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ADAPTIVE GLOBAL ASYMPTOTIC STABLE BASED ON MODEL REFERENCE AND DISTURBANCE ATTENUATION FOR MIMO UNCERTAIN LINEAR SYSTEMS WITH PERSISTENT DISTURBANCES

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Abstract. *This paper addresses an Adaptive Global Asymptotical Stable (adaptive GAS) controller for multi-input multi output (MIMO) uncertain linear systems based on the model reference adaptive control. There are many uncertainties, such as system parameter uncertainties, external load disturbance, friction force and model uncertainty. These are always reducing the performance quality of the system. To deal with these problems, we propose a control algorithm that is made by taking advantage of adaptive based model reference and state feedback. The Lyapunov theory is used to verify the asymptotical stable of the closed-loop system even in the present of persistent and other kind of disturbances. An application to control the permanent magnet linear synchronous motors and inverted pendulum systems is a good example for verifying this method.*
Keywords: Persistent disturbance, Model reference, Adaptive global asymptotical stable, State feedback, Disturbance compensator, Permanent magnet linear synchronous motors, Inverted cart pendulum

1. **Introduction.** Linear control area is a vast field that received a lot of attention by control experts and has benefited from extensive research. Furthermore, the linear control is the simplest control algorithm that will produce the desired results [1]. The methods used for designing a nonlinear system are based on the methods used for linear systems such as exact linearization, zero dynamics analysis in nonlinear systems. Furthermore, many real life systems can be modelled very well by linear system models. Often, the behaviour of a nonlinear system can, at least in the neighbourhood of an equilibrium solution, be approximately modelled by a linear system. In control area, the classification between linear and nonlinear is not clear in some situations. Therefore, in model control, the control theory is classified in classical and model control. Classical control typically deals with single-input single-output (SISO) systems using frequency domain tools. Modern control often deals with multi-output multi-input (MIMO) systems, and deals directly with the ordinary differential equations in the time domain. For nonlinear systems, the studies are only limited for SISO [2-6]. Some proposals are good examples such as using fuzzy combined with model reference adaptive law for the control of SISO nonlinear plants with unknown nonlinearities [7], or the dynamic adaptive backstepping and sliding mode control combined with chattering free mechanics for SISO uncertain nonlinear system with disturbance [8]. For MIMO nonlinear systems, the exact linearization is used to change nonlinear systems into linear systems [9]. Many real life systems can be modelled very well by linear system models. Actually, the behaviour of a nonlinear system can be approximately modelled by a linear system at least in the neighbourhood of an

equilibrium solution. Therefore, the control of MIMO linear systems has received a lot of attention from control specialists [10-14], and the uncertain parameters and disturbance problems have received the most attention.

The uncertainties always exist in the real plants. The uncertainties can be a mismatch between the parameters described in a catalogue by the producer and the actual parameters, friction forces and the external disturbances affected by the system. All of these uncertainties can be called persistent disturbance. If these persistent disturbances cannot be removed completely, they will have a significant effect on the performance of the system [15,16].

There have been many studies to deal with this problem, in recent years. These studies include conventional control techniques and intelligent control techniques [17,18] such as fuzzy control, neural network and adaptive fuzzy control. However, the drawbacks for these methods are that they require an exact mathematical model, inverse model [19], or some assumptions about the disturbance. However, in general cases, some assumptions are inaccurate. For example, the disturbance or uncertainties do not have a certain form, are not constant, or the bound of the disturbance cannot be defined in some situations. Furthermore, the use of such intelligent techniques takes a lot of time to learn and search for the optimal parameters [20-23]. This is not acceptable in industrial applications. A further drawback of intelligent methods is that it is based on the experience of the designers. For this reason, the robust control or adaptive control techniques are the best choices.

The robust control is a technique in which uncertainties can be considered as model error, and the main task of the control scheme is to keep the performance of the closed-loop system as a requirement of the change of model error [24-27]. The purpose of the adaptive control is also the same as the robust control that permits parameter variation while still allowing good control sensitivity and responsiveness. The main difference is that the robust control tends to the fix controller as well as constant parameter controller. However, the adaptive control tends to be a flexible controller, which means it can self-change the parameter or the structure to follow the correspondence changing in the plant aim to remain at the required performance of the system [28]. Adaptation also provides a rejection disturbance mechanism in response to the changing environmental disturbances [29]. Furthermore, in many situations, the use of adaptive control is the simplest choice.

The adaptive control has two types of structure: Self Tuning Regulators (STR) and Model Reference Adaptive Controllers (MRAC). In this paper, the MRAC techniques were used to reject disturbance [30]. The combination of MRAC with other control methods can be achieved high performance controllers. For example, the combination of MRAC and the integral sliding mode control (ISMC) for classes with uncertainties switched systems with time varying delay [31]. In this study, an adaptive control technique was used to adapt the unknown upper bounds of perturbations. The integral SMC did not reach the phase that made the controller robust with uncertainties and disturbances. The other study combines MRAC with type-2 fuzzy [32]. This control scheme guarantees the global stability of the resulting closed-loop system in the sense of unpredicted internal disturbance, data uncertainties.

The above analyses have shown that the study and proposal of new control methods for MIMO linear system is not only a useful class for MIMO linear systems but also for MIMO nonlinear systems. In this paper, we propose an adaptive global asymptotical stable controller for MIMO uncertain linear systems based on the MRAC. The controller guarantees the asymptotical stable of the closed-loop system with highly uncertainties and any form of disturbances.

