

## A BOUNDED POSITIVE NON-LINEAR PI CONTROLLER FOR DOUBLE-PIPE HEAT EXCHANGERS

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**ABSTRACT.** *In this work, temperature regulation of double-pipe heat exchangers is considered. The positive (unidirectional) and bounded features of the flow rate input variable are taken into account in the analysis. As a result, a bounded positive PI-type control scheme, which achieves the regulation objective avoiding input saturation, is proposed. It turns out to be a simple algorithm that does not need to feed back the whole state vector, that does not depend on the exact value of the system parameters, and whose stabilization character is global in the closed-loop system state-space domain. Moreover, it may be applied to both flow configuration cases, i.e., countercurrent and parallel-flow heat exchangers. The analytical developments are corroborated through experimental and simulation results.*

**Keywords:** Regulation, Nonlinear control, Output feedback, Bounded input, Heat exchangers

1. **Introduction.** Since long ago, heat exchangers have been part of a great number of industrial processes (see, for instance, [1, §1] and references therein) as well as academic environments [2, 3]. In view of their numerous applications, they have been the subject of many studies including, among others: steady-state, transient and frequency response analysis [4, 5, 6]; open-loop qualitative behavior characterization [1, 7]; numerical simulation [8, 9]; state reconstruction [10, 11], parameter identification [12, 13], or simultaneous estimation of both, states and parameters [14, 15]; and feedback control [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. Among these topics, the last mentioned one has proved to be essential in the solution formulation to cope with the operation demands imposed to current industrial processes. In particular, unexpected behaviors that deteriorate the closed-loop performance and/or prevent the pre-specified convergence goal are undesirable or unacceptable. Thus, a control scheme designed to avoid such unexpected or undesirable phenomena is always preferable.

Several control schemes for the outlet temperature regulation of heat exchangers have been developed in the literature through the application of various techniques. For instance, linearizing feedback algorithms have been proposed, based on a simple lumped-parameter model in [16], and considering a distributed-parameter model in [17]. Unfortunately, such a geometric control design methodology assumes the exact knowledge of the

structure and parameters of the system dynamics, involves all (or a considerable number of) the states of the considered model, and generally gives rise to complex expressions, neglecting the natural analytical-stability properties of the process. This is also the case for the partially linearized control variable methodology used in [18].

Other works like that in [19] – which proposes an optimal (LQI) control scheme, those in [20, 21] – where standard and particular Generalized Predictive Control (GPC) algorithms are respectively developed (the latter adopting a particular PID structure), and those in [22, 23] – where schemes based on Min-Max Model Predictive Control (MMMPC) were implemented, focus on the optimization of a cost function, disregarding the qualitative natural properties of the system. Moreover, the implementation of optimization-based methodologies (like optimal control, GPC and MMMPC) generally involve a considerable number of calculations in order to accomplish the considered optimization criterion at every sampling period. The computational burden becomes specially large in the MMMPC case in view of the considered double optimization. This problem was reduced through hinging hyperplanes in [22], and through a neural network solver in [23]. But the use of such supplementary techniques added up their own complications to the resulting design procedure.

Other methods, like the multi-loop algorithm tuned using game theory that was developed in [24], the PID-deadtime scheme (PID with time-delay compensation) proposed in [25], or the PI fuzzy controller developed in [26], base their efficiency on a tuning procedure or tuning formulae. Nevertheless, they lack formal stability proofs or stability region estimations.

On the other hand, as far as the authors are aware, previous works on control design for double-pipe heat exchangers do not simultaneously consider the positive (unidirectional) and bounded nature of the flow rate taken as input variable. Such controllers could eventually try to force the actuators to go beyond their natural capabilities, undergoing the well-known phenomenon of saturation. In a general context, the presence of such a non-linearity is not necessarily inconvenient, as long as it is taken into account in the control design and/or the closed loop analysis. Otherwise, it may give rise to undesirable behaviors as pointed out, for instance, in [27, §1] and [28, §I]. Thus, control design considering those input constraints turns out to be important in order to avoid such unexpected or undesirable closed-loop system behaviors. This is corroborated from the consideration of the input saturation phenomenon in the various contexts treated in the recent literature, as seen, for instance, in [29, 30, 31].

In this work, a non-linear PI-type controller for the hot fluid outlet temperature regulation of double-pipe heat exchangers is proposed, taking the cold fluid flow rate as control input. With respect to the above cited works, the designed algorithm takes into account the positive and bounded nature of the flow rate taken as input variable. Contrary to the exact and partial feedback linearizing methods like those in [16, 17, 18], it does not depend on the exact knowledge of the system parameters, does not need to feed back all (or a considerable number of) the process states, and takes into account the natural analytical and stability properties of the exchanger. As a result, the proposed algorithm turns out to have a simple structure that avoids the considerable number of calculations characterizing the optimization-based algorithms such as those in [19, 20, 21, 22, 23]. Moreover, the resulting controller guarantees stabilization to the desired outlet temperature for any initial condition within the system state-space domain avoiding input saturation, and is not restricted to a specific flow configuration but may be applied to both countercurrent and parallel-flow heat exchangers. These characteristics are corroborated through a formal closed-loop analysis which distinguishes the justification procedure of the proposed algorithm with respect to the tuning-procedure/formulae-based approaches like those in

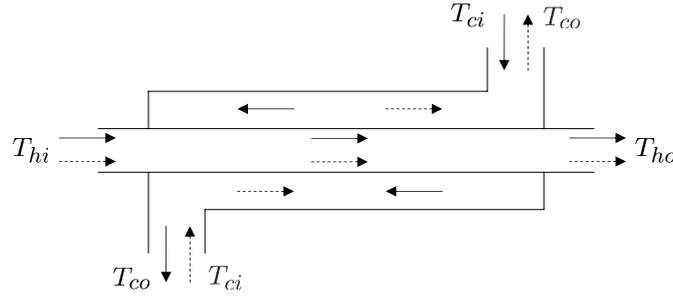


FIGURE 1. Counter/parallel-flow (solid/dashed resp.) double-pipe heat exchanger

[24, 25, 26]. Furthermore, the analytical developments are corroborated through experimental and simulation results.

