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EFFICIENT LMI-BASED QUADRATIC STABILITY AND STABILIZATION OF PARAMETER-DEPENDENT INTERVAL SYSTEMS WITH APPLICATIONS

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Abstract. *This paper is concerned with the problem of quadratic stability and stabilization for continuous-time linear parameter-dependent interval systems. Differing from previous results in the analysis and control design of interval systems, the new necessary and sufficient conditions proposed in this paper for the quadratic stability, quadratic stabilization and $\mathscr{D}$-stabilization are based on parameter-dependent model representation of interval systems. In the quadratic framework, an approach based on a vertex result on interval uncertain parameters is proposed. This allows the solvability conditions to be presented in terms of a set of parameterized linear matrix inequalities which can be efficiently solved by using standard numerical softwares. A linearized longitudinal dynamic model of the flight control system of a supersonic cruise missile is presented to illustrate the effectiveness and advantage of the proposed methods.*
Keywords: Parameter-dependent interval systems (PDISs), Quadratic stability, Quadratic stabilization, $\mathscr{D}$-stabilization, Linear matrix inequalities (LMIs), Flight control

1. Introduction.

It is well known that linear interval systems are a class of dynamic linear systems whose state-space matrices depend on a set of uncertain parameters which are not constant, but are variable on some fixed intervals. As one of the complex system models with uncertain parameters, robust control of dynamic interval systems have been studied intensively in the last two decades and significant progress has been made in this area (see, for example, [1-16]). This is mainly due to the fact that many real-world physical systems with various uncertainties are well characterized by dynamic interval systems. Moreover, both the stability analysis and the stabilization control are fundamental requirements of the most of designed control systems, certainly including interval control systems. Recently, lots of results about the robust stability and stabilization of interval systems are readily available in the existing literature (see, for example, [1, 2, 5-8, 10-16]). In [1], Mao and Chu presented effective, less conservative, necessary and sufficient conditions for the quadratic stability and stabilization of dynamic interval systems. Jetto and Orsini considered the efficient LMI-based quadratic stabilization of the interval LPV systems with noisy parameter measures [2]. In [4], a sufficient condition for quadratic stabilizability and root clustering was given via the way of an auxiliary convex problem. Myszkorowski discussed the stability of discrete-time linear interval systems [5]. Different conditions of robust stabilization of the linear time-invariant interval systems via constant state feedback control were presented in [8, 11]. An analysis on robust stability for interval descriptor systems could be found in [12]. Based on a novel LMI approach, a robust stability and stabilization of the fractional-order interval systems was proposed in [14].

Note that almost all the above mentioned results on checking the robust stability and designing the stabilizing controller for interval systems are only suitable for a class of parameter-independent interval systems, each of whose terms of system matrices varies independently in given intervals. However, in fact, the uncertain parameters in many physical systems are usually interdependent and interrelated with each other. Furthermore, for the dynamic system models based on the parameter-independent interval uncertainty descriptions, the existing approaches to interval systems may latently enlarge the envelope of original interval systems. This may lead to rather conservative results of robust stability and stabilization, even to some wrong conclusions.

Therefore, it is still of considerable theoretical and practical importance to seek a simple and effective criterion for robust stability of parameter-dependent interval systems (PDISs) whose system matrices are dependent on uncertain parameters and entries of the system matrices are linear combinations of these uncertain parameters. Also, the robust stabilization of PDISs is necessary to the synthesis problem for this kind of interval systems. This problem is obviously more difficult than that of robust stability analysis. Furthermore, for linear systems, the location of the closed-loop poles determines many control performance specifications such as stability, damping, and the speed of the time response. These specifications can be ensured by the placement of closed-loop poles in an suitable region of the complex plane, that is, regional pole placement or so-called $\mathscr{D}$-stabilization [17, 18]. It is well known that quadratic stability and stabilization approaches are convenient and effective in the analysis and synthesis of uncertain linear systems [1, 2, 20]. This is mainly due to the fact that quadratic approaches look for a fixed quadratic Lyapunov function for all admissible uncertainties. It is attractive to extend these approaches to dynamic interval systems, including parameter-dependent interval systems.

With the above motivations, the important issues of stability analysis and $\mathscr{D}$-stabilization of PDIS are investigated in this paper. Firstly, we propose a new PDIS model to represent a class of interval systems, which is different from the majority of existing parameter-independent interval systems. The uncertainty of the PDIS model appears in the form of affine parameters with the given interval values. By an illustrative example, it is shown that the new model form can be used to describe the PDISs and have a sense of practical background. Secondly, the key issues of quadratic stability, stabilization and $\mathscr{D}$-stabilization of such PDIS model are investigated, respectively. Necessary and sufficient conditions for quadratic stability and stabilization of such kinds of interval systems are presented in terms of linear matrix inequalities (LMIs), which can be efficiently solved by using standard numerical softwares. Furthermore, to take performance specifications into account, an LMI condition to implement quadratic $\mathscr{D}$-stabilization is presented by assigning the closed-loop poles in a specified LMI region. Finally, a linearized longitudinal dynamic model of the flight control system of a supersonic cruise missile is given to illustrate the effectiveness of the proposed method.

