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A FAMILY OF SET-MEMBERSHIP AFFINE PROJECTION ADAPTIVE FILTER ALGORITHMS

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Abstract. *In this paper, we extend the set-membership (SM) adaptive filtering approach to the various affine projection (AP) adaptive filter algorithms to propose the computationally efficient algorithms. Based on this, the SM-APA, SM selective regressor APA (SM-SR-APA), SM dynamic selection APA (SM-DS-APA) and SM selective partial update APA (SM-SPU-APA) are established. The SM-SR-APA reduces complexity by selecting a subset of input regressors at every iteration. In SM-DS-APA, the dynamic selection of input vectors is used during the adaptation. The filter coefficients are partially updated in SM-SPU-APA. Also by combination of SM and SPU approaches, the SM-SPU-SR-APA and SM-SPU-DS-APA are introduced. We demonstrate the good performance of the presented algorithms for system identification, line and acoustic echo cancellation applications.*
Keywords: Adaptive filter, Affine projection, Set-membership, Selective regressor, Partial update

1. **Introduction.** Most popular algorithms in adaptive filter signal processing are least mean squares (LMS) of Widrow and Hoff and normalized least mean squares (NLMS) algorithms [1]. These algorithms have low computational complexity, simplicity, robustness and slow convergence speed especially for correlated input data. To overcome the deteriorated convergence speed of LMS and NLMS for correlated input data, Ozeki and Umeda [2] developed the basic form of an affine projection algorithm (APA) using affine subspace projections [3,4]. While NLMS updates the weights based only on the current input vector, APA updates the weights based on current and previous input vectors. Affine projection algorithm (APA) is a useful family of adaptive filters which has numerous applications in digital signal processing [5-9].

To improve the performance of APAs, different APAs were presented in the literature. Especially, the computational complexity of the classical APA will be large for some applications such as line and acoustic echo cancellation. In these algorithms, a large number of filter coefficients will be needed to achieve good performance. Therefore, the large computational complexity is the main problem in these applications. To reduce the computational complexity, the selective regressor APA (SR-APA) was presented in [10]. In this algorithm, the optimal selection of input regressors was derived by comparing the cost functions based on minimum disturbance. In [11], a novel APA which dynamically selects input vectors (DS-APA) in order to improve convergence performance was established. Also, the selective partial update APA (SPU-APA) was presented in [12] to reduce the computational complexity. In this algorithm, the filter coefficients are partially updated at each iteration. It has been shown that the SPU-APA has close performance to ordinary APA. As with many other adaptive filter algorithms, the step-size determines the tradeoff between steady-state mean square error (MSE) and convergence rate in all these algorithms.

Having fast convergence, low steady-state MSE and low computational complexity at the same time is highly desirable in adaptive filter algorithms. The set-membership normalized LMS (SM-NLMS) is one of the algorithms that have these three features [13]. Based on [13], different SM adaptive algorithms have been developed. The SM affine projection algorithm (SM-APA) [14,15], the SM binormalized data-reusing LMS (SM-BNDRLMS) algorithms [16] and SM subband adaptive filters (SM-SAF) [17] are important examples of this family of adaptive filters. Also in [17,18], the SM-SPU-NLMS and SM-SPU-SAF were presented based on the combination of the partial updating and set-membership filtering approaches. In this paper, we extend the SM filtering approach to the various affine projection adaptive filters to establish the computationally efficient algorithms with good convergence speed and low steady-state mean square error.

What we propose in this paper can be summarized as follows:

- Extension of the set-membership filtering approach of [14] to the SPU-APA and the establishment of a novel SM-SPU-AP algorithm. This algorithm has low computational complexity, low steady-state mean-square deviation (MSD) and fast convergence speed compared with SPU-APA.
- Extension of the set-membership filtering concept to the SR-APA and DS-APA, and the establishment of a novel SM-SR-APA and SM-DS-APA. These introduced algorithms have better performance than SR-APA and DS-APA. Also, these algorithms have close performance to SM-APA.
- Combination of the SPU-APA with SR-APA and DS-APA to develop the SM-SPU-SR-APA and SM-SPU-DS-APA. The reduction of computational complexity of these proposed algorithms will be large due to SR, SPU and SM features.
- Demonstrating of the proposed algorithms in system identification, line and acoustic echo cancellation applications.

We have organized our paper as follows. In the following section, we briefly review the APA. In the next section, the SM affine projection algorithms are established. We conclude the paper by showing a comprehensive set of simulations in system identification, line and acoustic echo cancellation scenarios.

Throughout the paper, the following notations are adopted:

$|\,.\,|$: norm of a scalar;
$\|.\|^2$: squared Euclidean norm of a vector;
$(.)^T$: transpose of a vector or a matrix;
$\mathrm{Tr}(.)$: trace of a matrix.

2. **Background on Affine Projection Algorithm.** Figure 1 shows a typical adaptive filter setup, where $x(n)$, $d(n)$ and $e(n)$ are the input, the desired and the output error signals, respectively. Here, $\mathbf{h}(n)$ is the $M\times 1$ column vector of filter coefficients at iteration

n. Now, define the $M \times K$ matrix of the input signal as

$$\mathbf{X}(n) = [\mathbf{x}(n), \mathbf{x}(n-1), \ldots, \mathbf{x}(n-(K-1))], \tag{1}$$

and the $K \times 1$ vector of desired signal as

$$\mathbf{d}(n) = [d(n), d(n-1), \ldots, d(n-(K-1))]^T, \tag{2}$$

where $\mathbf{x}(n) = [x(n), x(n-1), \ldots, x(n-M+1)]^T$ is the input signal regressors and K is a positive integer (usually, but not necessarily $K \leq M$). The APA can be derived from the solution of the following optimization problem:

$$\min_{\mathbf{h}(n+1)} \|\mathbf{h}(n+1) - \mathbf{h}(n)\|^2 \tag{3}$$

subject to $\mathbf{d}(n) = \mathbf{X}^T(n)\mathbf{h}(n+1)$. Using the method of Lagrange multipliers to solve this optimization problem leads to the following recursion:

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu\mathbf{X}(n)(\mathbf{X}^T(n)\mathbf{X}(n))^{-1}\mathbf{e}(n), \tag{4}$$

where $\mathbf{e}(n)$ is the output error vector which is defined as

$$\mathbf{e}(n) = \mathbf{d}(n) - \mathbf{X}^T(n)\mathbf{h}(n). \tag{5}$$

3. Set-Membership Adaptive Filter Algorithms.

3.1. SM-APA.

From [14], we know that the SM-APA minimizes Equation (3) subject to $\mathbf{h} \in \Psi_n \cap \Psi_{n-1} \cap \ldots \cap \Psi_{n-K+1}$ where[1]

$$\Psi_{n-i} = \{\mathbf{h} \in \mathbf{R}^M : |d(n-i) - \mathbf{x}^T(n-i)\mathbf{h}| \leq \gamma\}. \tag{6}$$

In [14], it has been shown that the suitable update equation for SM-APA can be stated as

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mathbf{X}(n)(\mathbf{X}^T(n)\mathbf{X}(n))^{-1}\mathbf{q}\alpha(n)e(n) \tag{7}$$

where $\mathbf{q} = [1, 0, \ldots, 0]^T$ is $K \times 1$ column vector and $\alpha(n)$ can be obtained from Equation (8).

$$\alpha(n) = \begin{cases} 1 - \dfrac{\gamma}{|e(n)|} & \text{if } |e(n)| > \gamma \\ 0 & \text{otherwise} \end{cases}. \tag{8}$$

It is important to note that the SM-NLMS in [13] and SM-BNDR-LMS in [16] can also be established when $K = 1$ and $K = 2$ respectively.

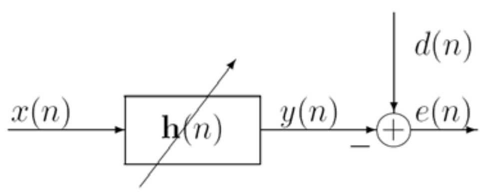

Figure 1. Prototypical adaptive filter setup

[1]The set Ψ_n is referred to as the constraint set, and its boundaries are hyperplanes. Also, γ is the magnitude of the error bound.

3.2. **SM-SPU-APA.** To reduce the computational complexity of SM-APA, the approach of SM filtering is extended to SPU-APA. By partitioning the input signal vector and the vector of filter coefficients into B blocks each of length L which are defined as[2]

$$\mathbf{x}(n) = [\mathbf{x}_1^T(n), \mathbf{x}_2^T(n), \ldots, \mathbf{x}_B^T(n)]^T \tag{9}$$

$$\mathbf{h}(n) = \left[\mathbf{h}_1^T(n), \mathbf{h}_2^T(n), \ldots, \mathbf{h}_B^T(n)\right]^T, \tag{10}$$

the SPU-APA solves the following optimization problem [12]

$$\min_{\mathbf{h}_F(n+1)} \|\mathbf{h}_F(n+1) - \mathbf{h}_F(n)\|^2, \tag{11}$$

subject to $\mathbf{d}(n) = \mathbf{X}^T(n)\mathbf{h}(n+1)$, where $F = \{j_1, j_2, \ldots, j_S\}$ denote the indices of the S blocks out of B blocks that should be updated at every adaptation. Again by using the Lagrange multiplier approach, the filter vector update equation is given by

$$\mathbf{h}_F(n+1) = \mathbf{h}_F(n) + \mu\mathbf{X}_F(n)(\mathbf{X}_F^T(n)\mathbf{X}_F(n))^{-1}\mathbf{e}(n), \tag{12}$$

where

$$\mathbf{X}_F(n) = [\mathbf{X}_{j_1}^T(n), \mathbf{X}_{j_2}^T(n), \ldots, \mathbf{X}_{j_N}^T(n)]^T, \tag{13}$$

is the $SL \times K$ matrix and $\mathbf{X}_i(n) = [\mathbf{x}_i(n), \mathbf{x}_i(n-1), \ldots, \mathbf{x}_i(n-K+1)]$ is the $L \times K$ matrix. The indices of F are obtained by the following procedure:

- Compute the following values for $1 \leq i \leq B$

$$\operatorname{Tr}\left(\mathbf{X}_i^T(n)\mathbf{X}_i^T(n)\right) \tag{14}$$

- The indices of F correspond to S largest values of Equation (14).

The SM-SPU-APA also minimizes $\|\mathbf{h}_F(n+1) - \mathbf{h}_F(n)\|^2$ but subject to $\mathbf{h} \in \Psi_n \cap \Psi_{n-1} \cap \ldots \cap \Psi_{n-K+1}$. This aim is obtained by following update equation

$$\mathbf{h}_F(n+1) = \mathbf{h}_F(n) + \mathbf{X}_F(n)(\mathbf{X}_F^T(n)\mathbf{X}_F(n))^{-1}\mathbf{q}\alpha(n)e(n). \tag{15}$$

The SM-SPU-APA has close performance to SM-APA. Also the performance of the introduced algorithm is better than SPU-APA. Furthermore, the complexity of SM-SPU-APA is lower than SPU-APA, and SM-APA due to SPU approach and applying the condition in Equation (8).

3.3. **SM-SR-APA.** In [10], another novel affine projection algorithm with selective regressors (SR) which was called (SR-APA) was presented. The SR-APA, minimizes (3) subject to

$$\mathbf{d}_{G_P}(n) = \mathbf{X}_{G_P}^T(n)\mathbf{h}(n), \tag{16}$$

where $G_P = \{i_1, i_2, \ldots, i_P\}$ denote the P subset (subset with P member) of the set $\{0, 1, \ldots, K-1\}$,

$$\mathbf{X}_{G_P}(n) = [\mathbf{x}(n-i_1), \mathbf{x}(n-i_2), \ldots, \mathbf{x}(n-i_P)], \tag{17}$$

is the $M \times P$ matrix of the input signal and

$$\mathbf{d}_{G_P}(n) = [d(n-i_1), d(n-i_2), \ldots, d(n-i_P)]^T, \tag{18}$$

is the $P \times 1$ vector of the desired signal. Using the method of Lagrange multipliers to solve this optimization problem leads to the following update equation

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu\mathbf{X}_{G_P}(n)(\mathbf{X}_{G_P}^T(n)\mathbf{X}_{G_P}(n))^{-1}\mathbf{e}_{G_P}(n), \tag{19}$$

where

$$\mathbf{e}_{G_P}(n) = \mathbf{d}_{G_P}(n) - \mathbf{X}_{G_P}^T(n)\mathbf{h}(n). \tag{20}$$

The indices of G_P are obtained by the following procedure:

[2]Note that $B = M/L$ and is an integer.

