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NONLINEAR ADAPTIVE DECOUPLING CONTROL BASED ON NEURAL NETWORKS AND MULTIPLE MODELS

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Abstract. *For a class of uncertain nonlinear multivariable discrete time dynamic systems, an adaptive decoupling controller (ADC) is presented, which can deal with the case that the zero dynamics (ZD) of the system is not asymptotically stable. The ADC developed is composed of a linear robust ADC, a neural network (NN) nonlinear ADC and a switching mechanism. The linear robust ADC can assure the bounded-input-bounded-output (BIBO) stability of the closed-loop system. The nonlinear NN ADC can improve the system performance. The switching mechanism is utilized to obtain the improved system performance and stability simultaneously. Theory analysis and simulation results are presented to show the effectiveness of the proposed method.*

Keywords: Adaptive decoupling control, Multiple models, Stability

1. **Introduction.** Multiple-input-multiple-output (MIMO) systems usually possess complicated dynamic coupling behaviors. The control schemes for single-input-single-output (SISO) systems, such as those reported in [1,2], are not easy to implement on complicated MIMO systems. Hence, how to achieve decoupling control of MIMO systems has become a topic of considerable research. Decoupling control is initially developed for deterministic linear systems, while, for uncertain linear systems, adaptive or active decoupling schemes are usually adopted [3-5]. In recent years, along with the introduction of neural networks (NNs), attempts have been made toward adaptive decoupling of uncertain nonlinear systems and NNs based nonlinear adaptive decoupling control has become the hotspot.

In [6,7], the nonlinear MIMO system is first decomposed into a linear model incorporating with an unmodelled dynamics. Then, by combining the one-step-ahead optimal weighting decoupling control law with a neural network (NN) compensator, an adaptive decoupling controller (ADC) based on NNs is proposed. However, due to the complexity of the NN structure and the nonlinear dependence of its map on the parameter values, stability and performance analysis of the closed-loop system are not provided. Although some relevant results for adaptive decoupling control using NNs have been presented in [8-10], some problems remain unsolved, and most of them suffer from one or more of the following drawbacks: (i) The zero dynamics (ZD) of the system is asymptotically stable. (ii) The linearized parameters of the system are known *a priori*. (iii) The NNs used are linearly parameterized.

To overcome the drawbacks mentioned above, this paper proposes an ADC for a class of nonlinear MIMO discrete time dynamic systems which are described by a nonlinear autoregressive moving average (NARMA) model. The proposed ADC is composed of a linear robust ADC, a nonlinear NN ADC and a switching mechanism. The controller structure is similar to that in [10,11]. However, in [10,11], it is assumed that the ZD of the system to be controlled is asymptotically stable. This paper relaxes the above assumption, realizes the adaptive decoupling control of uncertain nonlinear multivariable systems and shows that the proposed ADC can not only assure the bounded-input-bounded-output (BIBO) stability of the closed-loop system, but also improve the system performance.

The rest of the paper is organized as follows. The system under consideration is represented and the control problem is stated in Section 2. Section 3 develops the proposed ADC which is composed of a linear robust ADC, a nonlinear NN ADC and a switching mechanism. Global stability of the closed-loop system is analyzed in Section 4. Section 5 gives simulation results showing the effectiveness of the proposed method. Finally, some conclusions are drawn in Section 6.

2. **Nonlinear Decoupling Control for Known Systems.**

2.1. **Statement of the problem.**

The system to be controlled is an n-input-n-output nonlinear discrete time dynamic system which is described by the following NARMA model with unit delay:

$$y(t+1) = f[y(t), \cdots, y(t-n_a+1), u(t), \cdots, u(t-n_b)] \tag{1}$$

where $u(t) \in \mathbb{R}^n$ and $y(t) \in \mathbb{R}^n$ are the system input and output respectively; $f[\cdot] \in \mathbb{R}^n$ is a smooth nonlinear vector function; $n_a, n_b > 1$ are the system orders and the origin is an equilibrium point.

To control the nonlinear system (1) in a neighbourhood of the origin, the following Equation (2) is usually used as its equivalent system, which can be obtained by linearization using Taylor's formula around the origin.

$$A(z^{-1})y(t+1) = B(z^{-1})u(t) + v[y(t), \cdots, y(t-n_a+1), u(t), \cdots, u(t-n_b)] \tag{2}$$

where $A(z^{-1})$ and $B(z^{-1})$ are $n \times n$ matrix polynomials in the backward shift operator z^{-1} with the orders n_a and n_b respectively; $v[\cdot] \in \mathbb{R}^n$ is the remained high order nonlinear term. Without loss of generality, the matrix polynomial $A(z^{-1})$ is assumed to be monic and diagonal. In addition, $B(0)$ is assumed to be nonsingular. In the following, we will consider the system (2) and also make the following assumptions.

Assumption 2.1. *(i) The system orders n_a and n_b are known; (ii) The system parameter matrices forming $A(z^{-1})$ and $B(z^{-1})$ are unknown, but lie in a compact region $\sum$; (iii) The high order nonlinear term $v[\cdot]$ is globally bounded.*

Assumption 2.2. *For a nonlinear equations with the form*

$$C_0 \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} g_1(x_1, x_2, \cdots, x_n, X_0) \\ g_2(x_1, x_2, \cdots, x_n, X_0) \\ \vdots \\ g_n(x_1, x_2, \cdots, x_n, X_0) \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix} \tag{3}$$

where $x_1, x_2, \cdots, x_n$ are unknown variables; $X_0 \in \mathbb{R}^m$ is an arbitrary given vector; $g_i|i = 1, \cdots, n : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ are continuous bounded nonlinear functions and $C_0 \in \mathbb{R}^{n\times n}$ is a nonsingular matrix, there exist $x_1^*, x_2^*, \cdots, x_n^*$ satisfying (3) for arbitrary constants $r_1, r_2, \cdots, r_n$.

