22}$$

where $\bar{x}_{\mathrm{off}_i} = \begin{bmatrix} x_{c_i}^T & w_i^T \end{bmatrix}^T$, $\tilde{A}_i = \begin{bmatrix} A_i & \Xi_3 \\ -C_i & \mathbf{0} \end{bmatrix}$, $\tilde{B}_{x_i} = \begin{bmatrix} \Xi_1 \\ -D_{c_i} \end{bmatrix}$, $\tilde{B}_{u_i} = \begin{bmatrix} \Xi_4 \\ I \end{bmatrix}$, $\tilde{B}_{r_i} = \begin{bmatrix} \Xi_2 \\ \mathbf{0} \end{bmatrix}$. It follows from Remark 3.1 that the matrix $\tilde{A}_i$ is Hurwitz stable.

On the other hand, from (1) and (2), we obtain the augmented system for a controller $\mathcal{C}_j$, $j \neq i \in \mathcal{I}_2$ when it is online. That is

$$\dot{\bar{x}}_{\mathrm{on}_j}(t) = \hat{A}_j \bar{x}_{\mathrm{on}_j}(t) + \hat{B}_{r_j} r(t), \tag{23}$$

where $\bar{x}_{\mathrm{on}_j} = \begin{bmatrix} x^T & x_{c_j}^T \end{bmatrix}^T$, $\hat{A}_j = \begin{bmatrix} A + BD_{c_j} & BC_{c_j} \\ B_{c_{j_1}} & A_{c_j} \end{bmatrix}$, $\hat{B}_{r_j} = \begin{bmatrix} \mathbf{0} \\ B_{c_{j_2}} \end{bmatrix}$.

Remark 4.1. *Matrix $\hat{A}_j$ is supposed to be Hurwitz, i.e., in the absence of controller switching, the individual closed-loop systems under (1) and (2) would be globally stable.*

Consequently, when $\mathcal{C}_p$ $(j = p)$ is online and $\mathcal{C}_q$ $(i = q)$ is off-line, the following individual closed-loop system is obtained by augmenting (22) with (23):

$$\dot{x}_{\rm cl}(t) = A_{{\rm cl}_p} x_{\rm cl}(t) + B_{{\rm cl}_p} r(t),$$

where $x_{\rm cl} = \begin{bmatrix} \bar{x}_{{\rm on}_p}^T & \bar{x}_{{\rm off}_q}^T \end{bmatrix}^T$, matrices $A_{{\rm cl}_p} = \begin{bmatrix} \hat{A}_p & \mathbf{0} \\ \Xi_5 & \tilde{A}_q \end{bmatrix}$, $B_{{\rm cl}_p} = \begin{bmatrix} \hat{B}_{r_p} \\ \tilde{B}_{r_q} \end{bmatrix}$ and $\Xi_5 = \begin{bmatrix} \tilde{B}_{x_q} + \tilde{B}_{u_q} D_{c_p} & \tilde{B}_{u_q} C_{c_p} \end{bmatrix}$. In the same way, when $\mathcal{C}_q$ $(j = q)$ is online and $\mathcal{C}_p$ $(i = p)$ is off-line, we can have the other closed-loop system $\dot{x}_{\rm cl}(t) = A_{{\rm cl}_q} x_{\rm cl}(t) + B_{{\rm cl}_q} r(t)$. Thus, the switched closed-loop system when $r(t) = 0$ can be given by

$$\dot{x}_{\rm cl}(t) = A_{{\rm cl}_{\sigma(t)}} x_{\rm cl}(t), \quad \sigma(t) \in \mathcal{I}_2. \tag{24}$$

Definition 4.1. *System (24) is said to be globally exponentially stable under a switching signal $\sigma(t)$, if all solutions $x_{\rm cl}(t)$ of (24) starting from any initial condition $x_{\rm cl}(t_0)$ satisfy $\| x_{\rm cl}(t) \| \leq \kappa \| x_{\rm cl}(t_0) \| \, {\rm e}^{-\eta(t-t_0)}$, $\forall t \geq t_0$ for some constants $\kappa \geq 1$ and $\eta > 0$.*

The switching law design in terms of dwell time principles. We impose restrictions on the set of admissible switching signals by defining the set $\mathcal{S}_T = \{\sigma(t) : t_{k+1} - t_k \geq T\}$, where t_k are the commutation instants and $T > 0$. In other words, for the positive constant T, $\mathcal{S}_T$ denotes the set of all switching signals with interval between consecutive discontinuities no smaller than T. The constant T is called the *dwell time* [1]. Then find the minimum T for which (24) is exponentially stable for all possible $\sigma(t) \in \mathcal{S}_T$.