To verify our algorithm, we apply the proposal algorithm to control permanent magnet linear synchronous motor (PMLSM) and inverted card pendulum system (ICPS). PMSLM motors are widely used in automation systems, manufacture, transportation, transfer systems and the military, all of which require high accuracy, speed and quick-acting abilities [33,34]. The control of PMSLM is a challenging problem and has attracted a lot of attention. The ICPS is a very popular research subject because of the interesting dynamics they present and many practical applications they can model (particularly in the field of robotics). An ICPS is a complex multivariable, non-minimum phase and unstable, electromechanical system with severe nonlinearity [35]. Therefore, the control of ICPS is used to illustrate a new control theory. Its control process can reflect many significant problems, such as stabilization problems, nonlinear disturbance attenuation problems and robust problems.

The rest of this paper is organized as follows. The general MIMO linear systems with persistent disturbance and problem are described in Section 2. The process of designing a controller is discussed in Section 3. The application to control the permanent magnet linear synchronous motors is dealt with in Section 4. The application to control of the inverted pendulum system is dealt with in Section 5. And, conclusions are drawn in Section 6.

2. **Problem Statement and Preliminaries.** Consider an uncertain MIMO linear system in which the mathematical model is described as

$$\dot{x} = Ax + Bu + G(x)d \tag{1}$$

where $x(t) \in \Re^n$ is state variable vector, $u(t) \in \Re^m$ is input vector (control signal) and $d(t) \in \Re^p$ is uncertain parameters vector (uncertainty, immeasurable) of plant, A and B are constant matrices, $G(x)$ is matrix in which the elements depend on x.

Our aim is to design a feedback controller to control plant (1) so that the system performs as required and does not depend on disturbances $d(t)$.

The process of designing an adaptive GAS includes two steps

First, by using the transformed method $\begin{pmatrix} I_m & \Theta_{m\times n} \end{pmatrix}$ and feedback mechanism $u(\hat{u}, x)$ the plant (1) is transformed into new plant

$$\dot{x} = \hat{A}x + \hat{B}\left[\hat{u} + \hat{G}(x)\hat{d}\right] \tag{2}$$

where I_m is the identity matrix, $\Theta_{n\times m}$ is the zero matrix with $n \times m$ elements. $\hat{A}$, $\hat{B}$ are constant matrices. $\hat{A}$ is a stable matrix with arbitrarily given eigenvalues in the half plane. Hence, with any given symmetric positive defined matrix Q, the Lyapunov equation

$$P\hat{A} + \hat{A}^T P = -Q \tag{3}$$

always has a unique solution P, which is also symmetric positive definite.

Second, using the model reference technique to reject disturbance in the plant (2). The reference model is chosen

$$\dot{x}_r = \hat{A}x_r + \hat{B}w \tag{4}$$

By using the disturbance compensator

$$\begin{cases} \dot{\theta} = -H^{-1}\hat{G}(x)^T \hat{B}^T Pe \\ \xi = \hat{G}(x)\theta \end{cases} \tag{5}$$

where e is the errors between the plant outputs and reference outputs. H is the symmetric positive matrix which is chosen arbitrary.

3. Control Design.

3.1. Designing a transformative mechanism. The Equation (1) can be rewritten as below

$$\begin{aligned} \dot{x} &= Ax + \begin{pmatrix} B & I_n \end{pmatrix} \begin{pmatrix} u \\ G(x)d \end{pmatrix} \\ &= Ax + \hat{B}\left[\hat{u} + \hat{G}(x)\hat{d}\right] \end{aligned} \tag{6}$$

where $\hat{B}$ is the relevant matrix between inputs and state variables of a new plant and $\hat{G}(x)$ are defined below;

$$\begin{aligned} \hat{B} &= \begin{bmatrix} B & I_n \end{bmatrix} \\ \hat{G} &= \begin{bmatrix} \Theta_{(m+n)\times m} & G(x) \end{bmatrix} \end{aligned} \tag{7}$$

And $\hat{u}, \hat{d}$ are the new input and disturbance vectors

$$\hat{u} = \begin{bmatrix} u \\ 0_n \end{bmatrix}, \hat{d} = \begin{bmatrix} 0_m \\ d \end{bmatrix} \tag{8}$$

From $\hat{u} = \begin{bmatrix} u \\ 0_n \end{bmatrix}$ so the transformation is

$$u = \begin{pmatrix} I_m & \Theta_{m\times n} \end{pmatrix} \hat{u} \tag{9}$$

3.2. The feedback control design. Now, a feedback controller R for a new linear plant (6) to a conduct system to get new poles on the left complex plane with desired performance is designed. In essence, $R \in \Re^{n\times m}$ so that the eigenvalues of the matrix $A - \hat{B}R$ are given desired poles.

The aim of R ensures that the plant (6) has a desired kinetic performance without care for disturbance. So here, assuming $\hat{d} = 0$ and (6) can be:

$$\dot{x} = Ax + \hat{B}\hat{u} \tag{10}$$

The close-loop system as

$$\dot{x} = (A - \tilde{B}R)x + \tilde{B}w = \tilde{A}x + \tilde{B}w \tag{11}$$

The controller R has roots in the following equation

$$\det(sI - A + \tilde{B}R) = (s - s_1)(s - s_2)\ldots(s - s_n) \tag{12}$$

The methods for solving (12) were described in [36].

3.3. Designing adaptive controller based on model reference to reject disturbance. With the feedback controller R, the system (6) becomes

$$\dot{x} = \hat{A}x + \hat{B}\left[\hat{u} + \hat{G}(x)\hat{d}\right] \tag{13}$$

The aim of the rejected disturbance mechanics makes the plant (13) not be affected by disturbance $\hat{d}(t)$ or in other words tracking to model (13) omit $\hat{d}(t)$.

