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LOCAL FEEDBACK UNKNOWN INPUT OBSERVER FOR NONLINEAR SYSTEMS

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Abstract. *This paper proposes a new design of Functional unknown input observer for nonlinear systems. It represents an extension of the Luenberger observer with unknown inputs using the Taylor expansion for first order. The Local observer is characterized by its simplicity in the mathematical development. The aim of this observer is the estimate of the state function which represents the control law for example. The necessary and sufficient conditions for the existence of the observer are given. An numerical example is given to illustrate the attractiveness and the simplicity of the new design procedure.*
Keywords: Nonlinear systems, Unknown input observers, Stability, Robust state function estimation

1. **Introduction.** Control systems are used for regulating a great variety of machines, and processes. They control quantities such as motion, temperature, heat flow, fluid flow and fluid pressure. Most concepts in control theory are based on having sensors to measure the quantity under control. The performance of a feedback control system is of primary importance. Generally, we are interested in controlling the system with a control signal $u(t)$ that is a function of several measurable state variables:

$$u(t) = v(t) - r(t) \tag{1}$$

where $v(t) = Kx(t)$, therefore, K is a feedback matrix; $r(t)$ is the reference input vector to the system and $x(t)$ is the state vector. A block diagram of the closed loop system is seen in Figure 1.

If the entire state vector cannot be measured, in most complex systems, the control law cannot be implemented. However, it is usually not practical because it is not possible (in general) to measure all the states. In practice, only certain states are measured and provided as system outputs. In such a case, either a new approach that directly accounts for the no availability of the entire state vector must be devised, or a suitable approximation to the state vector must be determined. In this context, we can construct an unknown input observer which estimates a state function $v(t)$ as shown in Figure 2.

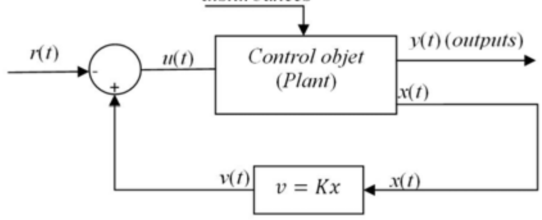

Figure 1. Closed loop system with full state feedback

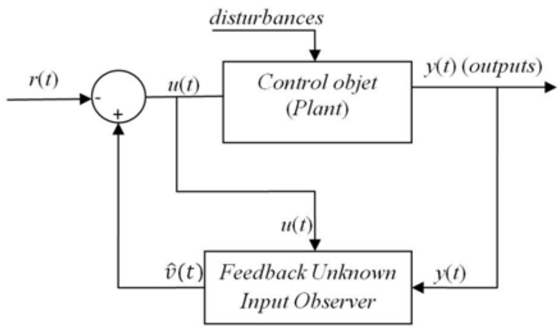

Figure 2. State feedback with full order observer

Feedback unknown input observer may be used to estimate the state function in presence of unknown input disturbances. We can cite some works in the context of linear systems as [1-7]. This observer is extended to nonlinear system. Most of the work uses a particular class of nonlinear system to build their observer; this class of system is obtained after a state transformation to change the original nonlinear systems into canonical forms. Some authors use the Lipschitz class of nonlinear systems for construction of the unknown input observer [8-11], and this dynamic system is described under the following form [8]:

$$\begin{cases} \dot{x} = Ax + Bu + g(x, u, t) \\ y = Cx + Df_D \end{cases} \tag{2}$$

where x, u, f_D and y are the state vector, the inputs, the sensor fault and the outputs of the system, therefore, $g(x, u, t)$ is assumed to be globally Lipschitz with respect to x:

$$\|g(x_1, u, t) - g(x_2, u, t)\| \leq \lambda \|x_1 - x_2\|, \quad \forall u, \forall t \geq 0. \tag{3}$$

It has proposed a functional observer for estimating the state function $v = C_2x$. This observer can be written as follows:

$$\begin{cases} \dot{w} = Nw + Ly_1 + Pg(\hat{v}, u, t) + PBu \\ \hat{v} = w + Qy_1 \end{cases} \tag{4}$$

In this paper, we present a new unknown input observer functional observer of nonlinear system. Its design is based on the linearization along a trajectory using Taylor development. The observer is able to trace the trajectory of state vector in presence of unknown inputs. The advantage of its design is that it is not based on the transformation of the nonlinear system into its canonical form. This paper presents new conditions for the existence of the robust nonlinear observer and describes step by step the design method. A numerical example is given to illustrate the attractiveness and simplicity of the new design procedure.