Remark 2.1. *The contents in Assumption 2.1 are usually made in literature on adaptive control, such as [10,11]. Assumption 2.2 is made to assure the existence of the control input in the designed nonlinear NN ADC in the sequel.*

Remark 2.2. *Since the delay of the system (2) is unit, the ZD exists [12,13]. In [10,11], the ZD of the controlled system is assumed to be asymptotically stable, so that the condition that an input sequence never grows faster than the output sequence is used when proceeding with stability analysis. In this paper, the stability can be guaranteed without the above condition, and then the ZD need not be asymptotically stable.*

2.2. **One-step-ahead optimal weighting decoupling control.** To realize decoupling control, the interaction between the input u_j and output y_i ($j \neq i$) is viewed as measurable disturbance. The matrix polynomial $B(z^{-1})$ is split into two terms: $B(z^{-1}) = \bar{B}(z^{-1}) + \bar{\bar{B}}(z^{-1})$ with $\bar{B}(z^{-1})$ being a diagonal matrix polynomial that contains the direct coupling terms of input-output pairs and $\bar{\bar{B}}(z^{-1})$ being a matrix polynomial with zeros on the diagonal containing the cross coupling terms. Then, the system (2) can be rewritten as

$$A(z^{-1})y(t+1) = \bar{B}(z^{-1})u(t) + \bar{\bar{B}}(z^{-1})u(t) + v[\cdot] \tag{4}$$

Introduce the following modified Clarke performance index [6-8]:

$$J(t) = \|\bar{e}(t+1)\|^2 \tag{5}$$

where

$$\bar{e}(t+1) = P(z^{-1})y(t+1) - R(z^{-1})w(t+1) + Q(z^{-1})u(t) + S(z^{-1})u(t) + K(z^{-1})v[\cdot] \tag{6}$$

is defined as the generalized tracking error of the system; $w(t) \in \mathbb{R}^n$ is the known reference input; $P(z^{-1})$, $Q(z^{-1})$, $R(z^{-1})$ and $K(z^{-1})$ are $n \times n$ diagonal weighting matrix polynomials; $S(z^{-1})$ is an $n \times n$ weighting matrix polynomial with zero diagonal elements. Introduce the following Diophantine equation:

$$P(z^{-1}) = FA(z^{-1}) + z^{-1}G(z^{-1}) \tag{7}$$

where F and $G(z^{-1})$ are respectively an $n \times n$ diagonal constant matrix and diagonal matrix polynomial with the order $n_g = \max\{n_a - 1, n_p - 1\}$, which are uniquely determined by (7) [14]. From (7), it is easy to known $F = P(0)$. By left-multiplying both sides of (4) using F, the one-step-ahead optimal weighting decoupling control law making the index (5) to be zero is obtained as

$$[FB(z^{-1}) + Q(z^{-1}) + S(z^{-1})]u(t) = R(z^{-1})w(t+1) - G(z^{-1})y(t) - [F + K(z^{-1})]v[\cdot] \tag{8}$$

If $v[\cdot]$ is small, it can be treated as a bounded disturbance and the following linear decoupling controller is adopted:

$$[FB(z^{-1}) + Q(z^{-1}) + S(z^{-1})]u(t) = R(z^{-1})w(t+1) - G(z^{-1})y(t) \tag{9}$$

In the following, a possible choice of the weighting matrix polynomials $P(z^{-1})$, $Q(z^{-1})$, $R(z^{-1})$, $S(z^{-1})$ and $K(z^{-1})$ is given, which is presented as:

$$P(z^{-1}) = I;\ Q(z^{-1}) = \bar{\Lambda} \cdot (1 - z^{-1});\ S(z^{-1}) = \bar{\bar{\Lambda}} \cdot (1 - z^{-1}) \tag{10}$$

$$R(z^{-1}) = I;\ K(z^{-1}) = \Gamma \cdot (1 - z^{-1}) \tag{11}$$

where I is the identity matrix; $\bar{\Lambda}$ is a diagonal constant matrix; $\bar{\bar{\Lambda}}$ is a constant matrix whose diagonal elements are all zero and $\Gamma = \bar{B}^{-1}(1)\bar{\Lambda}$. To this end, to guarantee the stability of the closed-loop system and the existence of the current input $u(t)$, the constant matrices $\bar{\Lambda}$ and $\bar{\bar{\Lambda}}$ should be selected by a trial and error method to satisfy the following inequalities:

$$\det\{B(z^{-1}) + (1 - z^{-1})A(z^{-1})(\bar{\Lambda} + \bar{\bar{\Lambda}})\} \neq 0, |z| \geq 1 \tag{12}$$

$$\det[B(0) + \bar{\Lambda} + \bar{\bar{\Lambda}}] \neq 0 \tag{13}$$

3. **ADC Based on Neural Networks and Multiple Models.** In this section, we consider the case that the system orders n_a and n_b are known, but Θ or the linearized parameter matrices forming $A(z^{-1})$, $B(z^{-1})$ and the nonlinear term $v[\cdot]$ are unknown. As preparation, the weighting matrix polynomials $P(z^{-1})$, $Q(z^{-1})$, $R(z^{-1})$, $S(z^{-1})$ and $K(z^{-1})$ are first off-line chosen possibly by (10) and (11). Left-multiplying both sides of (2) by F and using (7), the following equation is obtained:

$$\varphi(t+1) = \Theta^T X(t) + \zeta[X(t)] \tag{14}$$

where $\varphi(t+1) = P(z^{-1})y(t+1)$; $\Theta = [G_0, \cdots, G_{n_g}; H_0, \cdots, H_{n_b}]^T$ with $G_0 + G_1 z^{-1} + \cdots + G_{n_g} z^{-n_g} := G(z^{-1})$ and $H_0 + H_1 z^{-1} + \cdots + H_{n_b} z^{-n_b} := H(z^{-1})$; $X(t) = [y(t)^T, \cdots, y(t - n_g)^T, u(t)^T, \cdots, u(t - n_b)^T]^T$; $\zeta[X(t)] = Fv[X(t)]$ with $v[X(t)] := v[\cdot]$. Consequently, the linear decoupling controller (9) and the nonlinear optimal decoupling controller (8) can be rewritten respectively as

$$\Theta^T X(t) = R(z^{-1})w(t+1) - [Q(z^{-1}) + S(z^{-1})]u(t) \tag{15}$$

$$\Theta^T X(t) + \zeta[X(t)] = R(z^{-1})w(t+1) - [Q(z^{-1}) + S(z^{-1})]u(t) - K(z^{-1})v[X(t)] \tag{16}$$