The rest of this paper is organized as follows. The problem formulation and some preliminary results are presented in Section 2. Section 3 gives our main results of quadratic stability and stabilization of PDISs. A numerical example is given in Section 4 and we conclude this paper in Section 5.

2. Problem Formulation and Preliminaries.

Consider the following class of linear time-invariant uncertain systems:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ denote the state vector and the control input, respectively. The system matrices A and B are interval uncertain matrices in the sense that

$$\begin{aligned} A \in A_I = [\underline{A}, \overline{A}] = \left\{[a_{ij}] : \underline{a}_{ij} \le a_{ij} \le \overline{a}_{ij}, 1 \le i, j \le n\right\} \\ B \in B_I = [\underline{B}, \overline{B}] = \left\{[b_{ij}] : \underline{b}_{ij} \le b_{ij} \le \overline{b}_{ij}, 1 \le i \le n, 1 \le j \le m\right\} \end{aligned} \tag{2}$$

where $\underline{A} = \left[\underline{a}_{ij}\right]_{n\times n}$ and $\overline{A} = [\overline{a}_{ij}]_{n\times n}$ satisfy $\underline{a}_{ij} \le \overline{a}_{ij}$ for all $1 \le i, j \le n$, and $\underline{B} = \left[\underline{b}_{ij}\right]_{n\times m}$ and $\overline{B} = \left[\overline{b}_{ij}\right]_{n\times m}$ satisfy $\underline{b}_{ij} \le \overline{b}_{ij}$ for all $1 \le i \le n$, $1 \le j \le m$.

Generally, the system (1) and (2) is known as an interval system which is widely applied in many researches of robust stability and stabilization of dynamic interval system (see, for example, [1, 2, 5-8, 10-16]). However, this typical description can not be used to represent all of the interval system, such as the parameter-independent interval system. We can illustrate the reason by the following simple example, which is borrowed from a simplified and linearized longitudinal dynamic model of the flight control system of a supersonic cruise missile. The state-space equation of the parameter-dependent interval system is described as follows:

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} a-b & b & 0 \\ 0 & 0 & 1 \\ c & -c & 0 \end{bmatrix} x(t) + \begin{bmatrix} d \\ 0 \\ -e \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x(t) \end{cases} \tag{3}$$

where the ranges of parameters are

$$a = -0.005,\ 0.4 \le b \le 0.6,\ 8 \le c \le 15,\ 0.03 \le d \le 0.05,\ 35 \le e \le 70.$$

From the parameter values, we know that the system (3) is an interval system and the values of entries of system matrices are changed greatly. Following the representation of interval system (1) and (2), we denote parameter-dependent interval entries of the system matrix A as follows:

$$-0.605 \le a_{11} \le -0.405,\ 0.4 \le a_{12} \le 0.6,\ 8 \le a_{31} \le 15,\ -15 \le a_{32} \le -8.$$

Then, interval matrices A and B in the sense of (2) can be derived as follows:

$$\underline{A} = \begin{bmatrix} -0.605 & 0.4 & 0 \\ 0 & 0 & 1 \\ 8 & -15 & 0 \end{bmatrix},\ \overline{A} = \begin{bmatrix} -0.405 & 0.6 & 0 \\ 0 & 0 & 1 \\ 15 & -8 & 0 \end{bmatrix},\ \underline{B} = \begin{bmatrix} 0.03 \\ 0 \\ -70 \end{bmatrix},\ \overline{B} = \begin{bmatrix} 0.05 \\ 0 \\ -35 \end{bmatrix}.$$

However, the scope of $A_I = [\underline{A}, \overline{A}]$ is incorrect because the terms a_{11} and a_{12} are dependent on the common parameter b and the terms cannot reach to the extreme values simultaneously. Similarly, the terms a_{31} and a_{32} are dependent on the common parameter c. The actual relation between a_{31} and a_{32} is shown in Figure 1(a). However, if we follow the representation of interval system (1) and (2), the ranges of the terms a_{31} and a_{32} are expanded apparently as shown in Figure 1(b). So, the envelope curve of A_I is clearly expanded, which cannot be used to implement robust stability and stabilization of interval systems. Therefore, we should reformulate a new PDIS model for the system (3) and develop a new condition of quadratic stability and stabilization in the following.