**2. Nomenclature and Notation.** Throughout the paper, the system variables and parameters are denoted as follows:

$U$	heat transfer coefficient [J/(°C m <sup>2</sup> s)]	$A$	heat transfer surface area [m <sup>2</sup> ]
$C_p$	specific heat [J/(°C kg)]	$F$	mass flow rate [kg/s]
$M$	total mass inside the tube [kg]	$T$	temperature [°C]
$\Delta T$	temperature difference [°C]	$t$	time [s]
$\mathbb{R}$	set of real numbers	$\mathbb{R}_+$	set of positive real numbers
$\mathbb{R}^n$	set of $n$ -tuples $(x_j)$ with $x_j \in \mathbb{R}$	$\mathbb{R}_+^n$	set of $n$ -tuples $(x_j)$ with $x_j \in \mathbb{R}_+$
$0_n$	origin of $\mathbb{R}^n$		

*Subscripts:*

$u$	upper bound	$c$	cold	$i$	inlet
$l$	lower bound	$h$	hot	$o$	outlet

Let  $\Delta T_1$  and  $\Delta T_2$  stand for the temperature difference at each terminal side of the heat exchanger, i.e.,

$$\Delta T_1 \triangleq \begin{cases} T_{hi} - T_{co} & \text{if } \alpha = 1 \\ T_{hi} - T_{ci} & \text{if } \alpha = -1 \end{cases} \quad \text{and} \quad \Delta T_2 \triangleq \begin{cases} T_{ho} - T_{ci} & \text{if } \alpha = 1 \\ T_{ho} - T_{co} & \text{if } \alpha = -1 \end{cases}$$

where

$$\alpha \triangleq \begin{cases} 1 & \text{if counter flow} \\ -1 & \text{if parallel flow} \end{cases}$$

(see Figure 1). The *logarithmic mean temperature difference* (LMTD) among the fluids is typically expressed as (see, for instance, [32] and references therein):

$$\Delta T_\ell \triangleq \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}}$$

Nevertheless, this expression reduces to an indeterminate form when  $\Delta T_1 = \Delta T_2$ , which is specially problematic in the counterflow case. Such an indetermination is avoided if the LMTD is taken as

$$\Delta T_L \triangleq \begin{cases} \Delta T_\ell & \text{if } \Delta T_2 \neq \Delta T_1 \\ \Delta T_0 & \text{if } \Delta T_2 = \Delta T_1 = \Delta T_0 \end{cases} \quad (1)$$

This was proven in [32], together with the following analytical properties:

**Lemma 2.1.** [32, Lemma 2 and Remark 3]  $\Delta T_L$  in Equation (1) is continuously differentiable at every  $(\Delta T_1, \Delta T_2) \in \mathbb{R}_+^2$ . Moreover, it is positive on  $\mathbb{R}_+^2$ , while  $\lim_{\Delta T_1 \rightarrow 0} \Delta T_L = 0$  for any  $\Delta T_2 \in \mathbb{R}_+$ , and  $\lim_{\Delta T_2 \rightarrow 0} \Delta T_L = 0$  for any  $\Delta T_1 \in \mathbb{R}_+$ .

**Lemma 2.2.** [32, Lemma 3]  $\Delta T_L$  in Equation (1) is strictly increasing in its arguments, i.e.,  $\frac{\partial \Delta T_L}{\partial \Delta T_i} > 0$ ,  $i = 1, 2$ ,  $\forall (\Delta T_1, \Delta T_2) \in \mathbb{R}_+^2$ .

Finally, the interior and boundary of a set, say  $\mathcal{B}$ , will be respectively denoted as  $\text{int}(\mathcal{B})$  and  $\partial\mathcal{B}$ . The derivative of a scalar function depending on a single scalar variable, say  $\rho : \mathbb{R} \rightarrow \mathbb{R} : \varsigma \mapsto \rho(\varsigma)$ , will be denoted  $\rho'$ , i.e.,  $\rho'(\varsigma) = \frac{d\rho}{d\varsigma}(\varsigma)$ .

**3. The System Dynamics.** The following assumptions are considered:

- A1. The fluid temperatures and velocities are radially uniform.
- A2. The heat transfer coefficient is axially uniform and constant.
- A3. Constant fluid thermophysical properties.
- A4. No heat transfer with the surroundings (adiabatic operation).
- A5. Fluids are incompressible and single phase.
- A6. Negligible axial heat conduction.
- A7. There is no energy storage in the walls.
- A8. Inlet temperatures,  $T_{ci}$  and  $T_{hi}$ , are constant.
- A9. The flow rates are axially uniform and any variation is considered to take place instantaneously at every point along the whole length of the exchanger.
- A10. The hot fluid flow rate,  $F_h$ , is kept constant, while the value of the cold fluid flow rate,  $F_c$ , can be arbitrarily varied within a compact interval  $\mathcal{F}_c \triangleq [F_{cl}, F_{cu}]$ , for some positive constants  $F_{cl} < F_{cu}$ .

Under these assumptions, and taking the whole exchanger as a single *bi-compartmental cell*, a simplified but suitable lumped-parameter model that accurately captures the dynamical properties of the qualitative behavior of double-pipe heat exchanger, as thoroughly shown in [1], is:

$$\dot{T}_{co} = \frac{2}{M_c} \left[ F_c (T_{ci} - T_{co}) + \frac{UA}{C_{pc}} \Delta T_L(T_{co}, T_{ho}) \right] \quad (2a)$$

$$\dot{T}_{ho} = \frac{2}{M_h} \left[ F_h (T_{hi} - T_{ho}) - \frac{UA}{C_{ph}} \Delta T_L(T_{co}, T_{ho}) \right] \quad (2b)$$

where  $\Delta T_L(\cdot, \cdot)$  is the LMTD (complemented) expression (1), considered a function of  $(T_{co}, T_{ho})$ . A physically reasonable state-space domain for system in Equation (2) is [1, 7]

$$\mathcal{D} \triangleq \begin{cases} \{(T_{o1}, T_{o2}) \in \mathbb{R}^2 \mid T_{ci} < T_{oj} < T_{hi}, j = 1, 2\} & \text{if } \alpha = 1 \\ \{(T_{o1}, T_{o2}) \in \mathbb{R}^2 \mid T_{ci} < T_{o1} < T_{o2} < T_{hi}\} & \text{if } \alpha = -1 \end{cases}$$

The **control objective** consists in the regulation of the hot fluid outlet temperature,  $T_{ho}$ , towards a pre-specified desired value  $T_{hd}$  (the set-point), through the cold fluid flow rate  $F_c$  as input variable, taking into account the unidirectional nature and restricted range of such an input flow rate (according to Assumption 1). The use of a simple but qualitatively suitable model, like that in Equation (2), for the control design, aiming at the achievement of such an objective, is desirable as pointed out for instance in [33, 34]. Indeed, a high order process dynamics representation would end up in a complex scheme with complicated expressions, and would involve temperature measurements of intermediate points throughout the exchanger which are not always available. In particular, the model in Equation (2) has been used for control design for instance in [16, 18]. It

has actually been used as a suitable representation of the dynamics of double-pipe heat exchangers for numerous purposes, as pointed out in [1].

**Remark 3.1.** Notice that by considering  $\Delta T_L$  a function of  $(T_{co}, T_{ho})$  on  $\mathcal{D}$ , continuous differentiability and positivity hold for all  $(T_{co}, T_{ho})$  on  $\mathcal{D}$ , and 0 (zero) may be considered the value that  $\Delta T_L$  takes at any  $(T_{co}, T_{ho})$  on  $\partial\mathcal{D}$  such that  $\Delta T_1(T_{co}, T_{ho}) \cdot \Delta T_2(T_{co}, T_{ho}) = 0$  (see Lemma 2.1). Furthermore, strict monotonicity in its arguments holds as (applying the chain rule):  $\frac{\partial \Delta T_L}{\partial T_{ho}} > 0$  and  $\frac{\partial \Delta T_L}{\partial T_{co}} < 0$ ,  $\forall (T_{co}, T_{ho}) \in \mathcal{D}$  (see Lemma 2.2).