2. **Problem Statement.** Consider a nonlinear system described by state equation and measurements equations, augmented by a state function useful for control or diagnosis:

$$\begin{cases} \dot{x} = f(x) + Bu + Ed, \quad x(t=0) = x_0 \\ y = Cx \\ v = Mx \end{cases} \tag{5}$$

where $x \in \mathbb{R}^n$ describes the state of the system, $u \in \mathbb{R}^k$ the inputs, $d \in \mathbb{R}^p$ the unknown inputs, $v \in \mathbb{R}^s$ the state function to be estimated and $y \in \mathbb{R}^m$ the outputs of the system. $B \in \mathbb{R}^{n\times k}$, $E \in \mathbb{R}^{n\times p}$, $M \in \mathbb{R}^{s\times n}$ and $C \in \mathbb{R}^{m\times n}$ are known constant matrices of appropriate dimensions. $f(.)$ is supposed to be continuously differentiable. We assume that $rank[E] = p$ and $rank[C] = m$.

The aim of this paper is: Construct an observer such that it can estimate the state function of the considered nonlinear systems asymptotically without any knowledge of the input d.

In the following forms, such that $F_{\hat{x}}$, $K_{\hat{x}}$, $f_{\hat{x}}$, $h_{\hat{x}}$ and f_x represent respectively $F(\hat{x})$, $K(\hat{x})$, $f(\hat{x})$, $h(\hat{x})$ and $f(x)$.

3. **Main Results.** In this section, we propose an unknown input observer for nonlinear systems based on first order Taylor approximation.

Theorem 3.1. *We propose a local functional observer for the system described by (5), such that:*

$$\hat{v} = Nz + Ly \tag{6}$$

where z is given by:

$$\dot{z} = F_{\hat{x}}z + K_{\hat{x}}y + Tu + P(f_{\hat{x}} - D_x(f_{\hat{x}})\hat{x}) \tag{7}$$

$\hat{x}$ is defined by $z + Hy$ and different matrices of the observer checking:

$$F_{\hat{x}} = PD_x(f_{\hat{x}}) - K_{1\hat{x}}C \tag{8a}$$
$$K_{\hat{x}} = K_{1\hat{x}} + F_{\hat{x}}H \tag{8b}$$
$$PE = 0 \tag{8c}$$
$$T + PB = 0 \tag{8d}$$
$$L = MH \tag{8e}$$
$$N = M \tag{8f}$$

The following conditions are necessary for the existence of the observer:

$$rank[CE] = rank[E] = p \tag{9a}$$
$$PD_x(f_{\hat{x}}) - P_1^{-1}G_{\hat{x}}C^TQC < 0 \tag{9b}$$

where $z \in \mathbb{R}^n$, $\hat{v} \in \mathbb{R}^s$ and $D_x(f_{\hat{x}})$ are the state, the output vector of observer and the Jacobian matrix of f with respect to $\hat{x}$. $F_{\hat{x}} \in \mathbb{R}^{n\times n}$, $T \in \mathbb{R}^{n\times k}$, $K_{\hat{x}} \in \mathbb{R}^{n\times m}$, $H \in \mathbb{R}^{n\times m}$, $L \in \mathbb{R}^{s\times m}$, $N \in \mathbb{R}^{s\times n}$ and $P \in \mathbb{R}^{n\times n}$ are matrices which have to be designed such that $\hat{v}$ asymptotically converges to v, P_1 is a symmetric positive definite matrix, $G_{\hat{x}}$ is a diagonal positive definite matrix.

The nonlinear state feedback observer schema is presented by Figure 3.

Proof: Let the estimation error e_x:

$$e_x = x - \hat{x} = x - z - Hy = (I - HC)x - z \tag{10}$$

Let $P = I - HC$, with I is n identity matrix.