Assume the disturbance $\hat{d} = \tilde{d}$ of the MIMO plant (13) is known, so it can be rejected by the input compensational signal $\hat{u} = w - \hat{G}(x)\tilde{d}$, and the plant (13) will be:

$$\dot{x} = \hat{A}x + \hat{B}[w + \hat{G}(x)\theta] \tag{14}$$

But, in fact, the disturbance is uncertain, and $\theta = \hat{d} - \tilde{d} \neq 0$, i.e., the error between the plant output (1) and reference output always exist. Our aim is to adjust the uncertain parameter θ so that the error $e = x - x_m$ tends to zero.

Theorem 3.1. *The disturbance compensator (4) with H is arbitrary symmetrical positive definite, P is a symmetrical positive definite matrix obtained from Lyapunov equation that will drive errors between the plant output (14) and the reference model output (4) tending to zero.*

Proof: Tracking error between the plant output (14) and reference output (4)

$$e = x - x_m \tag{15}$$

The time derivative error

$$\dot{e} = \hat{A}e + \hat{B}\hat{G}(x)\theta \tag{16}$$

Using the Lyapunov function

$$V(e,\theta) = e^T Pe + \theta^T H\theta \tag{17}$$

The time derivative of V:

$$\begin{aligned} \dot{V} &= \dot{e}^T Pe + e^T P\dot{e} + \dot{\theta}^T H\theta + \theta^T H\dot{\theta} \\ &= -e^T Qe + 2\theta^T(\hat{G}^T\hat{B}^T Pe + H\dot{\theta}) \end{aligned} \tag{18}$$

If $\dot{V}$ is negatively defined, then

$$e \to 0$$

So, we adjust $\theta(t)$ as the following;

$$\dot{\theta} = -H^{-1}\hat{G}^T\hat{B}^T Pe \tag{19}$$

That means that signal ξ is compensated as in input u.

$$\begin{cases} \dot{\theta} = -H^{-1}\hat{G}(x)^T\hat{B}^T Pe \\ \xi = \hat{G}(x)\theta \end{cases} \tag{20}$$

The following diagram describes the control system.

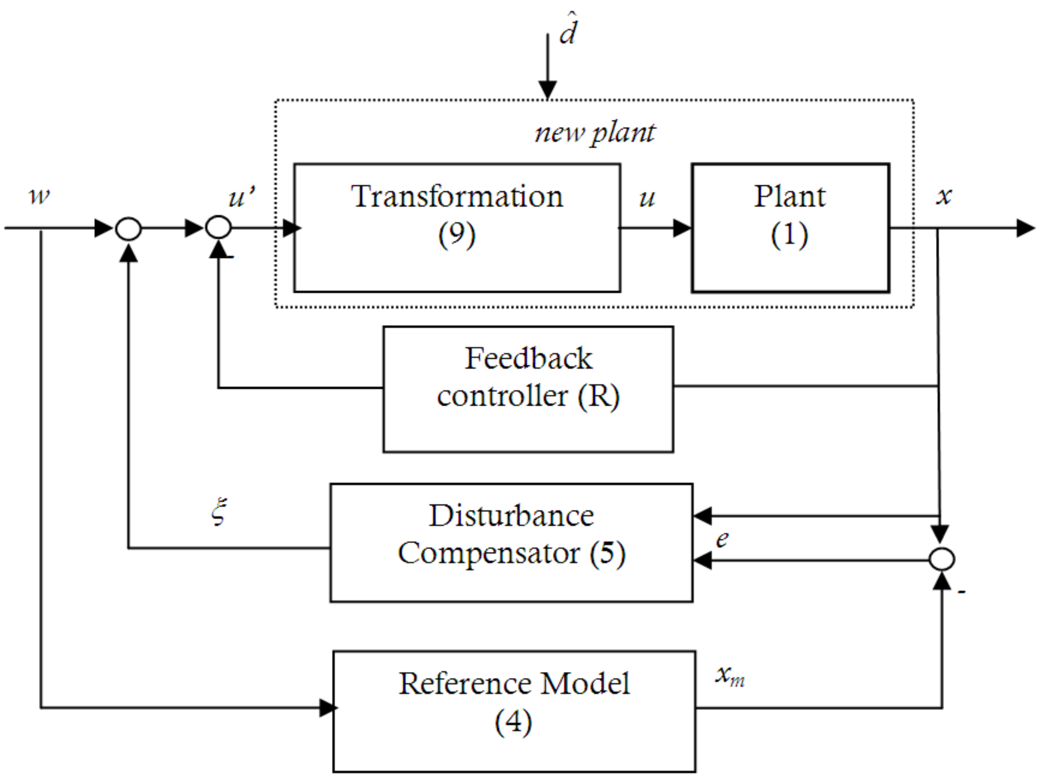


Figure 1. The structure of the linear adaptive GAS controller

4. Control of Permanent Linear Synchronous Motors.

4.1. Mathematical model of PMLSM. Recently, there are many models of PMLSM have been studied. Our purpose is to use the PMLSM to verify our control algorithm, so the dynamic model of a typical PMLSM is used here [37,38].

$$u_d = R_s i_d + L_d \frac{di_d}{dt} - \omega L_q i_q \tag{21}$$

$$u_q = R_s i_q + L_q \frac{di_q}{dt} - \omega L_d i_d + \omega \psi_{PM} \tag{22}$$

where u_d, u_q are d-q voltages, i_d, i_q are d-q currents, L_d, L_q are d-q inductances, R_s is resistance of armature wingding, ω is angular velocity, ψ_{PM} is permanent magnet flux linkage.

The power is calculated as the following:

$$P_e = F_e v = \frac{3}{2}\omega(\psi_d i_d - \psi_q i_q) \tag{23}$$

where v is the linear velocity of the mover.