Theorem 4.1. *(Bumpless Switching) Consider the system (24). Given controllers $\mathcal{C}_i$, $i \in \mathcal{I}_2$, their associated closed-loop system matrices $A_{{\rm cl}_i}$ and a scalar $\epsilon > 0$, if there exists a scalar $\beta > 0$ such that the following inequalities*

$$\left.\frac{dV_i(x_{\rm cl}(t))}{dt}\right| A_{{\rm cl}_i} x_{\rm cl} < -\beta V_i(x_{\rm cl}(t)), \quad i \in \mathcal{I}_2 \tag{25}$$

have positive definite matrices $P_i > 0$, then for every $\sigma \in S_{\tau_d}$ with the dwell time of $\mathcal{C}_j$, $j \neq i \in \mathcal{I}_2$ satisfying $\tau_d = \max\{T_s, T_\epsilon\}$, the switched closed-loop system (24) is globally exponentially stable and the controller $\mathcal{C}_j$ performs a bumpless switching.

Proof: (i) For any switching signal $\sigma \in S_T$, let $0 = t_0, t_1, t_2, \ldots$ be the corresponding switching time series. For any $t > 0$, there exists an integer i such that $t \in [t_i, t_{i+1})$. Let $N_\sigma(t_0, t)$ denote the number of switchings of σ over the internal $[t_0, t]$. Also suppose that during $[t_i, t_{i+1})$, mode i is active, where $i \in \mathcal{I}_2$.

For the given positive definite matrices P_i, $i \in \mathcal{I}_2$, let $V_i(x_{\rm cl}) = x_{\rm cl}^T P_i x_{\rm cl}$ be a Lyapunov candidate corresponding to mode i. It is obvious that there exist positive constants $a > 0$ and $b > 0$ such that

$$a \|x_{\rm cl}\|^2 \leq V_i(x_{\rm cl}) \leq b \|x_{\rm cl}\|^2, \tag{26}$$

where $a = \inf\limits_{i \in \mathcal{I}_2} \{\lambda_{\min}(P_i)\}$, $b = \sup\limits_{i \in \mathcal{I}_2} \{\lambda_{\max}(P_i)\}$. Then it follows from (25) that $\dot{V}_i(x_{\rm cl}) < -\beta V_i(x_{\rm cl})$, which yields $V_i(x_{\rm cl}(t)) \leq {\rm e}^{-\beta(t-t_i)} V_i(x_{\rm cl}(t_i))$. Therefore, from (26), we obtain

$$\|x_{\rm cl}(t)\|^2 \leq \chi {\rm e}^{-\beta(t-t_i)} \|x_{\rm cl}(t_i)\|^2, \tag{27}$$

where $\chi = \frac{b}{a}$. By this fact, a similar inequality can be derived as $\|x_{\rm cl}(t_i)\|^2 \leq \chi {\rm e}^{-\beta(t_i - t_{i-1})} \times \|x_{\rm cl}(t_{i-1})\|^2$. Iterating the inequality results in

$$\begin{aligned} \|x_{\rm cl}(t_i)\|^2 &\leq \chi^{N_\sigma(0,t)} \times {\rm e}^{-\beta(t_i - t_{i-1})} \times \ {\rm e}^{-\beta(t_{i-1} - t_{i-2})} \cdots {\rm e}^{-\beta(t_1 - t_0)} \times \|x_{\rm cl}(t_0)\|^2 \\ &\leq \chi^{N_\sigma(0,t)} {\rm e}^{-N_\sigma(0,t)\beta T} \|x_{\rm cl}(t_0)\|^2. \end{aligned} \tag{28}$$

