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STATIC SWITCHED OUTPUT FEEDBACK STABILIZATION FOR LINEAR DISCRETE-TIME SWITCHED SYSTEMS

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Abstract. *This paper focuses on the problem of switched static output feedback (SOF) control for discrete-time switched linear systems under arbitrary switching laws. The considered class of systems is characterized by a particular structure of system matrices. Our principle idea is addressed in the derivation of new sufficient linear matrix inequalities conditions for the synthesis of a switched controller for a particular class of switched systems. The adopted methodology is based on the using of a special congruence transformation and a switched quadratic Lyapunov function. We propose important sufficient LMI conditions for SOF stabilization in the general case which guarantee the switched-quadratically stability of the closed-loop system. The various conditions are given through a family of LMI (linear matrix inequalities) parameterized by a scalar variable which offers an additional degree of freedom, enabling, at the expense of a relatively small degree of complexity in the numerical treatment (one line search), to provide better results compared with previous ones in the literature. A numerical example is presented to illustrate the effectiveness of the proposed conditions.*
Keywords: Switched system, Static output feedback, LMI

1. **Introduction.** Switched linear systems are an important class of Hybrid Dynamical Systems (HDS) [1, 3, 16]. A switched system is represented by a set of continuous-time or discrete-time subsystems and a rule that orchestrates the switching among them. In this area, the suitable control problem is directed towards the determination of an adaptive switched control assuming the real time knowledge (possibly by identification) of the switching process. Switched systems have numerous applications in control of mechanical systems, the automotive industry, aircraft and air traffic control, switching power converters, and many other fields which include the modelling of communication networks, networked control systems, the modelling of bio-chemical reactions, the control of systems with large uncertainty using logic-based supervisors, etc. In recent years, an increasing interest in the study of stability analysis and control design for switched systems [4, 7, 26, 27] can be noticed. The stability and control synthesis issues for discrete switched systems under arbitrary switching sequences are addressed [28, 29]. In practice, switched systems can be applied to various modelling and control problems present in robotics, automotive systems, process control, power systems, air traffic control, switching power converters, and many other fields which include the modelling of communication networks, networked control systems, the modelling of bio-chemical reactions, the control of nonlinear systems that cannot be stabilized by continuous control laws, the control of systems with large uncertainty using logic-based supervisors, etc. [16, 17]. In recent years, particular efforts of researches have received an increasing interest and a growing attention in the study of the stability analysis and control design for switched systems [4, 7, 21, 22, 23]. The

stability issues and control synthesis for discrete switched systems under arbitrary switching have been addressed. Rapid progress in the field has generated many new ideas and powerful tools as multiple Lypunov functions (MLF), piecewise Lyapunov function (PLF) and switched Lyapunov function (SLF) [5, 12, 13]. The basic concepts and the main properties of this approach are based in the existence of a particular Lyapunov function which has the same switching signals as the switched system. The results provided in this paper are less conservative. We discussed the problem of stabilization of discrete switched systems by static output feedback. The main motivation for studying switched systems comes partly from the fact that static output feedback (SOF) control is very useful and more realistic, since it can be easily implemented with low cost [19, 20, 24]. A new LMI formulation that uses a scalar variable is proposed, which makes it useful and interesting for design problems. It is shown that the proposed method can work successfully in situations where the existing methods fail [8, 20, 21, 23]. The paper is organized as follows. Section 2 gives the problem statement. Section 3 is the main result of this paper. New sufficient LMI conditions are deduced to obtain stabilizing SOF controller gains based on a switched quadratic Lyapunov function. Section 4 gives numerical evaluation and an example to illustrate the effectiveness of the proposed approach. Finally, the paper is concluded in Section 5.

Notation. Notation used in the paper is standard. In general capital letters denote matrices. For two symmetric matrices, A and B, $A > B$ means that $A - B$ is positive definite. A' denotes the transpose of A, $diag(x; y; ...)$ denotes the diagonal matrix obtained from vectors or matrices $x, y, ...$. When no confusion is possible, identity and null matrices will be denoted respectively by I and 0. Furthermore, in the case of partitioned symmetric matrices, the symbol $\bullet$ denotes generically each of its symmetric blocks. N is the number of subsystems. $Conv\{\}$ stands for convex combination. $E = \{1, ..., N\}$ denotes the set of indexes.

