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DECENTRALIZED ADAPTIVE OUTPUT FEEDBACK ATTITUDE SYNCHRONIZATION TRACKING CONTROL OF SATELLITE FORMATION FLYING

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Abstract. *A quaternion-based attitude synchronization tracking problem is treated for satellite formation flying without using absolute and relative angular velocity measurements. More specifically, an adaptive control based output feedback attitude synchronization tracking control law is first developed under the requirement of communication links between each satellite in formation and the desired trajectory. The controller structure includes a low-pass linear filter state that is derived without explicit differentiation of attitude to synthesize angular velocity-like signals, and a parameter updating law is also involved to identify the satellite inertia matrix such that no knowledge of the inertia of the satellites in formation is required prior. Then, a modified control law is investigated by involving distributed finite-time sliding mode estimator to relax the requirement that every satellite has access to the desired angular velocity from the practical view of inexpensive online computations. Simulation results are presented to demonstrate the effectiveness of the control law, especially, potential advantages derived through the inclusion of the integral feedback term within the control law being evaluated by computing the attitude synchronization tracking error convergence in the presence of unknown disturbance torques.*
Keywords: Satellite formation flying, Attitude cooperative, Output feedback, Sliding-mode estimator

1. **Introduction.** Satellite formation flying (SFF) is a perfect concept providing for the distribution of a large satellite assignment to several simpler, cheaper and smaller satellites to get better space mission performance in the future. Precise formation of satellites or other spacecrafts makes applications such as large-scale distributed sensing (radar, interferometry, imaging, etc.) possible. Motivated by the development of the synthetic aperture technology, the satellites within a formation system are required to synchronize their attitudes and angular velocities while tracking the desired attitudes and angular velocities, especially, the time-varying cases. So, the SFF attitude synchronization problem receives more and more attentions in recent years.

However, the assumption that each spacecraft needs the feedback information of its own angular velocity and the angular velocity of its neighbor, but this cannot be always satisfied due to either cost limitation or implementation consideration. As a remedy for such situation, several researchers proposed attitude synchronization approaches without using angular velocity measurements, called output feedback based attitude synchronization. Based on the basic work of [1], the authors introduced an auxiliary system for each spacecraft and for each pair of spacecraft with a communication link in [2] such that no velocity information was used in the feedback loop. To this end, [3] extends the result of

[2], and a velocity-free control law was developed, but it required a single dynamic auxiliary system for each spacecraft. Under the control law, the system will not be affected by the number of neighbours in the formation and also can guarantee the same global asymptotic stability. In addition, based on passivity concept, [4] proposed a passivity-based output feedback approach for attitude synchronization and tracking of multiple spacecrafts with attitude dynamics represented by Euler-Lagrange equations of motion. To reduce the required control torque, the bounded functions were designed in [5] using the output feedback technique, but only synchronizing the angular velocity of each spacecraft to its neighbours without tracking the desired value was considered. In addition, the orbiting spacecraft also suffers from structured uncertainty and external disturbances practically. Especially, it is affected by constant external disturbance torques that are the main reason of increasing the steady-state attitude error. For a single spacecraft, there are various schedules to handle this problem [6-10]. However, for the case of multiple spacecrafts in the desired formation, there are few works in the literature, especially, only attitude information measurement. Hence, designing attitude based output feedback control for the SFF in the presence of unknown inertia parameters and disturbances is another challenge that needs to be tackled.

In the research of multiple spacecrafts system, another main challenge is how to design simply control rules with limited computing power and information interaction capability for each satellite to achieve a desired group behavior. In order to reduce the communicate computation, [11] proposed an algorithm for distributed estimation of the virtual reference's unaccepted state variables to each follower agent with first-order dynamic, and further [12] extends this result the case with a time-delay. [13,14] proposed distributed observers for the second-order follower-agents together with the neighbor-based control rules under the assumption that the velocity of the desired reference cannot be obtained by each follower agents. Also, [15] shows that the first-order decentralized sliding mode estimator can guarantee accurate position estimation and that the second-order decentralized sliding mode estimator can guarantee accurate position and velocity estimation in finite time when there exists a dynamic virtual reference in the absence of velocity or acceleration measurements. Nevertheless, all above papers discussed the distributed observer only for the linear dynamic systems, while, for the nonlinear system, it cannot be extended directly. Instead, in this paper, we investigate the control law design method with distributed sliding mode observers for nonlinear dynamic models and the disturbance is also considered.

In this work, the main contribution is that the distributed adaptive output feedback control law is investigated for cooperative attitude synchronization tracking of satellite formation flying, without requiring explicitly absolute and relative angular velocity feedback in formation and any sort of prior information on the body inertia matrix. First, a class of filter is derived to synthesize angular velocity-like signal, which is forced by the absolute and relative attitude errors, and the integral parts. Then, a distributed finite-time sliding-mode estimator is further developed for the control law such that the desired angular velocity is only available to a single satellite called leader which needs no absolute control. The conditions of the communication topology are also relaxed for the formation system under this modified control law such that the requirement for this undirected communication topology is a tree. The rest of the paper is organized as follows. In Section 2, the attitude tracking control problem of satellite formation flying is formulated using the unit quaternion to represent the attitude orientation. In Sections 3 and 4, the attitude synchronization tracking control algorithms and the passivity filter formulations are presented. Numerical simulations are presented in Section 5 to demonstrate the performance of the proposed control method. Finally, conclusions of the paper are given in Section 6.