Representing the basic interval parameters in the normalized form, the dynamic linear system is considered as follows:

$$\dot{x}(t) = A(\delta(t))\, x(t) + B(\delta(t))\, u(t) \tag{4}$$

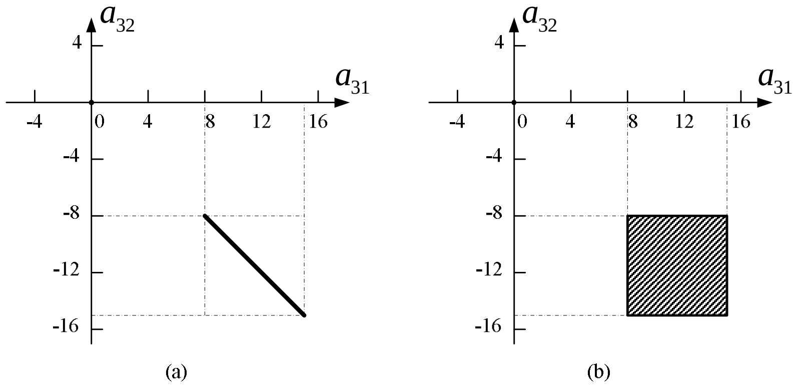

FIGURE 1. The relations between a_{31} and a_{32}

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the state vector and control input, respectively. The matrices $A(\delta)$ and $B(\delta)$ are functions of the interval variable $\delta(t) = [\delta_1(t), \cdots, \delta_p(t)]$. When the uncertain parameters are interval variables, we represent the system matrices as the following form

$$A(\delta(t)) = A_0 + \sum_{i=1}^{p} \delta_i(t) A_i, \quad B(\delta(t)) = B_0 + \sum_{i=1}^{p} \delta_i(t) B_i, \tag{5}$$

where A_0, B_0, A_i and B_i are the known real matrices, and $\delta_i(t)$, $i = 1, 2, \cdots, p$ are the interval variables which are in the following regular polyhedron:

$$\Delta_I = \left\{ \delta(t) = [\delta_1(t), \cdots, \delta_p(t)] \mid \delta_i(t) \in \left[\delta_i^-, \delta_i^+\right] \right\}. \tag{6}$$

Remark 2.1. *Denote $N = n \times (n+m)$, we derive that $p \leq N-1$. Based on the representations of the parameter-independent system (1) and (2) and the parameter-dependent interval system (4)-(6), we know that if $p = N-1$, the system (1) and (2) is equal to the system (4)-(6). Therefore, we can conclude that the parameter-independent interval system (1) and (2) is only a special case of parameter-dependent interval system (4)-(6).*

Remark 2.2. *It is important to note that the sense of the uncertain parameters $\delta_i(t)$ in (6) are different from that of the uncertain perturbations a_{ij} and b_{ij} in (2). The uncertain parameters $\delta_i(t)$ studied in this paper are arbitrary interval. However, the uncertain perturbations a_{ij} and b_{ij} presented in [1-16] are changed within certain relatively small ranges. Therefore, the model representation of the PDIS (4)-(6) is more suitable to the practical application than that of other model formulations.*

Similar to the previous quadratic stability and stabilization problems for interval systems in [1, 16], the problem of parameter-dependent interval system (4)-(6) will be investigated in this paper. In order to solve the problem, we present some preliminary results for later use.

Definition 2.1. *The dynamic interval system (4)-(6) without input is said to be quadratically stable if there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ satisfying*

$$A^{\mathrm{T}}(\delta(t)) P + PA(\delta(t)) < 0, \ \forall \delta(t) \in \Delta_I. \tag{7}$$

When the dynamic interval system (4)-(6) is quadratically stabilizable, we give the state-feedback control law as

$$u(t) = Kx(t). \tag{8}$$

Then, (4)-(6) and (8) form a closed-loop system

$$\dot{x}(t) = A_c(\delta(t)) x(t) = [A(\delta(t)) + B(\delta(t)) K] x(t). \tag{9}$$

Definition 2.2. *The dynamic interval system (4)-(6) is said to be quadratically stabilizable if there exist a matrix $K \in \mathbb{R}^{m\times n}$ and a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ satisfying*

$$A_c^{\mathrm{T}}(\delta(t))\,P + PA_c(\delta(t)) < 0,\ \forall\delta(t) \in \Delta_I. \tag{10}$$

From Definitions 2.1 and 2.2, it is easy to see that if the condition (10) is satisfied the closed-loop system (9) is quadratically stable. Hence, the gain matrix K is called as quadratically stabilizing state-feedback matrix.

In the most practical situations to control design of linear time-invariant systems, the good controllers should guarantee the stability of a desired closed-loop system, and also deliver sufficiently fast and well-damped time responses [17, 18]. A customary way to guarantee satisfactory performance is to place the closed-loop poles in a suitable region of the complex plane that embraces most practically useful stability regions. Therefore, based on the quadratic stability and stabilization of PDIS, we further research the robust pole placement in this paper.