2. **Problem Statement and Preliminaries.** Consider a linear switched system in the discrete time domain described by the following state equation:

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + Bu(k) \\ y(k) = C_{\sigma(k)}x(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector of the system at time k, $u(k) \in \mathbb{R}^m$ is the control input vector, $y(k) \in \mathbb{R}^p$ is the measured output vector. The switching rule $\sigma(k)$ takes values in the finite set E.

$$\sigma(k) \in \{1, ..., N\}$$

and it changes its value at an arbitrary discrete time. This means that the switched system is described by the following set of modes:

$$\{(A_i, B, C_i) \setminus i \in E\}$$

and that the evolution of $\sigma(k)$ gives the switching sequence between these modes.

As in [8, 20, 23], the following assumptions are made:

H_1**:** The pairs $(A_i,\ B)$ and $(A_i,\ C_i)$ are assumed to be stabilizable and detectable, respectively, and C_i of full rank.

H_2**:** The switching rule $\sigma(k)$ is not known a priori but its value is real-time available.

We assume also without loss of generality that:

$$B = \begin{bmatrix} 1_m \\ 0 \end{bmatrix}$$

In this paper, we investigate the switched output feedback stabilization problem, that means the problem of designing a switched static output feedback control law.

$$u(k) = K_{\sigma(k)}y(k) \tag{2}$$

where $K_{\sigma(k)} \in \{K_i \in \mathbb{R}^{m\times p} \backslash i \in E\}$.

Such that the resulting closed-loop system:

$$x(k+1) = (A_{\sigma(k)} + BK_{\sigma(k)}C_{\sigma(k)})x(k) \tag{3}$$

is asymptotically stable.

Defining the indicator function:

$$\alpha_i(k) = \begin{cases} 1 & \text{if the system is in the } i\text{-th mode} \\ 0 & \text{otherwise} \end{cases}$$

with $i = 1, ..., N$, the switched system matrices can also be written as:

$$A_{\sigma(k)} = \sum_{i=1}^{N} \alpha_i(k)A_i$$

$$C_{\sigma(k)} = \sum_{i=1}^{N} \alpha_i(k)C_i$$

and the closed-loop state matrix as:

$$A_{\sigma(k)} + BK_{\sigma(k)}C_{\sigma(k)} = \sum_{i=1}^{N} \alpha_i(k)(A_i + BK_iC_i)$$

In the following, we investigate new LMI-based conditions for the SOF stabilization problem by using the concept of switched Lyapunov function and the notion of congruence transformation. The influence of the state vector description and the characteristic matrix on the determination of the controller is also studied.

We introduce the concepts of switched Lyapunov function used later in order to develop the results given in this paper.

To check stability of the switched system (1) let the switched Lyapunov function be defined as [8]:

$$V(k, x(k)) = x(k)' P(\sigma(k))x(k) \tag{4}$$

$$V(k, x(k)) = x(k)' \left(\sum_{i=1}^{N} \alpha_i(k)P_i \right) x(k) \tag{5}$$

$P_1, ..., P_N$ are symmetric positive definite matrices. If such a positive definite Lyapunov function exists and its increment:

$$\Delta V(k, x_k) = V(k+1, x_{k+1}) - V(k, x_k) \tag{6}$$

is negative definite along the solution of (1) then the origin of the switched system is globally asymptotically stable.

According to Lyapunov stability theory, the closed-loop system (3) is asymptotically stable with a switched Lyapunov function of the form (4) if and only if

$$A'_{cl_i}P_jA_{cl_i} - P_i < 0, \ \forall(i,j) \in E \times E$$

where $A_{cl_i} = A_i + BK_iC_i$.

Let P_i positive-definite symmetric matrix partitioned into:

$$P_i = \begin{bmatrix} P_{1i} & P_{2i} \\ P_{3i}' & P_{4i} \end{bmatrix} \tag{7}$$

with $P_{1i} \in \mathbb{R}^{m\times m}$, $P_{2i} \in \mathbb{R}^{m\times(n-m)}$ and $P_{3i} \in \mathbb{R}^{(n-m)\times(n-m)}$ and define the matrix $D = \begin{bmatrix} 0 \\ 1_{(n-m)} \end{bmatrix}$ and the state $x(k) = \begin{bmatrix} x_D(k) \\ x_B(k) \end{bmatrix}$, where $x_D(k) = D'x(k)$ and $x_B(k) = B'x(k)$.