And put $e_v = Me_x$ such that $e_v = v - \hat{v}$ then the reconstruction of state function becomes possible. So, we proceed as follows:

$$e_v = v - \hat{v} = Mx - Nz - Ly \tag{11}$$

The estimation error e_v is also written:

$$e_v = Me_x = Mx - Mz - MHy \tag{12}$$

From (11) and (12), the estimation error $e_v = Me_x$ if only if:

$$N = M \text{ and } L = MH$$

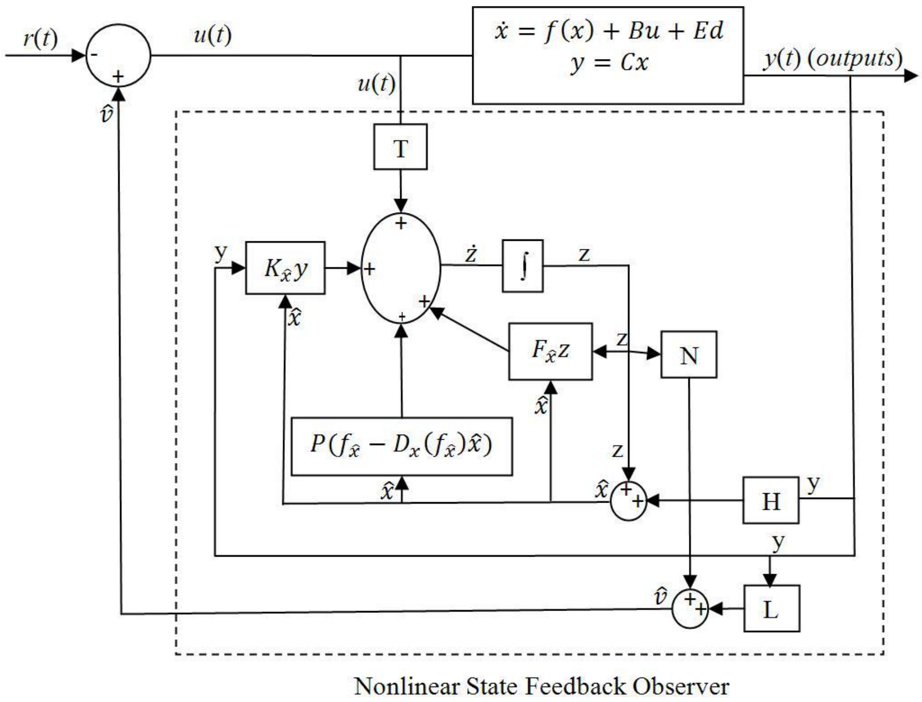

FIGURE 3. Nonlinear state feedback observer

In order to prove the stability and convergence of the observer, the state estimation error dynamic is analyzed. From the expression of the estimation error, we can write:

$$e_x = x - \hat{x} = x - z - Hy = -z + (I - HC)x \tag{13}$$

Let $P = I - HC$, (13) becomes:

$$e_x = Px - z \tag{14}$$

In order to investigate the stability and the convergence of the observer the state estimation error dynamics is analyzed:

$$\dot{e_x} = \dot{x} - \dot{\hat{x}} = P\dot{x} - \dot{z} \tag{15}$$

The local observer asymptotically reconstructs the state function of the system (5), if the estimation error $e_{\hat{x}}$ asymptotically converges to zero as t tends to infinity, whatever the initial state of the observer z_0 and the control of the system u. Then we can approximate the function $f(x)$ by its Taylor development to first order along of the trajectory, if $\hat{x}$ is become sufficiently close to x, we can write:

$$f_x = f_{\hat{x}+e_x} = f_{\hat{x}} + D_x(f_{\hat{x}})e_x + h_{\hat{x}} \tag{16}$$

where $h_{\hat{x}}$ the higher order terms and D_x is the differential operator defined by:

$$D_x(f_{\hat{x}}) = \left.\frac{\partial f_x}{\partial x}\right|_{x=\hat{x}} \tag{17}$$

Obviously, proving stability using this kind of the Taylor expansion works only if the nonlinearity is not too significant and higher order terms can be neglected.

We take into account the relation (15), we obtain:

$$\dot{e_x} = -F_{\hat{x}}z - Tu - K_{\hat{x}}y - Pf_{\hat{x}} + PD_x(f_{\hat{x}})\hat{x} + P(f_x + Bu + Ed) \tag{18}$$

The Taylor expansion approach leads to the following error dynamics:

$$\dot{e_x} \approx -F_{\hat{x}}z - Tu - K_{\hat{x}}y - Pf_{\hat{x}} + PD_x(f_{\hat{x}})\hat{x} + Pf_{\hat{x}} + PBu + PD_x(f_{\hat{x}})e_x + PEd \tag{19}$$