3.1. **Linear robust ADC.** A linear estimate model of (14) is first defined as

$$\varphi_1(t+1) = \hat{\Theta}_1(t)^T X(t) \tag{17}$$

where $\hat{\Theta}_1(t) = [\cdots, \hat{H}_{1,0}(t), \cdots]^T$ is an estimate of Θ at instant t and is updated as

$$\hat{\Theta}_1(t) = \text{proj}\{\hat{\Theta}'_1(t)\} \tag{18}$$

$$\hat{\Theta}'_1(t) = \hat{\Theta}_1(t-1) + \frac{a_1(t)X(t-1)e_1(t)^T}{1 + X(t-1)^T X(t-1)} \tag{19}$$

$$a_1(t) = \begin{cases} 1, & \text{if } \|e_1(t)\| > 2M \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

$$e_1(t) = \varphi(t) - \hat{\varphi}_1(t) \tag{21}$$

where M is a known upper bound of $\|\zeta[X(t)]\|$; $\hat{\Theta}'_1(t) = [\cdots, \hat{H}'_{1,0}(t), \cdots]^T$; proj$\{\cdot\}$ is a projection operator satisfying

$$\text{proj}\{\hat{\Theta}'_1(t)\} = \begin{cases} \hat{\Theta}'_1(t); & \hat{H}'_{1,0}(t) + Q(0) + S(0) \text{ is nonsingular} \\ [\cdots, \hat{H}'_{1,0}(t-1), \cdots]; & \text{otherwise} \end{cases} \tag{22}$$

Then, according to (15) and the certainty equivalent principle, the linear robust ADC $u_1(t)$ can be calculated from

$$\begin{aligned} \hat{\Theta}_1(t)^T X_1(t) = {} & R(z^{-1})w(t+1) - [Q(0) + S(0)]u_1(t) \\ & - z[Q(z^{-1}) + S(z^{-1}) - Q(0) - S(0)]u(t-1) \end{aligned} \tag{23}$$

where $X_1(t) = [\cdots, u_1(t), \cdots]^T$.

3.2. **Nonlinear NN ADC.** The nonlinear estimate model of (14) based on an NN is defined as

$$\hat{\varphi}_2(t+1) = \hat{\Theta}_2(t)^T X(t) + \hat{\zeta}[X(t)] \tag{24}$$

where $\hat{\Theta}_2(t) = [\cdots, \hat{H}_{2,0}(t), \cdots]^T$ is an another estimate of Θ at instant t, whose identification algorithm is similar as $\hat{\Theta}_1(t)$, i.e.,

$$\hat{\Theta}_2(t) = \text{proj}\{\hat{\Theta}_2'(t)\} \tag{25}$$

$$\hat{\Theta}_2(t) = \hat{\Theta}_2(t-1) + \frac{\mu(t)X(t-1)e_2(t)^T}{1 + X(t-1)^T X(t-1)} \tag{26}$$

$$\mu(t) = \begin{cases} 1, & \text{if } \|e_2(t)\| > 2d_0 \\ 0, & \text{otherwise} \end{cases} \tag{27}$$

$$e_2(t) = \varphi(t) - \hat{\varphi}_2(t) \tag{28}$$

where $\hat{\Theta}_2'(t) = [\cdots, \hat{H}_{2,0}'(t), \cdots]^T$; proj$\{\cdot\}$ is another projection operator satisfying

$$\text{proj}\{\hat{\Theta}_2'(t)\} = \begin{cases} \hat{\Theta}_2'(t); & \hat{H}'_{2,0}(t) + Q(0) + S(0) \text{ is nonsingular} \\ [\cdots, \hat{H}'_{2,0}(t-1), \cdots]; & \text{otherwise} \end{cases} \tag{29}$$

d_0 is a known upper bound of $\|\zeta[X(t-1)] - \hat{\zeta}[X(t-1)]\|$; $\hat{\zeta}[X(t-1)]$ is a bounded continuous nonlinear function, whose output is calculated according to

$$\hat{\zeta}[X(t-1)] = \hat{W}_2(t-1)^T \cdot \{\text{sigm}[\hat{W}_1(t-1)^T \cdot X(t-1) + \rho_1]\} + \rho_2 \tag{30}$$

where $X(t-1) \in \mathbb{R}^{n(n_a+n_b+1)}$ is the input of the NN; $\hat{W}_1(t-1)^T \in \mathbb{R}^{n(n_a+n_b+1)\times p}$, $\hat{W}_2(t-1)^T \in \mathbb{R}^{p\times n}$ are respectively the estimates of the optimal connection weight matrices between the input and hidden nodes, and the hidden and output nodes; $\rho_1 \in \mathbb{R}^{p\times 1}$ and $\rho_2 \in \mathbb{R}^{n\times 1}$ are the biases from the hidden layer and output layer respectively; p is the number of the hidden nodes; sigm$[\cdot]$ is a sigmoidal operator. It is worthy to emphasis that similar as in [10,11], no restriction is made on how the estimated weight matrix $\hat{W}_1(t-1), \hat{W}_2(t-1)$ are updated except they always lie inside some predefined compact region.

According to (16) and the certainty equivalent principle, the nonlinear NN ADC $u_2(t)$ can be calculated from

$$\begin{aligned} \hat{\Theta}_2(t)^T X_2(t) + \hat{\zeta}[X_2(t)] = {} & R(z^{-1})w(t+1) - [Q(0) + S(0)]u_2(t) \\ & - z[Q(z^{-1}) + S(z^{-1}) - Q(0) - S(0)]u(t-1) \\ & - K(0)\hat{v}[X_2(t)] - z[K(z^{-1}) - K(0)]\hat{v}[X(t-1)] \end{aligned} \tag{31}$$

where $X_2(t) = [\cdots, u_2(t), \cdots]^T$; $\hat{v}[X(t)] = F^{-1} \cdot \hat{\zeta}[X(t)]$.