1. Compute the following values for $0 \le i \le K-1$
$$\frac{e^2(n-i)}{\|\mathbf{x}(n-i)\|^2}, \tag{21}$$
where $\mathbf{e}(n) = [e(n), e(n-1), \ldots, e(n-(K-1))]^T$.
2. The indices of G_P correspond to P largest values of Equation (21).

It has been shown that the performance of SR-APA is better than APA with lower computational complexity [10]. To improve the performance of SR-APA, we extend the SM filtering to SR-APA to present more efficient algorithm with low steady-state MSD, fast convergence speed and lower computational complexity than SR-APA. The SM-SR-APA minimizes (3) subject to $\mathbf{h} \in \Psi_{n-i_1} \cap \Psi_{n-i_2} \cap \ldots \cap \Psi_{n-i_P}$. Following the same approach as SM-APA leads to the following update equation
$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mathbf{X}_{G_P}(n)(\mathbf{X}^T_{G_P}(n)\mathbf{X}_{G_P}(n))^{-1}\mathbf{u}\alpha(n-i_1)e(n-i_1), \tag{22}$$
where $\mathbf{u} = [1, 0, \ldots, 0]^T$ is $P \times 1$ column vector.

3.4. **SM-DS-APA.** In [11], the affine projection with dynamic selection of input vectors was presented. In this algorithm, the optimum selection of the input vectors is derived by the largest decrease of the mean-square deviation. This algorithm shows better performance than APA. Furthermore, the complexity of this algorithm is lower than APA. Let $G_{P(n)} = \{i_1, i_2, \ldots, i_{P(n)}\}$ denote a subset with $P(n)$ members of the set $\{0, 1, \ldots, K-1\}$, where $P(n)$ is defined as the number of the selected input vectors at iteration n. Then, the filter vector update equation of this algorithm which was called DS-APA for $P(n) \neq 0$ is given by [11]
$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu\mathbf{X}_{G_{P(n)}}(n)(\mathbf{X}^T_{G_{P(n)}}(n)\mathbf{X}_{G_{P(n)}}(n))^{-1}\mathbf{e}_{G_{P(n)}}(n). \tag{23}$$
For $P(n) = 0$, the filter coefficients do not change. The indices of $G_{P(n)}$ correspond to $P(n)$ members of $e(n-i)$ that satisfy the following condition:
$$e^2(n-i) > 2\sigma_v^2/(2-\mu) \tag{24}$$

By applying the SM approach to DS-APA, we can establish the computationally efficient algorithm with fast convergence speed and low steady-state error. The SM-DS-APA minimizes $\|\mathbf{h}(n+1) - \mathbf{h}(n)\|^2$ subject to $\mathbf{h} \in \Psi_{n-i_1} \cap \Psi_{n-i_2} \cap \ldots \cap \Psi_{n-i_{P(n)}}$. Therefore, the update equation can be stated as
$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mathbf{X}_{G_{P(n)}}(n)(\mathbf{X}^T_{G_{P(n)}}(n)\mathbf{X}_{G_{P(n)}}(n))^{-1}\mathbf{u}(n)\alpha(n-i_1)e(n-i_1), \tag{25}$$
where $\mathbf{u}(n) = [1, 0, \ldots, 0]^T$ is $P(n) \times 1$ column vector.

3.5. **SM-SPU-SR-APA and SM-SPU-DS-APA.** By combining the SPU approach with SR and DS affine projection, the SM-SPU-SR-APA and SM-SPU-DS-APA can be established. In SPU-SR-APA, the filter coefficients are partially updated, the input regressors are selected, and the adaptation is performed when the condition in Equation (8) is true. In SPU-DS-APA, the filter coefficients are partially updated, the input regressors are dynamically selected, and the adaptation is performed based on Equation (8). The proposed algorithms have three features of SM adaptive filters. The SM-SPU-SR-APA minimizes $\|\mathbf{h}_F(n+1) - \mathbf{h}_F(n)\|^2$ subject to $\mathbf{h} \in \Psi_{n-i_1} \cap \Psi_{n-i_2} \cap \ldots \cap \Psi_{n-i_P}$. Defining $SL \times P$ input signal matrix through
$$\mathbf{X}_{F,G_P}(n) = \begin{pmatrix} \mathbf{x}_{j_1}(n-i_1) & \ldots & \mathbf{x}_{j_1}(n-i_P) \\ \mathbf{x}_{j_2}(n-i_1) & \ldots & \mathbf{x}_{j_2}(n-i_P) \\ \vdots & \ddots & \vdots \\ \mathbf{x}_{j_S}(n-i_1) & \ldots & \mathbf{x}_{j_S}(n-i_P) \end{pmatrix} \tag{26}$$

the update equation for the SM-SPU-SR-APA is given by

$$\mathbf{h}_F(n+1) = \mathbf{h}_F(n) + \mathbf{X}_{F,G_P}(n)(\mathbf{X}^T_{F,G_P}(n)\mathbf{X}_{F,G_P}(n))^{-1}\mathbf{u}\alpha(n-i_1)e(n-i_1) \tag{27}$$

The SM-SPU-DS-APA can be established when G_P and $\mathbf{u}$ are replaced by $G_{P(n)}$ and $\mathbf{u}(n)$ respectively.