Now, we assume that $K_{\hat{x}} = K_{1\hat{x}} + K_{2\hat{x}}$ and we replace z by $(\hat{x} - Hy)$. The expression (19) becomes:

$$\dot{e_x} \approx -F_{\hat{x}}\hat{x} + (PD_x(f_{\hat{x}}) - K_{1\hat{x}}C)x + (-K_{2\hat{x}} + F_{\hat{x}}H)y + (PB - T)u + PEd \tag{20}$$

Our objective is to determine the conditions to guaranty that $\hat{x}$ converges asymptotically to x. So, we proceed as follows:

$$\begin{aligned} \dot{e_x} \approx\ & (PD_x(f_{\hat{x}}) - K_{1\hat{x}}C)e_x - (F_{\hat{x}} - (PD_x(f_{\hat{x}}) - K_{1\hat{x}}C)\hat{x} \\ & +(F_{\hat{x}}H - K_{2\hat{x}})y + (PB - T)u + PEd \end{aligned} \tag{21}$$

In Equation (21), $K_{1\hat{x}}$, $K_{2\hat{x}}$, T and P are matrices, which be determined to design an observer given by (8). It is very useful to choose an error given by an autonomous process, independent of variable d, y, u and $\hat{x}$. This leads to:

$$\begin{aligned} & F_{\hat{x}} + (K_{1\hat{x}}C - PD_x(f_{\hat{x}})) = 0 \\ & F_{\hat{x}}H - K_{2\hat{x}} = 0 \\ & PE = 0 \\ & PB - T = 0 \end{aligned}$$

We obtain the new equation $\dot{e_x} \approx (PD_x(f_{\hat{x}}) - K_{1\hat{x}}C)e_x$. The convergence of the estimator is more dependant of the term $F_{\hat{x}} = PD_x(f_{\hat{x}}) - K_{1\hat{x}}C$.

3.1. **Proof of the existence conditions.**

3.1.1. *Condition (9a).* From (8), we have:

$$PE = (I - HC)E = 0 \Longrightarrow HCE = E \tag{22}$$

The solution of this equation depends on the rank of matrix CE, So,

$$H \text{ exists } iff \ \ rank[CE] = rank[E] = p. \tag{23}$$

3.1.2. *Condition (9b).* The aim is to define the matrix $K_{1\hat{x}}$ so that the error of estimation converges asymptotically to zero. Let

$$V(e_x) = \frac{1}{2}e_x^T P_1 e_x \tag{24}$$

where P_1 is a symmetric positive definite matrix, the dynamic Lyapunov function can be writing:

$$\dot{V}(e_x) = e_x^T P_1(PD_x(f_{\hat{x}}) - K_{1\hat{x}}C)e_x \tag{25}$$

To ensure the asymptotic convergence of e to zero, we require time derivative of V to be negative.

In [13], he proposed an algorithm for determining the gain $K_{1\hat{x}}$ based on the assumption that $ker(C) \neq \{0\}$. The algorithm comprises two step:

Step (1): Considering the assumption that $ker(C) \neq \{0\}$, it can write (25) under the following form:

$$\dot{V}(e_x) = \bar{e_x}^T N^T P_1 PD_x(f_{\hat{x}})N\bar{e_x} \tag{26}$$

where N is right orthogonal to C and $e = N\bar{e}$. Tsinias showed in [13], that there exists a positif constant k_1 such that for any $e_x \in ker(C) - \{0\}$ there is a neighborhood S_{e_x} of e_x such that:

$$\bar{e_x}^T N^T P_1 PD_x(f_{\hat{x}})N\bar{e_x} \leq -k_1\|\bar{e_x}\|^2 \tag{27}$$

$\forall(\hat{x}, \bar{e_x}) \in \mathbb{R}^n \times S_{e_x}$.

Generally, P_1 is determined by solving inequalities using algebraic techniques to increase function. A solution exists if the nonlinearities of the considered system are bounded.

If $ker(C)$ is reduced to $\{0\}$, Step (1) is bypassed and P_1 taking equal to the identity matrix.

Step (2): The matrix P_1 is determined. Now, considered $e \in \mathbb{R}^n$ and research the matrix $K_{1\hat{x}}$ satisfying the condition of convergence of e to zero.

$$\dot{V}(e_x) = e_x^T P_1 P D_x(f_{\hat{x}}) e_x - e_x^T P_1 K_{1\hat{x}} C e_x < 0 \tag{28}$$

We know that:

$$e_x^T \lambda_{\min}(P_1) P D_x(f_{\hat{x}}) e_x \leq e_x^T P_1 P D_x(f_{\hat{x}}) e_x \leq e_x^T \lambda_{\max}(P_1) P D_x(f_{\hat{x}}) e_x \tag{29}$$

with λ is the eigenvalue of P_1, we assume that $G_{\hat{x}} = \lambda_{\max}(P_1) P D_x(f_{\hat{x}})$:

$$\begin{aligned} e_x^T G_{\hat{x}} e_x &= \sum_{i,j=1}^{n} g_{ij} e_i e_j \\ &\leq \sum_{i,j=1}^{n} |g_{ij}||e_i e_j| \end{aligned} \tag{30}$$

where g_{ij} represents the coefficients of the matrix $G_{\hat{x}}$.