The problem studied in this paper can be expressed as follows. Given the parameter-dependent interval system described by the model (4)-(6), judge whether the interval system is quadratically stable or not. If the system is quadratically stable, find a robust control law via linear state-feedback (8) and robust regional pole assignment, such that the closed-loop poles of (9) lie in the specified LMI region for all the interval values of the regular polyhedron (6). For the sake of conciseness, this problem will be named the quadratic stability and $\mathscr{D}$-stabilizability problem and our intent is to find the quadratic stability and $\mathscr{D}$-stabilizability conditions for PDISs and the associated $\mathscr{D}$-stabilizing state-feedback gains.

The following definition and lemmas are well known results in [17, 20], and will be essential for the proof in the next section.

Definition 2.3. *([17]) A subset $\mathscr{D}$ of the complex plane is called an LMI region if there exist a symmetric matrix L and a matrix M such that*

$$\mathscr{D} = \left\{\ z \in \mathbb{C} : f_{\mathscr{D}}(z) = L + zM + \bar{z}M^{\mathrm{T}} < 0\ \right\}.$$

Note that the characteristic function $f_{\mathscr{D}}$ takes values in the space of Hermitian matrices and that "< 0" stands for negative definite.

Lemma 2.1. *([20]) Let $y = f(x)$ be a convex function defined over the compact convex set Δ. Then $f(x) < 0$ holds in Δ if and only if it holds on all the extreme points of Δ_E, that is*

$$f(x) < 0,\ \forall x \in \Delta \iff f(x) < 0,\ \forall x \in \Delta_E.$$

Lemma 2.2. *([17]) The matrix A is $\mathscr{D}$-stable if and only if there exists a symmetric matrix X such that*

$$M_{\mathscr{D}}(A, X) := L \otimes X + M \otimes (XA) + M^{\mathrm{T}} \otimes \left(A^{\mathrm{T}}X\right) < 0.$$

Lemma 2.3. *([17]) Given two LMI regions $\mathscr{D}_1$ and $\mathscr{D}_2$, a matrix A is both $\mathscr{D}_1$-stable and $\mathscr{D}_2$-stable if and only if there exists a positive definite matrix X such that $M_{\mathscr{D}_1}(A, X) < 0$ and $M_{\mathscr{D}_2}(A, X) < 0$.*

3. Main Results.

In this section, necessary and sufficient conditions are derived for quadratic stability and stabilization of the linear uncertain systems described by the PDIS model (4)-(6) in terms of parameterized LMIs. Furthermore, an LMI-based approach is proposed for designing linear state-feedback control laws to quadratically $\mathscr{D}$-stabilize the uncertain PDIS model (4)-(6).

Theorem 3.1. *The linear uncertain system described by the PDIS model (4)-(6) is quadratically stable if and only if there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ satisfying the following set of LMIs:*

$$A^{\mathrm{T}}(\delta(t))P + PA(\delta(t)) < 0,\ \forall \delta_i(t) = \delta_i^- \ or\ \delta_i^+,\ i = 1, 2, \cdots, p. \tag{11}$$

Proof: Choose

$$V(x) = x^{\mathrm{T}}Px,$$

then

$$\dot{V}(\delta(t)) = x^{\mathrm{T}}\left[A^{\mathrm{T}}(\delta(t))P + PA(\delta(t))\right]x.$$

It is easy to see that $\dot{V}(\delta(t))$ is linear in $\delta(t)$, and thus convex with respect to $\delta(t)$. Thus, it follows from Lemma 2.1 that for $\forall x \neq 0$,

$$\dot{V}(\delta(t)) < 0,\ \forall \delta(t) \in \Delta_I \iff \dot{V}(\delta(t)) < 0,\ \forall \delta(t) \in \Delta_E$$

where

$$\Delta_E = \left\{\delta(t) = [\delta_1(t), \delta_2(t), \cdots, \delta_p(t)] \mid \delta_i(t) = \delta_i^- \text{ or } \delta_i^+\right\}.$$

This equivalently implies that (7) holds if and only if

$$A^{\mathrm{T}}(\delta(t))P + PA(\delta(t)) < 0,\ \forall \delta(t) \in \Delta_E$$

holds. The proof is then completed.

As in Section 2, we consider a set of closed-loop systems

$$\dot{x}(t) = A_c(\delta(t))x(t),\ \forall \delta(t) \in \Delta_I \tag{12}$$

with

$$A_c(\delta(t)) = A(\delta(t)) + B(\delta(t))K.$$

It follows from Theorem 3.1 that the closed-loop system (12), with $\delta(t) \in \Delta_I$, is quadratically stable if and only if there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$, such that

$$A_c(\delta(t))P + PA_c^{\mathrm{T}}(\delta(t)) < 0,\ \forall \delta(t) \in \Delta_E.$$

Following the typical treatment of this type of inequalities [20], we immediately have the following theorem.