4. **Computational Complexity.** To compare the computational complexity of the proposed algorithms, the number of multiplications, divisions and comparisons was calculated for one iteration. Tables 1 and 2 show the peak computational complexity of different SM-AP algorithms. For the SM-AP algorithms, the adaptation is related to the condition in Equation (8). If the condition in Equation (8) always becomes true (which in practice it does not), then the computational complexity of SM-AP algorithms is similar to the complexity of AP algorithms. However, the gains of applying the SM-AP algorithms comes through the reduced number of required updates, which cannot be accounted for a priori, and an increased performance as compared with different AP algorithms. In such applications as line and acoustic echo cancellation, the adaptive filter may be required to have a large number of coefficients in order to model the underlying physical system with sufficient accuracy. Therefore, by applying the SPU and SR approaches, the reduction in the computational complexity will be large for these applications. As we can see, the maximum number of multiplications in SM-SPU-APA is $(K^2+2K)SL+K^3+K^2$ which is lower than SM-APA. The complexity is depend on the number of filter coefficients to update (SL) in SM-SPU-APA. Also, in SM-SR-APA, the maximum number of multiplications is $(P^2+2P)M+P^3+P^2$ which is again less than SM-APA. In SM-SR-APA, the complexity is proportional to parameter P. When the parameter P increases, the complexity of SM-SR-APA will be closed to SM-APA. The number of multiplication in SM-SPU-SR-APA, and SM-SPU-DS-APA are $(P^2+2P)SL+P^3+P^2$ and $(P^2(n)+2P(n))SL+P^3(n)+P^2(n)$ respectively. In these algorithms, the complexity is proportional to number of filter coefficients to update and the number of recent regressors which is used at each iteration. The number of comparisons based on heapsort algorithm [19] have been also presented in Table 2. In the simulation results section, we present several applications to show the good performance of SM-AP algorithms to decrease the overall computational complexity.

Table 1. The peak computational complexity of the SM-AP algorithms for each iteration

Algorithm	Multiplications	Divisions
SM-APA	$(K^2+2K)M+K^3+K^2$	K
SM-SPU-APA	$(K^2+2K)SL+K^3+K^2$	–
SM-SR-APA	$(P^2+2P)M+P^3+P^2$	$2K$
SM-DS-APA	$(P^2(n)+2P(n))M+P^3(n)+P^2(n)$	$2K$
SM-SPU-SR-APA	$(P^2+2P)SL+P^3+P^2$	$2K$
SM-SPU-DS-APA	$(P^2(n)+2P(n))SL+P^3(n)+P^2(n)$	$2K$

5. **Simulation Results.** We justified the performance of the proposed algorithms by several computer simulations in system identification, line and acoustic echo cancellation scenarios.

TABLE 2. The peak computational complexity of the SM-AP algorithms for each iteration

Algorithm	Additional Multiplications	Comparisons
SM-APA	–	–
SM-SPU-APA	1	$B\log_2 S + O(B)$
SM-SR-APA	$(K-P)M+K+1$	$K\log_2 P + O(K)$
SM-DS-APA	–	$K\log_2 P(n) + O(K)$
SM-SPU-SR-APA	$(K-P)M+K+1$	$B\log_2 S + O(B) + K\log_2 P + O(K)$
SM-SPU-DS-APA	–	$B\log_2 S + O(B) + K\log_2 P(n) + O(K)$

5.1. **System identification.** In this simulation, the unknown system has 32 randomly selected taps ($M = 32$). The input signal $x(n)$ is a first order autoregressive (AR) signal generated by

$$x(n) = \rho x(n-1) + w(n) \tag{28}$$

where $w(n)$ is either a zero mean white Gaussian signal. The value of ρ is set to 0.9, generating a highly colored Gaussian signal. The measurement noise $v(n)$ with $\sigma_v^2 = 10^{-3}$ is added to the noise-free desired signal, $d(n) = \mathbf{h}_t^T\mathbf{x}(n)$, where $\mathbf{h}_t$ is the true unknown filter vector. The adaptive filter and the unknown channel are assumed to have the same number of taps. The mean square deviation (MSD) is taken and averaged over 200 independent trials. We set the parameters K, and γ to 4, and $\sqrt{5\sigma_v^2}$ [14] for all experiments. Also, the value of the step-size (μ) is set to 0.3.

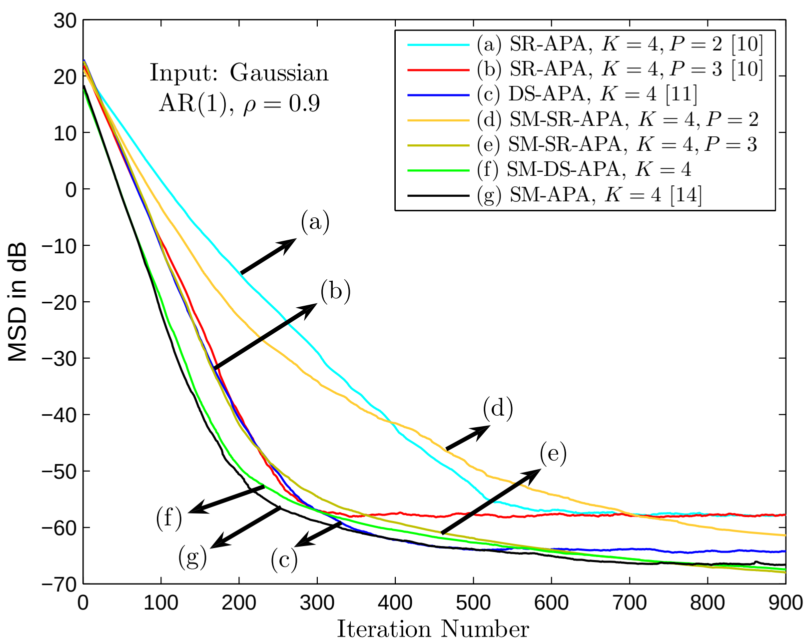

FIGURE 2. Comparison of SR-APA, DS-APA, SM-SR-APA, SM-DS-APA and SM-APA with $K = 4$, $P = 2$ and $P = 3$ (input: Gaussian AR(1), $\rho = 0.9$)