We know that:

$$|e_i e_j| \leq \frac{1}{2}(e_i^2 + e_j^2) \tag{31}$$

Then, (30) becomes:

$$\begin{aligned} e_x^T G_{\hat{x}} e_x &\leq \frac{1}{2} \sum_{i,j=1}^{n} \left(|g_{ij}|e_i^2 + |g_{ij}|e_j^2\right) < \sum_{i,j=1}^{n} \left(|g_{ij}|e_i^2 + |g_{ij}|e_j^2\right) \\ &\leq \sum_{k=1}^{n} \left(\sum_{j=1}^{n} |g_{kj}|\right) e_k^2 + \sum_{k=1}^{n} \left(\sum_{i=1}^{n} |g_{ik}|\right) e_k^2 \end{aligned} \tag{32}$$

So, we obtain:

$$e_x^T G_{\hat{x}} e_x \leq |e_x^T G_{\hat{x}} e_x| < \sum_{k=1}^{n} \left(\sum_{j=1}^{n} |g_{kj}| + \sum_{i=1}^{n} |g_{ik}|\right) e_k^2 \tag{33}$$

Following this latest development, we can conclude that for all $(e, \hat{x}) \in \mathbb{R}^n$, we find:

$$e_x^T G_{\hat{x}} e_x < e_x^T \overline{G}_{\hat{x}} e_x \tag{34}$$

where $\overline{G}_{\hat{x}}$ is a diagonal matrix, such that:

$$\overline{G}_{\hat{x}} = \begin{pmatrix} \alpha_1(\hat{x}) & 0 & 0 & \cdots & 0 \\ 0 & \alpha_2(\hat{x}) & 0 & \cdots & 0 \\ 0 & 0 & \alpha_3(\hat{x}) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \alpha_n(\hat{x}) \end{pmatrix} \tag{35}$$

with

$$\alpha_k(\hat{x}) = \sum_{j=1}^{n} |g_{kj}| + \sum_{i=1}^{n} |g_{ik}| \text{ such that } k = 1, 2, \ldots, n. \tag{36}$$

A sufficient condition to fulfill this inequality is that the matrix $PD_x(f_{\hat{x}}) - P_1 K_{1\hat{x}} C$ be negative semi-definite. We note that:

$$\dot{V}(e_x) \leq |e_x^T G_{\hat{x}} e_x| - e_x^T P_1 K_{1\hat{x}} C e_x \tag{37}$$

$\dot{V}(e_x)$ will be negative, if $K_{1\hat{x}}$ is defined such that the term $e_x^T P_1 K_{1\hat{x}} C e_x$ is a positive function and sufficiently large for the second member of (28) is negative.

A structure of $K_{1\hat{x}}$ to satisfy this conditions is:

$$K_{1\hat{x}} = P_1^{-1} G_{\hat{x}} C^T Q \tag{38}$$

where Q is a m dimensional square matrices to be determined, we obtain the following equality:

$$\dot{V}(e_x) = e_x^T P_1 P D_x(f_{\hat{x}}) e_x - e_x^T G_{\hat{x}} C^T Q C e_x \tag{39}$$

To increase the first term of the previous equality, we seek to determine a positif constant $\overline{G}_{\hat{x}}$ satisfying the following inequality:

$$|e_x^T P_1 P D_x(f_{\hat{x}}) e_x| < e_x^T \overline{G}_{\hat{x}} e_x \tag{40}$$

We choose then $G_{\hat{x}} = \overline{G}_{\hat{x}}$. Then, we obtain the following inequality:

$$\dot{V}(e_x) < e_x^T \overline{G}_{\hat{x}} e_x - e_x^T \overline{G}_{\hat{x}} C^T Q C e_x \tag{41}$$

Note here that the inequality can be rewritten as follows:

$$\dot{V}(e_x) < e_x^T \overline{G}_{\hat{x}} (I - C^T Q C) e_x < 0 \tag{42}$$

where Q is a matrix satisfying $C^T Q C - I \geq 0$. With this selection of the matrix $K_{1\hat{x}}$, the equilibrium $e_x = 0$ is asymptotically stable for the first order approximation.