In view of the inequalities (27) and (28), we deduce the following inequality

$$\|x_{\rm cl}(t)\| \leq \sqrt{\chi^{(N_\sigma(0,t)+1)}}\mathrm{e}^{-\frac{(N_\sigma(0,t)+1)\beta T}{2}}\|x_{\rm cl}(t_0)\|.$$

Hence, when $\sqrt{\chi^{(N_\sigma(0,t)+1)}}\mathrm{e}^{-\frac{(N_\sigma(0,t)+1)\beta T}{2}} < 1$, that is, the minimum dwell time satisfies

$$\tau_d \geq T_{\rm s} > T^* = \frac{\ln\chi}{\beta}, \tag{29}$$

where $T_{\rm s} = \inf\{T : T > T^*\}$, the system (24) is globally exponentially stable.

(ii) In addition, for steady-state switching, due to the fact that the signals $x(t)$, $u(t)$ and $r(t)$ have reached their steady states, then taking the derivative on both sides of Equation (22) with respect to time, we have

$$\ddot{\bar{x}}_{{\rm off}_i}(t) = \tilde{A}_i\dot{\bar{x}}_{{\rm off}_i}(t). \tag{30}$$

Since matrix $\tilde{A}_i$ is stable, the state of (30) must satisfy $\dot{\bar{x}}_{{\rm off}_i}(t) = \left[\begin{array}{cc} \dot{x}_{c_i}(t)^T & \dot{w}_i(t)^T \end{array}\right]^T \to 0$, $t \to \infty$ for $i \in \mathcal{I}_2$. Then by (3) and Definition 2.1, the online controller $\mathcal{C}_j$, $j \neq i \in \mathcal{I}_2$ performs a strictly bumpless switching. However, we also see that the switched system achieves strictly bumpless switching only when time approaches infinity. In order to facilitate implementation in practice, choose an appropriate scalar $\epsilon > 0$ that can meets the needs of $\mathcal{C}_j$ for performing bumpless switching in a finite time interval T_ϵ. Thus, we have $\dot{w}_i(t) \to \epsilon_{\tau_d}$, $t \to \tau_d$, where $0 < \epsilon_{\tau_d} \leq \epsilon$. This completes the proof.

5. **Algorithm of Bumpless Switching.** According to Theorem 4.1, the detailed algorithm is given as follows.

Step 1: Given a scalar $\beta > 0$. Find two positive definite matrices P_i, $i \in \mathcal{I}_2$ satisfying

$$A_{{\rm cl}_i}^T P_i + P_i A_{{\rm cl}_i} + \beta P_i < 0, \tag{31}$$

then calculate $\lambda_{\min}(P_i)$ and $\lambda_{\max}(P_i)$, to obtain a χ.

Step 2: Calculate T^* according to (29).

Step 3: Repeat Steps 1 to 2 until a satisfactory small value of T^* is obtained and then give an estimate $T_{\rm s} = \inf\{T : T > T^*\}$.

Step 4: Based on linear system theory, the state response of system (30) is $\dot{\bar{x}}_{{\rm off}_i}(t) = \mathrm{e}^{\tilde{A}_i t}\dot{\bar{x}}_{{\rm off}_{i_0}}$, where $\dot{\bar{x}}_{{\rm off}_{i_0}}$ is the initial state of (30) for the off-line controller $\mathcal{C}_i$. Accordingly, the error state is $\dot{w}_i(t) = \left[\begin{array}{ccc} \dot{w}_{i_1}(t) & \cdots & \dot{w}_{i_m}(t) \end{array}\right]^T$, where

$$\dot{w}_{i_h}(t) = C_h \times \left[\mathrm{e}^{\tilde{A}_i t}\dot{\bar{x}}_{{\rm off}_{i_0}}\right] \tag{32}$$

and C_h, $h = 1, 2, \ldots, m$ are appropriate dimensional matrices whose elements are either one or zero. Furthermore, choose a scalar $\epsilon > 0$ and solve $\dot{w}_{i_h}(T_{\epsilon_i}) - \varepsilon = 0$ to obtain m minimum time with $T_{\epsilon_1}, \cdots, T_{\epsilon_m}$ that guarantees bumpless switching for the m components of online controller $\mathcal{C}_j$, $j \neq i \in \mathcal{I}_2$ respectively. Thus, define $T_\epsilon = \sup\{T_{\epsilon_1}, \ldots, T_{\epsilon_m}\}$.