2. **Mathematical Model.** Attitude kinematics and dynamics of the i^{th} satellite using quaternion are given by [16-19]

$$\dot{q}_i = -\frac{1}{2}\omega_i^{\times} q_i + \frac{1}{2} q_{0i}\omega_i \tag{1a}$$

$$\dot{q}_{0i} = -\frac{1}{2}\omega_i^T q_i \tag{1b}$$

$$J_i\dot{\omega}_i = -\omega_i^{\times}(J_i\omega_i) + \tau_i + d_i \tag{1c}$$

where $\bar{q}_i = \begin{bmatrix} q_{0i} & q_i^T \end{bmatrix}^T$ is the quaternion denoting the rotation from the body frame of the i^{th} satellite to the inertial frame, $\bar{q}_i^* = \begin{bmatrix} \pm q_{0i} & -q_i^T \end{bmatrix}^T$ denotes the inverse of the quaternion, ω_i, J_i and τ_i are respectively the angular velocity, inertia tensor and control torque of the i^{th} satellite, d_i is the external disturbance torque, and the notation $\omega_i^{\times}$ represents the skew-symmetric matrix. For the latter analysis, let $\bar{q}_{ei} = \bar{q}_d^*\bar{q}_i$ and $\omega_{ei} = \omega_i - R_{ei}\omega_d$ denote, respectively, the attitude and angular velocity tracking error for the i^{th} satellite. Note that here $\bar{q}_d$ and ω_d denote, respectively, the desired attitude and the desired angular velocity. Without loss of generality, it is assumed that ω_d, $\dot{\omega}_d$ and $\ddot{\omega}_d$ are all bounded. Accordingly, denote $\bar{q}_{ij} = \bar{q}_j^*\bar{q}_i = \bar{q}_{ej}^*\bar{q}_{ei}$ and $\omega_{ij} = \omega_i - R_{ij}\omega_j = \omega_{ei} - R_{ij}\omega_{ej}$ respectively the relative attitude and velocity error, and here R_{ei} denotes the rotation from the body frame of the i^{th} satellite to the desired frame, and R_{ij} denotes the rotation from the body frame of the j^{th} satellite to the body frame of the i^{th} satellite. To this end, the satellite attitude tracking error kinematics and dynamics can be described as [16-19]:

$$\dot{q}_{ei} = -\frac{1}{2}\omega_{ei}^{\times} q_{ei} + \frac{1}{2} q_{0ei}\omega_{ei} \tag{2a}$$

$$\dot{q}_{0ei} = -\frac{1}{2}\omega_{ei}^T q_{ei} \tag{2b}$$

$$J_i\dot{\omega}_{ei} = \tau_i - \left[\omega_{ei} + (R_{ei}\omega_{di})\right]^{\times} J_i\left[\omega_{ei} + (R_{ei}\omega_{di})\right] - J_i\left[R_{ei}\dot{\omega}_{di} - \omega_{ei}^{\times}R_{ei}\omega_{di}\right] + d_i \tag{2c}$$

Note that it can be concluded from Equation (2) that the i^{th} satellite attitude tracking problem is equivalent to an stabilization problem for $\bar{q}_{ei}$ and ω_{ei}.

3. **Adaptive Output Feedback Attitude Synchronization Tracking Control Law Design.** In this section, a distributed control system design for attitude synchronization tracking problem without absolute and relative angular velocity is considered. Then, motivated by [8], the following filters are given as

$$\dot{x}_i = -x_i + 2k_i^x q_{ei} + k_i^i \int_0^t q_{ei}\,d\tau, \tag{3a}$$

$$\dot{x}_{ij} = -x_{ij} + 2h_{ij}^x q_{ij} + h_{ij}^i \int_0^t q_{ij}\,d\tau \tag{3b}$$

where the scalar constants $k_i^x \geq 0$ and $k_i^i \geq 0$; while for h_{ij}^x, h_{ji}^x, h_{ij}^i and h_{ji}^i, if the i^{th} satellite and j^{th} one communicate with one another, and then $h_{ij}^x = h_{ji}^x > 0$ and $h_{ij}^i = h_{ji}^i > 0$ is selected respectively, otherwise they equal to zero, i.e., $h_{ij}^x = h_{ji}^x = h_{ij}^i = h_{ji}^i = 0$.

Then, the control law for the i^{th} satellite in the formation is chosen as

$$\begin{aligned}
u_i = & -k_i^i \left(k_i^x - k_i^i/2\right)\left(q_{0ei} I_3 - q_{ei}^{\times}\right) \int_0^t q_{ei} d\tau - \left(k_i^x - k_i^i/2\right)\left(q_{ei}^{\times} - q_{0ei} I_3\right) x_i + W_i \hat{\theta}_i \\
& - \left[k_i^p + 2k_i^x \left(k_i^x - k_i^i\right) q_{0ei}\right] q_{ei} - \sum_{j=1}^n \left[h_{ij}^p + 4h_{ij}^x \left(h_{ij}^x - h_{ij}^i\right) q_{0ij}\right] q_{ij} \\
& - \sum_{j=1}^n h_{ij}^i \left(h_{ij}^x - h_{ij}^i/2\right)\left[\left(q_{0ij} I_3 - q_{ij}^{\times}\right) + R_{ij}\left(q_{0ji} I_3 - q_{ji}^{\times}\right)\right] \int_0^t q_{ij} d\tau \\
& - \sum_{j=1}^n \left(h_{ij}^x - h_{ij}^i/2\right)\left[\left(q_{ij}^{\times} - q_{0ij} I_3\right) x_{ij} - R_{ij}\left(q_{ji}^{\times} - q_{0ji} I_3\right) x_{ji}\right]
\end{aligned} \tag{4}$$