Figures 2 and 3 show the MSD curves of various SM-AP algorithms. Figure 2 compares the performance of SR-APA [5], DS-APA [6], SM-SR-APA, SM-DS-APA and SM-APA [9]. In SR-APA and SM-SR-APA, the parameter P was set to 2 and 3. The simulation

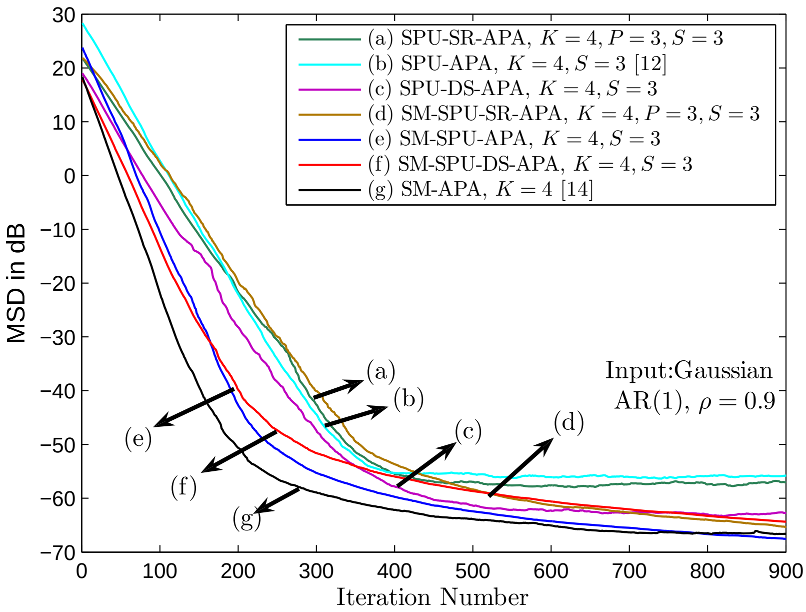

FIGURE 3. Comparison of SPU-APA, SPU-SR-APA, SPU-DS-APA, SM-SPU-SR-APA, SM-SPU-DS-APA and SM-APA with $K = 4$, $P = 3$, $B = 4$ and $S = 3$ (input: Gaussian AR(1), $\rho = 0.9$)

TABLE 3. The avearage number of updates in SM-AP algorithms for different applications

Algorithm	$M = 32$	$M = 128$	$M = 256$
SM-APA/APA	307/900	2420/14000	3724/12000
SM-SPU-APA/SPU-APA	352/900	2482/14000	4132/12000
SM-SR-APA/SR-APA	272/900	3790/14000	4180/12000
SM-DS-APA/DS-APA	298/900	2375/14000	3574/12000
SM-SPU-SR-APA/SPU-SR-APA	463/900	3841/14000	4831/12000
SM-SPU-DS-APA/SPU-DS-APA	324/900	2531/14000	4065/12000

results show that the SM-SR-APA has good convergence speed, low steady-state MSD and low computational complexity. By increasing the parameter P, the convergence speed of SM-SR-APA increases. Also, the steady-state MSD of SM-SR-APA is lower than SR-APA with $P = 2$ and 3. Table 3 presents the average number of updates for SM-SR-APA. This table shows that the average number of updates for SM-SR-APA is 272 instead of 900 for SR-APA. It means, we only need to update the filter coefficients for 272 iterations, and in other iterations, the condition in Equation (8) is not true and the filter coefficients do not change. For large values of P, the performance of SM-SR-APA will be closed to SM-APA. This performance is obtained by lower complexity because of using SR method.

This figure also compares the performance of SM-DS-APA with DS-APA. Simulation results show that the SM-DS-APA has faster convergence speed, and lower steady-state MSD than DS-APA. Also, the average number of updates for SM-DS-APA is 298 instead of 900 in DS-APA. This algorithm shows close performance to SM-APA. Furthermore, the complexity of SM-SR-APA is lower than SM-APA due to DS approach.

Figure 3 shows the MSD curves of SPU-APA [7], SPU-SR-APA, SPU-DS-APA, SM-SPU-SR-APA, SM-SPU-DS-APA and SM-APA. The parameter P was set to 3, and the values of B and S were set to 4 and 3 respectively. As we can see, the SM-SPU-APA has better performance than SPU-APA. The SM-SPU-APA has faster convergence speed, and lower steady-state MSD compared with SPU-APA. The average number of updates in SM-SPU-APA is 352 instead of 900 in SPU-APA. Furthermore, the filter coefficients are partially updated. As we see, the performance of SM-SPU-APA is close to SM-APA.

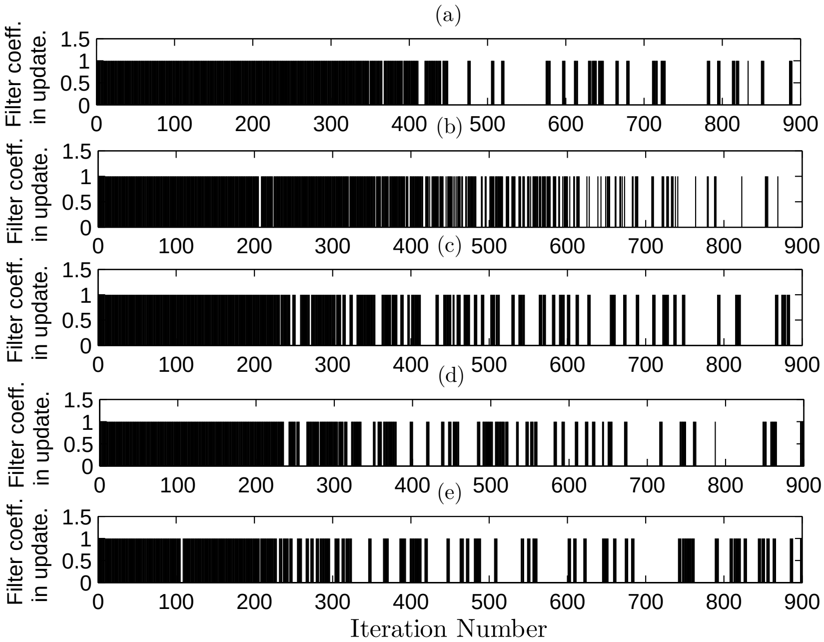

FIGURE 4. Filter coefficients in update for different SM affine projection algorithms in system identification application: (a) SM-SPU-SR-APA with $K = 4$, $S = 3$ and $P = 2$; (b) SM-SPU-APA with $K = 4$ and $S = 3$; (c) SM-SPU-DS-APA with $S = 3$; (d) SM-SR-APA with $P = 2$ and (e) SM-APA with $K = 4$