Step 5: Then $\tau_d = \max\{T_{\rm s}, T_\epsilon\}$ is a feasible minimum dwell time for both the switched closed-loop system (24) stability and the bumpless switching of $\mathcal{C}_j$.

Remark 5.1. *(i) In the algorithm, one can obtain $T_{\rm s}$ by the stability of $A_{{\rm cl}_i}$, $i \in \mathcal{I}_2$. Moreover, suppose that $\dot{\bar{x}}_{{\rm off}_{i_0}}$ is available, then for a given $\varepsilon > 0$, T_ε can always be solved. (ii) The key point of the algorithm is the selection of β in Step 1. Apparently, it dose not seem to be possible to know beforehand which selection features the best rate of convergence. One can practically give an initial value of $\beta > 0$ with a small step length and decide to stop the process if there is no significant variation of T^*. In addition, to select a scalar $\epsilon > 0$ experientially in Step 4, for instance, one can give* $0 < \epsilon \leq 10^{-2}$.

6. **Simulations.** In this section, the nonlinear longitudinal model of the generic hypersonic flight vehicle [23] is considered. The linearized equations at the trim state and trim input ($\mathcal{V}$ = 3000m/s, γ = 0rad, h = 30000m, α = 0.0056rad, $\bar{q}$ = 0rad/s, $\delta_{\mathcal{T}}$ = 0.0710, δ_e = –0.1588rad) are obtained as (1), where $x(t) = \left[\begin{array}{ccccc} \mathcal{V}^T & \gamma^T & h^T & \alpha^T & \bar{q}^T \end{array}\right]^T$, $u(t) = \left[\begin{array}{cc} \delta_{\mathcal{T}}^T & \delta_e^T \end{array}\right]^T$, $y(t) = \left[\begin{array}{cc} \mathcal{V}^T & h^T \end{array}\right]^T$, $\mathcal{V}$, γ, h, α and $\bar{q}$ are the velocity, flight path angle, altitude, angle of attack and pitch rate, respectively. Fuel equivalence ratio ($\delta_{\mathcal{T}}$) and pitch control surface deflection (δ_e). And

$$A = 1.0 \times 10^{-4} \times \left[\begin{array}{ccccc} -10.3070 & -315133 & -0.5413 & -583366 & 0 \\ 0.0018 & 0 & -0.0012 & 1047.5000 & 0 \\ 0 & 98426600 & 0 & 0 & 0 \\ -0.0018 & 0 & 0.0012 & -1047.5000 & 10000 \\ 1.2263 & 0 & -0.0185 & 66236 & 0 \end{array}\right],$$

$$B = \left[\begin{array}{ccccc} 94.7710 & 0.0212 & 0 & -0.0212 & -7.8789 \\ 5.9407 & -0.0072 & 0 & 0.0072 & 3.5803 \end{array}\right]^T, \quad C = \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right].$$

Remark 6.1. *Note that (1) is a general linear system. It can be simulated to illustrate the usefulness and effectiveness of the obtained results in general, and be verified numerically.*

Here we consider the situation that the physical system including two controllers (one for the nominal tracking control, one for the hot standby control). The control objective is to make the output $y(t)$ track the command $r(t)$ even with degraded control effectiveness such as actuator fault. Suppose that the loss of control effectiveness or failure of the actuator takes place at a certain time. Then the primary controller fails to work properly, meanwhile, a switching action arises and forces the standby controller to take over.

6.1. **Example: comparison between the non-bumpless switching and the proposed bumpless switching.** For system (1), controllers (33) and (34) are obtained by using the algorithm in [21]. We select two sets of weighting matrices as $Q_1 = \mathrm{diag}(10, 12, 10, 8000)$, $R_1 = \mathrm{diag}(2, 3)$ and $Q_2 = \mathrm{diag}(0.1, 0.1, 100, 8000)$, $R_2 = 10^{-6} \times \mathrm{diag}(2, 10000)$, respectively. Then by solving (17) and using Equation (20), CCs (35) and (36) are obtained, respectively. Consequently, we can easily have the individual closed-loop system matrices of (24). The switching strategy $\sigma(t)$ is: controller (33) is used first and then authority is switched to controller (34) after the first dwell time τ_{d_1} (suppose that the actuator fault happens at τ_{d_1}s). Furthermore, to verify the bidirectional switching performance, the authority is changed again, and (34) is switched to (33) after τ_{d_2}.