4. **Illustrative Example.** We consider the nonlinear dynamic system with unknown input writing under the following form:

$$\begin{cases} \dot{x} = f_x + Bu + Ed \\ y = Cx \\ v = Mx \end{cases}$$

where $f_x = \begin{pmatrix} x_2 \\ -\sin(x_1) - 0.4x_2x_3 \\ \frac{1}{1+x_2^2} - x_3 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 0.4 \\ 0 \end{pmatrix}$, $E = \begin{pmatrix} -x_1 \\ 0 \\ 0 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$, $M = \begin{pmatrix} -2 & 1 & -4 \\ 0 & 1 & -2 \end{pmatrix}$, $u \in [\ -1 \quad 1\ ]$ and

$$\begin{array}{ll} d = 0 & \text{for } [\ 0 \quad 100s\ ] \text{ and } [\ 150s \quad 300s\ ], \\ d = 1 & \text{for } [\ 100s \quad 150s\ ]. \end{array}$$

We choose a projection matrix P such that $PE = 0$, we take P equal to:

$$P = \begin{pmatrix} 0 & -0.1 & 0.5 \\ 0 & -0.1 & -1 \\ 0 & 1 & 0.5 \end{pmatrix}$$

So, the matrix $PD_x(f_{\hat{x}})$ can be written:

$$PD_{\hat{x}}(f_x) = \begin{pmatrix} 0.1\cos(\hat{x}_1) & 0.04\hat{x}_3 - \frac{\hat{x}_2}{(\hat{x}_2^2+1)^2} & 0.04\hat{x}_2 - 0.5 \\ 0.1\cos(\hat{x}_1) & 0.04\hat{x}_3 + \frac{2x_2}{(\hat{x}_2^2+1)^2} & 0.04\hat{x}_2 + 1 \\ \cos(\hat{x}_1) & 0.4\hat{x}_3 - \frac{\hat{x}_2}{(\hat{x}_2^2+1)^2} & 0.4\hat{x}_2 - 0.5 \end{pmatrix}$$

Now, we seek a matrix P_1 such that, we obtain:

$$\dot{V}(e_x) = \bar{e_x}^T N^T P_1 P D_x(f_{\hat{x}}) N \bar{e_x}$$

where N is right orthogonal to C:

$$N = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \Longrightarrow e_x = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} e_1$$

If we choose $P_1 = \begin{pmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \end{pmatrix}$ then the dynamic of the Lyapunov function can be written:

$$\begin{aligned}\dot{V}(e_x) = \Bigg(p_1\left(0.1\cos(\hat{x}_1) + 0.04\hat{x}_3 - \frac{\hat{x}_2}{(\hat{x}_2^2+1)^2} - 0.04\hat{x}_2 + 0.5\right) \\ + p_2\left(0.1\cos(\hat{x}_1) + 0.04\hat{x}_3 + \frac{2\hat{x}_2}{(\hat{x}_2^2+1)^2} - 0.04\hat{x}_2 - 1\right) \\ + p_3\left(0.1\cos(\hat{x}_1) + 0.4(\hat{x}_3+\hat{x}_2) + \frac{\hat{x}_2}{(\hat{x}_2^2+1)^2} + 0.5\right)\Bigg) e_1^2 < 0\end{aligned}$$

$$\begin{aligned}\dot{V}(e_x) \leq \Bigg(p_1|0.1\cos(\hat{x}_1)| + 0.04p_1|\hat{x}_3+\hat{x}_2| + p_1\left|\frac{\hat{x}_2}{(\hat{x}_2^2+1)^2}\right| + 0.5p_1 \\ + p_2|0.1\cos(\hat{x}_1)| + 0.04p_2|\hat{x}_3+\hat{x}_2| + p_2\left|\frac{2\hat{x}_2}{(\hat{x}_2^2+1)^2}\right| - p_2 \\ + p_3|0.1\cos(\hat{x}_1)| + 0.4p_3|\hat{x}_3+\hat{x}_2| + p_3\left|\frac{\hat{x}_2}{(\hat{x}_2^2+1)^2}\right| + 0.5p_3\Bigg) e_1^2 < 0\end{aligned}$$

The state x_1 and $x_3 \in [\ -1 \quad 1\ ]$ and $x_2 \in [\ -0.8 \quad 1\ ]$.