Remark 3.1. *From (29) and Assumption 2.2, it is easy to know the current control input $u_2(t)$ in the nonlinear adaptive controller Equation (31) exists. However, since $u_2(t)$ is nonlinearly parameterized, the compact analytic solution for $u_2(t)$ is difficult to obtained. However, in practice, $u_2(t)$ can be generally calculated using an recursive procedure such as the Newton numerical method etc. Due to the uncertainty of the training of the NN, the performance or even stability of the closed-loop system can hardly be guaranteed when the nonlinear ADC (31) is used alone. Therefore, the switching mechanism in the sequel is indispensable.*

3.3. **Switching mechanism.** In this section, the problem of adaptive decoupling control by switching between the linear robust ADC $u_1(t)$ and the nonlinear NN ADC $u_2(t)$ is considered. At every instant t, by comparing $J_1(t)$ and $J_2(t)$, the ADC $u^*(t)$ corresponding to the smaller $J^*(t)$ is chosen to be applied to the system, i.e.,

$$u(t) = \begin{cases} u_1(t), & \text{if } J_1(t) \leq J_2(t) \\ u_2(t), & \text{otherwise} \end{cases} \tag{32}$$

The switching rule function $J_j(t)$, $j = 1, 2$ is described by

$$J_j(t) = \sum_{l=1}^{t} \frac{a_j(l)(\| e_j(l) \|^2 - 4M^2)}{2(1 + X(l-1)^T X(l-1))} + c \sum_{l=t-N+1}^{t} (1 - a_j(l)) \| e_j(l) \|^2 \tag{33}$$

where $a_j(t) = 1$ if $\|e_j(t)\| > 2M$, otherwise $a_j(t) = 0$; the identification errors $e_j(t)$, $j = 1, 2$ are calculated by (21) and (28) respectively; $j = 1$ denotes linear, $j = 2$ denotes nonlinear; N is an integer and $c \geq 0$ is a predefined constant.

4. **Stability and Performance Analysis.** We now present the results on system stability and performance analysis in the following Theorem 4.2 which is analyzed based on the following Lemma 4.1, Lemma 4.2 and Theorem 4.1.

Lemma 4.1. *Consider the following n-dimension time-invariant system:*

$$\begin{aligned} x(t+1) &= \mathcal{A}x(t) + \mathcal{B}_1(t)u(t) + \mathcal{B}_2 f_1[x(t), u(t)] + \mathcal{B}_3 f_2(t), \ x(0) = x_0 \\ y(t) &= \mathcal{C}x(t) + \mathcal{D}_1 u(t) + \mathcal{D}_2 f_1[x(t), u(t)] + \mathcal{D}_3 f_2(t) \end{aligned} \tag{34}$$

where the origin is an equilibrium point; $y(t)$, $u(t)$ are respectively the p-dimension output and m-dimension input; $x(t)$ is the n-dimension state; $f_1[\cdot]$ and $f_2(t)$ are n-dimension bounded nonlinear functions satisfying $\|f_1[\cdot]\| \leq \Delta$ with $\Delta > 0$ being known and $\|f_2(t)\| \leq \Delta_1$ with $\Delta_1 > 0$. If the system (34) is asymptotically stable, there exist constants c_1 and c_2 such that $\|y(t)\| \leq c_1 + c_2 \max_{0 \leq \tau \leq t} \|u(\tau)\|$.

Proof: From (34), we have

$$\begin{aligned} y(t) = {} & \mathcal{C}\mathcal{A}^t x_0 + \mathcal{D}_1 u(t) + \sum_{j=1}^{t} \mathcal{C}\mathcal{A}^{j-1}\mathcal{B}_1 u(t-j) + \sum_{j=1}^{t} \mathcal{C}\mathcal{A}^{j-1}\mathcal{B}_2 f_1[x(t-j), u(t-j)] \\ & + \sum_{j=1}^{t} \mathcal{C}\mathcal{A}^{j-1}\mathcal{B}_3 f_2(t-j) + \mathcal{D}_2 f_1[x(t), u(t)] + \mathcal{D}_3 f_2(t) \end{aligned} \tag{35}$$

Since the system (34) is asymptotically stable, for any j ($j = 1, \cdots, t$), we also have

$$\|\mathcal{A}^j\| \leq c_0 \lambda^j, 0 \leq \lambda < 1 \text{ with } 0 \leq c_0 < \infty \tag{36}$$

Therefore, from (35) and (36), it can be shown that

$$\begin{aligned} \|y(t)\| \leq {} & \|\mathcal{C}x_0\| c_0 \lambda^t + \|\mathcal{D}_1\| \cdot \|u(t)\| + c_0 \sum_{j=1}^{t} \|\mathcal{C}\lambda^{j-1}\mathcal{B}_1\| \cdot \|u(t-j)\| \\ & + \Delta \cdot c_0 \sum_{j=1}^{t} \|\mathcal{C}\lambda^{j-1}\mathcal{B}_2\| + \Delta_1 \cdot c_0 \sum_{j=1}^{t} \|\mathcal{C}\lambda^{j-1}\mathcal{B}_3\| + \Delta \cdot \|\mathcal{D}_2\| + \Delta_1 \|\mathcal{D}_3\| \end{aligned} \tag{37}$$

$$\leq \|\mathcal{C}x_0\|c_0\lambda^t + \|\mathcal{D}_1\| \max_{0\leq\tau\leq t} \|u(\tau)\| + \max_{0\leq\tau\leq t} \|u(\tau)\| \cdot c_0 \sum_{j=1}^{t} \|\mathcal{C}\lambda^{j-1}\mathcal{B}_1\|$$

$$+ \Delta \cdot c_0 \sum_{j=1}^{t} \|\mathcal{C}\lambda^{j-1}\mathcal{B}_2\| + \Delta_1 \cdot c_0 \sum_{j=1}^{t} \|\mathcal{C}\lambda^{j-1}\mathcal{B}_3\| + \Delta \cdot \|\mathcal{D}_2\| + \Delta_1 \cdot \|\mathcal{D}_3\|$$

Denote $c_1 = \|\mathcal{C}x_0\| \cdot c_0 + \Delta \cdot c_0\|\mathcal{C}\mathcal{B}_2\|/(1-\lambda) + \Delta_1 \cdot c_0\|\mathcal{C}\mathcal{B}_3\|/(1-\lambda) + \Delta\|\mathcal{D}_2 + \Delta_1\|\mathcal{D}_3\|$, $c_2 = \|\mathcal{D}_1\| + c_0\|\mathcal{C}\mathcal{B}_1\|/(1-\lambda)$, then from (37), the result is obtained.