According to the algorithm in Section 5, for $\beta = 0.176$, solving (31) can give two positive definite matrices P_1 and P_2. Then calculating (29) obtains $T_s > T^* = \frac{\ln\chi}{\beta} = 122.66$. Let $T^* \leq T_s = 123$. Furthermore, choosing $\epsilon = 0.01$, a simple calculation shows that the dwell time for bumpless switching in the two stages are $T_{\epsilon^1} = 184.02$ and $T_{\epsilon^2} = 100.64$, by letting the initial state of (32) in the two switching moments be $\left[5\ 5\ 0.48\ 0.01\right]^T$ and $\left[811.87\ 1603.62\ -566.57\ 61.12\right]^T$ respectively. Consequently, the minimum dwell time for the above two goals is given as $\tau_{d_1} = 184.02$ and $\tau_{d_2} = 123$.

The simulation results by direct implementation of switch are depicted in Figure 2(a). The results indicate that the system experiences an undesirable transient in control input, which seriously deteriorates the tracking performance after switching. Figure 2(b) shows the results by using the proposed bumpless switching scheme. By forcing the difference between off-line controller and plant input to a very small size, a "smooth" transition is allowed and the tracking performance is assured with minimized undesirable transients.

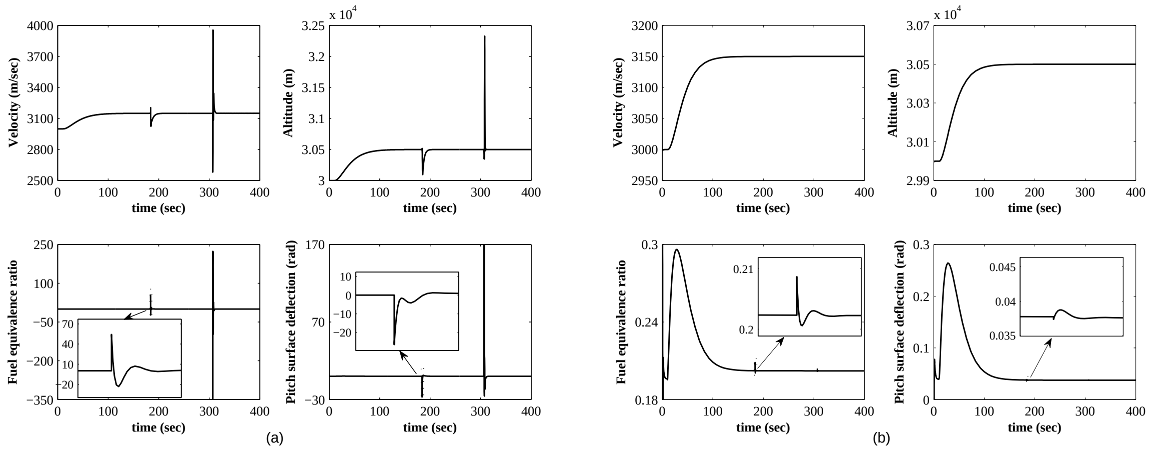

FIGURE 2. Response to the 500(m) altitude step order and 150(m/s) velocity step order (upper figure); control inputs (lower figure): (a) directly switch and (b) the proposed bumpless switching scheme

6.2. **Example: comparison between the scheme of [14] and the proposed bumpless switching.** For an accurate comparison between the method of [14] and the proposed method, we define a *bumpless performance index* that is a measurement of the bump size of plant control input. $\varrho = |u(t_{\max}) - u(t_{\rm s})|/u(t_{\rm s})$, where $u(t_{\max})$ is the first peak value of control input at the switching moment, and $u(t_{\rm s})$ is the steady-state value of control input. The smaller the index is, the better the bumpless performance will be. Without loss of generality, we make the performance analysis only at the first switch.