Let

$$A_i = \begin{bmatrix} A_{1i} & A_{2i} \\ A_{3i} & A_{4i} \end{bmatrix} \tag{8}$$

and

$$C_i = \begin{bmatrix} C_{1i} & C_{2i} \end{bmatrix}$$

partitioned according to the partition of P_i.

Let $T_i \in \mathbb{R}^{n\times n}$, be nonsingular matrices such that:

$$T_i = \begin{bmatrix} I & Y_i \\ 0 & Z \end{bmatrix} \tag{9}$$

Let $T_s \in \mathbb{R}^{n\times n}$, be nonsingular matrices such that:

$$T_s = \begin{bmatrix} I & 0 \\ 0 & \epsilon I \end{bmatrix} \tag{10}$$

Using the matrix T_s as a similarity transformation for system 1, we have system (4) is stable if and only if $\tilde{A}_{cl_i} = \tilde{A}_i + \tilde{B}K_i\tilde{C}_i$ is stable, where $\tilde{A}_i = T_sA_iT_s^{-1}$, $\tilde{B} = T_sB = B$, and $\tilde{C}_i = C_iT_s^{-1}$.

We present now a useful lemma used in the proofs later in the paper.

Lemma 2.1. *Let Φ a symmetric matrix and N, M matrices of appropriate dimensions. The following statements are equivalent:*

(i) $\Phi < 0$ *and* $\Phi + NM' + MN' < 0$

(ii) *There exists a matrix F such that:*

$$\begin{pmatrix} \Phi & M + NF \\ M' + F'N' & -F - F' \end{pmatrix} < 0$$

Proof: See [25].

We present in the following a sufficient condition for the static output feedback stabilization (SOF).

3. Main Results.

The next theorems formulate new sufficient LMI conditions for the synthesis of a stabilizing switched SOF controller.

Theorem 3.1. *System (1) is switched-quadratically stabilizable by a static output feedback (2) if there exist positive definite symmetric matrices P_i portioned as in (7) and gains $K_i \in \mathbb{R}^{m\times p}$ such that $\forall(i,j) \in E \times E$:*

$$\begin{pmatrix} -P_i + \tilde{A}_i{}'DP_{3j}D'\tilde{A}_i & 0 \\ \bullet & -P_{1j} \end{pmatrix} + \begin{pmatrix} \tilde{A}_i{}'B + \tilde{C}_i{}'K_i' \\ 0 \end{pmatrix} \tag{11}$$

$$\begin{pmatrix} P_{2j}D'\tilde{A}_i & P_{1j} \end{pmatrix} + \begin{pmatrix} \tilde{A}_i{}'DP_{2j}' \\ P_{1j} \end{pmatrix} \begin{pmatrix} B'\tilde{A}_i + K_i\tilde{C}_i & 0 \end{pmatrix} < 0$$

Proof: Let $V(k) = \tilde{x}'(k)P_i\tilde{x}(k)$, where $\tilde{x}(k) = T_s x(k)$. System (1) is switched-quadratically stabilizable by a switched static output feedback (2) if and only inequality $A'_{cl_i}P_jA_{cl_i} - P_i < 0, \forall(i,j) \in E \times E$ holds.

Hence:

$$\begin{aligned} V(k+1) - V(k) = &\tilde{x}'(k)[(\tilde{A}_i{}' + \tilde{C}_i{}'K_i'B')BP_{1j}B'(\tilde{A}_i + BK_i\tilde{C}_i) \\ &+ (\tilde{A}_i{}' + \tilde{C}_i{}'K_i'B')BP_{2j}D'(\tilde{A}_i + BK_i\tilde{C}_i) \\ &+ (\tilde{A}_i{}' + \tilde{C}_i{}'K_i'B')DP_{2j}'B'(\tilde{A}_i + BK_i\tilde{C}_i) \\ &+ (\tilde{A}_i{}' + \tilde{C}_i{}'K_i'B')DP_{3j}D'(\tilde{A}_i + BK_i\tilde{C}_i)]\tilde{x}(k) < 0 \end{aligned}$$