Because $\psi_d = L_d i_d + \psi_{PM}$, $\psi_q = L_q i_q$, $v = \frac{\tau}{\pi}\omega$, then thrust force can be derived as the following:

$$F_e = \frac{3\pi}{2\tau}[\psi_{PM} i_q + (L_d - L_q) i_d i_q] \tag{24}$$

where τ is pole pitch.

In this paper, the mover current vector is perpendicular to the permanent magnet of the stator. In the inner loop of the current, the excitation component $i_d = 0$ is adopted. Accordingly, the thrust force equation can be expressed as

$$F_e = \frac{3\pi}{2\tau}\psi_{PM} i_q = K_t i_q \tag{25}$$

The motion equation of the PMLSM can be described as

$$F_e = K_t i_d = F_d + Bv + M\frac{dv}{dt} \tag{26}$$

where B is friction factor, M is mover mass, F_d is the external uncertainty and disturbance term that may be comprised of dry and viscous friction.

The Equation (26) can be rewritten as follows

$$\dot{v} = -\frac{B}{M}v + \frac{K_t}{M}i_d - \frac{1}{M}F_d \tag{27}$$

To facilitate the derivation of adaptive GAS, (22) and (27) are written into the following state-space form

$$\begin{cases} \dot{x} = Ax + Bu + GF_d \\ y = Cx \end{cases} \tag{28}$$

where $x = \begin{bmatrix} i_q \\ v \end{bmatrix}$ is the state variable, $u = u_q$ is the input variable, and the matrices are:

$$A = \begin{bmatrix} -\frac{R_s}{L_q} & -\frac{K}{L_q} \\ \frac{1.5K}{M} & -\frac{B}{M} \end{bmatrix}, \tag{29}$$

$$B = \begin{bmatrix} \dfrac{1}{L_q} \\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & 1 \end{bmatrix},$$
$$G = \begin{bmatrix} 0 \\ -\dfrac{1}{M} \end{bmatrix},$$
$$K = \frac{\pi \psi_{PM}}{\tau}.$$

The parameter of PMLSM and its quality required are shown in the Table 1.

Table 1. The parameter of PMLSM

Parameter	Value
Mover mass (Kg)	1.97
Friction factor (N/(m/s))	5.2982
Flux linkage (Wb)	0.4849
Pole pitch (m)	0.06096
Phase resistance (Ω)	11.8
Inductance (mH)	4.8

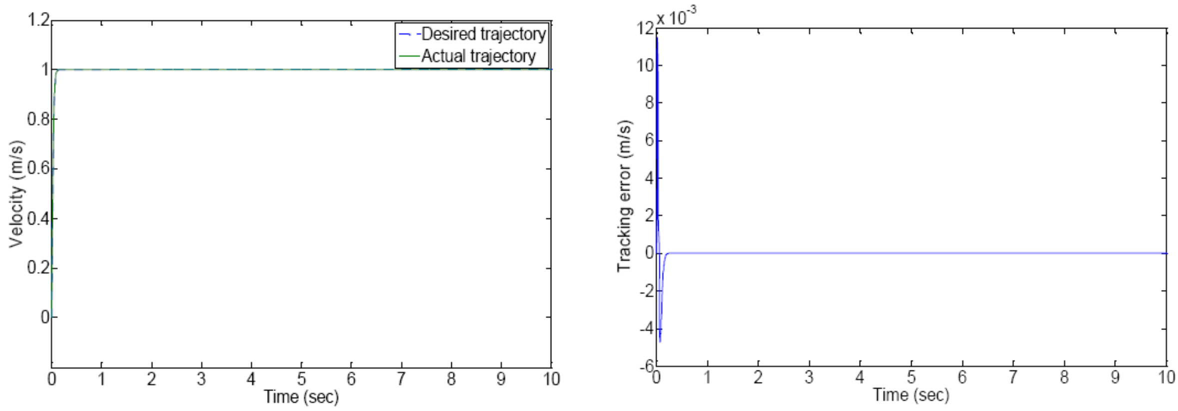
The system model for the control input is estimated by applying a factor to the corresponding parameter matrices of the original system, i.e., $\hat{M} = 0.95M$, $\hat{B} = 0.95B$, $\hat{K} = K$, $\hat{L}_q = 0.95L_q$.

4.2. **Simulation and results.** The Matlab/Simulink software is used for simulation. To do so, the parameters of the motor control system had to be identified. The related parameters of the PMLSM are listed in Table 1. White noise is chosen for disturbance. The simulation results are shown in Figure 2.

We also compare our proposal with recent proposals in the control of PMLSM such as using model-based disturbance attenuation (MBDA) and PI, PID to reject disturbances [39], using the combining adaptive backstepping and sliding mode (ABSMC) [40]. In [39], two methods of MBDA were proposed, which are MBDA-PI and MBDA-PID. In these methods, the MBDA-PID had the fast tracking and strong disturbance attenuation. The simulation results are shown in Figures 3 and 4.