where $\hat{\theta}_i$ is the estimated parameter of θ_i and updated by the following adaptive law

$$\begin{aligned}
\hat{\theta}_i = & \hat{\theta}_i(0) + 2\int_0^t \Lambda_i \frac{d\left(W_i^T \left(q_{ei}^{\times} + q_{0ei}\right)^{-1}\right)}{dt} q_{ei} \\
& -2\Lambda_i W_i^T \left(q_{ei}^{\times} + q_{0ei}\right)^{-1} q_{ei} + 2\Lambda_i W_i^T \left(q_{ei}^{\times} + q_{0ei}\right)^{-1} q_{ei}(0)
\end{aligned} \tag{5}$$

with $W_i = (R_{ei}\omega_d)^{\times} L(R_{ei}\omega_d) + L(R_{ei}\dot{\omega}_d)$, $\Lambda_i > 0$ and the parameter θ_i is defined as $\theta_i = \left[\begin{array}{cccccc} (J_i)_{11} & (J_i)_{12} & (J_i)_{13} & (J_i)_{22} & (J_i)_{23} & (J_i)_{33} \end{array}\right]^T$. Note that the matrix $L(x)$ is defined as

$$L(x) = \left[\begin{array}{cccccc} (x)_1, & (x)_2, & (x)_3, & 0, & 0, & 0 \\ 0, & (x)_1, & 0, & (x)_2, & (x)_3, & 0 \\ 0, & 0, & (x)_1, & 0, & (x)_2, & (x)_3 \end{array}\right] \tag{6}$$

Then, the following statement can be concluded as:

Theorem 3.1. *Consider the system given in Equation (2) with the control law in Equation (4) and Equation (5) under the ideal case $d(t) = 0$. Assume that there exist scalar constants $k_i^x \geq 0$, $k_i^i \geq 0$, $k_i^p \geq \left(k_i^i\right)^2/2$, and $h_{ij}^x = h_{ji}^x > 0$, $h_{ij}^i = h_{ji}^i > 0$, $h_{ij}^p \geq \left(h_{ij}^i\right)^2/2$, when the i^{th} satellite and j^{th} one communicate with one another, otherwise, they are set to zero, i.e., $h_{ij}^x = h_{ji}^x = h_{ij}^i = h_{ji}^i = 0$. If the control gains also satisfy*

$$P_i \geq 2\sum_{j=1}^n a_{ij} \tag{7}$$

where $P_i = k_i^p - q_{0ei}\left(k_i^i\right)^2/2$ and $a_{ij} = h_{ij}^p - q_{0ij}\left(h_{ij}^i\right)^2$ for $i, j = 1, \cdots, n$. Then $q_i \to q_j \to q_d$, $\omega_i \to \omega_j \to \omega_d$ as $t \to \infty$.

Proof: Consider the following candidate of Lyapunov function

$$\begin{aligned}
V = & \frac{1}{2}\sum_{i=1}^n \omega_{ei}^T J_i \omega_{ei} + \frac{1}{2}\sum_{i=1}^n \tilde{\theta}_i^T \Lambda_i^T \tilde{\theta}_i + \sum_{i=1}^n k_i^p \left[\left(q_{0ei} - 1\right)^2 + q_{ei}^T q_{ei}\right] \\
& + \frac{1}{2}\sum_{i=1}^n \dot{x}_i^T \dot{x}_i - \sum_{i=1}^n k_i^i q_{ei}^T \dot{x}_i + \frac{1}{2}\sum_{i=1}^n \sum_{j=1}^n h_{ij}^p \left[\left(q_{0ij} - 1\right)^2 + q_{ij}^T q_{ij}\right] \\
& + \frac{1}{2}\sum_{i=1}^n \sum_{j=1}^n \dot{x}_{ij}^T \dot{x}_{ij} - \sum_{i=1}^n \sum_{j=1}^n h_{ij}^i q_{ij}^T \dot{x}_{ij}
\end{aligned} \tag{8}$$

with the estimation error $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. From Equation (8), it can be shown that the Lyapunov function is positive definite so long as k_i^p and h_{ij}^p are selected according to the constrain $k_i^p \geq \left(k_i^i\right)^2/2$ and $h_{ij}^p \geq \left(h_{ij}^i\right)^2/2$. In view of Equation (8), the time derivative of V taken along trajectories of the closed-loop system can be evaluated through laborious yet relatively straightforward algebra and then is given by

$$\dot{V} = -\sum_{i=1}^n \left\|\dot{x}_i - k_i^i q_{ei}\right\|^2 - \sum_{i=1}^n \sum_{j=1}^n \left\|\dot{x}_{ij} - h_{ij}^i q_{ij}\right\|^2 \leq 0 \tag{9}$$

