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EXPONENTIAL STABILITY ANALYSIS AND STABILIZATION OF DISCRETE-TIME NONLINEAR SWITCHED SYSTEMS WITH TIME DELAYS

Guojiang Zhang[1], Chunsong Han[2], Yu Guan[3] and Ligang Wu[2,*]

[1]Control and Simulation Center
[2]Space Control and Inertial Technology Research Center
[3]Research Center of Satellite Technology
Harbin Institute of Technology
No. 92, West Da-Zhi Street, Harbin 150001, P. R. China
[*]Corresponding author: ligangwu@hit.edu.cn

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Abstract. *This note considers the problems of stability and stabilization for discrete-time switched nonlinear systems with time-varying delay. The nonlinearity is assumed to satisfy a special constraint. The purpose of the robust stability problem is to give conditions such that the discrete-time switched nonlinear delay system is exponentially stable, while the purpose of stabilization is to design a state feedback control law such that the resulting closed-loop system is exponentially stable. By applying the average dwell time approach together with the piecewise Lyapunov function technique, also by constructing a proper Lyapunov-Krasovskii functional and employing the free-weighting matrix method, some delay-dependent stability conditions are proposed. A strict linear matrix inequality (LMI) design approach is developed. An explicit expression for the desired state feedback control law is also given. Finally, two numerical examples are provided to demonstrate the application of the proposed methods.*

Keywords: Exponential stability, Discrete-time, Switched systems, Average dwell time, Piecewise Lyapunov function

1. **Introduction.** Switched systems represent an important class of hybrid systems, and they consist of a family of subsystems and a switching rule specifying which subsystem will be activated along the system trajectory at a certain time instant [9, 10, 21, 23]. There are two important factors which can be seen as the main motivation for studying such hybrid systems, that is, first, from a practical point of view, switching among different system structures is an essential feature of many real-world systems such as chemical processes, transportation systems, computer controlled systems and communication industries; second, from a control point of view, multi-controller switching provides an effective mechanism to cope with highly complex systems and/or systems with large uncertainties. For example, many intelligent control strategies are designed based on the idea of switching controllers to improve the system performances.

Discrete time switched systems have received increasing attention in recent years, and a large amount of results have been reported. In particular, Daafouz et al. investigated the stability analysis and control synthesis problems for switched systems by using a switched Lyapunov function approach [1]; Du et al. considered the generalized $\mathscr{H}_2$ output feedback controller design for uncertain discrete-time switched systems via switched Lyapunov functions; Geromel and Colanri studied the stability and stabilization problems for discrete time switched systems [5]; Ji et al. addressed the quadratic stabilization problem for uncertain discrete-time switched systems via output feedback [8]; Saif et al. considered a parameterized delay-dependent approach to the control of switched discrete-time systems with time-delay [11]; Sun et al. investigated the delay-dependent robust stability and stabilization problems for discrete-time switched systems with mode-dependent time-varying delays [13]; Wang and Zhao concerned with the exponential stability analysis for discrete-time switched linear systems with time-delay [14]. On the other hand, in the past decades, considerable attention has been paid to the problems of robust stability analysis and synthesis for discrete-time systems with time-delay, see [3, 4, 15, 19, 20] and references therein.

Stability analysis is an important problem in system science and control engineering. Recently, the dwell time approach is applied widely to deal with switched systems, see, for example, [6, 7, 12, 16, 17, 18, 22]. Given a positive constant τ_d called 'dwell time' and that let $\mathscr{S}(\tau_d)$ denote the set of all switching signals with interval between consecutive discontinuities no smaller than τ_d, it has been shown that one can pick τ_d sufficiently large such that the switched system considered is exponentially stable for any switching signal belonging to $\mathscr{S}(\tau_d)$. Hespanha and Morse investigated the stability of switched systems with average dwell time approach [6]; Ishii and Francis consider the stabilization problem for a linear system by switching control with dwell time [7]; Sun et al. addressed the stability and $\mathscr{L}_2$-gain analysis for switched delay systems by the average dwell time approach [12]; Wu et al. addressed the sliding mode control, guaranteed cost control and model reduction problems for continuous time switched systems with time delays [16, 17, 18].

In this paper, we are interested in investigating the stability analysis and stabilization problems for discrete-time switched nonlinear systems with time delays. The purpose of the stability problem is to develop conditions such that the discrete-time switched nonlinear delay system is exponentially stable. To reduce the overdesign in the quadratic framework, this paper also proposes a parameter-dependent analysis procedure, which is much less conservative than the quadratic approach. By using the average dwell time approach and the piecewise Lyapunov function technique, a delay-dependent sufficient condition is proposed to guarantee the exponential stability of the considered system. Here, to reduce the conservatism of the delay-dependent condition, we shall introduce some slack matrix variables to seek the relationship between the Newton-Leibniz formula, instead of applying model transformation. Similarly, the purpose of stabilization is the design of memoryless state feedback control laws such that the resultant closed-loop system is exponentially stable. A strict LMI design approach is proposed and an explicit expression for the desired state feedback control law is given. Finally, two numerical examples are provided to illustrate the effectiveness of the proposed theory.