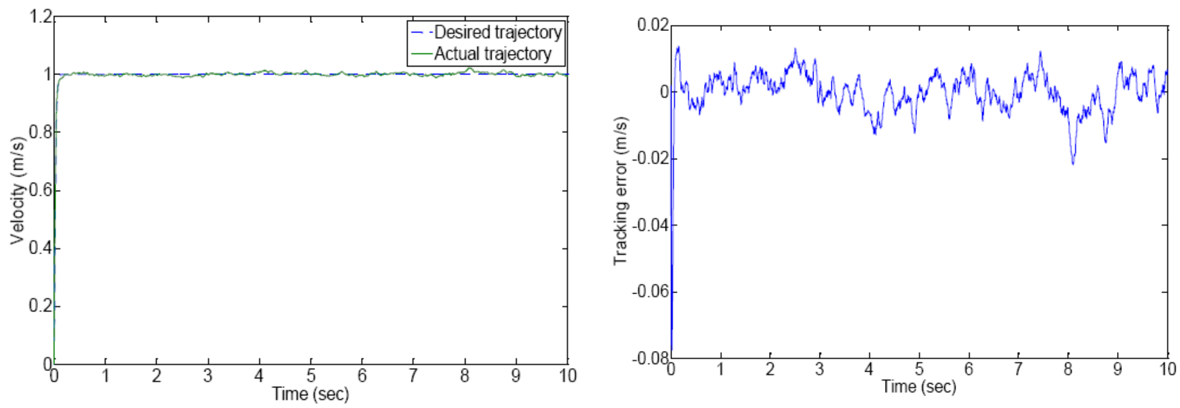
Through simulation, the MBDA-PI and DABSMC are good disturbance removers when the noise power is small (smaller than 0.1). In order to show the best performance of our proposal, in the comparison of MBDA-PI and DAB-SMC, the noise power is increased 10 times.

The DABSMC is better with attenuation in comparison to MBDA-PI (Figures 3(a) and 4(a)) and the tracking problem of DABSMC is also better than MBDA-PI (Figures 3(b) and 4(b)). The disturbance attenuation and tracking problem of adaptive GAS are better than DABSMC (Figures 2 and 3). Therefore, the performance of adaptive GAS is better than DABSMC and MBDA-PI. This is clearer when the noise power is increased 10 times, since the performance of system is still good (Figure 5).



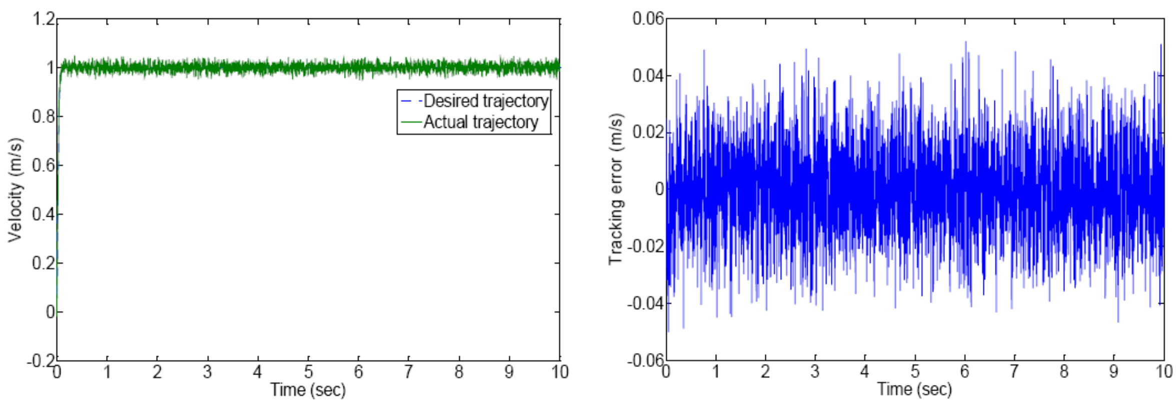
(a) Velocity (b) Tracking error

FIGURE 2. Simulation results with adaptive GAS



(a) Velocity (b) Tracking error

FIGURE 3. Simulation results with DABSMC

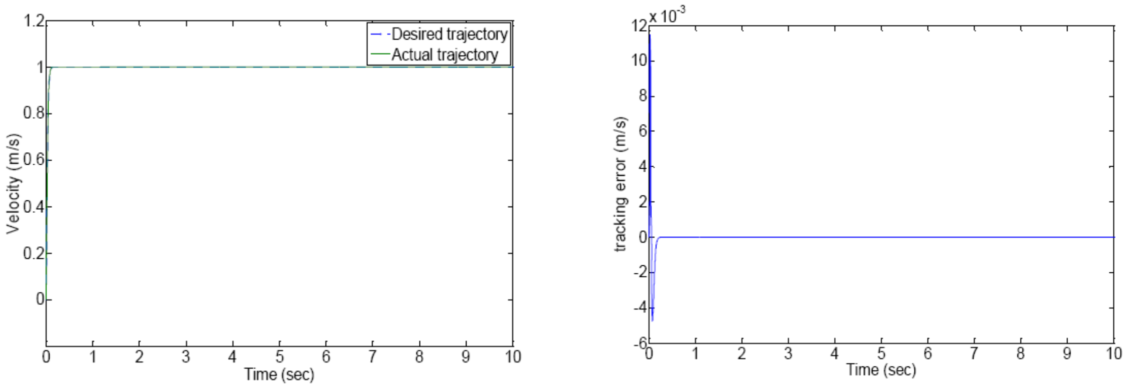


(a) Velocity (b) Tracking error

FIGURE 4. Simulation results with MBDA-PID

5. Control of Inverted-Cart Pendulum System.

5.1. **Mathematical model of inverted cart pendulum system.** The ICPS [41] is graphically depicted in the Figure 6. The goal is for the cart track and the reference trajectory to be precise and to maintain the pendulum angle $\theta(t) = 0$ by using a feedback controller with a sensor (encoder or potentiometer) at $\theta(t)$ and with an actuator to produce



(a) Velocity (b) Tracking error

FIGURE 5. Simulation results with adaptive GAS (disturbance power increase 10 times)