Thus, V_i, ω_{ei} and $\dot{x}_i$, $\dot{x}_{ij}$ are all bounded. Recalling that q_{ei} is always bounded because of unit norm constraint on the quaternion, and also, because $\dot{V}_i \le 0$ and $V_i \ge 0$, then from Equation (9), it can be obtained

$$\int_0^\infty \sum_{i=1}^{n} \left\| \dot{x}_i - k_i^i q_{ei} \right\|^2 + \int_0^\infty \sum_{i=1}^{n} \sum_{j=1}^{n} \left\| \dot{x}_{ij} - h_{ij}^i q_{ij} \right\|^2 = V(0) - V(\infty) \tag{10}$$

then using Barbalat's lemma [20], it can be concluded that $\dot{x}_i - k_i^i q_{ei} \to 0$ and $\lim\limits_{t\to\infty} \ddot{x}_i - k_i^i \dot{q}_{ei} = 0$ as $t \to \infty$. Then we have $\lim\limits_{t\to\infty} \dot{q}_{ei} = 0$ when $k_i^x - k_i^i/2 \neq 0$. Further considering the unit norm constraint on the quaternion vector $q_{0ei}^2 + q_{ei}^T q_{ei} = 1$ and using the Barbalat's lemma, $\lim\limits_{t\to\infty} \dot{q}_{0ei} = 0$ can be concluded. To this end, Equation (2c) can be differentiated again with time to show the fact that $\ddot{\omega}_{ei} \in L_\infty$, which implies that $\dot{\omega}_{ei}$ is uniformly continuous, and by applying Barbalat's lemma once more time, then we can conclude $\dot{\omega}_{ei} \to 0$, as $t \to \infty$.

Then, just like the process of the proof for $\omega_{ei} \to 0$ previously, we can get the result that $\lim\limits_{t\to\infty} \dot{q}_{ij} = 0$, $\lim\limits_{t\to\infty} \dot{q}_{0ij} = 0$, $\omega_{ij} \to 0$ and $\dot{\omega}_{ij} \to 0$, since the relative attitude q_{ij} and relative angular velocity ω_{ij} still scarify the satellite attitude error kinematics and dynamics as given in Equation (2). Then using the above result that $\dot{\omega}_{ei} \to 0$ and $\omega_{ei} \to 0$ as $t \to \infty$, the closed-loop system in Equation (2c) with Equations (3) and (5) reduces to

$$\begin{aligned}
&-\left(k_i^p - k_i^i k_i^x q_{0ei}\right) q_{ei} - \left(k_i^x - k_i^i/2\right)\left(q_{0ei} I_3 - q_{ei}^\times\right) \dot{x}_i - \sum_{j=1}^{n} \left(h_{ij}^p - h_{ij}^i h_{ij}^x q_{0ij}\right) q_{ij} \\
&- \sum_{j=1}^{n} \left(h_{ij}^x - h_{ij}^i/2\right) \cdot \left[\left(q_{0ij} I_3 - q_{ij}^\times\right) + R_{ij}\left(q_{ij}^\times + q_{0ij} I_3\right)\right] \dot{x}_{ij} \\
&\underset{t\to\infty}{\Rightarrow} -\left[k_i^p - q_{0ei}\left(k_i^i\right)^2/2\right] q_{ei} - \sum_{j=1}^{n} \left[h_{ij}^p - q_{0ij}\left(h_{ij}^i\right)^2\right] q_{ij} \underset{t\to\infty}{\to} 0
\end{aligned} \tag{11}$$

With the notion $P_i = k_i^p - q_{0ei}\left(k_i^i\right)^2/2$ and $a_{ij} = h_{ij}^p - q_{0ij}\left(h_{ij}^i\right)^2$, the above Equation (11) can be rewritten as

$$P_i q_{ei} + \sum_{j=1}^{n} a_{ij} q_{ij} \underset{t\to\infty}{\to} 0, \quad i, j = 1, \cdots, n. \tag{12}$$

Accordingly, Equation (12) can be further rewritten in matrix form [21]

$$(M \otimes I_3)\, Q \to 0 \tag{13}$$

where $Q = \left[q_{e1}^T, \cdots, q_{en}^T\right]^T$ is the column vector composed of q_{ei}, $i = 1, \cdots, n$, $\otimes$ denotes the Kronecker product, and $M = [m_{ij}] \in R^{n\times n}$ is given by $m_{ii} = P + \sum\limits_{j=1}^{n} a_{ij} q_{0ej}$, $m_{ij} = -a_{ij} q_{0ei}$. Then we can see that the formation will be convergence only when $Q = 0$. While, a sufficient condition for $Q = 0$ is that the matrix M has full rank; the matrix M is strictly diagonally dominant if $P_i \ge 2\sum\limits_{j=1}^{n} a_{ij}$. Then the attitude tracking error $q_{ei} \to 0$ is guaranteed. Thus we are able to show that $\lim\limits_{t\to\infty} [q_{ei}, \omega_{ei}] \to 0$, thereby completing the proof of achieving the stated attitude synchronization tracking stabilization objective.