Lemma 4.2. *For the specified matrix polynomials $P(z^{-1})$, $Q(z^{-1})$ and $S(z^{-1})$ chosen by (10), provided (12) is satisfied, there exist positive constants d_1 and d_2, such that*

$$\|X(t)\| \leq d_1 + d_2 \max_{0\leq\tau\leq t} \|\bar{e}(\tau+1)\| \tag{38}$$

Proof: Combining (2) with (6) and eliminating $u(t)$ and $y(t+1)$ respectively, we can obtain

$$\begin{aligned} &\{P(z^{-1})B(z^{-1}) + A(z^{-1})[Q(z^{-1}) + S(z^{-1})]\}u(t) \\ =&A(z^{-1})\bar{e}(t+1) + A(z^{-1})R(z^{-1})w(t+1) - [P(z^{-1}) + A(z^{-1})K(z^{-1})]v[X(t)] \end{aligned} \tag{39}$$

$$\begin{aligned} &[\tilde{B}(z^{-1})P(z^{-1}) + \tilde{Q}(z^{-1})A(z^{-1})]y(t+1) \\ =&\tilde{B}(z^{-1})\bar{e}(t+1) + \tilde{B}(z^{-1})R(z^{-1})w(t+1) + [\tilde{Q}(z^{-1}) - \tilde{B}(z^{-1})K(z^{-1})]v[X(t)] \end{aligned} \tag{40}$$

where $\tilde{B}(z^{-1})$, $\tilde{Q}(z^{-1})$ are determined by

$$\tilde{B}(z^{-1})[Q(z^{-1}) + S(z^{-1})] = \tilde{Q}(z^{-1})B(z^{-1}) \tag{41}$$

$$\det \tilde{B}(z^{-1}) = \det B(z^{-1}) \tag{42}$$

Since $P(z^{-1})B(z^{-1}) + A(z^{-1})[Q(z^{-1}) + S(z^{-1})]$ is stable, for (39), $u(t)$ can be viewed as the output of the following asymptotically stable system:

$$\begin{aligned} x(t+1) &= \mathcal{E}x(t) + \mathcal{F}_1\bar{e}(t+1) + \mathcal{F}_2 w(t+1) + \mathcal{F}_3 v[X(t)] \\ u(t) &= \mathcal{G}x(t) + \mathcal{H}_1\bar{e}(t+1) + \mathcal{H}_2 w(t+1) + \mathcal{H}_3 v[X(t)] \end{aligned} \tag{43}$$

where $\mathcal{E}$ is stable; $\bar{e}(t+1)$ is the input; $w(t)$ is the known bounded disturbance; $v[X(t)]$ is the nonlinear disturbance. Therefore, from (iii) in Assumption 2.1 and Lemma 4.1, there exist constants d_3 and d_4 such that

$$\|u(t)\| \leq d_3 + d_4 \max_{0\leq\tau\leq t} \|\bar{e}(\tau+1)\| \tag{44}$$

Introducing matrix polynomials $\tilde{\tilde{A}}(z^{-1})$, $\tilde{\tilde{B}}(z^{-1})$ which satisfy

$$A(z^{-1})\tilde{\tilde{B}}(z^{-1}) = B(z^{-1})\tilde{\tilde{A}}(z^{-1}) \tag{45}$$

$$\det \tilde{\tilde{B}}(z^{-1}) = \det B(z^{-1}) \tag{46}$$

then from (41), (42), (45) and (46), we can obtain

$$\begin{aligned} &\det[\tilde{B}(z^{-1})P(z^{-1}) + \tilde{Q}(z^{-1})A(z^{-1})] \\ =&\det\{P(z^{-1})B(z^{-1}) + A(z^{-1})[Q(z^{-1}) + S(z^{-1})]\} \end{aligned} \tag{47}$$

Consequently, from (40) and (47), similar as (44), it can be easily obtained that there exist positive constants d_5 and d_6, such that

$$\|y(t)\| \leq d_5 + d_6 \max_{0\leq\tau\leq t} \|\bar{e}(\tau+1)\| \tag{48}$$

Since $X(t) = [y(t)^T, \cdots, u(t)^T, \cdots]^T$, from (44) and (48), there exist positive constant d_1 and d_2, such that (38) is obtained.

Theorem 4.1. *For the specified matrix polynomials $P(z^{-1})$, $Q(z^{-1})$ and $S(z^{-1})$ chosen by (10), provided (12) is satisfied, the linear robust adaptive decoupling control algorithm (17)-(23) when applied to the system (2) leads to a BIBO stable closed-loop system.*

Proof: Define $\tilde{\Theta}_1(t) = \hat{\Theta}_1(t) - \Theta$, then by (14), (17) and (21), we have

$$e_1(t) = -\tilde{\Theta}_1(t-1)^T X(t-1) + \zeta[X(t-1)] \tag{49}$$

Consequently, from (18), (19), (22) and (49), we can obtain

$$\begin{aligned} \|\tilde{\Theta}_1(t)\|^2 &\le \|\tilde{\Theta}_1(t-1)\|^2 - \frac{a_1(t)[\|e_1(t)\|^2 - 4\|\zeta[X(t-1)]\|^2]}{2(1+X(t-1)^T X(t-1))} \\ &\le \|\tilde{\Theta}_1(t-1)\|^2 - \frac{a_1(t)[\|e_1(t)\|^2 - 4M^2]}{2(1+X(t-1)^T X(t-1))} \end{aligned} \tag{50}$$

From (18), since $a_1(t) = 1$ for $\|e_1(t)\| > 2M$ and is 0 otherwise, $\{\|\tilde{\Theta}_1(t)\|^2\}$ is a nonincreasing sequence. Hence from (ii) in Assumption 2.1, $\|\hat{\Theta}_1(t)\|$ is bounded. Moreover,

$$\lim_{N\to\infty} \sum_{t=1}^{N} \frac{a_1(t+1)(\|e_1(t+1)\|^2 - 4M^2)}{2(1+X(t)^T X(t))} < \infty \tag{51}$$

$$\lim_{t\to\infty} \frac{a_1(t+1)(\|e_1(t+1)\|^2 - 4M^2)}{2(1+X(t)^T X(t))} \to 0 \tag{52}$$