In this figure, the comparison of the performance for SPU-SR-APA and SM-SPU-DS-APA has been also presented. Again better performance for SM-SPU-SR-APA is observed. This fact can be seen in SM-SPU-DS-APA. In these algorithms, by combining the SPU with DS and SR approaches, the computational complexity is reduced. Furthermore, by applying the SM approach, the average number of updates is reduced. Figure 4 shows that when the filter coefficients are updated during the adaptation for different SM affine projection algorithms. Binary numbers have been used in this figure where 1 means that the filter coefficients are updated and 0 means that the adaptation is not performed. As we can see, in some iteration, the filter coefficients are not updated.

In Figure 5, we demonstrated the tracking performance of the presented algorithms. The parameters K, P and S were set to 4, 3 and 3 respectively. At iteration 1000, the unknown system changed randomly. As we can see, the presented SM adaptive algorithms have good tracking performance ability. The results show that the SM-SR-APA and SM-DS-APA have close performance to SM-APA.

5.2. **Line echo cancellation.** In communications over phone lines, a signal traveling from a far-end point to a near-end point is usually reflected in the form of an echo at the near-end due to mismatches in circuity. The purpose of a line echo canceller (LEC) is to eliminate the echo from received signal. In this experiment, the input signal is a speech

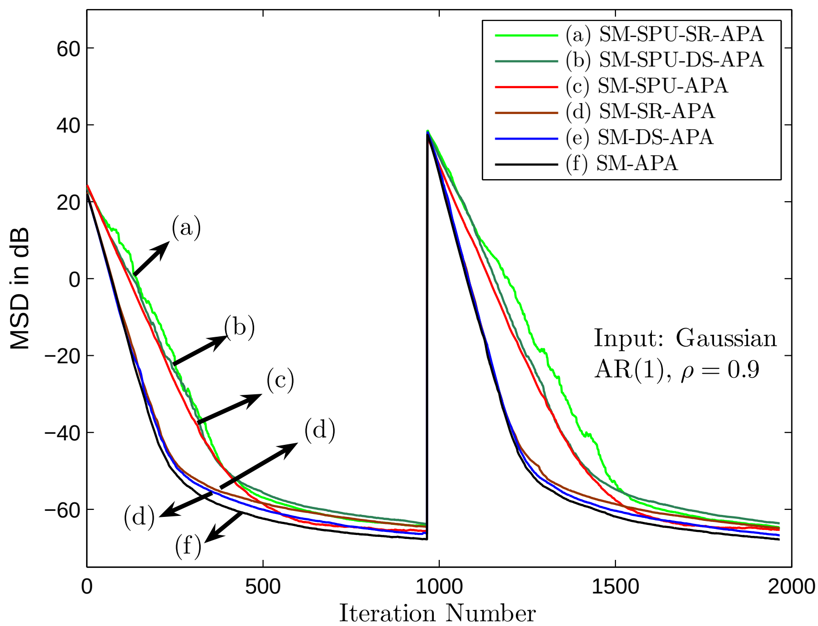

FIGURE 5. Comparison of tracking performance in SM-APA, SM-SPU-APA, SM-SR-APA, SM-DS-APA, SM-SPU-SR-APA and SM-SPU-DS-APA for $K = 4$, $P = 3$ and $S = 3$ (input: Gaussian AR(1), $\rho = 0.9$)

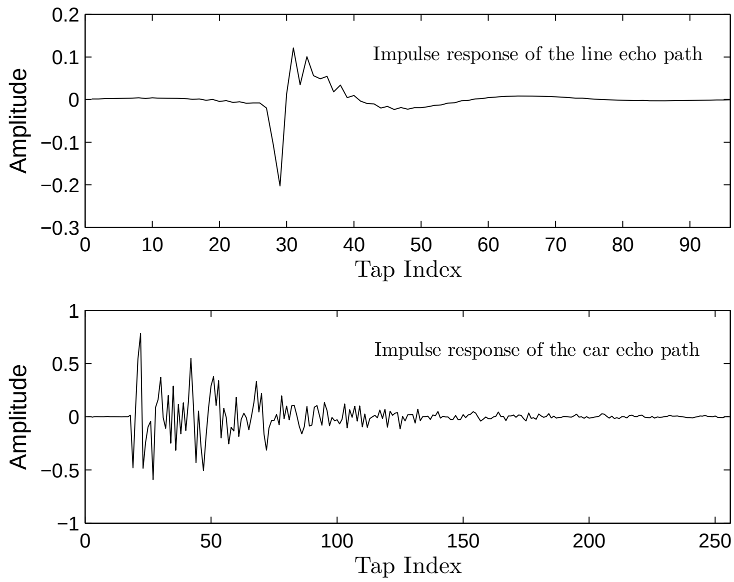

FIGURE 6. Impulse responses of the line and car echo paths

signal. Also, Figure 6(a) shows the impulse response sequence of a typical echo path[3]. In this simulation, the length of adaptive filter is 128. The parameters B, S and P were set to 4, 3 and 2 respectively. Figure 7(a) shows the Far-end signal samples. This signal is a synthetic signal that emulates the properties of speech [20]. Figure 7(b) shows the Echo signal. Figures 8(a)-8(e) show the error signals that are obtained by SM-SPU-SR-APA,

[3]The impulse response of the line echo path and the input speech signal is from [20], page 347.

SM-SPU-APA, SM-SPU-DS-APA, SM-SR-APA and SM-APA respectively. As we can see, the performance of SM-SR-APA in Figure 8(d) is close to the SM-APA in Figure 8(e). Table 3 presents the average number of updates in different SM-AP algorithms. Furthermore, the overall computational complexity of SM-AP algorithms is reduced due to SPU and SR approaches. Figure 9 shows that when the filter coefficients are updated during the adaptation for different SM affine projection algorithms in line echo cancellation application. According to this figure, the adaptation is not performed in some iteration which leads to reduction in overall computational complexity.