**Remark 3.2.** Let  $y$  denote the open-loop state vector, i.e.,  $y \triangleq (T_{co}, T_{ho})^T$ , and let  $\dot{y} = \bar{f}(y; \theta)$  represent the open-loop system dynamics in Equation (2) assuming constant flow rates, where  $\theta \in \mathbb{R}^p$  (for some positive integer  $p$ ) is the system parameter vector. Considering Lemma 2.1, one sees (from Equation (2)) that  $\bar{f}$  is continuously differentiable in  $(y; \theta)$  on  $\mathcal{D} \times \mathbb{R}_+^p$ . Then, the system solutions,  $y(t; y_0, \theta)$  with  $y_0 \triangleq y(0) \in \mathcal{D}$ , do not only exist and are unique, but are also continuously differentiable with respect to initial conditions and parameters, for all  $y_0 \in \mathcal{D}$  and all  $\theta$  sufficiently close to any nominal parameter vector  $\theta_0 \in \mathbb{R}_+^p$  (see, for instance, [35, §3.3]).

In [1], it was shown that, considering constant flow rates, the system dynamical model in Equation (2) possesses a unique equilibrium point  $(T_{co}^*, T_{ho}^*) \in \mathcal{D}$ , where

$$\begin{pmatrix} T_{co}^* \\ T_{ho}^* \end{pmatrix} = \begin{pmatrix} 1 - P & P \\ RP & 1 - RP \end{pmatrix} \begin{pmatrix} T_{ci} \\ T_{hi} \end{pmatrix} \triangleq \begin{pmatrix} g_c(F_c) \\ g_h(F_c) \end{pmatrix} \tag{3}$$

with  $R = \frac{F_c C_{pc}}{F_h C_{ph}}$ ,

$$P = \begin{cases} \frac{1 - S}{1 + (-S)^\beta R} & \text{if } R \neq \alpha \\ \frac{UA}{UA + F_c C_{pc}} & \text{if } R = \alpha = 1 \end{cases}$$

$S = \exp\left(\frac{\alpha UA}{F_h C_{ph}} - \frac{UA}{F_c C_{pc}}\right)$  and  $\beta \triangleq \frac{\alpha + 1}{2}$ .

**Claim 3.1.**  $g_h$  in Equation (3) is a one-to-one strictly decreasing continuously differentiable function of  $F_c$ .

**Proof:** Continuous differentiability of  $g_h$  with respect to  $F_c$  follows from the arguments given in Remark 3.2. Hence, from Equation (3),  $g'_h(F_c) = \frac{dg_h}{dF_c}(F_c)$  is given by

$$g'_h(F_c) = \begin{cases} \frac{RS [1 + \gamma - e^\gamma] (T_{hi} - T_{ci})}{F_c (1 + (-S)^\beta R)^2} & \text{if } R \neq \beta \\ -\frac{C_{pc} U^2 A^2 (T_{hi} - T_{ci})}{2C_{ph} F_h (UA + C_{ph} F_h)^2} & \text{if } R = \beta = 1 \end{cases}$$

where  $\gamma \triangleq \frac{UA}{C_{pc} F_c} - \frac{\alpha UA}{C_{ph} F_h}$ . Thus, from Formula 4.2.30 in [36]<sup>1</sup>, it follows that  $g'_h(F_c) < 0$ ,  $\forall F_c > 0$ , showing that  $g_h(F_c)$  is strictly decreasing on its domain. This, in turn, corroborates its one-to-one character.

**Remark 3.3.** Observe that through Claim 3.1, two important facts are concluded:

1.  $T_{ho}^*$  is restricted to a reachable steady-state space defined by

$$\mathcal{R}_h \triangleq [g_h(F_{cu}), g_h(F_{cl})] \tag{4}$$

<sup>1</sup>Formula 4.2.30 in [36] states the following well-known inequality:  $e^x > 1 + x$ ,  $\forall x \neq 0$ .

2. Any value of  $T_{ho}^* \in \mathcal{R}_h$  is uniquely defined by a specific flow rate value  $F_c^* \in \mathcal{F}_c$  (see Assumption 1), which in turn defines a unique value of  $T_{co}^*$  according to Equation (3).

**4. The Proposed Controller.** The analysis developed in [1], under the consideration of constant flow rates, showed that the vector field in Equation (2) has a normal component pointing to the interior of  $\mathcal{D}$  at every point on  $\partial\mathcal{D}$ . Consequently, for all initial state vectors in  $\mathcal{D}$ , the system trajectories remain in  $\mathcal{D}$  globally in time, and are bounded since  $\mathcal{D}$  is bounded. Moreover,  $\mathcal{D}$  was proven to contain a sole invariant composed by a unique equilibrium point  $(T_{co}^*, T_{ho}^*)$  (see Equation (3)). Therefore, every trajectory of the dynamical model in Equation (2) converges to  $(T_{co}^*, T_{ho}^*)$ . The idea is then to propose a controller defined in terms of a dynamic internal (auxiliary) state,  $\phi$ , in such a way that the closed-loop system keeps the same analytical features, with  $F_c$  forced to evolve within  $\text{int}(\mathcal{F}_c)$  (i.e., such that  $F_{cl} < F_c(t) < F_{cu}$ ,  $\forall t \geq 0$ ), and forcing the existence of a sole invariant composed by a unique equilibrium point  $(T_{co}^*, T_{ho}^*, \phi^*)$  where  $T_{ho}^* = T_{hd}$ , the pre-specified desired set-point (according to the control objective, stated in Section 3). This is achieved through the following control scheme.

**Proposition 4.1.** Consider the dynamical system in Equation (2) with  $F_c$  being able to take values exclusively on  $\mathcal{F}_c = [F_{cl}, F_{cu}]$ . Let the value of  $F_c$  be continuously computed as

$$F_c(\phi, T_{ho}) = k_p \eta(\phi) (T_{ho} - T_{hd}) + \phi \quad (5)$$

for any constant  $T_{hd}$  taken on  $\text{int}(\mathcal{R}_h)$  (see (4)), where

$$\eta(\phi) \triangleq (\phi - F_{cl})(F_{cu} - \phi)$$

$\phi$  is an auxiliary state whose instantaneous value is dynamically calculated according to the following auxiliary dynamics

$$\dot{\phi} = k_i \eta(\phi) (T_{ho} - T_{hd}) \quad (6)$$

$k_p$  is a nonnegative scalar satisfying

$$k_p < \frac{1}{(F_{cu} - F_{cl})(T_{hi} - T_{ci})} \quad (7)$$

and  $k_i$  is a positive constant. Then, provided that  $k_i$  is sufficiently small, for any initial closed-loop (extended) state vector  $(T_{co}, T_{ho}, \phi)(0) \in \mathcal{D} \times \text{int}(\mathcal{F}_c)$ :  $T_{ho}(t) \rightarrow T_{hd}$  as  $t \rightarrow \infty$ , with  $F_c(t) \in \text{int}(\mathcal{F}_c)$ ,  $\forall t \geq 0$ , and  $(T_{co}, T_{ho})(t) \in \mathcal{D}$ ,  $\forall t \geq 0$ .