From (17), (21) and (23),

$$\begin{aligned} e_1(t+1) =& \varphi(t+1) - \hat{\varphi}_1(t+1) \\ =& P(z^{-1})y(t+1) - \hat{\Theta}_1(t)^T X(t) \\ =& P(z^{-1})y(t+1) - R(z^{-1})w(t+1) + [Q(z^{-1}) + S(z^{-1})]u(t) \end{aligned} \tag{53}$$

then according to (6) and (53),

$$\bar{e}(t+1) = e_1(t+1) + K(z^{-1})v[X(t)] \tag{54}$$

By (38) and (54), and the boundedness of $v[X(t)]$, there exist positive constants c_3, c_4 such that

$$\|X(t)\| \le c_3 + c_4 \max_{0\le\tau\le t} \|e_1(\tau+1)\| \tag{55}$$

Then from (52), (55) and adopting the similar line as Theorem 6.1 in [11], we obtain that $\|e_1(t+1)\|$ is bounded, therefore the closed-loop system is BIBO stable.

Theorem 4.2. *For the specified matrix polynomials $P(z^{-1})$, $Q(z^{-1})$, $R(z^{-1})$, $S(z^{-1})$ and $K(z^{-1})$ chosen by (10) and (11), provided (12) is satisfied, the proposed multiple model adaptive decoupling control algorithm (14)-(30) when applied to the system (2) leads to a BIBO stable closed-loop switching system. Moreover, for arbitrary small positive number ε, the steady state generalized tracking error always satisfies $\|\bar{e}(t)\| < 2M + \varepsilon$. Especially, if the nonlinear model is chosen eventually, $\|\bar{e}(t)\| \le 3d_0$, at steady state.*

Proof: Define $\tilde{\Theta}_2(t) = \hat{\Theta}_2(t) - \Theta$, then by (14), (24) and (28), we have

$$e_2(t) = -\tilde{\Theta}_2(t-1)^T X(t-1) + \zeta[X(t-1)] - \hat{\zeta}[X(t-1)] \tag{56}$$

Consequently, from (25), (26), (29) and (56), we can obtain

$$\begin{aligned} \|\tilde{\Theta}_2(t)\|^2 &\le \|\tilde{\Theta}_2(t-1)\|^2 - \frac{\mu(t)[\|e_2(t)\|^2 - 4\|\zeta[X(t-1)] - \hat{\zeta}[X(t-1)]\|^2]}{2(1+X(t-1)^T X(t-1))} \\ &\le \|\tilde{\Theta}_2(t-1)\|^2 - \frac{\mu(t)[\|e_2(t)\|^2 - 4d_0^2]}{2(1+X(t-1)^T X(t-1))} \end{aligned} \tag{57}$$

From (27), since $\mu(t) = 1$ for $\|e_2(t)\| > 2d_0$ and is 0 otherwise, $\{\|\tilde{\Theta}_2(t)\|^2\}$ is a nonincreasing sequence. Hence from (ii) in Assumption 2.1, $\|\hat{\Theta}_2(t)\|$ is bounded.

According to (24), (28), (31), (54) and the certainty equivalence principle, at every instant t,

$$\bar{e}(t+1) = e_1(t+1) + K(z^{-1})v[X(t)], \text{ or} \tag{58}$$

$$\bar{e}(t+1) = e_2(t+1) + K(z^{-1})\{v[X(t)] - \hat{v}[X(t)]\} \tag{59}$$

Since at every instant, the identification error of the system $e(t+1) = e_1(t+1)$ or $e_2(t+1)$, then by (38) in Lemma 4.2, the boundedness of $\hat{v}[X(t)]$ and (iii) in Assumption 2.1, there exist positive constants c_5 and c_6 such that

$$\|X(t)\| \le c_5 + c_6 \max_{0\le\tau\le t} \|e(\tau+1)\| \tag{60}$$

From (51), it is easy to know $J_1(t)$ is always bounded. For $J_2(t)$, there exist two cases:

(i) $J_2(t)$ is bounded. By the switching law (32), it follows that

$$\lim_{t\to\infty} \frac{a_2(t+1)(\|e_2(t+1)\|^2 - 4M^2)}{2(1+X(t)^T X(t))} \to 0 \tag{61}$$

Therefore, the identification error $e(t)$ of the system satisfies

$$\lim_{t\to\infty} \frac{a(t+1)(\|e(t+1)\|^2 - 4M^2)}{2(1+X(t)^T X(t))} \to 0 \tag{62}$$

where

$$a(t+1) = \begin{cases} 1, & \text{if } \|e(t+1)\| > 2M \\ 0, & \text{otherwise} \end{cases}$$

(ii) $J_2(t)$ is unbounded. Since $J_1(t)$ is bounded, there exists instant t_0 such that $J_1(t) \le J_2(t)$, $\forall t \ge t_0$. Therefore, the identification error $e(t+1) = e_1(t+1)$ also satisfies (60) at $\forall t \ge t_0 + 1$.

By (60) and (62), and following the similar line as Theorem 4.1, the BIBO stability of the closed-loop switching system is obtained. From (62) and the boundedness of $X(t)$, the identification error $e(t)$ of the system satisfies

$$\lim_{t\to\infty} a(t+1)(\|e(t+1)\|^2 - 4M^2) = 0 \tag{63}$$

i.e., for arbitrary small $\varepsilon > 0$, there exists time instant T, such that when $t > T$, $\|e(t)\| \le 2M + \varepsilon$. From (58) and (59), since $K(z^{-1})|_{z=1} = 0$, the steady-state generalized tracking error $\bar{e}(t) = e(t)$, which satisfies $\|\bar{e}(t)\| \le 2M + \varepsilon$.