given that $D'B = B'D = 0$ and $B'B = I$, the last inequality is equivalent to:

$$\begin{aligned} &- P_i + \tilde{A}_i{}'DP_{3j}D'\tilde{A}_i + (\tilde{A}_i{}'B + \tilde{C}_i{}'K_i')P_{1j}(B'\tilde{A}_i + K_i\tilde{C}_i) \\ &+ (\tilde{A}_i{}'B + \tilde{C}_i{}'K_i')P_{2j}D'\tilde{A}_i + \tilde{A}_i{}'DP_{2j}(B'\tilde{A}_i + K_i\tilde{C}_i) < 0 \end{aligned}$$

and then by Schur complement we leads to (11).

Remark 3.1. *The condition of Theorem 3.1 is nonlinear in the unknown variables P_i and K_i. The problem of solving numerically (11) is non-convex. Using Lemma 2.1 and a suitable congruence transformation, the nonlinearity of the condition 11 can be eliminated and sufficient LMI conditions can be obtained for particular class of systems. These conditions have the advantage to be convex and are numerically well tractable.*

In the following, we provide the main result of this paper. First, two sufficient scaling LMI conditions for particular class of systems are given. Then, a sufficient condition is presented for the general case. All the results are proved using condition of the Theorem 3.1, Lemma 2.1 and suitable congruence transformation. Indeed, the proposed condition (11) of Theorem 3.1 is linear in the unknown variables and it can be so easily solved using convex optimization techniques using the Lemma 2.1. This can be achieved only by verifying the following condition:

$$-P_i + \tilde{A}_i'DP_{3j}D'\tilde{A}_i < 0;\ \forall(i,j) \in E \times E$$

and thus, sufficient LMI conditions can be obtained. The controller synthesis method presented in the Theorem 3.1 is applied to some particular class of switched systems.

3.1. **Case $A_{i4} = 0$.** The basic idea is to guarantee the negativity of the matrix in order to apply the Lemma 2.1 and to deduce thereafter the stability of the switched system. To reach this objective, we proceed by imposing some hypothesis like the partitioning of the Lyapunov matrices P_i and the blocks A_i of the state matrices which composed the switched system.

In the following section, we propose a new method for the synthesis of stabilizing switched SOF controllers which is formulated as a feasibility problem of a set of sufficient LMI conditions for system (1) with $A_{i4} = 0$ and partitioned as (8).

Theorem 3.2. *System (1) with $A_{i4} = 0$ is switched-quadratically stabilizable by a switched static output feedback (2) if there exist positive definite symmetric matrices P_i partitioned as in (7), matrices G_i and R_i, and a real ϵ sufficiently small such that $\forall(i,j) \in E \times E$:*

$$\begin{pmatrix} -P_i + \varepsilon^2\Phi_1'P_{3j}\Phi_1 & \Phi_2G_i + \Phi_3R_i + \varepsilon\Phi_4P_{2j}' \\ \bullet & P_{1j} - G_i - G_i' \end{pmatrix} < 0 \tag{12}$$

where

$$\Phi_1 = \begin{bmatrix} A_{3i} & 0 \end{bmatrix}, \Phi_2 = \begin{bmatrix} A_{1i}' \\ \varepsilon^{-1}A_{2i}' \end{bmatrix}, \Phi_3 = \begin{bmatrix} C_{1i}' \\ \varepsilon^{-1}C_{2i}' \end{bmatrix}, \Phi_4 = \begin{bmatrix} A_{3i}' \\ 0 \end{bmatrix}$$

The stabilizing output feedback controller gains are given by:

$$K_i = (R_i G_i^{-1})' \tag{13}$$

Proof: Applying similarity transformation T_s (10) (ϵ sufficiently small) to the switched system (1) with $A_{i4} = 0$, the stabilization by SOF (2) is equivalent by Lemma 2.1 to:
$\begin{pmatrix} -P_i + \varepsilon^2 \Phi_1' P_{3j} \Phi_1 & 0 \\ \bullet & -P_{1j} \end{pmatrix} + sym \begin{pmatrix} \Phi_2 + \Phi_3 K_i' \\ 0 \end{pmatrix} \begin{pmatrix} \varepsilon P_{2j} \Phi_1 & P_{1j} \end{pmatrix} < 0$