The rest of this paper is organized as follows. The stability analysis and stabilization problems for discrete-time switched delay systems is formulated in Sections 2. Section 3 presents our main results. Numerical examples are given in Section 4 and we conclude this paper in Section 5.

Notations. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $P > 0$ means that P is real symmetric and positive definite; I and 0 represent an identity matrix and a zero matrix, respectively; and $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix. In symmetric block matrices or long matrix expressions, we use a star $(\star)$ to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. **System Description and Preliminaries.** Consider a discrete-time nonlinear switched system with time-delays, which can be described by the following dynamical equation:

$$\begin{aligned} x(k+1) &= A(\alpha_k)x(k) + A_d(\alpha_k)x(k-d(k)) + A_\tau(\alpha_k)f\left(x(k-\tau)\right) + B(\alpha_k)u(k), \\ x(\theta) &= \phi(\theta), \quad -\max\{\tau, d_2\} \le \theta \le 0, \end{aligned} \tag{1}$$

for $k = 1, 2, \ldots$, where $x(k) \in \mathbb{R}^n$ is the state vector; $u(k) \in \mathbb{R}^m$ represents the control input; $\phi(\theta)$, $-\max\{\tau, d_2\} \le \theta \le 0$ are the initial conditions; $\{(A(\alpha_k), A_d(\alpha_k), A_\tau(\alpha_k), B(\alpha_k)) : \ \alpha_k \in \mathcal{N}\}$ is a family of matrices parameterized by an index set $\mathcal{N} = \{1, 2, \ldots, N\}$ and $\alpha_k : \mathbb{Z}^+ \to \mathcal{N}$ is a piecewise constant function of time, called a switching signal, which takes its values in the finite set $\mathcal{N}$. At an arbitrary discrete time k, the value of α_k, denoted by α for simplicity, might depend on k or $x(k)$, or both, or may be generated by any other hybrid scheme. We assume that the sequence of subsystems in switching signal α_k is unknown *a priori*, but its instantaneous value is available in real time. For the switching time sequence $k_0 < k_1 < k_2 < \cdots$ of switching signal α, the holding time between $[k_l, k_{l+1})$ is called the dwell time of the currently engaged subsystem, where $l \in \mathcal{N}$. The delay $d(k)$ satisfying $1 \le d_1 \le d(k) \le d_2$, where d_1 and d_2 are constant positive scalars representing the minimum and maximum delays, respectively. In addition, $f(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is nonlinear function, which satisfies the following assumption.

Assumption 2.1. *For the nonlinear function $f(\cdot)$, there exist matrices Π_1 and Π_2 such that*

$$\left(f(x) - \Pi_1 x\right)^T \left(f(x) - \Pi_2 x\right) \le 0, \quad x \in \mathbb{R}^n. \tag{2}$$

Remark 2.1. *For each possible value $\alpha_k = i$, $i \in \mathcal{N}$, we will denote the system matrices associated with mode i by $A(i) = A(\alpha_k), A_d(i) = A_d(\alpha_k), A_\tau(i) = A_\tau(\alpha_k)$ and $B(i) = B(\alpha_k)$, where $A(i)$, $A_d(i)$, $A_\tau(i)$ and $B(i)$ are constant matrices. Corresponding to the switching signal α, we have the switching sequence $\{(i_0, k_0), (i_1, k_1), \ldots, (i_l, k_l), \ldots, |i_l \in \mathcal{N},\ l = 0, 1, \ldots\}$ with $k_0 = 0$, which means that the i_lth subsystem is activated when $k \in [k_l, k_{l+1})$.*

In this paper, we design a stabilization controller with the following general structure:

$$u(k) = K\left(\alpha_k\right)x(k), \tag{3}$$

where $K\left(\alpha_k\right) \in \mathbb{R}^{m\times n}$ are parameter matrices switching with the same switching signal as the original system.

Substituting $u(k)$ in (3) into the system (1), we obtain the closed-loop stabilization system as

$$\begin{aligned} x(k+1) &= \hat{A}(\alpha_k)x(k) + A_d(\alpha_k)x(k-d(k)) + A_\tau(\alpha_k)f\left(x(k-\tau)\right), \\ x(\theta) &= \phi(\theta), \quad -\max\{\tau, d_2\} \le \theta \le 0. \end{aligned} \tag{4}$$

where $\hat{A}(\alpha_k) = A(\alpha_k) + B(\alpha_k)K\left(\alpha_k\right)$.

The following definitions and lemma are introduced, which will play key roles in deriving our main results.

Definition 2.1. *The discrete-time switched time-delay system in (1) with $u(k) = 0$ is said to be exponentially stable under $\alpha(k)$ if the solution $x(k)$ satisfies*

$$\|x(k)\| \le \eta\rho^{(k-k_0)}\left\|x(k_0)\right\|_C, \quad \forall k \ge k_0,$$

for constants $\eta \ge 1$ and $0 < \rho < 1$, and

$$\left\|x(k_0)\right\|_C \triangleq \underbrace{\left\{\left\|x(k+\theta)\right\|, \left\|\xi(k+\theta)\right\|, \left\|f(\xi(k+\theta))\right\|\right\}}_{\sup_{-\max\{\tau, d_2\} \le \theta \le 0}},$$

where $\xi(\theta) \triangleq x(\theta+1) - x(\theta)$.