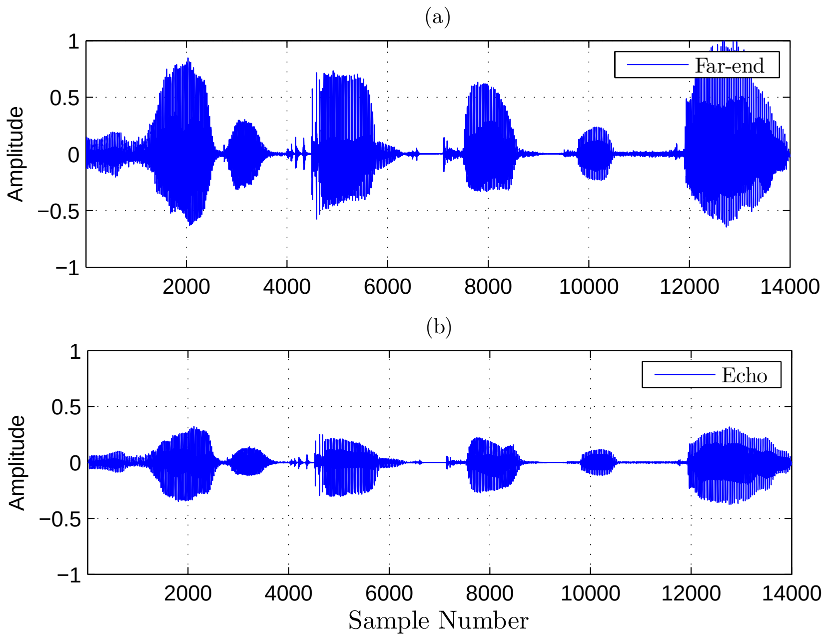

FIGURE 7. (a) Far-end signal and (b) echo signal

5.3. **Acoustic echo cancellation.** In acoustic echo cancellation, the exact impulse response of the echo path in Figure 6(b) with 256 taps has been used[4]. The input signal is colored Gaussian. The parameters B, S and P were set to 4, 3 and 3 respectively. Figure 10 shows the MSD curves of SM-AP algorithms when the impulse response of the car echo path should be identified. As we can see the SM-AP algorithms have good and close performance to SM-APA. Also, the reduction in computational complexity is large in this application. The average number of updates has been presented in Table 3. Figure 11 shows that when the filter coefficients are updated during the adaptation for different SM affine projection algorithms in acoustic echo cancellation application. Again this figure shows that in some iteration, the filter coefficients do not change which will be efficient feature in this application.

6. **Conclusions.** In this paper, we presented various SM affine projection adaptive filter algorithms. The SM-SPU-APA, SM-SR-APA, SM-DS-APA, SM-SPU-SR-APA and SM-SPU-DS-APA were established. These algorithms had good convergence speed, low steady-state MSE and low computational complexity. The good performance of the presented algorithms were demonstrated in system identification, line and acoustic echo cancellation scenarios.

[4]The impulse response of the car echo path is from [12].

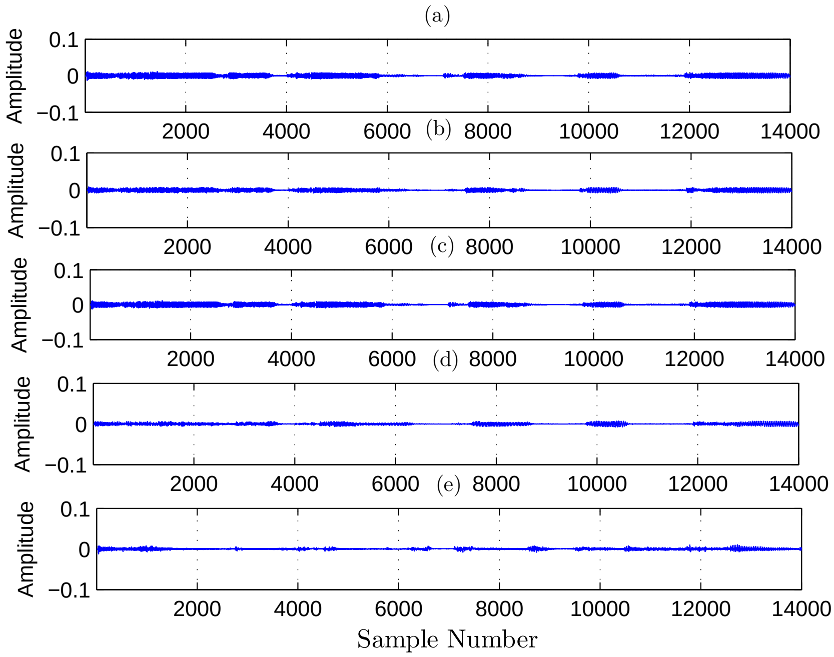

FIGURE 8. (a) Error obtained by SM-SPU-SR-APA with $K = 4$, $S = 3$ and $P = 2$; (b) error obtained by SM-SPU-APA with $K = 4$ and $S = 3$; (c) error obtained by SM-SPU-DS-APA with $S = 3$; (d) error obtained by SM-SR-APA with $P = 2$ and (e) error obtained by SM-APA with $K = 4$