Thus, we obtain:

$$\dot{V}(e_x) \leq (0.93p_1 - 0.32p_2 + 1.65p_3)e_1^2 < 0$$

We can choose the matrix P_1 follows, to satisfy the previous inequality:

$$P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 0.1 \end{pmatrix}$$

We can then determinate $\overline{G}_{\hat{x}} = diag(\alpha_i(\hat{x}))$, $i = 1, 2, 3$, whatever $e_x(t) \in \mathbb{R}^n$, such that:

$$\alpha_1(\hat{x}) = 0.1|\cos(\hat{x}_1)| + 0.5\left|0.04\hat{x}_3 + 0.1\cos(\hat{x}_1) - \frac{\hat{x}_2}{(\hat{x}_2^2+1)^2}\right| + 0.5\left|\cos(\hat{x}_1) - 0.04\hat{x}_2 + 0.5\right|$$

$$\begin{aligned}\alpha_2(\hat{x}) = \left|0.04\hat{x}_3 + \frac{2\hat{x}_2}{(\hat{x}_2^2+1)^2}\right| + 0.5\left|\frac{\hat{x}_2}{(\hat{x}_2^2+1)^2} - 0.1\cos(\hat{x}_1) - 0.04\hat{x}_3\right| \\ + 0.5\left|0.04\hat{x}_2 - 0.4\hat{x}_3 - \frac{\hat{x}_2}{(\hat{x}_2^2+1)^2} + 1\right|\end{aligned}$$

$$\alpha_3(\hat{x}) = |0.4\hat{x}_2 + 0.5| + 0.5|\cos(\hat{x}_1) - 0.04\hat{x}_2 + 0.5| + 0.5\left|0.04\hat{x}_2 - 0.4\hat{x}_3 - \frac{\hat{x}_2}{(\hat{x}_2^2+1)^2} + 1\right|$$

The next step is to find a matrix Q rending $(C^TQC - I)$ positive semi-definite. We find:

$$Q = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$$

4.1. **Simulation results.** Figure 4 represents the evolution of the state function vector during the interval of time t = [0   300s], with a command $u = 1$. We have introduced a disturbance between times $t_1 = 100s$ and $t_2 = 150s$ and we added in the output system a gaussian noise variance equals to 0.001 and average value of zero.