There exists ϵ, such that:

$$\Phi = \begin{pmatrix} -P_i + \varepsilon^2 \Phi_1' P_{3j} \Phi_1 & 0 \\ \bullet & -P_{1j} \end{pmatrix} < 0$$

and by Lemma 2.1, for ϵ sufficiently small, this equivalent to the existence of matrix G_i such that:

$$\begin{pmatrix} -P_i + \varepsilon^2 \Phi_1' P_{3j} \Phi_1 & 0 & \Phi_2 G_i + \Phi_3 K_i' G_i + \varepsilon \Phi_4 P_{2j}' \\ \bullet & -P_{1j} & P_{1j} \\ \bullet & \bullet & -G_i - G_i' \end{pmatrix} < 0$$

where

$$\Phi_1 = \begin{bmatrix} A_{3i} & 0 \end{bmatrix}, \Phi_2 = \begin{bmatrix} A_{1i}' \\ \varepsilon^{-1} A_{2i}' \end{bmatrix}, \Phi_3 = \begin{bmatrix} C_{1i}' \\ \varepsilon^{-1} C_{2i}' \end{bmatrix}, \Phi_4 = \begin{bmatrix} A_{3i}' \\ 0 \end{bmatrix}$$

Letting $R_i = K_i' G_i$, we obtain by Schur complement (12).

3.2. **Case A_{3i} of full row rank.** The following theorem is deduced by taking matrices A_{3i} of full row rank and applying a suitable congruence transformation which transforms the state matrices $\tilde{A}_i$ into $\tilde{A}_i = \begin{bmatrix} \tilde{A}_{1i} & \tilde{A}_{2i} \\ \tilde{A}_{3i} & 0 \end{bmatrix}$ and then we obtain a sufficient condition for SOF by using the result of the Theorem 3.2. Now, we give a new sufficient LMI conditions to compute the controller gains K_i.

Theorem 3.3. *System (1) with A_{3i} full row rank, is switched-quadratically stabilizable by a switched static output feedback (2) if there exist positive definite symmetric matrices P_i partitioned as in (7), matrices G_i and R_i, and a real ϵ sufficiently small such that $\forall (i,j) \in E \times E$:*

$$\begin{pmatrix} -T_i^{(-1)'} P_i T_i^{-1} + T_i^{(-1)'} \Phi_5' P_{3j} \Phi_5 T_i^{-1} & T_i^{(-1)'} \Phi_2 G_i + T_i^{(-1)'} \Phi_3 R_i + T_i^{(-1)'} \Phi_5 P_{2j}' \\ \bullet & P_{1j} - G_i - G_i' \end{pmatrix} < 0 \tag{14}$$

with: $T_i = \begin{bmatrix} I & -\tilde{A}_{4i} \left(\tilde{A}_{3i} \tilde{A}_{3i}' \right)^{-1} \tilde{A}_{3i} \\ 0 & I \end{bmatrix}$, $\Phi_2 = \begin{bmatrix} A_{1i}' \\ \varepsilon^{-1} A_{2i}' \end{bmatrix}$, $\Phi_3 = \begin{bmatrix} C_{1i}' \\ \varepsilon^{-1} C_{2i}' \end{bmatrix}$, $\Phi_5 = \begin{bmatrix} \epsilon A_{3i} & A_{4i} \end{bmatrix}$.