3. **Main Results.** First, we will use the piecewise Lyapunov technique and the average dwell time approach to propose a sufficient condition for the exponential stability of the discrete-time switched time-delay system in (1) with $u(k) = 0$.

Theorem 3.1. *Given a constant* $0 < \beta < 1$*, supposed that there exist matrices* $P(i) > 0$*,* $Q(i) > 0$*,* $R(i) > 0$*,* $Z(i) > 0$*,* $S_1(i) > 0$*,* $S_2(i) > 0$ *and matrices* $L(i)$*,* $M(i)$*,* $N(i)$ *such that for* $i \in \mathcal{N}$*,*

$$\begin{bmatrix} \beta^{-(d_2+1)}\Phi(i) & d_2 L(i) & (d_2-d_1)\, M(i) & d_2 N(i) \\ \star & -d_2 S_1(i) & 0 & 0 \\ \star & \star & -(d_2-d_1)\, S_1(i) & 0 \\ \star & \star & \star & -d_2 S_2(i) \end{bmatrix} < 0, \tag{5}$$

where

$$\begin{aligned} \Phi(i) \triangleq & \begin{bmatrix} \Phi_{11}(i) & 0 & 0 & \dfrac{\Pi_1^T + \Pi_2^T}{2} & 0 \\ \star & -\beta^{d_2+1} Q(i) & 0 & 0 & 0 \\ \star & \star & -\beta^{d_2+1} R(i) & 0 & 0 \\ \star & \star & \star & \beta Z(i) - I & 0 \\ \star & \star & \star & \star & -\beta^{\tau+1} Z(i) \end{bmatrix} \\ & + \begin{bmatrix} A^T(i) \\ A_d^T(i) \\ 0 \\ 0 \\ A_\tau^T(i) \end{bmatrix} P(i) \begin{bmatrix} A^T(i) \\ A_d^T(i) \\ 0 \\ 0 \\ A_\tau^T(i) \end{bmatrix}^T \\ & + \begin{bmatrix} A^T(i) - I \\ A_d^T(i) \\ 0 \\ 0 \\ A_\tau^T(i) \end{bmatrix} d_2\beta\left(S_1(i) + S_2(i)\right) \begin{bmatrix} A^T(i) - I \\ A_d^T(i) \\ 0 \\ 0 \\ A_\tau^T(i) \end{bmatrix}^T \\ & + 2\beta^{d_2+1} \left\{ L(i) \begin{bmatrix} I \\ -I \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + M(i) \begin{bmatrix} 0 \\ I \\ -I \\ 0 \\ 0 \end{bmatrix}^T + N(i) \begin{bmatrix} I \\ 0 \\ I \\ 0 \\ 0 \end{bmatrix}^T \right\}, \end{aligned}$$

$$\Phi_{11}(i) \triangleq -\beta P(i) + \beta R(i) + \beta(d_2 - d_1 + 1) Q(i) - \frac{\Pi_1^T \Pi_2 + \Pi_2^T \Pi_1}{2}. \tag{6}$$

Then, the discrete-time switched time-delay system in (1) with $u(k) = 0$ *is exponentially stable for any switching signal with average dwell time satisfying* $T_a > T_a^* = \operatorname{ceil}\left(-\dfrac{\ln\mu}{\ln\beta}\right)$*, where* $\mu \geq 1$ *satisfies*

$$\begin{aligned} & P(i) \leq \mu P(j),\ Q(i) \leq \mu Q(j),\ R(i) \leq \mu R(j), \\ & Z(i) \leq \mu Z(j),\ S_1(i) \leq \mu S_1(j),\ S_2(i) \leq \mu S_2(j),\ \forall i, j \in \mathcal{N}. \end{aligned} \tag{7}$$

Proof: Choose a Lyapunov function candidate of the form:

$$\begin{cases} V(x_k,\alpha_k) \triangleq \sum\limits_{i=1}^{6} V_i(x_k,\alpha_k), \\ V_1(x_k,\alpha_k) \triangleq x^T(k)P(\alpha_k)x(k), \\ V_2(x_k,\alpha_k) \triangleq \sum\limits_{l=k-d(k)}^{k-1} \beta^{k-l}x^T(l)Q(\alpha_k)x(l), \\ V_3(x_k,\alpha_k) \triangleq \sum\limits_{l=k-d_2}^{k-1} \beta^{k-l}x^T(l)R(\alpha_k)x(l), \\ V_4(x_k,\alpha_k) \triangleq \sum\limits_{s=-d_2+1}^{-d_1} \sum\limits_{l=k+s}^{k-1} \beta^{k-l}x^T(l)Q(\alpha_k)x(l), \\ V_5(x_k,\alpha_k) \triangleq \sum\limits_{s=-d_2}^{-1} \sum\limits_{l=k+s}^{k-1} \beta^{k-l}\xi^T(l)\left(S_1(\alpha_k)+S_2(\alpha_k)\right)\xi(l), \\ V_6(x_k,\alpha_k) \triangleq \sum\limits_{l=k-\tau}^{k-1} \beta^{k-l}f^T(x(l))Z(\alpha_k)f(x(l)), \end{cases} \tag{8}$$

where $\xi(k) \triangleq x(k+1)-x(k)$, and $P(\alpha_k)>0$, $Q(\alpha_k)>0$, $R(\alpha_k)>0$, $Z(\alpha_k)>0$, $S_1(\alpha_k)>0$ and $S_2(\alpha_k)>0$ are real matrices to be determined.