Remark 3.1. *In this work, all design parameters have been set to scalar values to permit ease in algebra and analysis. They can, however, be easily replaced by appropriate order matrices without sacrificing any theoretical assurances.*

Remark 3.2. *From Equation (13), it can be seen that if $q_{0ej} > 0$ for all $t \geq 0$ is satisfied under the initial conditions for $j = 1, \cdots, n$, then matrix M will always be strictly diagonally dominant, and the proposed distributed coordinated attitude control law will enable us to prioritize goal-seeking and formation-keeping behaviors.*

4. **Adaptive Output Feedback Control Design with Finite-Time Sliding-Mode Estimator.** From above analysis, the attitude synchronization tracking convergence is achieved under the control law designed in Equation (4). While the burden of communication links between each satellite in formation and the desired trajectory is heavy, and will involve a time-consuming design procedure. A distributed finite-time sliding-mode estimator is introduced into the designed control law such that only one satellite called leader can obtain angular velocity in the formation.

For the synthesis of the control law, it is assumed that ω_d, $\dot{\omega}_d$ and $\ddot{\omega}_d$ are all bounded with $\|\dot{\omega}_d\|_\infty < \delta$. Since ω_d cannot be accepted by the formation spacecrafts except the leader, its value cannot be used directly in the control design. Instead, we have to estimate ω_d during the evolution. That is, each follower has to estimate ω_d only by the information obtained from its neighbors in a decentralized way. The distributed finite-time sliding-mode estimator is designed as

$$\dot{\hat{x}}_i = -\beta \mathrm{sgn}\left[\sum_{j \in N_i^B} b_{ij}\left(\hat{x}_i - \hat{x}_j\right) + D_i\left(\hat{x}_i - \omega_d\right)\right] \tag{14}$$

where $b_{ij} \geq 0$, $D_i \geq 0$ and N_n^B denotes the neighbor set of the i^{th} satellite, in which each pair of the satellites knows the estimate of the desired angular velocity of each other associated with the undirected graph G_n^B. In view of Equation (4), the new control law for the i^{th} satellite it can be rewritten as

$$\begin{aligned} u_i = {} & W_i^* \hat{\theta} - \left[k_i^p + 2k_i^x\left(k_i^x - k_i^i\right) q_{0ei}\right] q_{ei} - k_i^i\left(k_i^x - k_i^i/2\right)\left(q_{0ei} I_3 - q_{ei}^\times\right) \int_0^t q_{ei} d\tau \\ & - \sum_{j \in N_i^A} h_{ij}^i\left(h_{ij}^x - h_{ij}^i/2\right) \cdot \left[\left(q_{0ij} I_3 - q_{ij}^\times\right) + R_{ij}\left(q_{0ji} I_3 - q_{ji}^\times\right)\right] \int_0^t q_{ij} d\tau \\ & - \left(k_i^x - k_i^i/2\right)\left(q_{ei}^\times - q_{0ei} I_3\right) x_i - \sum_{j \in N_i^A}\left[h_{ij}^p + 4h_{ij}^x\left(h_{ij}^x - h_{ij}^i\right) q_{0ij}\right] q_{ij} \\ & - \sum_{j \in N_i^A}\left(h_{ij}^x - h_{ij}^i/2\right)\left[\left(q_{ij}^\times - q_{0ij} I_3\right) x_{ij} - R_{ij}\left(q_{ji}^\times - q_{0ji} I_3\right) x_{ji}\right] \end{aligned} \tag{15}$$

where $W_i^* = (R_{ei}\hat{x}_i)^\times L(R_{ei}\hat{x}_i) + L(R_{ei}\dot{\hat{x}}_i)$, and $k_i^x > 0$, $k_i^i > 0$, $k_i^p \geq \left(k_i^i\right)^2/2 > 0$, if the i^{th} satellite is the leader which is the only one has absolute control part, but $k_i^x = 0$, $k_i^i = 0$, $k_i^p = 0$ if it is a follower. Note that N_n^A denotes the neighbor set of the ith satellite, in which each pair of the satellites can transmit the knowledge of the attitude to each other associated with the undirected graph G_n^A. Then, we have the following conclusion.

Theorem 4.1. *Consider the formation given in Equation (2) under the control law Equation (15) with Equation (5), under the ideal case that $d(t) = 0$. Assume there is only one satellite can obtain the desired angular velocity in the formation, and there exist undirected graph G_n^B that is connected and $\beta > \delta$ as well as the undirected graph G_n^A which is a tree, then $q_i \to q_j \to q_d$, $\omega_i \to \omega_j \to \omega_d$ asymptotically as $t \to \infty$.*

Proof: To first prove that the sliding mode estimator in Equation (14) can guarantee $\hat{x}_i \to \omega_d$ in finite time, the following Lyapunov function candidate is considered

$$V_o = \frac{1}{2}\bar{x}^T\left(W \otimes I_3\right)\bar{x} \tag{16}$$

where $\bar{x} = \hat{x} - 1_n \otimes I_3\omega_d$, 1_n is an $n \times 1$ vector with all one, and $\hat{x} = \left[\hat{x}_1^T, \cdots, \hat{x}_n^T\right]^T$.

Let the Laplacian matrix $\ell_B = [b_{ij}] \in R^{n\times n}$ associated with G_n^B be defined as $l_{ij} = \begin{cases} \sum\limits_{j\in N_n^B} b_{ij}, i = j \\ -b_{ij}, i \neq j \end{cases}$, and $W = \ell_B + diag\left(D_1, \cdots, D_n\right)$.