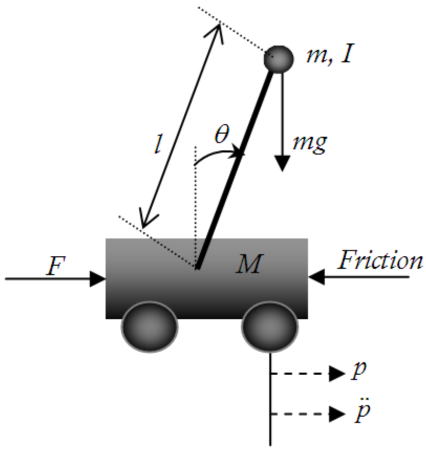


FIGURE 6. Inverted cart pendulum system and force diagram

an input force $F(t)$. There are two possible outputs, the pendulum angle $\theta(t)$ or the cart displacement $p(t)$. The classical inverted pendulum has only one input, the force $F(t)$ where one single input force has to control both the angle of the pendulum and the position of the cart at the same time. To derive its equations of motion, we can use Newton's Second Law or Lagrange dynamics approach. Here, Lagrange dynamics approach was used.

The Lagrange equation

$$\begin{aligned} &L = T - V, \\ &\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{p}}\right) - \frac{\partial L}{\partial p} = F + d \\ &\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \end{aligned} \tag{30}$$

where d includes external disturbance, and unmodeled dynamics such as viscous and coulomb friction forces.

The kinetic energy (T) of the system is

$$T = \frac{1}{2}m\left(\frac{d}{dt}(p - l\sin\theta)\right)^2 + \frac{1}{2}m\left(\frac{d}{dt}(l\cos\theta)\right)^2 + \frac{1}{2}M\dot{p}^2 + \frac{1}{2}I\dot{\theta}^2 \tag{31}$$

The potential (V) of the system with the up-left configuration of the pendulum at zero is:

$$V = mgl\cos\theta$$

The Lagrange of the system is constructed and the differential equations are obtained as [26,27]

$$\begin{aligned}&(M+m)\ddot{p} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F + d\\&(I+ml^2)\ddot{\theta} + ml\ddot{p}\cos\theta - mgl\sin\theta = 0\end{aligned} \tag{32}$$

where p (m) is the cart position, θ (rad) is the rod angle, and F (N) is the input force provided by the motor, M and m (kg) are the masses of the cart and, respectively, the rod, l (m) and I (kgm^2) are the rod semi-length and inertia respectively. d includes parameter uncertainties, external disturbance, and unmodeled dynamics such as viscous and coulomb friction forces.

While the ICPS is running, normally θ hardly changes at the equilibrium point or near zero. Therefore, approximation processing can be made: $\cos\theta \approx 1$, $\sin\theta \approx \theta$, $(d\theta/dt)^2 \approx 0$. The expression (32) can be simplified as follows:

$$\begin{cases}(M+m)\ddot{p} + ml\ddot{\theta} = F + d\\ ml\ddot{p} + (I+ml^2)\ddot{\theta} - mgl\theta = 0\end{cases} \tag{33}$$

Set the state-variables:

$$\begin{aligned}&x_1(t) = p(t)\\&x_2(t) = \dot{p}(t) = \dot{x}_1(t)\\&x_3(t) = \theta(t)\\&x_4(t) = \dot{\theta}(t) = \dot{x}_3(t)\end{aligned} \tag{34}$$

The expression (33) can be rewritten as:

$$\begin{cases}\dot{x} = Ax + Bu + Gd\\ y = Cx\end{cases}$$

where $u = F$, and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0\\ 0 & 0 & -\dfrac{m^2l^2g}{I(M+m)+Mml^2} & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & \dfrac{(M+m)mgl}{I(M+m)+Mml^2} & 0\end{bmatrix},$$
$$B = \begin{bmatrix} 0\\ \dfrac{I+ml^2}{I(M+m)+Mml^2}\\ 0\\ -\dfrac{ml}{I(M+m)+Mml^2}\end{bmatrix}, \tag{35}$$
$$G = \begin{bmatrix} \dfrac{I+ml^2}{I(M+m)+Mml^2}\\ -\dfrac{ml}{I(M+m)+Mml^2}\end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

Stability: From $\det(sI\text{-}A) = 0$ follows: $s_1 = 0$, $s_2 = 0$, $s_3 = 5.5869$ and $s_4 = -5.5869$, therefore, the linear model of the ICPS is unstable.

Controllability: To check the controllability, we check the rank of the controllability matrix $Q_C = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix}$ and $\text{rank}(Q_C) = 4$, so the linear model of the inverted pendulum is controllable. The parameters are listed in the Table 2.