For $k\in[k_l,k_{l+1})$, as in the previous section, we define $\Delta V_j(x_k,\alpha_k) \triangleq V_j(x_{k+1},\alpha_k) - V_j(x_k,\alpha_k)$, $j=1,2,3,4,5,6$, thus $\Delta V(x_k,\alpha_k)=\sum_{i=1}^{6}\Delta V_i(x_k,\alpha_k)$ with

$$\Delta V_1(x_k,\alpha_k) = x^T(k+1)P(\alpha_k)x(k+1) - x^T(k)P(\alpha_k)x(k), \tag{9}$$

$$\begin{aligned}\Delta V_2(x_k,\alpha_k) \le\; & -(1-\beta)\sum_{l=k-d(k)}^{k-1}\beta^{k-l}x^T(l)Q(\alpha_k)x(l) + \sum_{l=k+1-d_2}^{k-d_1}\beta^{k+1-l}x^T(l)Q(\alpha_k)x(l) \\ & +\beta x^T(k)Q(\alpha_k)x(k) - \beta^{d_2+1}x^T(k-d(k))Q(\alpha_k)x(k-d(k)),\end{aligned} \tag{10}$$

$$\begin{aligned}\Delta V_3(x_k,\alpha_k) =\; & -(1-\beta)\sum_{l=k-d_2}^{k-1}\beta^{k-l}x^T(l)R(\alpha_k)x(l) \\ & +\beta x^T(k)R(\alpha_k)x(k) - \beta^{d_2+1}x^T(k-d_2)R(\alpha_k)x(k-d_2),\end{aligned} \tag{11}$$

$$\begin{aligned}\Delta V_4(x_k,\alpha_k) =\; & -(1-\beta)\sum_{s=-d_2+1}^{-d_1}\sum_{l=k+s}^{k-1}\beta^{k-l}x^T(l)Q(\alpha_k)x(l) \\ & +\beta(d_2-d_1)x^T(k)Q(\alpha_k)x(k) - \sum_{l=k+1-d_2}^{k-d_1}\beta^{k+1-l}x^T(l)Q(\alpha_k)x(l),\end{aligned} \tag{12}$$

$$\begin{aligned}\Delta V_5(x_k,\alpha_k) \le\; & -(1-\beta)\sum_{s=-d_2}^{-1}\sum_{l=k+s}^{k-1}\beta^{k-l}\xi^T(l)\left(S_1(\alpha_k)+S_2(\alpha_k)\right)\xi(l) \\ & +d_2\beta\xi^T(k)\left(S_1(\alpha_k)+S_2(\alpha_k)\right)\xi(k) - \beta^{d_2+1}\sum_{l=k-d_2}^{k-1}\xi^T(l)S_2(\alpha_k)\xi(l) \\ & -\beta^{d_2+1}\sum_{l=k-d(k)}^{k-1}\xi^T(l)S_1(\alpha_k)\xi(l) - \beta^{d_2+1}\sum_{l=k-d_2}^{k-d(k)-1}\xi^T(l)S_1(\alpha_k)\xi(l),\end{aligned} \tag{13}$$

$$\Delta V_6(x_k,\alpha_k) \le -(1-\beta)\sum_{l=k-\tau}^{k-1}\beta^{k-l}f^T(x(l))Z(\alpha_k)f(x(l)) + \beta f^T(x(k))Z(\alpha_k)f(x(k))$$

$$-\beta^{\tau+1}f^T(x(k-\tau))Z(\alpha_k)f(x(k-\tau)). \tag{14}$$

Moreover, for $\zeta(k) \triangleq \begin{bmatrix} x^T(k) & x^T(k-d(k)) & x^T(k-d_2) & f^T(x(k)) & f^T(x(k-\tau)) \end{bmatrix}^T$ and any appropriately dimensioned matrices $L(\alpha_k)$, $M(\alpha_k)$ and $N(\alpha_k)$, $\alpha_k \in \mathcal{N}$, the following equations are true:

$$2\beta^{d_2+1}\zeta^T(k)L(\alpha_k)\left[x(k)-x(k-d(k))-\sum_{l=k-d(k)}^{k-1}\xi(l)\right]=0, \tag{15}$$

$$2\beta^{d_2+1}\zeta^T(k)M(\alpha_k)\left[x(k-d(k))-x(k-d_2)-\sum_{l=k-d_2}^{k-d(k)-1}\xi(l)\right]=0, \tag{16}$$

$$2\beta^{d_2+1}\zeta^T(k)N(\alpha_k)\left[x(k)-x(k-d_2)-\sum_{l=k-d_2}^{k-1}\xi(l)\right]=0. \tag{17}$$