From (57), it is easy to know

$$\lim_{t\to\infty} \frac{\mu(t+1)(\|e_2(t+1)\|^2 - 4d_0^2)}{2(1+X(t)^T X(t))} \to 0 \tag{64}$$

From the boundedness of $X(t)$, we can obtain that the identification error $e_2(t)$ satisfies

$$\lim_{t\to\infty} \mu(t+1)(\|e_2(t+1)\|^2 - 4d_0^2) \to 0 \tag{65}$$

Choose $\varepsilon = d_0$, then when $t > T$, $\|e_2(t)\| \le 3d_0$. Consequently, if the nonlinear model is chosen eventually, at steady state, the generalized tracking error satisfies $\|\bar{e}(t)\| \le 3d_0$.

5. **Simulation.** In this section, simulations are conducted and the results are presented. Since comparing with [10,11], one of the main contributions of this paper is the relaxation of the assumption that the ZD of the controlled system is asymptotically stable, to illustrate the effectiveness of the proposed method, examples should be selected such that the ZD is not asymptotically stable. To this end, the following two-input-two-output discrete time nonlinear system is considered:

$$y_1(t+1) = 1.1y_1(t)+0.2u_1(t)+u_2(t)+u_1(t-1)+1.2\sin(u_1(t)+y_2(t))-\frac{1.2(u_1(t)+y_2(t))}{1+u_1(t)^2+y_2(t)^2}$$

$$y_2(t+1) = 0.2y_2(t)+0.25u_1(t)+0.2u_2(t)+u_2(t-1)+\sin(u_2(t)+y_1(t))-\frac{(u_2(t)+y_1(t))}{1+u_2(t)^2+y_1(t)^2}$$

The reference trajectories $w_1 = 0.1\text{sign}(\sin(\pi t/100))$ and $w_2 = 0.1$ are chosen to be followed. It is easy to know the origin is one of the equilibrium points and the ZD of the above system is not asymptotically stable around the origin.

According to what is stated in Section 2, the weighting matrix polynomials $P(z^{-1})$, $Q(z^{-1})$, $R(z^{-1})$, $S(z^{-1})$ and $K(z^{-1})$ are offline chosen as

$$P(z^{-1}) = R(z^{-1}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad Q(z^{-1}) = \begin{pmatrix} 1.2-1.2z^{-1} & 0 \\ 0 & 0.2-0.2z^{-1} \end{pmatrix}$$

$$S(z^{-1}) = \begin{pmatrix} 0 & 0.6-0.6z^{-1} \\ -7+7z^{-1} & 0 \end{pmatrix}, \quad K(z^{-1}) = \begin{pmatrix} 0.3-0.3z^{-1} & 0 \\ 0 & 0.5-0.5z^{-1} \end{pmatrix}$$

The architecture of the NN described in Section 3 is adopted with the sigmoidal operator $\text{sigm}[x] = [e^x - e^{-x}]/[e^x + e^{-x}]$. The back-propagation with adaptive learning rate in batch mode is chosen. In order to determine the optimum number of hidden nodes, a series of different topologies are used. It is achieved that the square error value is least when the number of hidden nodes is 23. So number 23 is chosen as the number of hidden nodes. In addition the parameters of the network are chosen as follows: Learning ratio $\text{lr} = 0.01$ and Momentum factor $\text{mc} = 0.001$. It is worthy to emphasis that the nonlinear NN ADC $u_2(t)$ is computed by the Newton iterative procedure within a given tolerance $\text{tl} = 0.001$.

As comparison, simulations are first conducted by using the method in [10]. The results are illustrated in Figure 1. It can be seen that using the method in [10], the system can not be controlled at all. Figure 2 shows the performance when the ADC proposed in this paper is used. It is obvious that the good tracking performance of the output signals and small amplitude of the input signals are all achieved. Figure 3 is the switching sequence between the linear robust ADC $u_1(t)$, and the nonlinear NN ADC $u_2(t)$. From Figure 3, in most of the time, the nonlinear NN ADC $u_2(t)$ works, and only when it degrades, in order to guarantee the stability of the system, the linear robust ADC $u_1(t)$ begins to work until the NN ADC $u_2(t)$ recovers.

6. **Conclusion.** In this paper, a nonlinear MIMO discrete time dynamic system is expressed as a linear model incorporating with a high order nonlinear term around the origin. A linear robust ADC and a nonlinear NN ADC are designed respectively. By the switching mechanism, without the assumption that the ZD is asymptotically stable, the BIBO stability of the closed-loop switching system is assured and the performance is improved.

The ADC method is proposed under the condition that the high order nonlinear term of the controlled system is globally bounded. The problem becomes extremely challenging when the nonlinearity is not globally bounded. Just as what is pointed out in [11], two directions of research in this respect are possible: one is to establish some global results