**Proof:** First note that, from the satisfaction of inequality (7), it follows that

$$\begin{aligned} 0 < -k_p(F_{cu} - F_{cl})(T_{hi} - T_{ci}) + 1 &\leq k_p(F_{cu} + F_{cl} - 2\phi)(T_{ho} - T_{hd}) + 1 \\ &= k_p \eta'(\phi)(T_{ho} - T_{hd}) + 1 = \frac{\partial F_c}{\partial \phi} \end{aligned}$$

$\forall (\phi, T_{ho}) \in \mathcal{F}_c \times \mathcal{T}$ , where

$$\mathcal{T} \triangleq \{T_o \in \mathbb{R} \mid T_{ci} \leq T_o \leq T_{hi}\},$$

i.e.,

$$\frac{\partial F_c}{\partial \phi} > 0 \quad \forall (\phi, T_{ho}) \in \mathcal{F}_c \times \mathcal{T} \quad (8)$$

Moreover,  $\frac{\partial F_c}{\partial T_{ho}} = k_p \eta(\phi) \geq 0$ ,  $\forall (\phi, T_{ho}) \in \mathcal{F}_c \times \mathcal{T}$ , with strict inequality on  $\text{int}(\mathcal{F}_c) \times \text{int}(\mathcal{T})$  if  $k_p > 0$ . Then  $F_c(F_{cl}, T_{ci}) = F_{cl} < F_c(\phi, T_{ho}) < F_{cu} = F_c(F_{cu}, T_{hi})$ ,  $\forall (\phi, T_{ho}) \in \text{int}(\mathcal{F}_c) \times \text{int}(\mathcal{T})$ , or equivalently

$$F_c(\phi, T_{ho}) \in \text{int}(\mathcal{F}_c) \quad \forall (\phi, T_{ho}) \in \text{int}(\mathcal{F}_c) \times \text{int}(\mathcal{T}) \quad (9)$$

Now, the closed-loop dynamics takes the form

$$\frac{d}{dt} \begin{pmatrix} T_{co} \\ T_{ho} \\ \phi \end{pmatrix} = \begin{pmatrix} \frac{2}{M_c} \left[ F_c(\phi, T_{ho}) (T_{ci} - T_{co}) + \frac{UA}{C_{pc}} \Delta T_L(T_{co}, T_{ho}) \right] \\ \frac{2}{M_h} \left[ F_h(T_{hi} - T_{ho}) - \frac{UA}{C_{ph}} \Delta T_L(T_{co}, T_{ho}) \right] \\ k_i \eta(\phi) (T_{ho} - T_{hd}) \end{pmatrix} \tag{10}$$

with  $F_c(\phi, T_{ho})$  as expressed in Equation (5). Let  $x$  denote the closed-loop (extended) state vector, i.e.,  $x \triangleq (T_{co}, T_{ho}, \phi)^T$ , and let  $\dot{x} = f(x)$  represent the closed-loop system dynamics (in Equation (10)). Based on Lemma 2.1 (see also Remark 3.1), and in view of the satisfaction of (9), it can be verified that, with  $\alpha = 1$ :

$$\begin{aligned} f_1(T_{hi}, T_{ho}, \phi) &= \frac{2F_c(\phi, T_{ho})}{M_c} (T_{ci} - T_{hi}) < 0 & \forall (T_{ho}, \phi) \in \text{int}(\mathcal{T}) \times \text{int}(\mathcal{F}_c) \\ f_2(T_{co}, T_{ci}, \phi) &= \frac{2F_h}{M_h} (T_{hi} - T_{ci}) > 0 & \forall (T_{co}, \phi) \in \text{int}(\mathcal{T}) \times \text{int}(\mathcal{F}_c) \end{aligned}$$

with  $\alpha = -1$ :

$$\begin{aligned} f_1(T_{co}, T_{co}, \phi) &= \frac{2F_c(\phi, T_{co})}{M_c} (T_{ci} - T_{co}) < 0 & \forall (T_{co}, \phi) \in \text{int}(\mathcal{T}) \times \text{int}(\mathcal{F}_c) \\ f_2(T_{ho}, T_{ho}, \phi) &= \frac{2F_h}{M_h} (T_{hi} - T_{ho}) > 0 & \forall (T_{ho}, \phi) \in \text{int}(\mathcal{T}) \times \text{int}(\mathcal{F}_c) \end{aligned}$$

and for any  $\alpha \in \{-1, 1\}$ :

$$\begin{aligned} f_1(T_{ci}, T_{ho}, \phi) &= \frac{2UA}{M_c C_{pc}} \Delta T_L(T_{ci}, T_{ho}) > 0 & \forall (T_{ho}, \phi) \in \text{int}(\mathcal{T}) \times \text{int}(\mathcal{F}_c) \\ f_2(T_{co}, T_{hi}, \phi) &= -\frac{2UA}{M_h C_{ph}} \Delta T_L(T_{co}, T_{hi}) < 0 & \forall (T_{co}, \phi) \in \text{int}(\mathcal{T}) \times \text{int}(\mathcal{F}_c) \\ f_3(T_{co}, T_{ho}, F_{cl}) &= f_3(T_{co}, T_{ho}, F_{cu}) = 0 & \forall (T_{co}, T_{ho}) \in \mathcal{D} \end{aligned}$$

This shows that there is no point on the boundary of  $\mathcal{D} \times \mathcal{F}_c$  where the vector field  $f$  has a normal component pointing outwards. Consequently, for any initial extended state vector in  $\mathcal{D} \times \text{int}(\mathcal{F}_c)$ , the closed-loop system solution cannot leave the system state-space domain  $\mathcal{D} \times \text{int}(\mathcal{F}_c)$ . Moreover, it is clear that the points on  $\partial\mathcal{D} \times \text{int}(\mathcal{F}_c)$  cannot even be approached. On the other hand, from expression (8), it can be seen that, for any given cold fluid flow rate steady-state value  $F_c^* \in \mathcal{F}_c$ , corresponding to a specific hot fluid outlet steady-state temperature  $T_{ho}^* \in \mathcal{T}$  (according to Remark 3.3), there corresponds a unique auxiliary state equilibrium value  $\phi^* \in \mathcal{F}_c$ . With this and Remark 3.3 in mind, it can be seen from Equation (6) that the closed-loop system has a unique equilibrium point  $x^* = (T_{co}^*, T_{ho}^*, \phi^*)$  in  $\mathcal{D} \times \text{int}(\mathcal{F}_c)$ , where  $T_{ho}^* = T_{hd}$ , and  $\phi^*$  takes the unique value on  $\mathcal{F}_c$  through which  $T_{ho}^*$  adopts the desired value  $T_{hd}$ . Besides, letting  $x_l^* \triangleq (g_c(F_{cl}), g_h(F_{cl}), F_{cl})$  and  $x_u^* \triangleq (g_c(F_{cu}), g_h(F_{cu}), F_{cu})$  (see Equation (3); observe further that  $F_c(F_{cl}, g_h(F_{cl})) = F_{cl}$  and  $F_c(F_{cu}, g_h(F_{cu})) = F_{cu}$ ) – with  $g_h(F_{cl}) = \max\{T_{ho}^* \in \mathcal{R}_h\}$  and  $g_h(F_{cu}) = \min\{T_{ho}^* \in \mathcal{R}_h\}$  (see Remark 3.3), it follows that  $f(x_l^*) = f(x_u^*) = 0_3$ . Actually,  $x_l^*$  and  $x_u^*$  are the only equilibrium points on the boundary of  $\mathcal{D} \times \mathcal{F}_c$ . The