In view of Equation (16), taking the time derivative of V_o leads to

$$\begin{aligned} \dot{V}_o &= \bar{x}^T \left(W \otimes I_3\right)\left[-\beta \text{sgn}\left(W \otimes I_3\bar{x}\right) - 1_n \otimes I_3\dot{\omega}_d\right] \\ &\leq -\left(\beta - \delta\right)\left\|W \otimes I_3\bar{x}\right\|_1 \\ &\leq -\left(\beta - \delta\right)\left\|W \otimes I_3\bar{x}\right\|_2 \\ &\leq -\left(\beta - \delta\right)\lambda_{\min}\left(W\right)\left\|\bar{x}\right\|_2 \\ &\leq -\left(\beta - \delta\right)\frac{\sqrt{2}\lambda_{\min}\left(W\right)}{\sqrt{\lambda_{\max}\left(W\right)}}V_o^{\frac{1}{2}} \end{aligned} \tag{17}$$

where W is symmetric positive definite when at least one $D_i > 0$, and $\beta > \delta$. Then by using Lyapunov stability theory, V_o tend to zero after the settling time T_1 given by

$$T_1 = \frac{\sqrt{\bar{x}^T\left(0\right)\left(W \otimes I_3\right)\bar{x}\left(0\right)}\sqrt{\lambda_{\max}\left(W\right)}}{\left(\beta - \rho\right)\lambda_{\min}\left(W\right)} \tag{18}$$

which implies that $\hat{x}_i \to \omega_d$ in finite time as $t \geq T_1$. Thus, $\hat{x}_i$ can be used to replace ω_d, and accordingly $\dot{\hat{x}}_i$can be used to replace $\dot{\omega}_d$ when $t \geq T_1$. Assume that only one satellite, such as the lth one, called the leader which is $D_l > 0$, $k_l^x > 0$, $k_l^i > 0$ and $k_l^p \geq \left(k_l^i\right)^2/2 > 0$ with $D_i = 0$, $i \neq l$, $k_i^x = 0$, $k_i^i = 0$ and $k_i^p = 0$ for $i \neq l$. Using the similar development in Theorem 3.1, Equation (12) can be rewritten as

$$P_l q_{ei} + \sum_{j\in N_n^A} a_{ij}q_{ij} \underset{t\to\infty}{\to} 0 \tag{19}$$

where $P_l = k_l^p - q_{0ei}\left(k_l^i\right)^2/2$, and $a_{ij} = h_{ij}^p - q_{0ij}\left(h_{ij}^i\right)^2$ for $i, j = 1, \cdots, n$.

For further analyzing of Equation (19), we assign a direction to the undirected links of the communication graph G_n^A, by considering one of the nodes of each edge to be the positive end of the link, and then we can obtain the directed graph $\tilde{G}_n^A = \left(N_n, \tilde{E}_n, W_n\right)$, with $\tilde{E}_n$ being the set of ordered edges of the graph. To this end, let $m = \left|\tilde{E}_n\right|$ be the total number of edges in the graph $\tilde{G}_n^A$, which is also equal to the total number of undirected links in G_n^A. With the assumption that the communication graph G_n^A is a tree, the obtained directed graph $\tilde{G}_n^A$ is weakly connected and acyclic, that is $m = n - 1$. In addition, we consider that the desired attitude is transmitted to the leader by a fictitious satellite, described by an additional $(n+1)^{\text{th}}$ node in the communication graph $\tilde{G}_n^A$, via a directed communication link constituting a new nth edge in $\tilde{G}_n^A$, with weight P_l that we assume $a_{l,n+1} = a_{n+1,l}$. Then, with these assumptions and definitions, a new directed graph $\tilde{G}_n^{A*} = \left(N_n^*, \tilde{E}_n^*, W_n^*\right)$, with $(n+1)$ nodes and n edges, can be obtained.

Let the weighted incidence matrix of $H^* \in R^{(n+1)\times n}$ be defined as

$$d_{il(u,v)} = \begin{cases} a_{uv}, & \text{if node } i \text{ is the source of the directed edge } (u,v) \\ -a_{uv}, & \text{if node } i \text{ is the sink of the directed edge } (u,v) \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

where $l^{(u,v)}$: $\tilde{E}_n^* \to \{1, \cdots, n\}$ is a function that associates a single number from the set from the set $\{1, \cdots n\}$ to each edge $(u, v) \in \tilde{E}_n^*$. It can easily be verified that the directed graph $\tilde{G}_n^{A*}$ is also weakly connected and acyclic, and hence the rank of H^* is n [21].

To this end, let $Q_u \in R^{3n}$ be the column vector containing the vectors q_{ij}, for $\forall(i,j) \in \tilde{E}_n^*$, and the vector q_{el}. Then using the fact that $q_{ij} = -q_{ji}$, Equation (20) can be written in matrix form

$$(H \otimes I_3)\, Q_u \to 0 \tag{21}$$

where the matrix H is constructed by deleting the last row of the H^*, and is full rank. Thus, the only solution to Equation (21) is $Q_u \to 0$, that is $q_{ij} \to 0$, $\forall(i,j) \in \tilde{E}_n^*$, and $q_{el} \to 0$, or $q_i \to q_j \to q_d$.