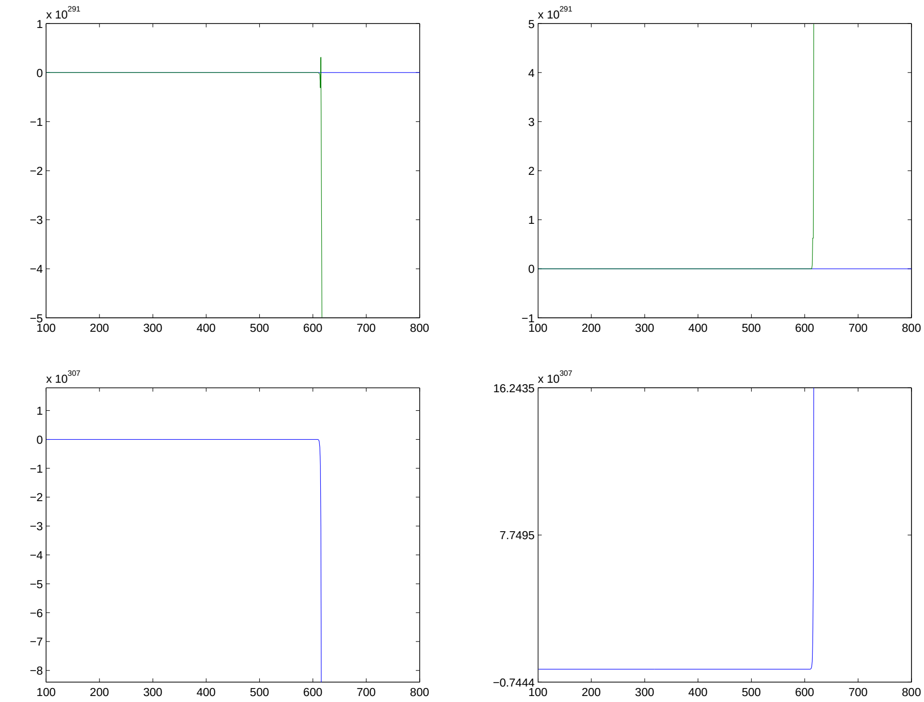

FIGURE 1. System performance when the method proposed in [10] is used

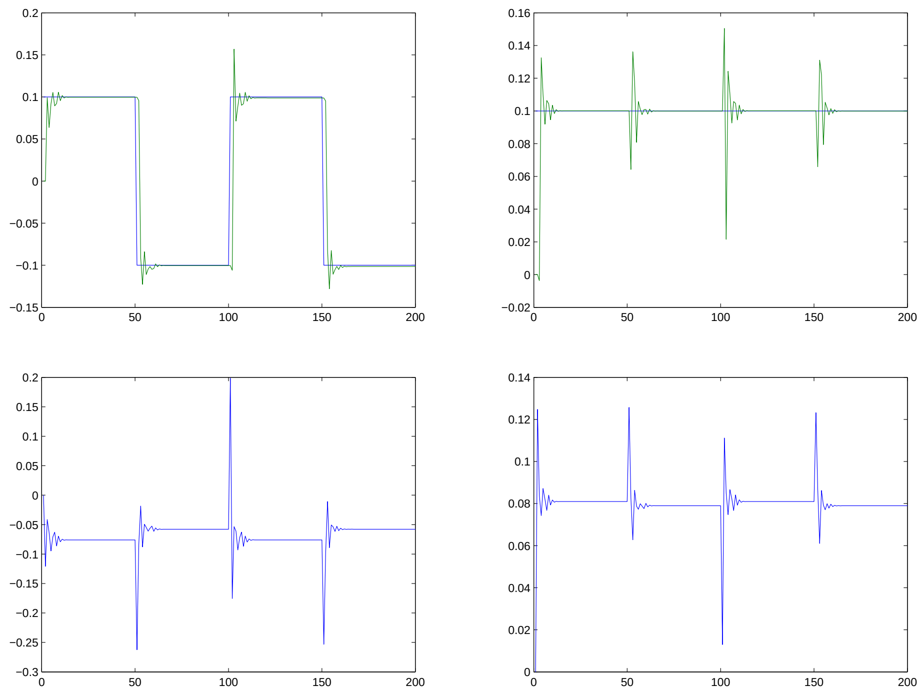

FIGURE 2. System performance when the ADC proposed in this paper is used

for the closed-loop system by imposing more structural constraints on the system. The other is to establish some local results using continuity arguments.

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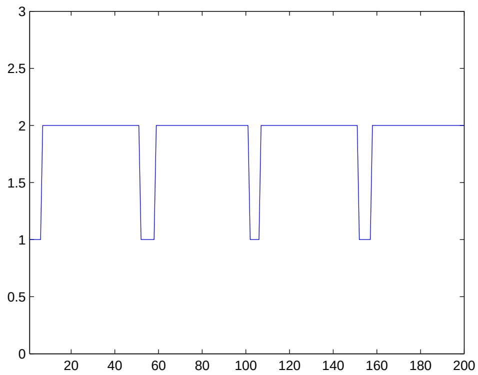

FIGURE 3. Switching sequence: 1 denotes linear, 2 denotes nonlinear

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REFERENCES

[1] T. Sato, Design of a generalized minimum variance control in sampled-data control systems, *International Journal of Innovative Computing, Information and Control*, vol.5, no.10(B), pp.3295-3302, 2009.
[2] Z. W. Lin, Y. Xia, P. Shi and H. Wu, Robust sliding mode control for uncertain linear discrete systems independent of time-delay, *International Journal of Innovative Computing, Information and Control*, vol.7, no.2, pp.869-880, 2011.
[3] B. Wittenmark, R. Midleton and G. C. Goodwin, Adaptive decoupling of multivariable systems, *International Journal of Control*, vol.46, no.6, pp.1993-2009, 1987.
[4] B. Liu, Y. Xia, M. Fu, M. Wang and H. Wu, Active decoupling for double-loop delayed systems with ADRC techniques, *International Journal of Innovative Computing, Information and Control*, vol.5, no.10(B), pp.3367-3376, 2009.
[5] T. Chai, Direct adaptive decoupling control for general stochastic multivariable systems, *International Journal of Control*, vol.51, no.5, pp.885-909, 1990.
[6] H. Yue and T. Chai, Adaptive decoupling control of multivariable nonlinear nonminimun phase systems using neural networks, *Proc. of the American Control Conference*, Philadelphia, PA, pp.513-514, 1998.
[7] T. Chai and H. Yue, Multivariable intelligent decoupling control system and its application, *Acta Automatica Sinica*, vol.31, no.1, pp.123-131, 2005.
[8] T. Chai, L. Zhai and H. Yue, Multiple models and neural networks based decoupling control of ball mill coal-pulverizing systems, *Journal of Process Control*, vol.21, no.3, pp.351-366, 2011.
[9] Y. Fu and T. Chai, Neural-network-based nonlinear adaptive dynamical decoupling control, *IEEE Transactions on Neural Networks*, vol.18, no.3, pp.921-925, 2007.
[10] Y. Fu and T. Chai, Nonlinear multivariable adaptive control using multiple models and neural networks, *Automatica*, vol.43, no.6, pp.1101-1110, 2007.
[11] L. J. Chen, *Nonlinear Adaptive Control of Discrete-time Systems Using Neural Networks and Multiple Models*, Yale University, Berkeley, 2001.
[12] B. D. Joao and K. S. Narendra, Issues in the application of neural networks for tracking based on inverse control, *IEEE Transactions on Automatic Control*, vol.44, no.11, pp.2007-2027, 1999.
[13] L. J. Chen and K. S. Narendra, Identification and control of a nonlinear discrete-time system based on its linearization: A unified framework, *IEEE Transactions on Neural Networks*, vol.15, no.3, pp.663-673, 2004.
[14] K. J. Astrom and B. Wittenmark, *Adaptive Ontrol*, Addison-Wesley Publishing Company, 1989.