Jacobian matrix of  $f$ , i.e.,

$$\frac{\partial f}{\partial x} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ -\frac{2UA}{M_h C_{ph}} \frac{\partial \Delta T_L}{\partial T_{co}} & -\frac{2}{M_h} \left[ F_h + \frac{UA}{C_{ph}} \frac{\partial \Delta T_L}{\partial T_{ho}} \right] & 0 \\ 0 & k_i \eta(\phi) & k_i \eta'(\phi)(T_{ho} - T_{hd}) \end{pmatrix}$$

where

$$\begin{aligned} \frac{\partial f_1}{\partial x_1} &= \frac{2}{M_c} \left[ -F_c(\phi, T_{ho}) + \frac{UA}{C_{pc}} \frac{\partial \Delta T_L}{\partial T_{co}} \right] \\ \frac{\partial f_1}{\partial x_2} &= \frac{2}{M_c} \left[ k_p \eta(\phi)(T_{ci} - T_{co}) + \frac{UA}{C_{pc}} \frac{\partial \Delta T_L}{\partial T_{ho}} \right] \\ \frac{\partial f_1}{\partial x_3} &= \frac{2}{M_c} (k_p \eta'(\phi)(T_{ho} - T_{hd}) + 1)(T_{ci} - T_{co}) \end{aligned}$$

and  $\eta'(\phi) = F_{cu} + F_{cl} - 2\phi$  - evaluated at  $x_l^*$  and  $x_u^*$ , i.e.,  $\frac{\partial f}{\partial x}|_{x=x_l^*}$  and  $\frac{\partial f}{\partial x}|_{x=x_u^*}$ , have eigenvalues  $k_i(F_{cu} - F_{cl})(g_h(F_{cl}) - T_{hd}) > 0$  and  $k_i(F_{cl} - F_{cu})(g_h(F_{cu}) - T_{hd}) > 0$ , respectively. Then  $x_l^*$  and  $x_u^*$  are unstable and consequently the points on  $\mathcal{D} \times \partial \mathcal{F}_c$  cannot be asymptotically approached from the interior of the system state-space domain either. Consequently, for any  $x_0 \in \mathcal{D} \times \text{int}(F_c)$ ,  $x(t; x_0) \in \mathcal{D} \times \text{int}(F_c)$ ,  $\forall t \geq 0$ , or equivalently  $(T_{co}, T_{ho})(t) \in \mathcal{D}$  and  $\phi(t) \in \text{int}(\mathcal{F}_c)$ ,  $\forall t \geq 0$ . This and expression (9) prove that  $F_c(\phi, T_{ho})(t) = F_c(\phi(t), T_{ho}(t)) \in \text{int}(\mathcal{F}_c)$ ,  $\forall t \geq 0$ . Now, consider the Jacobian matrix of  $f$  at  $x^*$ , i.e.,  $\frac{\partial f}{\partial x}|_{x^*}$ . Its characteristic polynomial is  $P(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$ , where

$$\begin{aligned} a_2 &\triangleq \left[ \frac{2F_c(\phi, T_{ho})}{M_c} + \frac{2F_h}{M_h} - \frac{2UA}{M_c C_{pc}} \frac{\partial \Delta T_L}{\partial T_{co}} + \frac{2UA}{M_h C_{ph}} \frac{\partial \Delta T_L}{\partial T_{ho}} \right]_{x=x^*} \\ a_1 &\triangleq \left[ \frac{4F_h F_c(\phi, T_{ho})}{M_c M_h} + \frac{4U A F_c(\phi, T_{ho})}{M_c M_h C_{ph}} \frac{\partial \Delta T_L}{\partial T_{ho}} - \frac{4U A F_h}{M_h M_c C_{pc}} \frac{\partial \Delta T_L}{\partial T_{co}} \right. \\ &\quad \left. + \frac{4U A k_p \eta(\phi)(T_{ci} - T_{co})}{M_c M_h C_{ph}} \frac{\partial \Delta T_L}{\partial T_{co}} \right]_{x=x^*} \end{aligned}$$

and  $a_0 \triangleq k_i \bar{a}_0$  with

$$\bar{a}_0 \triangleq \left[ \frac{4U A \eta(\phi)(T_{ci} - T_{co})}{M_c M_h C_{ph}} \frac{\partial \Delta T_L}{\partial T_{co}} \right]_{x=x^*}$$

From these expressions and Lemma 2.2 (see also Remark 3.1), it follows that

$$\begin{aligned} a_2 > b_2 &\triangleq \frac{2F_{cl}}{M_c} + \frac{2F_h}{M_h} > 0 \\ a_1 > b_1 &\triangleq -\frac{4F_h U A}{M_h M_c C_{pc}} \left[ \frac{\partial \Delta T_L}{\partial T_{co}} \right]_{x=x^*} > 0 \\ 0 < \bar{a}_0 < \bar{b}_0 &\triangleq \frac{4U A \eta\left(\frac{F_{cl} + F_{cu}}{2}\right)(T_{ci} - T_{hi})}{M_c M_h C_{ph}} \left[ \frac{\partial \Delta T_L}{\partial T_{co}} \right]_{x=x^*} \end{aligned}$$

where the fact that  $\eta(\phi) \leq \eta\left(\frac{F_{cl} + F_{cu}}{2}\right)$ ,  $\forall \phi \in \mathcal{F}_c$ , has been taken into account. Furthermore, assume that  $k_i$  satisfies

$$k_i \leq \frac{b_1 b_2}{\bar{b}_0} = \frac{8F_h C_{ph}(F_{cl} M_h + F_h M_c)}{M_h M_c C_{pc}(F_{cu} - F_{cl})^2(T_{hi} - T_{ci})} \tag{11}$$