By method [14], we select two sets of weighting matrices as $W_{u_1} = 10^5 \times \mathrm{diag}(1, 10)$, $W_{e_1} = 10^3 \times \mathrm{diag}(10, 1)$ and $W_{u_2} = 10^5 \times \mathrm{diag}(1, 10)$, $W_{e_2} = 10^3 \times \mathrm{diag}(2, 1)$, respectively. Then, the bumpless switching compensatory gains (37) and (38) corresponding to (33) and (34) are obtained, respectively. The gain corresponds to the vector $\Theta = \left[\begin{array}{ccc} x_{c_i}^T & x^T & u^T \end{array}\right]^T$.

$$B_{c_{1_1}} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}, \quad B_{c_{1_2}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_{c_1} = \begin{bmatrix} 0.0051 & -0.0132 \\ 0.0076 & 0.0029 \end{bmatrix}, \tag{33}$$

$$D_{c_1} = \begin{bmatrix} -0.0256 & 310.0800 & 0.0507 & 12.5450 & 2.3023 \\ -0.0382 & -64.0150 & -0.0112 & -3.0220 & -0.8751 \end{bmatrix};$$

$$B_{c_{2_1}} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}, \quad B_{c_{2_2}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_{c_2} = \begin{bmatrix} 0.0176 & -0.1300 \\ 0.0237 & 0.0321 \end{bmatrix}, \tag{34}$$

$$D_{c_2} = \begin{bmatrix} -0.0344 & 555.9000 & 0.1283 & 18.3950 & 3.0869 \\ -0.0450 & -121.1200 & -0.0294 & -4.5301 & -1.1499 \end{bmatrix}.$$

$$K_{r_1} = 1.0 \times 10^{-14} \times \begin{bmatrix} 0 & 0.0220 \\ 0 & -0.0220 \end{bmatrix}, \quad K_{c_1} = \begin{bmatrix} -2.6433 & -0.4784 \\ -0.3189 & -2.5622 \end{bmatrix}, \tag{35}$$

$$K_{w_1} = \begin{bmatrix} 1.6105 & 43.8760 \\ -1.2666 & 37.1930 \end{bmatrix}, \quad K_{x_1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad K_{u_1} = \begin{bmatrix} 20.1300 & 96.4360 \\ -13.8540 & 23.6950 \end{bmatrix};$$

$$K_{r_2} = 1.0 \times 10^{-13} \times \begin{bmatrix} 0 & 0.8470 \\ 0 & 0 \end{bmatrix}, \quad K_{c_2} = \begin{bmatrix} -2238.2000 & -548.6100 \\ 0 & -0.9614 \end{bmatrix}, \tag{36}$$

$$K_{w_2} = \begin{bmatrix} 9870 & 626260 \\ -20 & 20 \end{bmatrix}, \quad K_{x_2} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad K_{u_2} = \begin{bmatrix} 20720 & 279950 \\ -60 & 40 \end{bmatrix}.$$

$$F_1 = \begin{bmatrix} 0.2266 & -0.0285 & 1.4312 & -10.6160 & 0.0094 & 3.9339 & 3.8581 & 1.8071 & 8.2063 \\ -0.2847 & -1.6472 & 0.5298 & 6750.7000 & 2.5840 & 232.8200 & 46.1120 & -8.2063 & 18.0710 \end{bmatrix}, \tag{37}$$

$$F_2 = \begin{bmatrix} -0.1708 & -0.0238 & 1.8564 & 457.3500 & 0.0888 & 27.3920 & 11.6200 & 2.7719 & 20.5710 \\ -0.0477 & -0.1578 & 0.2373 & 3646.2000 & 1.6048 & 152.8700 & 32.0280 & -9.1996 & 12.3960 \end{bmatrix}. \tag{38}$$

The simulation results by method [14] are shown in Figure 3. Note that the tracking performance is not bad, which, unfortunately, is at the cost of serious deterioration of the plant control input. Furthermore, we can obtain the detailed characterization. As shown in Tables 1 and 2, the bumpless performance indices of fuel equivalence ratio and pitch surface deflection got by [14] are 44.24% and 242.02%, respectively. However, the indices obtained by our method are much smaller, which are 3.17% and 2.93%, respectively. Thus, it clearly demonstrates that the proposed method achieves a better bumpless performance.