Assumption 2.1 gives

$$\begin{bmatrix} x^T(k) & f^T(x(k)) \end{bmatrix}\begin{bmatrix} \dfrac{\Pi_1^T\Pi_2+\Pi_2^T\Pi_1}{2} & -\dfrac{\Pi_1^T+\Pi_2^T}{2} \\ -\dfrac{\Pi_1+\Pi_2}{2} & I \end{bmatrix}\begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix} \le 0. \tag{18}$$

Consider (9)-(18), we have

$$\begin{aligned} &\Delta V(x_k,\alpha_k)+(1-\beta)V(x_k,\alpha_k) \\ \le\ &\zeta^T(k)\left\{\Phi(\alpha_k)+\beta^{d_2+1}\left[d_2L(\alpha_k)S_1^{-1}(\alpha_k)L^T(\alpha_k)\right.\right. \\ &\left.\left.+\ (d_2-d_1)\,M(\alpha_k)S_1^{-1}(\alpha_k)M^T(\alpha_k)+d_2N(\alpha_k)S_2^{-1}(\alpha_k)N^T(\alpha_k)\right]\right\}\zeta(k) \\ &-\beta^{d_2+1}\left[\sum_{l=k-d(k)}^{k-1}\Gamma_1^TS_1^{-1}(\alpha_k)\Gamma_1+\sum_{l=k-d_2}^{k-d(k)-1}\Gamma_2^TS_1^{-1}(\alpha_k)\Gamma_2+\sum_{l=k-d_2}^{k-1}\Gamma_3^TS_2^{-1}(\alpha_k)\Gamma_3\right], \end{aligned} \tag{19}$$

where $\Phi(\alpha_k)$ is defined in (6) and

$$\begin{aligned} &\Gamma_1 \triangleq \left[S_1(\alpha_k)\xi(l)+L^T(\alpha_k)\zeta(k)\right], \quad \Gamma_2 \triangleq \left[S_1(\alpha_k)\xi(l)+M^T(\alpha_k)\zeta(k)\right], \\ &\Gamma_3 \triangleq \left[S_2(\alpha_k)\xi(l)+N^T(\alpha_k)\zeta(k)\right]. \end{aligned}$$

Moreover, from (5), it follows that

$$\begin{aligned} \Phi(\alpha_k)+\beta^{d_2+1}\left[d_2L(\alpha_k)S_1^{-1}(\alpha_k)L^T(\alpha_k)+(d_2-d_1)\,M(\alpha_k)S_1^{-1}(\alpha_k)M^T(\alpha_k)\right. \\ \left.+d_2N(\alpha_k)S_2^{-1}(\alpha_k)N^T(\alpha_k)\right]\ <\ 0. \end{aligned}$$

Then one can easily achieve

$$\Delta V(x_k,\alpha_k)+(1-\beta)V(x_k,\alpha_k)<0, \quad \forall k\in[k_l,k_{l+1}). \tag{20}$$

Now, for an arbitrary piecewise constant switching signal α_k, and for any $k>0$, we let $k_0<k_1<\cdots<k_l<\cdots$, $l=1,\ldots$, denote the switching points of α_k over the interval $(0,k)$. As mentioned earlier, the i_lth subsystem is activated when $k\in[k_l,k_{l+1})$. Therefore, for $k\in[k_l,k_{l+1})$, it holds from (20) that

$$V(x_k,\alpha_k)<\beta^{k-k_l}V(x_{k_l},\alpha_{k_l}). \tag{21}$$

Using (7) and (8), at switching instant t_k, we have

$$V(x_{k_l},\alpha_{k_l})\le\mu V(x_{k_l},\alpha_{k_{l-1}}). \tag{22}$$

Therefore, it follows from (21) and (22) and the relationship $\vartheta = N_\alpha(0,k) \le (k-k_0)/T_a$ that

$$\begin{aligned} V(x_k,\alpha_k) &\le \beta^{k-k_l}\mu V(x_{k_l},\alpha_{k_{l-1}}) \\ &\le \cdots \\ &\le \beta^{(k-k_0)}\mu^{\vartheta} V(x_{k_0},\alpha_{k_0}) \\ &\le (\beta\mu^{1/T_a})^{(k-k_0)} V(x_{k_0},\alpha_{k_0}). \end{aligned} \tag{23}$$

Notice from (8) that there exist two positive constants a and b ($a \le b$) such that

$$V(x_k,\alpha_k) \ge a\,\|x(k)\|^2, \quad V(x_{k_0},\alpha_{k_0}) \le b\,\|x(k_0)\|_C^2. \tag{24}$$

Combining (23) and (24) yields

$$\|x(k)\|^2 \le \frac{1}{a}V(x_k,\alpha_k) \le \frac{b}{a}(\beta\mu^{1/T_a})^{(k-k_0)}\,\|x(k_0)\|_C^2. \tag{25}$$

Furthermore, letting $\rho \triangleq \sqrt{\beta\mu^{1/T_a}}$, it follows that

$$\|x(k)\| \le \sqrt{\frac{b}{a}}\rho^{(k-k_0)}\,\|x(k_0)\|_C. \tag{26}$$

By Definition 2.1, we know that if $0 < \rho < 1$, that is, $T_a > T_a^* = \operatorname{ceil}\left(-\dfrac{\ln\mu}{\ln\beta}\right)$, the discrete-time switched time-delay system in (1) with $u(k) = 0$ is exponentially stable, where function ceil(h) represents rounding real number h to the nearest integer greater than or equal to h. The proof is completed.