Remark 4.1. *The decentralized estimator adopted here is the first-order, although both the desired reference and the formation satellites are described by the second-order nonlinear dynamic models. In fact, it is preferred to have a first-order sliding mode "observer" instead of second-order "observers" (corresponding to the second-order nonlinear dynamic models), regarding the selected observer in Equation (23) which can significantly simplify the construction of the proper Lyapunov function.*

5. Simulation Result and Comparison.

To study the effectiveness and performance of the proposed formation control strategies, the detailed response is numerically simulated using the set of governing equations of motion (1) in conjunction with the proposed control law. Note that because the control law in Equation (15) is an extension of control law given in Equation (4), here only the control law in Equation (15) is conducted in the simulation to achieve attitude synchronization and tracking among three satellites. The satellite formation flying system parameters, initial conditions and controller parameters used in the numerical simulations are given in Table 1.

Table 1. Parameters of satellites

Inertia Matrix (kgm^2)	$J_1=\begin{bmatrix}24.31 & 0.2 & -0.5\\ 0.2 & 24.37 & 0.3\\ -0.5 & 0.3 & 23.64\end{bmatrix}$; $J_2=\begin{bmatrix}20.25 & 0.1 & -0.2\\ 0.1 & 20.33 & 0.14\\ -0.2 & 0.14 & 20.66\end{bmatrix}$; $J_3=\begin{bmatrix}30.35 & 0.3 & -0.6\\ 0.3 & 30.17 & 0.46\\ -0.6 & 0.46 & 30.61\end{bmatrix}$
Initial Attitude	$\bar{q}_1^T(0)=[0.993, 0.2, 0.3, 0.1]$; $\bar{q}_2^T(0)=[0.5774, 0,\ 0.5774,\ 0.5774]$; $\bar{q}_3^T(0)=[0.91, 0.3, 0.2, 0.2]$
Initial Angular Velocity (rad/s)	$\omega_1^T(0)=[0.15, 0, 0]$; $\omega_2^T(0)=[-0.11, 0, 0]$; $\omega_3^T(0)=[-0.12, 0, 0]$
Desired Attitude	$\bar{q}_d^T=[0.3772, 0.4329, 0.6645, 0.4783]$
Desired Angular Velocity (rad/s)	$\omega_d^T=0.1\left[-\cos(t/40), -\sin(t/50), \cos(t/60)\right]$
Initial Estimator	$\hat{\eta}_1(0)=[25,0,0,25,0,25]^{\mathrm{T}}$; $\hat{\eta}_2(0)=[20,0,0,20,0,20]^{\mathrm{T}}$; $\hat{\eta}_3(0)=[30,0,0,30,0,30]^{\mathrm{T}}$
External Disturbance (N·m)	$d_1=d_2=d_3=\left[-0.14, 0.21, -0.13\right]$
Control Parameter	$k_1^{\mathrm{p}}=28; k_1^{\mathrm{d}}=3;\ \ k_2^{\mathrm{p}}=25; k_2^{\mathrm{d}}=3;\ \ k_3^{\mathrm{p}}=30; k_3^{\mathrm{d}}=3;$ $k_{12}^{\mathrm{p}}=6; k_{12}^{\mathrm{d}}=0.6;\ \ k_{21}^{\mathrm{p}}=5; k_{21}^{\mathrm{d}}=0.4;\ \ k_{31}^{\mathrm{p}}=8; k_{31}^{\mathrm{d}}=0.8;$ $k_{13}^{\mathrm{p}}=6; k_{13}^{\mathrm{d}}=0.6\ \ k_{23}^{\mathrm{p}}=5; k_{23}^{\mathrm{d}}=0.4\ \ k_{32}^{\mathrm{p}}=8; k_{32}^{\mathrm{d}}=0.8$ $\lambda_1=6; \lambda_2=5;$ $\lambda_3=5;$

For comparison, two cases are considered in the simulation. In the first case, the integral terms are introduced in the filters and control law, that is the control gains $k_1^1 \neq 0$, $h_{ij}^i \neq 0$. Taking satellite 3 for example, the results of the attitude and angular velocity tracking errors are shown in Figure 2. It can be observed from these simulations that the distributed attitude synchronization control is able to reject the constant external disturbance torque, and the satellites 3 can track the desired attitude and angular velocity within 25s even if only the satellite 1 can obtain the reference angular velocity value. Accordingly, the integral terms in filters (3b) driving inputs reject the constant external disturbance torque when satellites 3 are only controlled by relative control part. For the second case, there is no integral terms considered, that is, the control gains $k_1^1 = 0$ and $h_{ij}^i = 0$. All other control gains and initial conditions remain the same for a fair comparison. From the comparison Figure 2 with Figure 3, respectively, it can be observed that adding the integral feedback introduces smaller steady state error for the coordinated attitude tracking without paying much penalty in terms of either increases in control torque magnitudes or reduction in control speeds.