Observe that, under this consideration, it follows that  $a_0 = k_i \bar{a}_0 < k_i \bar{b}_0 \leq b_1 b_2 < a_1 a_2$ , i.e.,  $a_0 < a_1 a_2$  which is a necessary and sufficient condition for the three roots of  $P(\lambda)$  to have negative real part (see, for instance, in [37, Example 6.2]). Thus,  $x^*$  is asymptotically stable. Its attractivity is global on  $\mathcal{D} \times \text{int}(\mathcal{F}_c)$  if  $\{x^*\}$  is the only invariant in  $\mathcal{D} \times \text{int}(\mathcal{F}_c)$ , which is the case for small enough values of  $k_i$ . Indeed, from boundedness of  $\mathcal{D} \times \text{int}(\mathcal{F}_c)$  and its positive invariance with respect to the closed-loop system dynamics, every solution  $x(t; x_0 \in \mathcal{D} \times \text{int}(\mathcal{F}_c))$  has a nonempty, compact, and invariant positive limit set  $\mathcal{L}^+$ , and  $x(t; x_0) \rightarrow \mathcal{L}^+$  as  $t \rightarrow \infty, \forall x_0 \in \mathcal{D} \times \text{int}(\mathcal{F}_c)$  [35, Lemma 4.1]. Then, the global attractivity of  $x^*$  on  $\mathcal{D} \times \text{int}(\mathcal{F}_c)$  is subject to the absence of periodic orbits on  $\mathcal{D} \times \text{int}(\mathcal{F}_c)$  (implying  $\mathcal{L}^+ = \{x^*\}$ ). Sufficiently small values of  $k_i$  render the closed loop a *slowly varying system* [35, §9.6]. Then, the 3rd-order closed-loop dynamics can be approximated by the 2nd-order system in Equation (2) with (quasi) constant  $F_c$ . Since under such representation no closed orbits can take place<sup>2</sup>, the absence of periodic solutions of the closed-loop (3rd-order) system on  $\mathcal{D} \times \text{int}(\mathcal{F}_c)$  is deduced. Thus,  $T_{ho}(t) \rightarrow T_{hd}$  as  $t \rightarrow \infty$ .

**Remark 4.1.** Notice, from the proof of Proposition 4.1, that inequality (11) may be taken as an a priori control gain tuning criterion for  $k_i$ . The right-hand-side expression may be calculated using available system parameter (average) estimations; one may trust that a value of  $k_i$  quite smaller than the calculated bound would satisfy inequality (11). Further, one could in general expect that upper and lower bound reliable estimations of each parameter are available, e.g.,  $C_{ph} \in [C_{phl}, C_{phu}]$ ,  $C_{pc} \in [C_{pcl}, C_{pcu}]$ ,  $M_h \in [M_{hl}, M_{hu}]$ ,  $M_c \in [M_{cl}, M_{cu}]$  and  $F_h \in [F_{hl}, F_{hu}]$  (the inlet temperatures are assumed to be measurable); then by choosing  $k_i \leq \frac{8F_{hl}C_{phl}(F_{cl}M_{hl}+F_{hl}M_{cl})}{M_{hu}M_{cu}C_{pcu}(F_{cu}-F_{cl})^2(T_{hi}-T_{ci})}$ , the satisfaction of inequality (11) is ensured.

**Remark 4.2.** The sufficiently small restriction imposed to  $k_i$ , as well as to  $k_p$  through inequality (7), may be seen as a limitation of the proposed algorithm. However, the sufficient character of the conditions stated through inequality (7) and Remark 4.1 permits the consideration of control gain values higher than the concerned bounds (up to certain limit) without destabilizing the closed loop or saturating the input.

**Remark 4.3.** Observe that the proposed approach does not need to feed back the whole extended state vector. No measurements of  $T_{co}$  are required for its implementation. Furthermore, the exact knowledge of the accurate values of the system parameters is not needed. Such features characterize the proposed algorithm as a simple controller that gives rise to a control signal evolving within its physical limits. This way, undesirable phenomena, such as windup, are avoided.

**Remark 4.4.** Observe that the proposed controller may be equivalently expressed in the following PI form

$$F_c(t) = k_p e_\phi(t) + k_i \int_0^t e_\phi(\tau) d\tau + \phi_0 \tag{12}$$

where  $e_\phi$  is a weighted error variable defined as  $e_\phi \triangleq \eta(\phi)e$ , with  $e \triangleq T_{ho} - T_{hd}$  (the standard error variable) and  $\phi$  is the current value of the integral action, i.e., of (the addition of) the last two terms in the right-hand side of (12), and  $\phi_0$  is the value of  $\phi$  at the initial time  $t = 0$ , i.e.,  $\phi_0 = \phi(0) \in \text{int}(\mathcal{F}_c)$ . Figure 2 shows a graphical representation of the computation procedure giving rise to the controller implementation. Note that the (weight) function  $\eta(\phi)$ , involved in the proposed scheme, states the difference with respect to a conventional PI control law. It is actually thanks to such a non-linear term,  $\eta(\phi)$ ,

<sup>2</sup>This is verified through Bendixon's Criterion (see, for instance, [35, Lemma 2.2]), since  $\frac{\partial \bar{f}_1}{\partial y_1} + \frac{\partial \bar{f}_2}{\partial y_2} = -a_2 < 0, \forall y \in \mathcal{D}$ , as was stated and shown in [1].

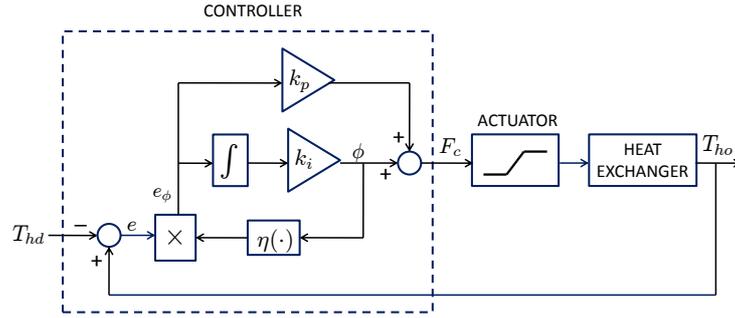


FIGURE 2. Block diagram: controller implementation

that the input flow rate,  $F_c$ , is kept within its physical limits. Indeed, observe that, in view of expression (9) (guaranteed through the satisfaction of inequality (7)), for any  $\phi(0) \in \text{int}(\mathcal{F}_c)$ ,  $F_c$  is not able to go beyond the lower and upper bounds of  $\mathcal{F}_c$  since, at  $F_{cl}$  or  $F_{cu}$ ,  $\phi$  stops evolving. Moreover, due to the unstable nature of the consequent equilibrium points,  $x_l^*$  and  $x_u^*$ , appearing on the boundary of  $\mathcal{D} \times \mathcal{F}_c$ , such limit values of  $F_c$ , i.e.,  $F_{cl}$  and  $F_{cu}$ , cannot even be asymptotically approached. This way,  $F_c$  is guaranteed to evolve within  $\text{int}(\mathcal{F}_c) = (F_{cl}, F_{cu})$ .