Theorem 3.2. *The linear uncertain system described by the PDIS model (4)-(6) is quadratically stabilizable if and only if there exist a matrix $W \in \mathbb{R}^{m\times n}$ and a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ satisfying the following set of LMIs:*

$$A(\delta(t))P + PA^{\mathrm{T}}(\delta(t)) + B(\delta(t))W + W^{\mathrm{T}}B^{\mathrm{T}}(\delta(t)) < 0,\quad \forall \delta(t) \in \Delta_E \tag{13}$$

where

$$\Delta_E = \left\{\delta(t) = [\delta_1(t), \delta_2(t), \cdots, \delta_p(t)] \mid \delta_i(t) = \delta_i^- \ or\ \delta_i^+\right\}.$$

Moreover, the quadratically stabilizing state-feedback matrix is given by

$$K = WP^{-1}.$$

Next, a $\mathscr{D}$-stabilization result of the PDISs is to be established.

It is known that the transient response of a linear system is related to the location of its poles (see [17] and the references therein). By constraining poles to lie in a prescribed region, specific bounds can be put on these quantities to ensure a satisfactory transient response. As shown in Figure 2, a region for control purposes is the set $S(\alpha, r, \theta)$ of complex numbers $x + jy$ such that

$$x < -\alpha < 0,\quad |x + jy| < r,\quad |y| < -x\tan\theta. \tag{14}$$

Confining the closed-loop poles to $S(\alpha, r, \theta)$ region ensures a minimum decay rate α, a minimum damping ratio $\zeta = \cos\theta$, and a maximum undamped natural frequency $\omega_d =$

$r \sin\theta$. This in turn bounds the maximum overshoot, the frequency of oscillatory modes, the delay time, the rise time and the settling time.

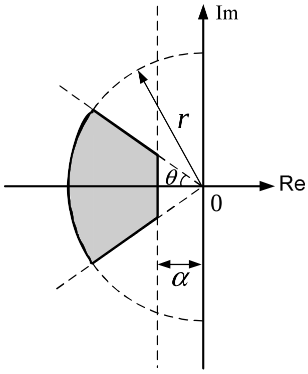

FIGURE 2. The LMI region: $S(\alpha, r, \theta)$

As for the realization of quadratic $\mathscr{D}$-stabilizability of system (4)-(6) with a given level of performance of $S(\alpha, r, \theta)$ region, a necessary and sufficient condition is presented in the following theorem.

Theorem 3.3. *Given the LMI region $S(\alpha, r, \theta)$, the linear uncertain system described by the PDIS model (4)-(6) is quadratically $\mathscr{D}$-stabilizable if and only if there exist a matrix $W \in \mathbb{R}^{m\times n}$ and a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$, satisfying the following set of LMIs:*

$$A(\delta(t))P + PA^{\mathrm{T}}(\delta(t)) + B(\delta(t))W + W^{\mathrm{T}}B^{\mathrm{T}}(\delta(t)) + 2\alpha P < 0,\ \forall\delta(t) \in \Delta_E \tag{15}$$

$$\begin{bmatrix} -rP & A(\delta(t))P + B(\delta(t))W \\ PA^{\mathrm{T}}(\delta(t)) + W^{\mathrm{T}}B^{\mathrm{T}}(\delta(t)) & -rP \end{bmatrix} < 0,\ \forall\delta(t) \in \Delta_E \tag{16}$$

$$\begin{bmatrix} \Omega_1(\delta(t))\sin\theta & \Omega_2(\delta(t))\cos\theta \\ \Omega_2^{\mathrm{T}}(\delta(t))\cos\theta & \Omega_1(\delta(t))\sin\theta \end{bmatrix} < 0,\ \forall\delta(t) \in \Delta_E \tag{17}$$

where

$$\Omega_1(\delta(t)) = A(\delta(t))P + PA^{\mathrm{T}}(\delta(t)) + B(\delta(t))W + W^{\mathrm{T}}B^{\mathrm{T}}(\delta(t))$$
$$\Omega_2(\delta(t)) = A(\delta(t))P - PA^{\mathrm{T}}(\delta(t)) + B(\delta(t))W - W^{\mathrm{T}}B^{\mathrm{T}}(\delta(t))$$
$$\Delta_E = \left\{\delta(t) = [\delta_1(t), \cdots, \delta_p(t)] \mid \delta_i(t) = \delta_i^- \text{ or } \delta_i^+\right\}.$$

In this case, the quadratically $\mathscr{D}$-stabilizing state-feedback matrix is given by

$$K = WP^{-1}. \tag{18}$$

Proof: Note that the region $S(\alpha, r, \theta)$ in (14) can be represented in the intersection of three LMI-based subregions, that is,

$$\mathscr{D} = \mathscr{D}_\alpha \cap \mathscr{D}_{(r,q)} \cap \mathscr{D}_\theta \tag{19}$$

where their associated characteristic functions

$$f_{\mathscr{D}_\alpha}(z) = z + \bar{z} + 2\alpha$$
$$f_{\mathscr{D}_{(r)}}(z) = \begin{bmatrix} -r & z \\ \bar{z} & -r \end{bmatrix}$$
$$f_{\mathscr{D}_\theta}(z) = \begin{bmatrix} (z+\bar{z})\sin\theta & (z-\bar{z})\cos\theta \\ (\bar{z}-z)\cos\theta & (z+\bar{z})\sin\theta \end{bmatrix}.$$