Remark 3.1. *In Theorem 3.1, we propose a sufficient condition for the exponential stability condition for the considered the discrete-time switched time-delay system in (1) with* $u(k) = 0$*. Here,* β *plays a key role in controlling the low bound of the average dwell time, which can be seen from* $T_a > T_a^* = \operatorname{ceil}\left(-\dfrac{\ln\mu}{\ln\beta}\right)$*, specifically, if* β *is given a smaller value, the low bound of the average dwell time becomes smaller with a fixed* μ*, which may result in the unstability of the system.*

Remark 3.2. *Note that when* $\mu = 1$ *in* $T_a > T_a^* = \operatorname{ceil}\left(-\dfrac{\ln\mu}{\ln\beta}\right)$ *we have* $T_a > T_a^* = 0$*, which means that the switching signal* $\alpha(k)$ *can be arbitrary. In this case, (7) turns out to be* $P(i) = P(j) = P$*,* $Q(i) = Q(j) = P$*,* $R(i) = R(j) = P$*,* $Z(i) = Z(j) = Z$*,* $S_1(i) = S_1(j) = S_1$*,* $S_2(i) = S_2(j) = S_2$*,* $\forall i,j \in \mathcal{N}$*, and the proposed approach becomes quadratic one thus conservative. In this case, the system in (1) with* $u(k) = 0$ *turns out to be*

$$\begin{aligned} x(k+1) &= Ax(k) + A_d x(k-d(k)) + A_\tau f\left(x(k-\tau)\right), \\ x(\theta) &= \phi(\theta), \quad -\max\{\tau, d_2\} \le \theta \le 0. \end{aligned} \tag{27}$$

And we have the following result for the system in (27).

Corollary 3.1. *The discrete-time time-delay system in (27) is asymptotically stable if there exist matrices* $P > 0$*,* $Q > 0$*,* $R > 0$*,* $Z > 0$*,* $S_1 > 0$*,* $S_2 > 0$*, and matrices* L*,* M*,* N *such that*

$$\begin{bmatrix} \Psi & d_2 L & (d_2-d_1)\,M & d_2 N \\ \star & -d_2 S_1 & 0 & 0 \\ \star & \star & -(d_2-d_1)\,S_1 & 0 \\ \star & \star & \star & -d_2 S_2 \end{bmatrix} < 0,$$

where

$$\Psi \triangleq \begin{bmatrix} \Psi_{11} & 0 & 0 & \frac{\Pi_1^T+\Pi_2^T}{2} & 0 \\ \star & -Q & 0 & 0 & 0 \\ \star & \star & -R & 0 & 0 \\ \star & \star & \star & Z-I & 0 \\ \star & \star & \star & \star & -Z \end{bmatrix} + \begin{bmatrix} A^T \\ A_d^T \\ 0 \\ 0 \\ A_\tau^T \end{bmatrix} P \begin{bmatrix} A^T \\ A_d^T \\ 0 \\ 0 \\ A_\tau^T \end{bmatrix}^T$$
$$+ \begin{bmatrix} A^T-I \\ A_d^T \\ 0 \\ 0 \\ A_\tau^T \end{bmatrix} d_2\left(S_1+S_2\right) \begin{bmatrix} A^T-I \\ A_d^T \\ 0 \\ 0 \\ A_\tau^T \end{bmatrix}^T$$
$$+2\left\{ L \begin{bmatrix} I \\ -I \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + M \begin{bmatrix} 0 \\ I \\ -I \\ 0 \\ 0 \end{bmatrix}^T + N \begin{bmatrix} I \\ 0 \\ I \\ 0 \\ 0 \end{bmatrix}^T \right\},$$

and $\Psi_{11} \triangleq -P + R + (d_2 - d_1 + 1)Q - \dfrac{\Pi_1^T\Pi_2 + \Pi_2^T\Pi_1}{2}$.

To prove the above result, the following Lyapunov function is chosen:

$$\begin{aligned} \hat{V}(x_k) \triangleq\ & x^T(k)Px(k) + \sum_{l=k-d(k)}^{k-1} x^T(l)Qx(l) + \sum_{l=k-d_2}^{k-1} x^T(l)Rx(l) \\ & + \sum_{s=-d_2+1}^{-d_1}\sum_{l=k+s}^{k-1} x^T(l)Qx(l) + \sum_{s=-d_2}^{-1}\sum_{l=k+s}^{k-1} \xi^T(l)\left(S_1+S_2\right)\xi(l) \\ & + \sum_{l=k-\tau}^{k-1} f^T(x(l))Zf(x(l)), \end{aligned}$$

where $\xi(k) \triangleq x(k+1) - x(k)$, and $P > 0$, $Q > 0$, $R > 0$, $Z > 0$, $S_1 > 0$ and $S_2 > 0$ are real matrices to be determined. The remainder processes can be followed by the same lines of the proof of Theorem 3.1, and we omit the details.