The stabilizing switched output feedback controller gains are given by

$$K_i = (R_i G_i^{-1})' \tag{15}$$

Proof: Let $V(k) = \tilde{x}'(k) P_i \tilde{x}(k)$, where $\tilde{x}(k) = T_s x(k)$. System (1) is switched-quadratically stabilizable by a switched static output feedback (2) if and only inequality $A_{cl_i}' P_j A_{cl_i} - P_i < 0$, $\forall (i,j) \in E \times E$ holds. Hence:

$$\begin{aligned} V(k+1) - V(k) = & \tilde{x}'(k)[(\tilde{A}_i{}' + \tilde{C}_i{}' K_i' B') B P_{1j} B' (\tilde{A}_i + B K_i \tilde{C}_i) \\ & + (\tilde{A}_i{}' + \tilde{C}_i{}' K_i' B') B P_{2j} D' (\tilde{A}_i + B K_i \tilde{C}_i)(\tilde{A}_i{}' + \tilde{C}_i{}' K_i' B') D P_{2j}' B' (\tilde{A}_i \\ & + B K_i \tilde{C}_i) + (\tilde{A}_i{}' + \tilde{C}_i{}' K_i' B') D P_{3j} D' (\tilde{A}_i + B K_i \tilde{C}_i)] \tilde{x}(k) < 0 \end{aligned}$$

given that $D'B = B'D = 0$ and $B'B = I$, the last inequality is equivalent to:

$$\begin{aligned} &- P_i + \tilde{A}_i{}' D P_{3j} D' \tilde{A}_i + (\tilde{A}_i{}' B + \tilde{C}_i{}' K_i') P_{1j} (B' \tilde{A}_i + K_i \tilde{C}_i) \\ &+ (\tilde{A}_i{}' B + \tilde{C}_i{}' K_i') P_{2j} D' \tilde{A}_i + \tilde{A}_i{}' D P_{2j} (B' \tilde{A}_i + K_i \tilde{C}_i) < 0 \end{aligned}$$

and then by Schur complement we leads to (12).

By applying 2.1 this is equivalent to the existence of matrices G_i such that:

$$\begin{bmatrix} -P_i + \tilde{A}_i' D P_{3j} D' \tilde{A}_i & 0 & \tilde{A}_i' B G_i + \tilde{C}_i' K_i' G_i + \tilde{A}_i' D P_{2j}' \\ \bullet & -P_{1j} & P_{1j} \\ \bullet & \bullet & -G_i - G_i' \end{bmatrix} < 0$$

By schur complement formula, these inequalities are equivalent to:

$$\begin{bmatrix} -P_i + \tilde{A}_i' D P_{3j} D' \tilde{A}_i & \tilde{A}_i' B G_i + \tilde{C}_i' K_i' G_i + \tilde{A}_i' D P_{2j}' \\ \bullet & P_{1j} - G_i - G_i' \end{bmatrix} < 0$$

Using a congruence transformation and multiplying these inequalities with $\begin{bmatrix} T_i^{-1} & 0 \\ 0 & I \end{bmatrix}$ to the right and its transpose to the left yields to:

$$\begin{bmatrix} -T_i^{(-1)'} P_i T_i^{-1} + T_i^{(-1)'} \Phi_5' P_{3j} \Phi_5 T_i^{-1} & T_i^{(-1)'} \Phi_2 G_i + T_i^{(-1)'} \Phi_3 K_i' G_i + T_i^{(-1)'} \Phi_5 P_{2j}' \\ \bullet & P_{1j} - G_i - G_i' \end{bmatrix} < 0$$

By taking, $T_i = \begin{bmatrix} I & -\tilde{A}_{4i} \left(\tilde{A}_{3i} \tilde{A}_{3i}' \right)^{-1} \tilde{A}_{3i} \\ 0 & I \end{bmatrix}$ and $\Phi_2 = \begin{bmatrix} A_{1i}' \\ \varepsilon^{-1} A_{2i}' \end{bmatrix}$, $\Phi_3 = \begin{bmatrix} C_{1i}' \\ \varepsilon^{-1} C_{2i}' \end{bmatrix}$, $\Phi_5 = \begin{bmatrix} \epsilon A_{3i} & A_{4i} \end{bmatrix}$, the condition (14) is satisfied $\forall (i,j) \in E \times E$. We can compute so the switched output feedback gains as $K_i = (R_i G_i^{-1})' \forall i \in E$.

Remark 3.2. *By similarity transformation matrix B can be in the following form:*

$$B = \begin{bmatrix} 0 \\ 1_m \end{bmatrix}$$

and then a sufficient conditions for SOF stabilization similar to those of Theorem 3.2 and 3.3 can be deduced, when $A_{1i} = 0$ or A_{2i} is of full row rank.