From Lemma 2.2 and Theorem 3.1, the closed-loop system (12) is quadratically $\mathscr{D}_{\alpha}$-stable, $\mathscr{D}_{(r,q)}$-stable and $\mathscr{D}_{\theta}$-stable, if and only if there exists a symmetric positive definite matrix $P \in R^{n\times n}$ such that

$$A_c(\delta(t))P + PA_c^{\mathrm{T}}(\delta(t)) < 0,\ \forall\delta(t) \in \Delta_E \tag{20}$$

$$\begin{bmatrix} -rP & A_c(\delta(t))P \\ PA_c^{\mathrm{T}}(\delta(t)) & -rP \end{bmatrix} < 0,\ \forall\delta(t) \in \Delta_E \tag{21}$$

and

$$\begin{bmatrix} \left[A_c(\delta(t))P + PA_c^{\mathrm{T}}(\delta(t))\right]\sin\theta & \left[A_c(\delta(t))P - PA_c^{\mathrm{T}}(\delta(t))\right]\cos\theta \\ \left[PA_c^{\mathrm{T}}(\delta(t)) - A_c(\delta(t))P\right]\cos\theta & \left[A_c(\delta(t))P + PA_c^{\mathrm{T}}(\delta(t))\right]\sin\theta \end{bmatrix} < 0,\ \forall\delta(t) \in \Delta_E \tag{22}$$

are satisfied.

Applying Lemma 2.3 to (19), we have that the system (4)-(6) is quadratically $\mathscr{D}$-stabilizable if and only if the conditions (20)-(22) are tenable simultaneously.

Substituting $A_c(\delta(t)) = A(\delta(t)) + B(\delta(t))K$ into (20)-(22) and setting $W = KP$, we can derive that the inequalities (20)-(22) are equivalent to (15)-(17), respectively. This ends the proof.

Remark 3.1. *The conditions (11) given in Theorem 3.1 and conditions (13) given in Theorem 3.2 are the set of parameterized LMIs, which can be solved by the standard LMI-Toolbox in the Matlab environment* [21]. *Thus, by Theorem 3.1, we can easily examine whether a parameter-dependent interval system is quadratically stable. Also, by Theorem 3.2, the LMI Control Toolbox in Matlab makes it easy to examine whether a parameter-dependent interval system is quadratically stabilizable. Similar to those in Theorems 3.1 and 3.2, (15)-(17) in Theorems 3.3 are also the parameterized LMIs. More importantly, for a quadratically stabilizable parameter-dependent interval system, the LMI-Toolbox can calculate the solutions W and P satisfying quadratic $\mathscr{D}$-stabilizability with a given level of performance of $S(\alpha, r, \theta)$ region. Then, the quadratically $\mathscr{D}$-stabilizing state-feedback matrix is obtained directly from (18). That means Theorems 3.1-3.3, respectively, provide tractable analysis and synthesis approaches for parameter-dependent interval system (4)-(6).*

4. **Application to Flight Control of Supersonic Cruise Missiles.** Recall the example presented in Section 2. Consider the simplified and linearized longitudinal dynamic model of the flight control system of a supersonic cruise missile as follows:

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} a(t)-b(t) & b(t) & 0 \\ 0 & 0 & 1 \\ c(t) & -c(t) & 0 \end{bmatrix} x(t) + \begin{bmatrix} d(t) \\ 0 \\ -e(t) \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x(t) \end{cases} \tag{23}$$

where the system parameters $a(t)$, $b(t)$, $c(t)$, $d(t)$, $e(t)$ all change with the variation of the flying height and flying speed, and their varying rules are very complicated and hard to express analytically. To deal with this complicated time-varying system, as is done in practice, we choose ten operating points on the whole trajectory, which correspond to serval important flight moments as shown in Table 1. The values of the parameters $a(t)$, $b(t)$, $c(t)$, $d(t)$, $e(t)$ at the ten operating points are known and are given in Table 1.