Notice that there exist two product terms between the Lyapunov matrices (that is, $P(i)$ and $S_1(i) + S_2(i)$) and the system dynamic matrix $A(i)$ in the LMI condition proposed in Theorem 3.1, which will bring some difficulties in the solution of the stabilization control synthesis problem. In the following, a subsequent result is given in order to facilitate the control design procedure.

Corollary 3.2. *Given a constant* $0 < \beta < 1$*, supposed that there exist matrices* $P(i) > 0$*,* $Q(i) > 0$ *and* $Z(i) > 0$ *such that for* $i \in \mathcal{N}$*,*

$$\begin{bmatrix} \Phi_{11}(i) & 0 & \frac{\Pi_1^T+\Pi_2^T}{2} & 0 \\ \star & -\beta^{d_2+1}Q(i) & 0 & 0 \\ \star & \star & \beta Z(i)-I & 0 \\ \star & \star & \star & -\beta^{\tau+1}Z(i) \end{bmatrix} + \begin{bmatrix} A^T(i) \\ A_d^T(i) \\ 0 \\ A_\tau^T(i) \end{bmatrix} P(i) \begin{bmatrix} A^T(i) \\ A_d^T(i) \\ 0 \\ A_\tau^T(i) \end{bmatrix}^T < 0, \quad (28)$$

where $\Phi_{11}(i) \triangleq -\beta P(i) + \beta(d_2 - d_1 + 1)Q(i) - \dfrac{\Pi_1^T\Pi_2 + \Pi_2^T\Pi_1}{2}$*. Then, the discrete-time switched time-delay system in (1) with* $u(k) = 0$ *is exponentially stable for any switching signal with average dwell time satisfying* $T_a > T_a^* = \mathrm{ceil}\left(-\dfrac{\ln\mu}{\ln\beta}\right)$*, where* $\mu \geq 1$ *satisfies*

$$P(i) \leq \mu P(j),\ Q(i) \leq \mu Q(j),\ Z(i) \leq \mu Z(j),\ \forall i, j \in \mathcal{N}. \tag{29}$$

Next, we are in a position to consider the stabilization of the system (1) and design the desired controller.

Theorem 3.2. *Given a constant* $0 < \beta < 1$*, the system in (1) is stabilizable, that is, the closed system in (4) is exponentially stable under the control input* $u(t)$ *in (3), if there exist matrices* $X(i) > 0$*,* $\hat{Q}(i) > 0$*,* $Z(i) > 0$ *and* $Y(i)$ *such that for* $i \in \mathcal{N}$*,*

$$\begin{bmatrix} \tilde{\Phi}_{11}(i) & 0 & X(i)\dfrac{\Pi_1^T + \Pi_2^T}{2} & 0 & X(i)A^T(i) + Y^T(i)B^T(i) \\ \star & -\beta^{d_2+1}\hat{Q}(i) & 0 & 0 & X(i)A_d^T(i) \\ \star & \star & \beta Z(i) - I & 0 & 0 \\ \star & \star & \star & -\beta^{\tau+1}Z(i) & A_\tau^T(i) \\ \star & \star & \star & \star & -X(i) \end{bmatrix} < 0, \tag{30}$$

where $\tilde{\Phi}_{11}(i) \triangleq -\beta X(i) + \beta(d_2 - d_1 + 1)\hat{Q}(i) - 2X(i) + \left(\dfrac{\Pi_1^T\Pi_2 + \Pi_2^T\Pi_1}{2}\right)^{-1}$*. Then, the discrete-time switched time-delay system in (1) is exponentially stabilizable for any switching signal with average dwell time satisfying* $T_a > T_a^* = \mathrm{ceil}\left(-\dfrac{\ln\mu}{\ln\beta}\right)$*, where* $\mu \geq 1$ *satisfies*

$$X(i) \leq \mu X(j),\ \hat{Q}(i) \leq \mu\hat{Q}(j),\ Z(i) \leq \mu Z(j),\ \forall i, j \in \mathcal{N}. \tag{31}$$

In this case, a robustly stabilizing state feedback control law can be chosen by

$$u(k) = Y(i)X^{-1}(i)x(k). \tag{32}$$

Proof: By suing Schur complement and considering the closed-loop system in (4), we have that the closed-loop system in (4) is exponentially stable if there matrices $P(i) > 0$, $Q(i) > 0$ and $Z(i) > 0$ such that for $i \in \mathcal{N}$,

$$\begin{bmatrix} \Phi_{11}(i) & 0 & \dfrac{\Pi_1^T + \Pi_2^T}{2} & 0 & \hat{A}^T(i)P(i) \\ \star & -\beta^{d_2+1}Q(i) & 0 & 0 & A_d^T(i)P(i) \\ \star & \star & \beta Z(i) - I & 0 & 0 \\ \star & \star & \star & -\beta^{\tau+1}Z(i) & A_\tau^T(i)P(i) \\ \star & \star & \star & \star & -P(i) \end{bmatrix} < 0, \tag{33}$$

where the switching signal has an average dwell time satisfying $T_a > T_a^* = \mathrm{ceil}\left(-\dfrac{\ln\mu}{\ln\beta}\right)$, where $\mu \geq 1$ satisfies

$$P(i) \leq \mu P(j),\ Q(i) \leq \mu Q(j),\ Z(i) \leq \mu Z(j),\ \forall i, j \in \mathcal{N}.$$