**5. Closed Loop Tests.** In order to verify the efficiency of the proposed controller, experiments were carried out on a bench-scale pilot plant consisting of a completely instrumented double-pipe heat exchanger<sup>3</sup>. The plant operates as a water-cooling process – with the hot and cold water respectively flowing through the internal and external tubes – and may be configured in either countercurrent or parallel flow configuration. Engelhard Pyro-Control Pt-100 temperature transmitters measure the temperatures at one extreme of the pipes (the one coinciding with the hot fluid outlet in both flow configurations) while RIY-Moore temperature transmitters measure the temperatures at the other extreme (the one coinciding with the hot fluid inlet in both flow configurations). The current signals produced by the transmitters (in the range of 4 – 20 mA) are fed to current-to-voltage converters, and the resulting voltage signals are then read through a data acquisition card (AT-MIO-16E-1 by National Instruments). Both fluid flow rates are measured *via* Platon flowmeters, and the cold fluid flow rate is regulated through a pneumatic valve (Research Control Valve by Badger Meter, Inc.). A monitoring interface, designed using LabVIEW®, displays the controlled output  $T_{ho}$  and the manipulated variable  $F_c$ .

For the developed experimental tests, the inlet temperatures were kept constant at  $T_{ci} = 30^\circ\text{C}$  and  $T_{hi} = 66^\circ\text{C}$ . The hot fluid flow rate was fixed at  $F_h = 16.7 \times 10^{-3}$  kg/s. The cold fluid flow rate  $F_c$  was made vary between  $F_{cl} = 0.8 \times 10^{-3}$  kg/s and  $F_{cu} = 10.8 \times 10^{-3}$  kg/s, respectively the lower and upper input bounds. On the other hand, after numerous trial-and-error experimental tests, a control gain combination giving rise to good closed loop responses was determined to be:  $k_p = 2.5$  s/( $^\circ\text{C} \cdot \text{kg}$ ) and  $k_i = 0.72$  [1/( $^\circ\text{C} \cdot \text{kg}$ )].

Experiments were carried out in both – countercurrent and parallel– flow configurations. In all the performed tests, the controller gain values were defined as mentioned above, i.e.,  $k_p = 2.5$  s/( $^\circ\text{C} \cdot \text{kg}$ ) and  $k_i = 0.72$  [1/( $^\circ\text{C} \cdot \text{kg}$ )]. The experiments were run departing from *steady-state* conditions corresponding to a constant cold fluid flow rate value of  $F_c = 2 \times 10^{-3}$  kg/s. At  $t = 50$  s, the loop was closed taking  $T_{hd} = 62.5^\circ\text{C}$ ; at  $t = 600$  s, the set-point was changed to  $T_{hd} = 61^\circ\text{C}$ ; finally, at  $t = 1100$  s, the hot fluid flow rate was perturbed by changing its value from  $F_h = 16.7 \times 10^{-3}$  to  $20 \times 10^{-3}$  kg/s.

<sup>3</sup>A study on the calculation of the system parameters and model validation of such an experimental device (where the dynamical model in Equation (2) was validated) has been developed in [38].

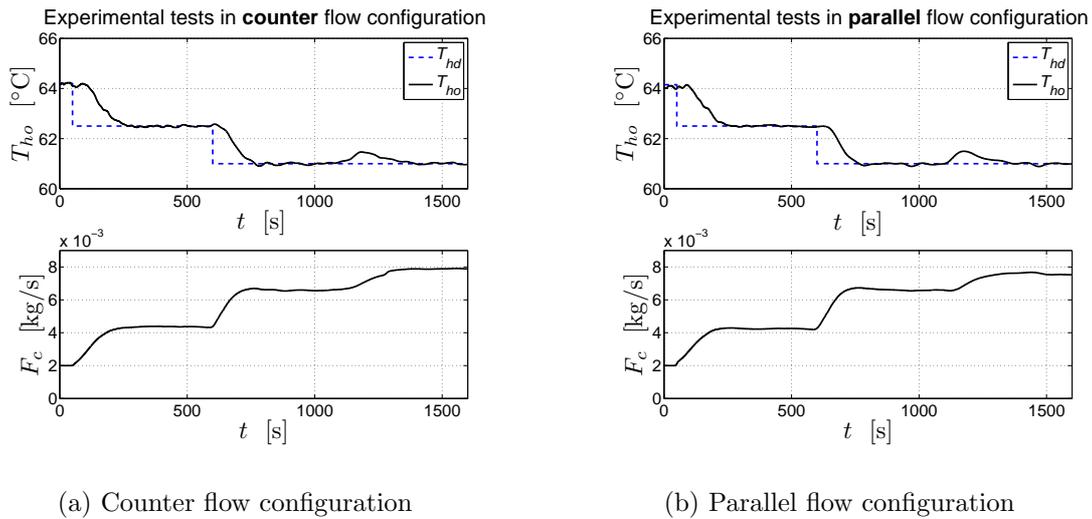


FIGURE 3. Experimental results with the proposed scheme

Figure 3 shows the experimental results for the countercurrent and the parallel flow configurations. Observe that the control objective is achieved in every case; in particular, the outlet desired temperature was reached when the loop was closed and the reference was changed, and successful outlet temperature stabilization was performed under perturbation; moreover such achievements took place avoiding input saturation. Note further that the closed-loop system takes comparable times to recover from a perturbation than from a reference change; for instance, one sees, from the graphs on the figure, that (in both flow configuration cases) a stabilization time of around 200 s takes place for both the reference change produced at  $t = 600$  s and the perturbation arisen at  $t = 1100$  s.

For comparison purposes, the linearizing feedback approach developed in [16] was implemented in counter flow configuration. The reader may corroborate the complexity of the control expression developed in [16] with respect to the simplicity of the algorithm in Proposition 4.1. The controller parameter values were tuned as suggested in [16]. In view of the slow closed-loop responses produced by this controller (as will be seen and commented below), two tests were performed. The first test departed from the same steady-state initial conditions of the previous experiments, i.e., those corresponding to a cold fluid flow rate constant value of  $F_c = 2 \times 10^{-3}$  kg/s; at  $t = 50$  s, the loop was closed taking  $T_{hd} = 62.5$  °C; afterwards, at  $t = 2000$  s, the reference was changed to  $T_{hd} = 61$  °C. With the system in closed loop, the second test departed from the steady-state conditions produced at the end of the first test; at  $t = 300$  s, the hot fluid flow rate was perturbed by changing its value from  $F_h = 16.7 \times 10^{-3}$  to  $20 \times 10^{-3}$  kg/s.