3.3. **General case.**

Theorem 3.4. *System (1) is switched-quadratically stabilizable by a switched static output feedback (2) if there exist positive definite symmetric matrices P_i portioned as in (7), matrices G_i and R_i, and a real ϵ sufficiently small such that $\forall (i,j) \in E \times E$:*

$$\begin{pmatrix} -P_i + \Phi_1' P_{3j} \Phi_1 & \Phi_2 G_i + \Phi_3 R_i + \epsilon \Phi_4 P_{2j}' \\ \bullet & P_{1j} - G_i - G_i' \end{pmatrix} < 0 \tag{16}$$

where $\Phi_1 = \begin{bmatrix} \epsilon A_{3i} & (I - \frac{A_{3i} A_{3i}'}{\lambda_{max}(A_{3i} A_{3i}')}) A_{4i} \end{bmatrix}$, $\Phi_2 = \begin{bmatrix} A_{1i}' \\ \epsilon^{-1} A_{2i}' \end{bmatrix}$, $\Phi_3 = \begin{bmatrix} C_{1i}' \\ \epsilon^{-1} C_{2i}' \end{bmatrix}$ *and* $\Phi_4 = \begin{bmatrix} A_{3i}' \\ A_{4i}' \end{bmatrix}$*. The stabilizing switched output feedback controller gains are given by*

$$K_i = (R_i G_i^{-1})' \tag{17}$$

Proof: Follows in a direct way from the LMI given in Section 2 applied on the switched system (1) obtained after similarity transformation T_s. The resulting LMIs are treated by congruence transformation by taking $T_i = \begin{bmatrix} I & \frac{A'_{3i}A_{4i}}{\lambda_{\max}(A_{3i}A'_{3i})} \\ 0 & \epsilon I \end{bmatrix}$ and change of variables $R_i = K'_i G_i$.

Remark 3.3. *By applying 2.1, it is clear that in the general case*

$$\Phi = \begin{bmatrix} -P_i + \Phi'_1 P_{3j} \Phi_1 & 0 \\ \bullet & -P_{1j} \end{bmatrix},$$

the stability condition depends on the negativity of Φ.

4. Numerical Example.

4.1. Numerical evaluation.

To prove the efficiency of the proposed conditions, a numerical evaluation is given in this section. The problem considered here is the design of a static output feedback controller stabilizing the switched system.The result obtained using the Theorem 3.3 is compared to the three methods developed in [8, 20, 23] and summarize in the Table 1. The switched system is characterized by: the number of modes (N), the system order (n), number of inputs (m) and the number of outputs (p). For fixed values of (N, n, m, p), we generate randomly 100 switched systems of the form (1). So the purpose is to design by using four methods a feedback controller in the form (2) such that the closed-loop system (3) is stable.

1. Method1: This corresponds to conditions in Theorem 3.3 of our paper.
2. Method2: uses the conditions given in Theorem 4 [8].
3. Method3: uses the conditions given in Theorem 4.1 [20].
4. Method4: uses the conditions given in Theorem 1 [23] .

For each switched system, we try to compute a stabilizing output feedback control using four methods. By using the matlab LMI Control Toolbox to check the feasibility of the LMI conditions, we introduce a counter (Success Method1, Success Method2, Success Method3 and Success Method4) which is increased if the corresponding method succeeds in providing an output feedback stabilizing control. One can see that our proposed static feedback synthesis conditions given in 3.3 reduce significantly the conservatism. The table 1 summarizes the obtained results.

4.2. Numerical example.

To illustrate the applicability of our approach, we present a numerical example. This example provide a comparison of our result to the result presented in [8, 20, 23]. This example show that our synthesis method works successfully in situations where the methods developed in [8, 20, 23] do not. Therefore, we can consider these approaches as alternative approaches for the class of switched systems of the form (1) under arbitrary switching law.