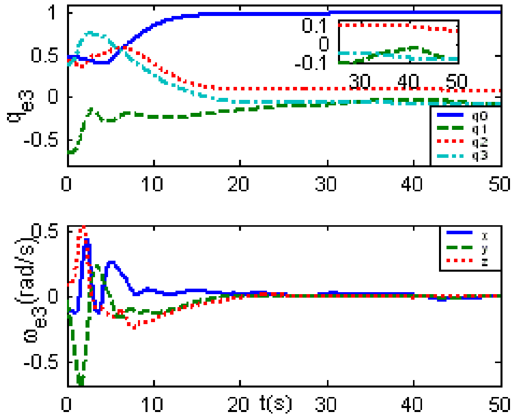

FIGURE 1. Absolute attitude and angular velocity errors of sat. 3

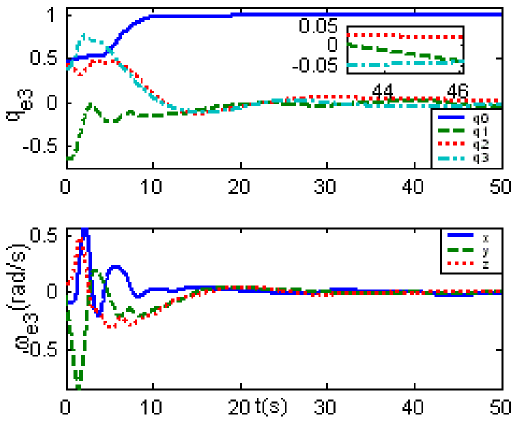

FIGURE 2. Absolute attitude and angular velocity errors of sat. 3 ($k_3^i = 0, h_{32}^i = 0$)

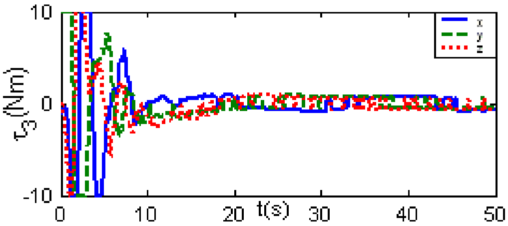

FIGURE 3. Control torque of sat. 3

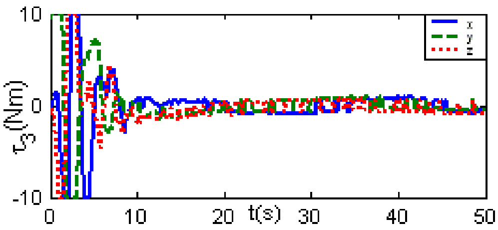

FIGURE 4. Control torque of sat. 3 ($k_3^i = 0, h_{32}^i = 0$)

In addition, Figures 5 and 6 show, respectively, the attitudes, angular velocities tracking path of satellites 1, 2, 3 from different initial states. From the simulations, it can be obtained that the distributed control law with sliding-mode estimator could guarantee attitude synchronization without the requirement for absolute angular velocity measurement and relative angular velocity measurement.

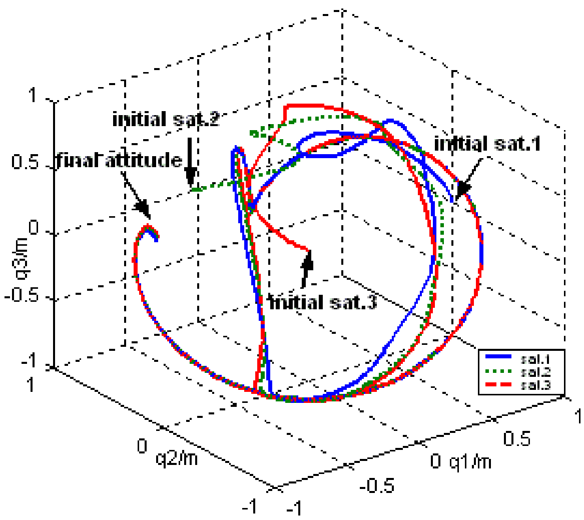

FIGURE 5. Attitude tracking path of sat. 1∼3

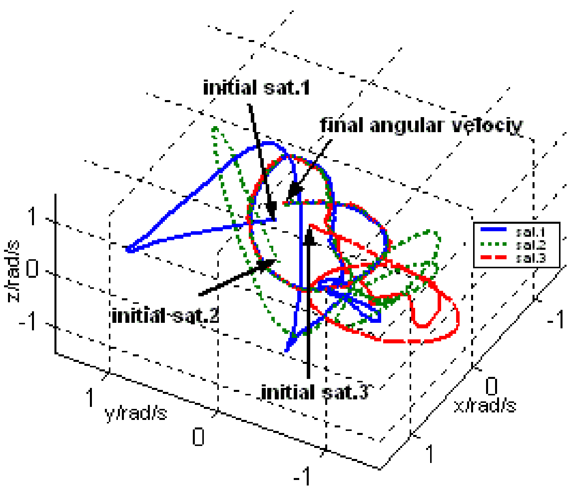

FIGURE 6. Angular velocity tracking path of sat. 1∼3

6. **Conclusions.** Adaptive control based attitude synchronization tracking scheme is proposed for satellite formation flying (SFF) without using explicitly absolute and relative angular velocity feedback. To generate the necessary damping that would have been generated by the angular velocity and the relative one, a class of filter providing an additional provision for integral feedback action is derived. Then to relax the requirement that all satellite has access to the desired angular velocity, a modified control law is developed by introducing a finite-time sliding mode estimator for each formation satellite to obtain an accurate estimate of the desired angular velocity. Numerical implementation of these new results provides the assurance of significantly improved steady-state attitude error convergence in the presence of constant external disturbances as a result of integral feedback action.

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