Figure 4 shows the closed-loop outlet temperature response and control signal arisen with such a linearizing feedback scheme at both performed tests: the results of the first test (reference change) are shown in Figure 4(a) while those of the second test (perturbation rejection) are presented in Figure 4(b). Note that notoriously longer stabilization times take place compared with those previously observed with the proposed scheme. Indeed, note from the graphs on the figures that while a regulation time of about 950 s took place when the loop was closed (at  $t = 50$  s during the first test), the system took more than 2000 s to get stabilized from the reference change (at  $t = 2000$  s during the first test) and more than 1500 s to recover from the perturbation (arisen at  $t = 300$  s during the second test). Moreover, responses with overshoot are observed during the first test,

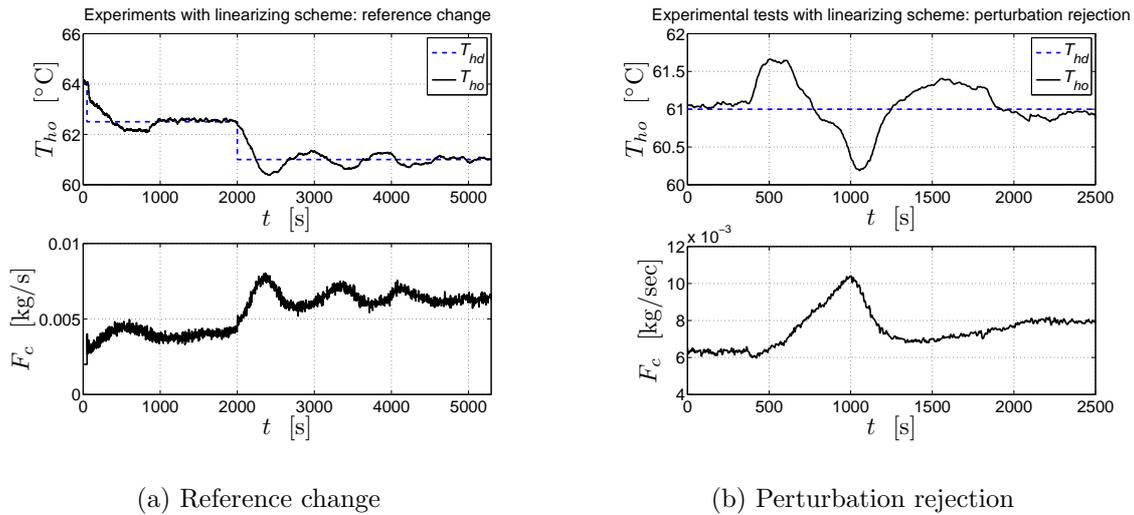


FIGURE 4. Experimental results with the linearizing feedback scheme

and oscillating convergence takes place after the reference was changed (during the first test) and when the perturbation was produced (during the second test). Furthermore, observe that the resulting control signals are noisy. This may be a consequence of the high dependence of the linearizing feedback controller on the system states (entailing a high degree of measurement noise corruption).

Further tests were performed through computer simulation using model (2) in counter flow configuration ( $\alpha = 1$ ). The system parameter values were taken from [15] as follows:  $U = 1050 \text{ J}/(\text{°C m}^2 \text{ s})$ ,  $A = 0.014 \text{ m}^2$ ,  $M_c = 0.134 \text{ kg}$ ,  $M_h = 0.015 \text{ kg}$ ,  $C_{pc} = 4174 \text{ J}/(\text{°C kg})$  and  $C_{ph} = 4179 \text{ J}/(\text{°C kg})$ . The rest of the involved values were taken as in the previously considered experimental setup, i.e., of  $T_{ci} = 30\text{°C}$ ,  $T_{hi} = 66\text{°C}$ ,  $F_h = 16.7 \times 10^{-3} \text{ kg/s}$ ,  $F_{cl} = 0.8 \times 10^{-3} \text{ kg/s}$  and  $F_{cu} = 10.8 \times 10^{-3} \text{ kg/s}$ . The proposed approach was compared with the fuzzy PI controller recently developed in [26]. The control parameters on the fuzzy PI algorithm were taken such that the integral of the square of the error (among the output,  $T_{ho}$ , and the desired value,  $T_{hd}$ ) was minimized; the resulting values (using the notation used in [26], and expressed in the appropriate units which are omitted) were  $Ge = 5 \times 10^{-6}$ ,  $Gde = 4 \times 10^{-6}$  and  $Gu = 2000$ . As for the proposed approach, the control gain values were fixed as  $k_p = 2.7 \text{ s}/(\text{°C kg})$  and  $k_i = 3.8 [1/(\text{°C kg})]$ . A test similar to the one performed through the experimental setup was implemented for both simulated controllers: same departing conditions, reference changes and perturbation. The results are shown in Figure 5. One sees that both controllers achieve the regulation objective. Moreover, the closed-loop variable responses obtained with the fuzzy PI are faster than those that took place with the proposed controller. However, observe that the fuzzy PI scheme produces control signals that force the actuator to react *sweeping* large flow rate ranges in very short periods of time. More importantly, input saturation could not be avoided, which is more significantly seen when the hot fluid flow rate value was perturbed at  $t = 1100 \text{ s}$ . On the contrary, the proposed scheme achieves acceptable stabilization times through smooth signals avoiding input saturation.

**6. Conclusions.** In this work, temperature regulation of double-pipe heat exchangers was considered. The positive (unidirectional) and bounded nature of the flow rate taken as input variable was taken into account in the analysis. This has been proved to be important in several senses. For instance, in view of such input constraints – inherent to

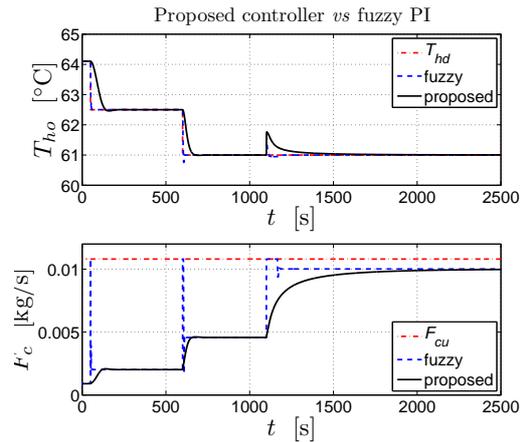


FIGURE 5. Proposed controller vs fuzzy PI

real actuators typically used for the control of heat exchangers, the analysis proved that the *reachable steady-state space* is restricted to a subset of the system state-space domain. Any intention to stabilize the process towards a *desired* outlet temperature outside such a reachable steady-state space would fail, and would give rise to an unexpected phenomenon, such as a steady-state error. Furthermore, a bounded positive PI-type regulation scheme that avoids input saturation was proposed. It turns out to be a simple algorithm that does not need to feed back the whole closed-loop state vector, does not depend on the exact knowledge of the system parameters, and whose stabilization character is global in the closed-loop system state-space domain. Through such a regulation scheme, unexpected or undesirable phenomena, such as windup, are avoided. Moreover, it was shown to be applicable in both flow configuration cases, i.e., to countercurrent and parallel heat exchangers. Experimental and simulation tests corroborated the theoretical developments and showed good results compared to the implementation of a linearizing feedback scheme previously proposed in the literature and to a recently published fuzzy PI controller.

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