Consider the discrete-time switched system (1) with 2 modes described by the following matrices:

$$A_1 = \begin{bmatrix} 0.4970 & 0.1913 & 0.9737 & 0.8015 \\ 0.0547 & 0.7246 & 0.7045 & 0.2062 \\ 0.7727 & 0.8047 & 0.9046 & 0.6584 \\ 0.8727 & 0.7566 & 0.4210 & 0.0013 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.1563 & 0.9613 & 0.7349 & 0.4966 \\ 0.5578 & 0.9513 & 0.2226 & 0.0070 \\ 0.3162 & 0.7963 & 0.3999 & 0.0919 \\ 0.7384 & 0.4003 & 0.2517 & 0.6022 \end{bmatrix}$$

TABLE 1. Numerical evaluation

Switched System	Success	N=2
n=2 m=1 p=1	Method1 Method2 Method3 Method4	**51** 45 32 45
n=3 m=2 p=2	Method1 Method2 Method3 Method4	**99** 97 92 97
n=4 m=2 p=2	Method1 Method2 Method3 Method4	**36** 31 4 31
n=5 m=3 p=2	Method1 Method2 Method3 Method4	**36** 13 1 28
n=6 m=3 p=3	Method1 Method2 Method3 Method4	**27** 10 0 9
n=7 m=4 p=3	Method1 Method2 Method3 Method4	**20** 1 0 1

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.9820 & 0.3476 & 0.4437 & 0.5295 \\ 0.4123 & 0.6682 & 0.8399 & 0.2571 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0.8324 & 0.4655 & 0.0857 & 0.7154 \\ 0.2935 & 0.0455 & 0.5131 & 0.0354 \end{bmatrix}$$

Note that both A_1 and A_2 are unstable. For this switched system, the methods in [8, 20, 23] does not allow to compute a switched SOF controller. The condition in Theorem 3.3 provides the following controller gains:

$$K_1 = \begin{bmatrix} -0.8862 & -0.6978 \\ -0.5793 & -1.1387 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -0.2636 & -1.0142 \\ -0.3506 & -0.1892 \end{bmatrix}$$

We allow the system to switch arbitrarily between these two modes according to the switching rule $\sigma(k)$. For a switching sequence as depicted in Figure 2, an initial condition $x(0) = [-0.4\ -0.3\ -0.2\ 0.1]^T$, we can see in Figure 1 that by using our switched controller synthesis procedure, that the trajectory of the closed-loop system is stable.

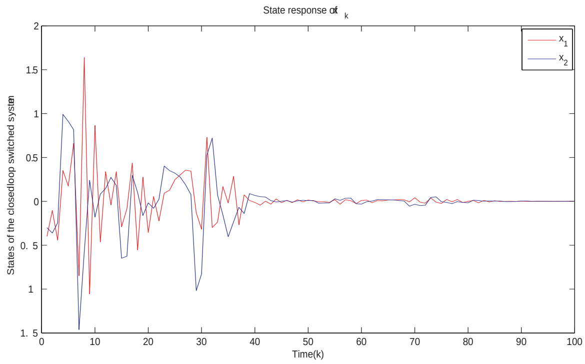

Figure 1. States of the switched system in the closed-loop

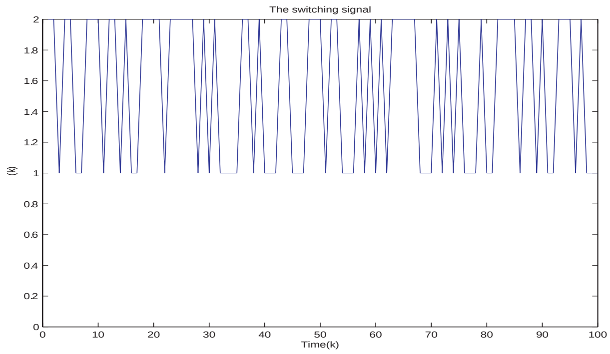

Figure 2. The switching rule $\sigma(k)$

5. **Conclusion.** In this paper, the problem of synthesis of switched SOF controller for discrete-time switched linear systems under arbitrary switching laws has been investigated. Our main contribution consists in providing a new sufficient LMI conditions for the SOF control method for a particular class of switched systems. A numerical evaluation is presented to illustrate the effectiveness of the proposed approach. As shown in the numerical example, our method can work successfully in situations where the methods in [8, 20, 23]. In addition, we will consider the extension of our result in the future for the studying of robust SOF control.

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