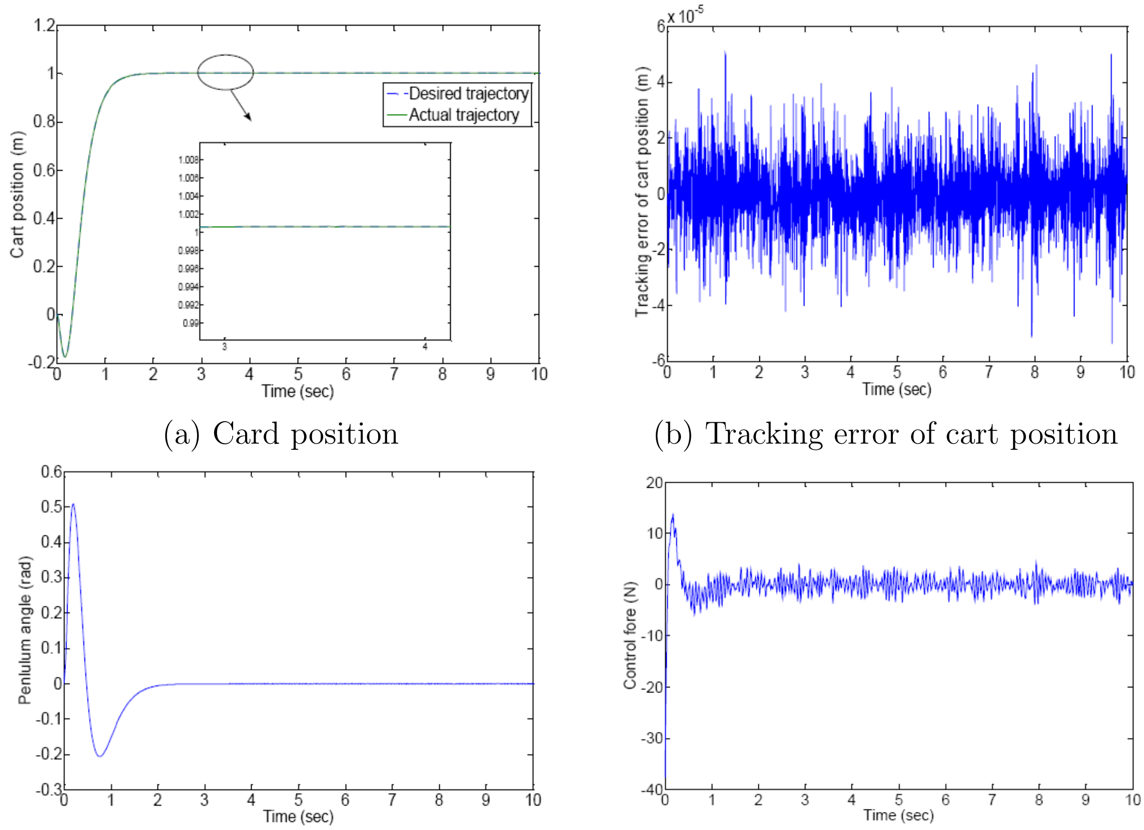
TABLE 2. The parameter of inverted pendulum

Parameter	Value
Mass of the cart (Kg)	0.5
Mass of the pendulum (Kg)	0.2
Length to pendulum center (m)	0.3
Inertia of the pendulum (Kg*m^2)	0.006
Gravity constant (m/s^2)	9.8

The desired poles are –4, –6, –7 and –10. The choice of the pole locations is somewhat arbitrary. These depend on our desired the system performance.

The system model for the control input is estimated by applying a factor to the corresponding parameter matrices of original system, i.e., $\hat{M} = 0.95M$, $\hat{m} = 0.95m$, $\hat{I} = I$.

5.2. **Simulation and results.** The Matlab/Simulink Software is used for simulation. The parameters of the ICPS are chosen as Table 2. The simulation results are shown in Figure 7.

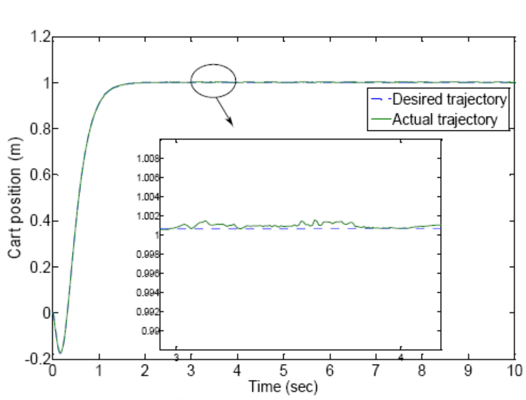


(a) Card position (b) Tracking error of cart position

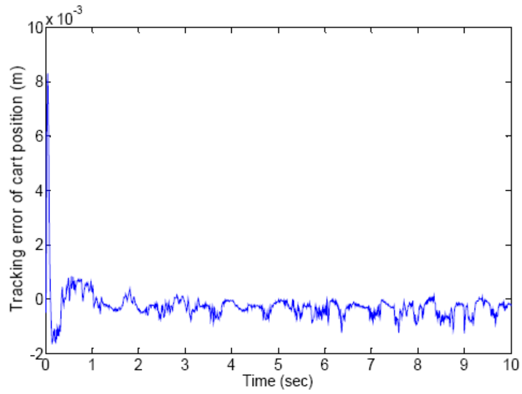
(c) Pendulum angle (d) Control force

FIGURE 7. Simulation results with adaptive GAS

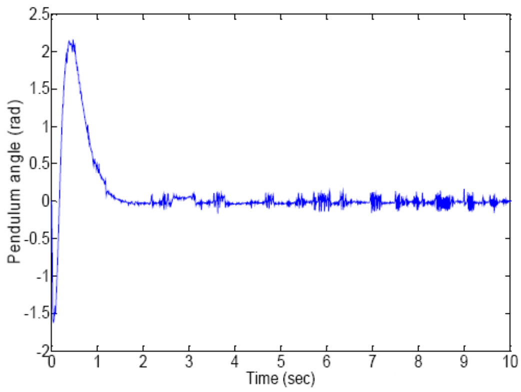
We compare our proposal with other recent studies in the control of ICPS [44,45]. In [44], the LQG is used for the control of ICPS. LQG is a combination of linear quadratic regulator (LQR) and kalman filter. It is now well-known that the linear quadratic regulator (LQR) has very impressive robustness properties, and thus, it can be considered as robust control. It is even performs better than other robust controls, such as the loop shaping controller [46]. However, LQR control does not assure robust stability when the system is highly uncertain, and effected by disturbances. To overcome this drawback, the kalman filter is used to receive the pure state (without disturbances) and there is no phase difference between actual states and estimated states. The drawback of the LQG method is it can only be used for systems with white noise. In [45], the integral sliding mode control (ISMC) is used for control of ICPS.