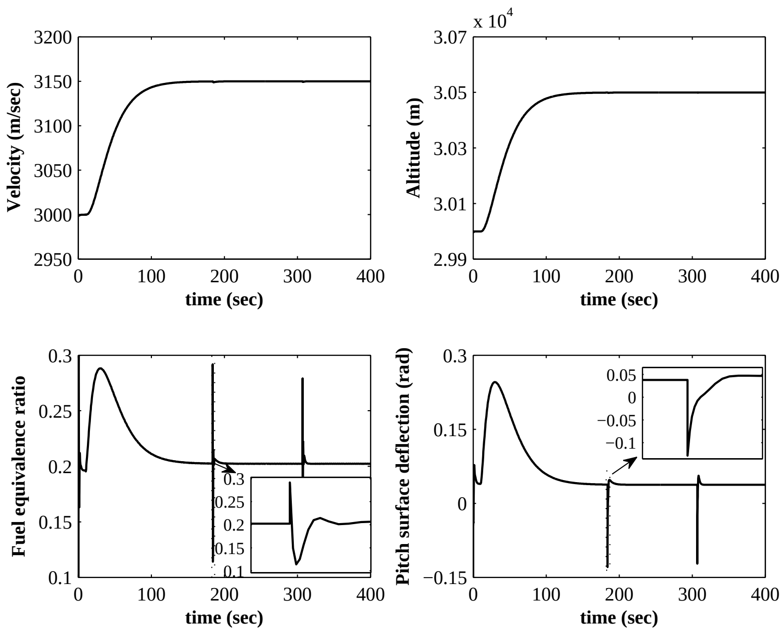

FIGURE 3. Response to the 500(m) altitude step order and 150(m/s) velocity step order with the bumpless switching scheme of [14] (upper figure); control inputs with the bumpless switching scheme of [14] (lower figure)

TABLE 1. Comparison of $\delta_{\mathcal{T}}$

Method	$u(t_{\max})$	$u(t_{\text{s}})$	ϱ
Non-bumpless switching	54.2050	0.2085	25897.60%
Turner and Walker [14]	0.2918	0.2023	44.24%
The proposed method	0.2086	0.2022	3.17%

7. **Conclusions.** In this paper, we have dealt with the problem of bumpless switching performance optimal control for continuous-time switched linear systems. Regarding the bumpy phenomenon, firstly, a new extended bumpless switching scheme has been proposed

Table 2. Comparison of δ_e (rad)

Method	$u(t_{\max})$	$u(t_{\mathrm{s}})$	ϱ
Non-bumpless switching	–26.4745	0.0376	70310.90%
Turner and Walker [14]	–0.1286	0.0376	242.02%
The proposed method	0.0387	0.0376	2.93%

based on the optimal theory and the internal model principle. After that, we adopt multiple Lyapunov functions method and dwell time theory to obtain an estimate of the minimum dwell time and derive a sufficient condition for both the exponential stability of switched closed-loop systems and the bumpless switching accordingly. Finally, two examples have confirmed the effectiveness of the proposed bumpless switching approach.

Acknowledgment. This work is partially supported by the National Nature Science Foundation of China under Grant No. 90716012 and also by China Aerospace Support Foundation No. 2008-HT-HGD-03. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