Based on the results developed in the previous sections, the interval system (23) can be written as the following PDIS model:

$$\begin{cases} \dot{x}(t) = A(\delta)x(t) + B(\delta)u(t) \\ y(t) = Cx(t) \end{cases} \tag{24}$$

TABLE 1. The parameters in the ten operating points

Operating points	The time-varying parameters				
t, time, s	$a(t)$	$b(t)$	$c(t)$	$d(t)$	$e(t)$
0	–0.005	0.4∼0.6	8.0∼15.0	0.03∼0.05	35.0∼70.0
20	–0.003	0.3∼0.4	6.0∼13.0	0.01∼0.03	19.0∼38.0
40	–0.002	0.1∼0.3	4.0∼10.0	0.01∼0.02	11.0∼21.0
60	–0.001	0.1∼0.3	3.0∼8.0	0.01∼0.08	8.0∼17.0
100	–0.0002	0.05∼0.1	2.0∼7.0	0.03∼0.06	5.0∼10.0
150	–8.34e-5	0.05∼0.1	1.0∼5.0	0.03∼0.06	5.0∼10.0
300	–7.57e-6	0.05∼0.1	1.0∼4.0	0.03∼0.06	5.0∼10.0
600	0.006	0.2∼0.3	16.0∼35.0	0.02∼0.03	22.0∼46.0
900	0.008	0.5∼1.0	49.0∼92.0	0.05∼0.08	45.0∼97.0
1000	0.01	1.0∼1.5	66.0∼130.0	0.09∼ 0.2	86.0∼170.0

where

$$A(\delta(t)) = A_0 + \sum_{i=1}^{4} \delta_i(t) A_i,\ B(\delta(t)) = B_0 + \sum_{i=1}^{4} \delta_i(t) B_i,\ C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$A_0 = \begin{bmatrix} a_0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},\ A_1 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \end{bmatrix},\ A_3 = A_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_0 = B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ B_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ B_4 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

and $\delta_i(t)$, $i = 1, \cdots, 4$ are the interval variables which are the regular polyhedrons as shown in Table 1. For example, we can denote the first and tenth operating points as the following form:

Case 1:
$\Delta_{I_1} = \{a_0 = -0.005,\ 0.4 \le \delta_1 \le 0.6,\ 8 \le \delta_2 \le 15,\ 0.03 \le \delta_3 \le 0.05,\ 35 \le \delta_4 \le 70\}$

Case 2:
$\Delta_{I_2} = \{a_0 = 0.01,\ 1.0 \le \delta_1 \le 1.5,\ 66 \le \delta_2 \le 130,\ 0.09 \le \delta_3 \le 0.2,\ 86 \le \delta_4 \le 170\}$.

If the interval system (24) in the above two cases is represented to be a parameter-independent interval model just as system (1) and (2), we can find that the system is not asymptotically stable and stabilizable by using Theorems 1 and 2 in [14]. Likewise, by using Theorems 3 and 4 in [1], the system (1) and (2) is not quadratically stable and stabilizable. However, by using Theorems 3.1 and 3.2 in this paper, the above system is determined to be quadratically stable and stabilizable. The comparative results of stability analysis is shown in Table 2. The about results show that the PDIS model in (24) cannot be represented to be a parameter-independent interval model in (1) and (2), and it may be incorrect in practice to implement the existing approaches in both analysis and synthesis for such kind of the PDISs. Thus, the results presented in this paper may be encouraging in some practical applications.

Furthermore, by Theorem 3.3 of this paper, a quadratically $\mathscr{D}$-stabilizing state-feedback control law for the quadratically stabilizable parameter-dependent interval system (24) can be obtained with a given level of performance. For example, we need to design a state-feedback control law in (8) satisfying the following specifications:

1) the closed-loop system is quadratically stable;

Table 2. Results obtained by different methods

Cases	Both Cases 1 and Cases 2		
Methods	Theorem 1 in [14]	Theorem 3 in [1]	Theorem 3.1
Results	Not asymptotically stable	Not quadratically stable	Quadratically stable

2) the overshoot σ of the system's unit step response of the system is less than 2%;
3) the setting time t_s of the system's unit step response of the system is lower than 0.5s.

The feasible solutions of (15)-(17) are as follows:

$$\text{Case 1:} \begin{cases} P_1 = 10^{-8} \times \begin{bmatrix} 0.0475 & -0.0010 & 0.0019 \\ -0.0010 & 0.0049 & -0.0287 \\ 0.0019 & -0.0287 & 0.4205 \end{bmatrix} \\ W_1 = 10^{-6} \times \begin{bmatrix} 0.0000 & -0.0001 & 0.1411 \end{bmatrix} \\ \alpha_1 = 0.39, \quad r_1 = 4000, \quad \theta_1 = \pi/31.2\,\mathrm{deg}. \end{cases}$$

$$\text{Case 2:} \begin{cases} P_2 = 10^{-4} \times \begin{bmatrix} 0.0032 & -0.0003 & 0.0010 \\ -0.0003 & 0.0011 & -0.0062 \\ 0.0010 & -0.0062 & 0.1162 \end{bmatrix} \\ W_2 = 10^{-3} \times \begin{bmatrix} 0.0001 & -0.0002 & 0.1771 \end{bmatrix} \\ \alpha_2 = 0.7, \quad r_2 = 4000, \quad \theta_2 = \pi/10\,\mathrm{deg}. \end{cases}$$

In two cases, the quadratically $\mathscr{D}$-stabilizing state-feedback gain matrices are

$$K_1 = \begin{bmatrix} 4.4548 & 330.7532 & 56.1389 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 4.6751 & 120.5057 & 21.6521 \end{bmatrix},$$

and the corresponding unit step response results for Cases 1 and 2 are shown in Figure 3. More importantly, thanks to the $\mathscr{D}$-stabilization of $S(\alpha, r, \theta)$ region, the above specifications are perfectly achieved with $t_{s1} = t_{s2} = 0.48$s, $\sigma_1 = 0.75\%$ and $\sigma_2 = 1.21\%$.