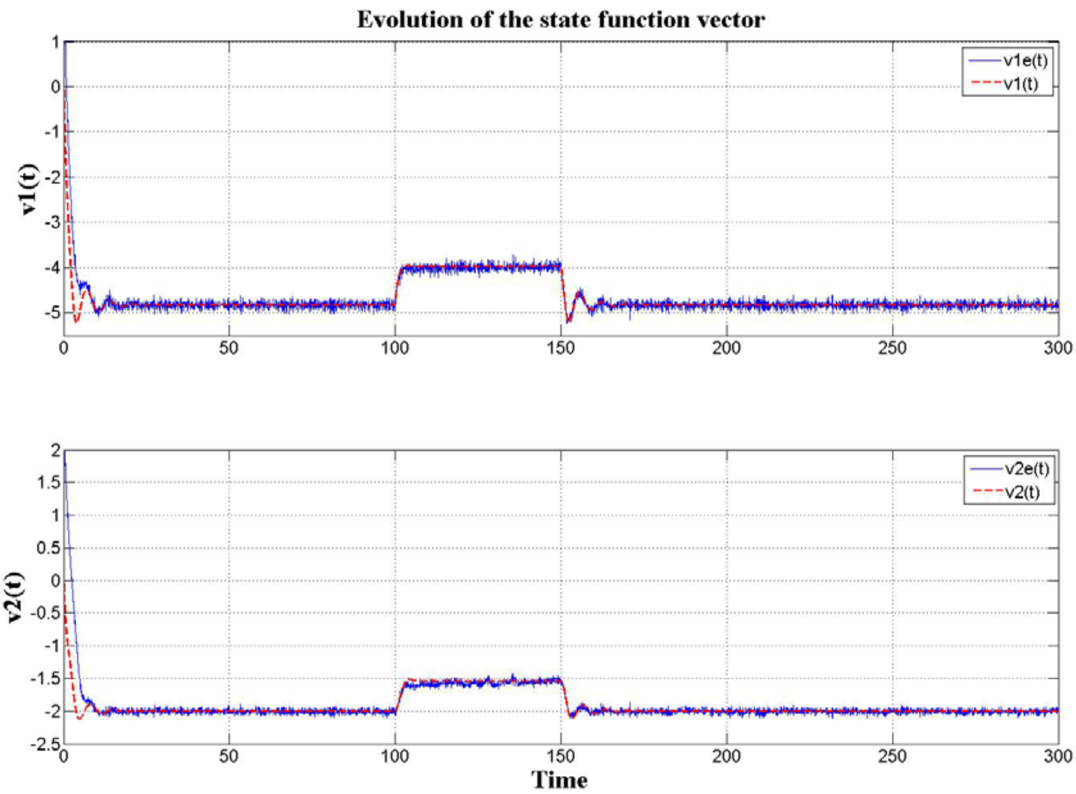

FIGURE 4. Evolution of the state function vector

We note that the output observer converges quickly to the state function system even in the presence of the disturbance.

To see the behavior of the output observer $\hat{v}$ relative to the state function v. We study the evolution of the estimation error $e_v = v - \hat{v}$.

In Figure 5, we see that the estimation error converges rapidly to zero.

5. **Conclusion.** This paper has proposed a novel functional unknown input observer (an extension of the Luenberger observer) and existence conditions for the synthesis of full-order state feedback observer for nonlinear systems. The attractive feature of the proposed observer is the simplicity with which the design process can be accomplished and it can be used in a large class of nonlinear systems. Numerical example has been given to illustrate the attractiveness and simplicity of the new design procedures.

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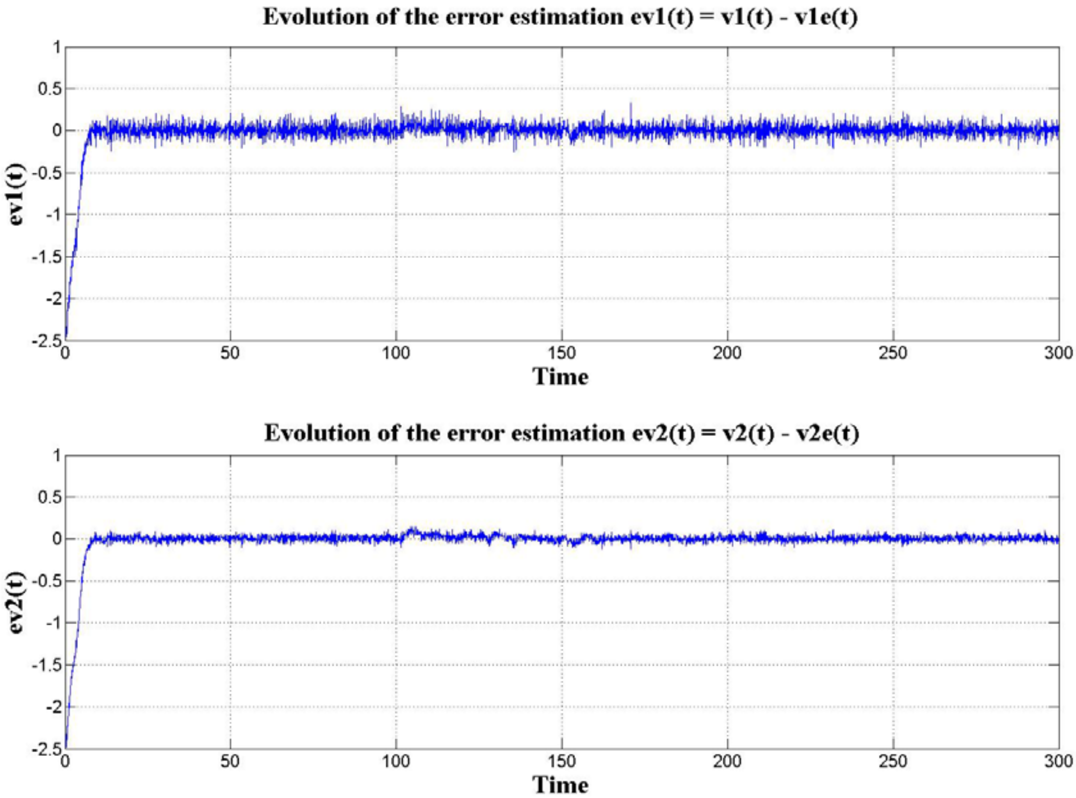

Figure 5. Estimation error e_v

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