[1] D. Liberzon, *Switching in Systems and Control*, Birkhauser, Boston, 2003.
[2] J. Zhao and G. M. Dimirovski, Quadratic stability of a class of switched nonlinear systems, *IEEE Transactions on Automatic Control*, vol.49, no.4, pp.574-578, 2004.
[3] D. Cheng, Stabilization of planar switched systems, *Systems and Control Letters*, vol.51, no.2, pp.79-88, 2004.
[4] R. Wang and J. Zhao, Exponential stability analysis for discrete-time switched linear systems with time-delay, *International Journal of Innovative Computing, Information and Control*, vol.3, no.6(B), pp.1557-1564, 2007.
[5] J. Zhao and D. J. Hill, On stability, L_2-gain and H_∞ control for switched systems, *Automatica*, vol.44, no.5, pp.1220-1232, 2008.
[6] H. Yang, X. Sun and X. Liu, Stabilization and switched control for a class of switched fuzzy systems, *International Journal of Innovative Computing, Information and Control*, vol.5, no.10(A), pp.3099-3108, 2009.
[7] A. Benzaouia and F. Tadeo, Stabilization of positive switching linear discrete-time systems, *International Journal of Innovative Computing, Information and Control*, vol.6, no.6, pp.2427-2437, 2010.
[8] L. Zaccarian and A. R. Teel, A common framework for anti-windup, bumpless transfer and reliable designs, *Automatica*, vol.38, no.10, pp.1735-1744, 2002.
[9] J. D. Bendtsen, J. Stoustrup and K. Trangbaek, Bumpless transfer between observer-based gain scheduled controllers, *International Journal of Control*, vol.78, no.7, pp.491-504, 2005.
[10] M. C. Turner, N. Aouf, D. G. Bates et al., Switched control of a vertical/short take-off land aircraft: An application of linear quadratic bumpless transfer, *Proc. of IMechE, Part I: Journal of Systems and Control Engineering*, vol.220, no.3, pp.157-170, 2006.
[11] J. Stoustrup and M. Komareji, A parameterization of observer-based controllers: Bumpless transfer by covariance interpolation, *Proc. of American Control Conf.*, St. Louis, USA, pp.1871-1875, 2009.
[12] K. Zheng, T. Basar and J. Bentsman, H_∞ bumpless transfer under controller uncertainty, *IEEE Transactions on Automatic Control*, vol.54, no.7, pp.1718-1723, 2009.
[13] R. Hanus, M. Kinnaert and J. L. Henrotte, Conditioning technique, a general anti-windup and bumpless transfer method, *Automatica*, vol.23, no.6, pp.729-739, 1987.
[14] M. C. Turner and D. J. Walker, Linear quadratic bumpless transfer, *Automatica*, vol.36, no.8, pp.1089-1101, 2000.
[15] L. Zaccarian and A. R. Teel, The L_2 bumpless transfer problem for linear plants: Its definition and solution, *Automatica*, vol.41, no.7, pp.1718-1723, 2005.
[16] A. B. Arehart and W. A. Wolovich, Bumpless switching controllers, *Proc. of the 35th IEEE Conf. on Decision & Control*, Kobe, Japan, pp.1654-1655, 1996.

[17] K. Zheng and J. Bentsman, Input/output structure of the infinite horizon LQ bumpless transfer and its implications for transfer operator synthesis, *International Journal of Robust and Nonlinear Control*, vol.20, no.8, pp.923-938, 2010.

[18] K. Zheng, J. Bentsman and C. W. Taft, Full operating range robust hybrid control of a coal-fired boiler/turbine unit, *ASME Journal of Dynamic Systems, Measurement, and Control*, vol.130, no.4, pp.041011.1-041011.14, 2008.

[19] A. Smirnov, K. Tolsa and R. P. Jastrzebski, Implementation of a bumpless switch in axial magnetic bearings, *Proc. of International Symposium on Industrial Embedded Systems*, Trento, Italy, pp.63-68, 2010.

[20] I. Malloci, L. Hetel, J. Daafouz et al., Bumpless transfer for discrete-time switched systems, *Proc. of American Control Conf.*, St. Louis, USA, pp.1078-1083, 2009.

[21] Y. Huo, M. Mirmirani, P. Ioannou et al., Altitude and velocity tracking control for an airbreathing hypersonic cruise vehicle, *Proc. of AIAA Guidance, Navigation, and Control Conf. and Exhibit*, Keystone, USA, pp.4690-4699, 2006.

[22] B. D. O. Anderson and J. B. Moore, *Linear Optimal Control*, Prentice Hall, Englewood Cliffs, New Jersey, 1971.

[23] Z. H. Yao, W. Bao, J. T. Chang et al., Modeling for couplings of an airframe-propulsion integrated hypersonic vehicle with engine safety boundarie, *Proc. of IMechE, Part G: Journal of Aerospace Engineering*, vol.224, no.1, pp.43-55, 2010.