Similar to Cases 1 and 2, we can also derive the quadratically $\mathscr{D}$-stabilizing state-feedback gain matrices of other operating points by Theorem 3.3. Then, using the obtained controllers of ten operating points and the general gain-scheduled approach [22], we present the result of the output attitude tracking control on the whole trajectory. The attitude tracking results and the error curve are shown in Figures 4 and 5, respectively. It can be seen from the simulation results that the tracking performance is satisfactory and the tracking error is lower than 0.5 in spite of the existing complicated time-varying parameters in the system matrices, which show that the designed time varying flight control system works perfectly and offers good dynamical performances.

5. **Conclusions.** This paper has presented the necessary and sufficient conditions for the quadratic stability, stabilization and $\mathscr{D}$-stabilization of parameter-dependent interval systems. The reformulation of parameter-dependent interval systems in this paper are more extensively dynamic interval systems than that of the parameter-independent ones in the existing literature. In spite of the implicit conservativeness of quadratic stability and stabilization, the conditions can be efficiently solved by using standard numerical softwares because the results are based on vertex values of interval uncertain parameters and the conditions are established in terms of a set of parameterized linear matrix inequalities. The design of flight control systems of the supersonic cruise missile has been provided to illustrate the effectiveness and advantage of the proposed methods.

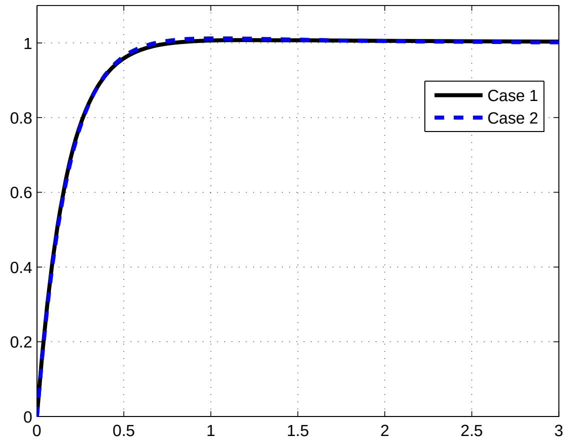

FIGURE 3. The unit step responses of Cases 1 and 2

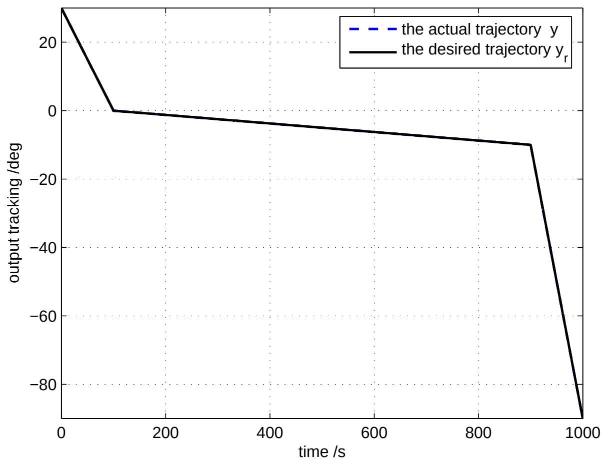

FIGURE 4. The output tracking results

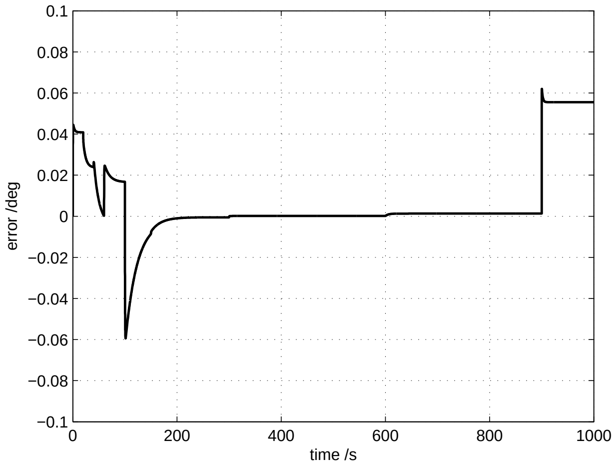

FIGURE 5. The tracking error

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