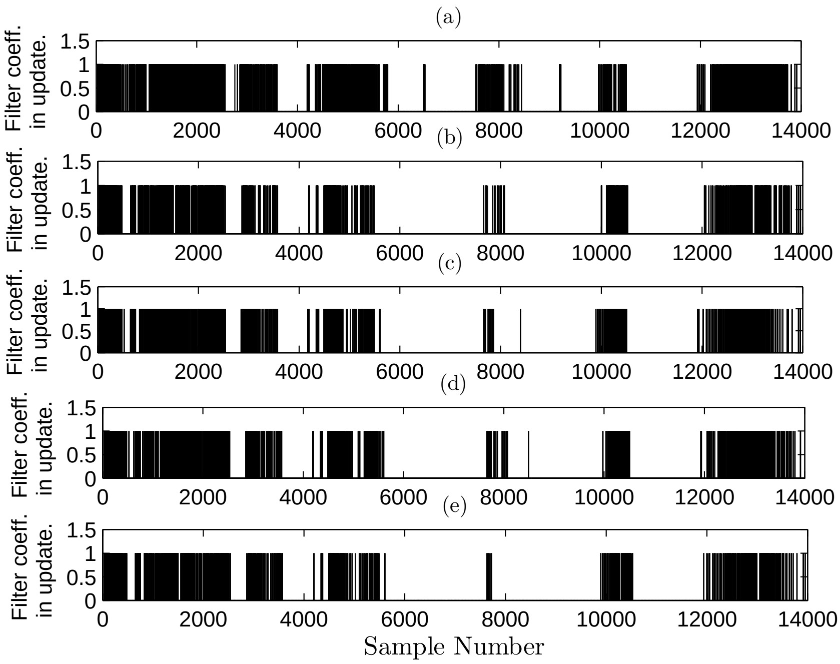

FIGURE 9. Filter coefficients in update for different SM affine projection algorithms in line echo cancellation application: (a) SM-SPU-SR-APA with $K = 4$, $S = 3$ and $P = 2$; (b) SM-SPU-APA with $K = 4$ and $S = 3$; (c) SM-SPU-DS-APA with $S = 3$; (d) SM-SR-APA with $P = 2$ and (e) SM-APA with $K = 4$

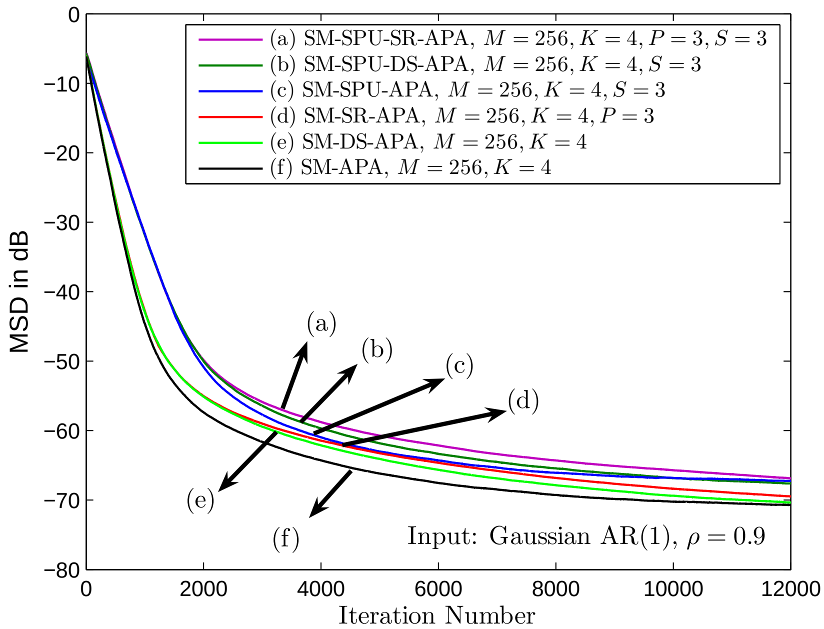

Figure 10. Comparison of SM-APA, SM-SPU-APA, SM-SR-APA, SM-DS-APA, SM-SPU-SR-APA and SM-SPU-DS-APA when the impulse response of the car echo path should be identified (input: Gaussian AR(1), $\rho = 0.9$)

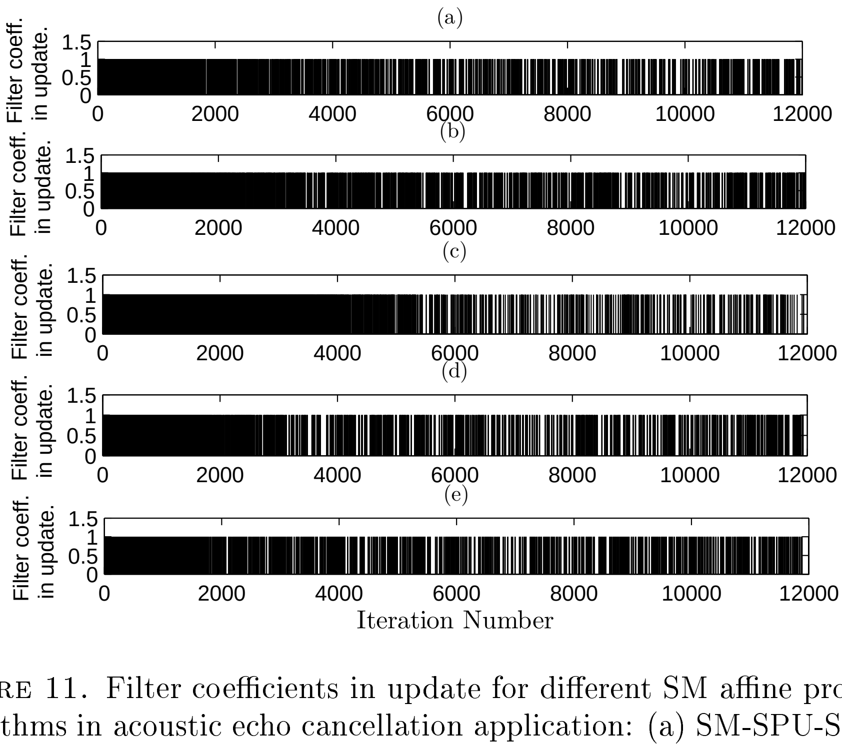

Figure 11. Filter coefficients in update for different SM affine projection algorithms in acoustic echo cancellation application: (a) SM-SPU-SR-APA with $K = 4$, $S = 3$ and $P = 2$; (b) SM-SPU-APA with $K = 4$ and $S = 3$; (c) SM-SPU-DS-APA with $S = 3$; (d) SM-SR-APA with $P = 2$ and (e) SM-APA with $K = 4$

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