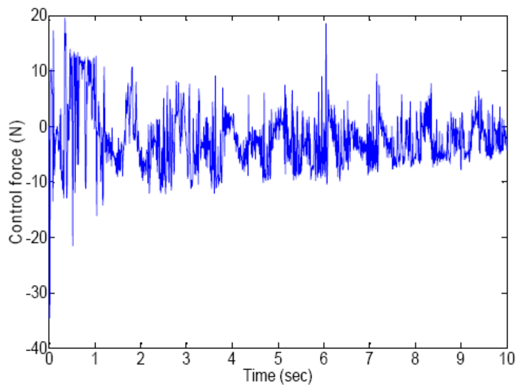
(a) Card position



(b) Tracking error of cart position



(c) Pendulum angle



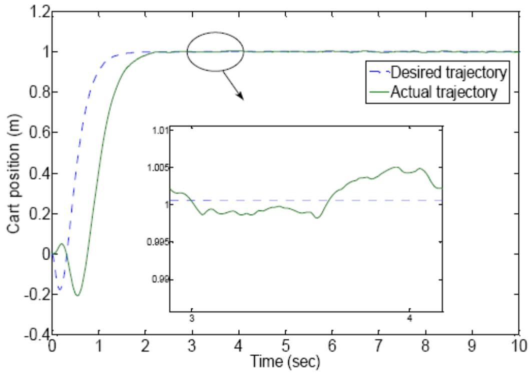
(d) Control force

Figure 8. Simulation results with ISMC

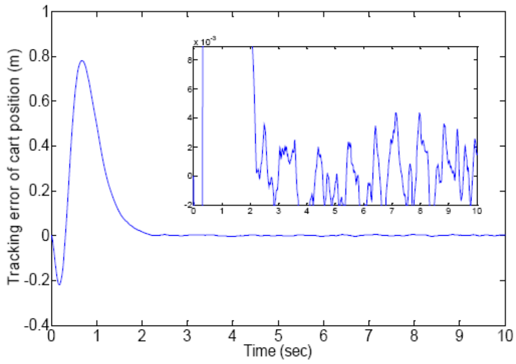
The simulation results are shown in Figures 8 and 9. From the simulation result, we have some the following comments:

- Regarding to disturbance attenuation, the ISMC is better than LQG (Figures 8(a) and 9(a)). Furthermore LQG is only used with white noise disturbance. Neither of the methods completely remove disturbance. But, the adaptive GAS is completely removes disturbance (Figure 7(a)).
- Regarding the tracking problem. For the LQG method, during the time from 0 to 1.8 seconds, the output signal is not tracking a reference signal (Figure 9(a)). With both of the adaptive GAS and ISMC methods, the output signal tracks the reference signal precisely (Figures 7(a) and 8(a)). However, the tracking error of the adaptive GAS is very small in comparison to ISMC and LQG (Figures 7(b), 8(b) and 9(b)). The maximum tracking error of the adaptive GAS is 5×10^{-5}m, and the ISMC is 8.3×10^{-3}m.

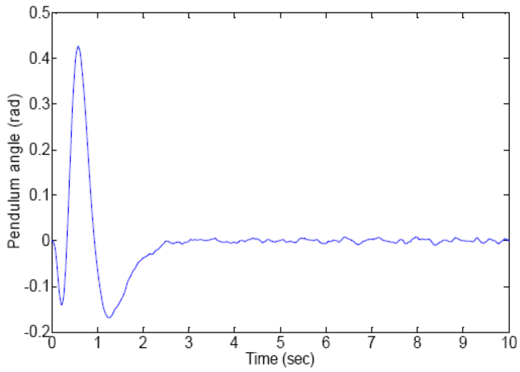
- Regarding the pendulum angle, the adaptive GAS is the most stable in comparison to the ISMC and LQG (Figures 7(c), 8(c) and 9(c)). The pendulum angle of the adaptive GAS method is asymptotical tending to zero (Figure 7(c)).



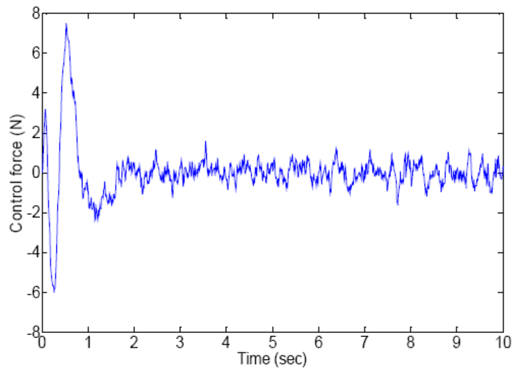
(a) Card position



(b) Tracking error of cart position



(c) Pendulum angle



(d) Control force

FIGURE 9. Simulation results with LQG

So the adaptive GAS performs best. Furthermore, it can remove any form of disturbance. The performance of the system can be achieved arbitrarily when we choose the poles.

6. **Conclusions.** In this paper, we proposed an adaptive GAS for controlling MIMO uncertain linear plant with persistent disturbance. In this method, the controller includes two parts: firstly, the transformed method, and state feedback make the plant (stable or unstable) achieve the required performance. Secondly, a nominal plant is designed parallel to the plant. Through the output feedback, the error signal, which is the difference between the plant output and the nominal plant output, is achieved by means of a disturbance compensator. Accordingly, disturbance attenuation can be attained. The method is simple and thus permits easy implementation and application to industry.

Simulation studies performed with a PMLSM and with an inverted cart pendulum indicate that the proposed algorithm assures global asymptotic stability. The simulation results indicate that the proposed controller can achieve the superior tracking performance with any form of disturbance, and does not require exact plant model and robust performance which is contrary to other recent methods. Furthermore by choosing poles, we can achieve performance arbitrarily.

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