Performing a congruence transformation to (33) by $\mathrm{diag}\{X(i), X(i), I, I, X(i)\}$ (where $X(i) = P^{-1}(i)$), it follows that

$$\begin{bmatrix} \hat{\Phi}_{11}(i) & 0 & X(i)\dfrac{\Pi_1^T + \Pi_2^T}{2} & 0 & X(i)\hat{A}^T(i) \\ \star & -\beta^{d_2+1}\hat{Q}(i) & 0 & 0 & X(i)A_d^T(i) \\ \star & \star & \beta Z(i) - I & 0 & 0 \\ \star & \star & \star & -\beta^{\tau+1}Z(i) & A_\tau^T(i) \\ \star & \star & \star & \star & -X(i) \end{bmatrix} < 0, \tag{34}$$

where $\hat{\Phi}_{11}(i) \triangleq -\beta X(i) + \beta(d_2 - d_1 + 1)\hat{Q}(i) - X(i)\dfrac{\Pi_1^T\Pi_2 + \Pi_2^T\Pi_1}{2}X(i)$ and $\hat{Q}(i) = X(i)Q(i)X(i)$.

On the other hand, the following matrix inequality holds:

$$\left[X(i) - \left(\frac{\Pi_1^T\Pi_2 + \Pi_2^T\Pi_1}{2}\right)^{-1}\right]\left(\frac{\Pi_1^T\Pi_2 + \Pi_2^T\Pi_1}{2}\right)\left[X(i) - \left(\frac{\Pi_1^T\Pi_2 + \Pi_2^T\Pi_1}{2}\right)^{-1}\right] \geq 0$$

thus,

$$X(i)\frac{\Pi_1^T\Pi_2 + \Pi_2^T\Pi_1}{2}X(i) \geq 2X(i) - \left(\frac{\Pi_1^T\Pi_2 + \Pi_2^T\Pi_1}{2}\right)^{-1}$$

Therefore, matrix inequality (34) holds if the following LMI holds:

$$\begin{bmatrix} \tilde{\Phi}_{11}(i) & 0 & X(i)\dfrac{\Pi_1^T + \Pi_2^T}{2} & 0 & X(i)\hat{A}^T(i) \\ \star & -\beta^{d_2+1}\hat{Q}(i) & 0 & 0 & X(i)A_d^T(i) \\ \star & \star & \beta Z(i) - I & 0 & 0 \\ \star & \star & \star & -\beta^{\tau+1}Z(i) & A_\tau^T(i) \\ \star & \star & \star & \star & -X(i) \end{bmatrix} < 0, \tag{35}$$

where $\tilde{\Phi}_{11}(i)$ is defined in (30). Moreover, we define $Y(i) = K(i)X(i)$, we have (30), and we know that $K(i) = Y(i)X^{-1}(i)$. The proof is completed.

4. Illustrative Examples.

In this section, we will give two numerical examples to illustrate the effectiveness of the proposed analysis and design methods.

Example 4.1 (Stability Analysis). *Consider the system in (1) with $N = 2$, and its parameters are given as follows:*

Subsystem 1.

$$A(1) = \begin{bmatrix} 0.20 & 0.10 & -0.01 \\ 0.10 & 0.20 & -0.10 \\ 0.20 & -0.06 & -0.13 \end{bmatrix}, \quad A_d(1) = \begin{bmatrix} 0.06 & -0.20 & -0.15 \\ 0.04 & -0.01 & 0.36 \\ 0.20 & 0.10 & -0.07 \end{bmatrix}, \quad A_\tau(1) = A_d(1).$$

Subsystem 2.

$$A(2) = \begin{bmatrix} 0.30 & -0.10 & -0.30 \\ -.04 & 0.20 & 0.20 \\ 0.10 & -0.05 & -0.20 \end{bmatrix}, \quad A_d(2) = \begin{bmatrix} -0.04 & 0.05 & -0.20 \\ -0.20 & 0.10 & -0.10 \\ 0.06 & -0.10 & -0.03 \end{bmatrix}, \quad A_\tau(2) = A_d(2),$$

and $f(x) = \begin{bmatrix} f_1^T(x) & f_2^T(x) & f_3^T(x) \end{bmatrix}^T$ *with*

$$\begin{cases} f_1(x) = -\tanh(x_1) + 0.2x_1 + 0.1x_2 + 0.1x_3, \\ f_2(x) = 0.1x_1 - \tanh(x_2) + 0.2x_2, \\ f_3(x) = 0.1x_1 + 0.2x_3 - \tanh(x_3) \end{cases}$$

It is easy to verify that

$$\Pi_1 = \begin{bmatrix} -0.8 & 0.1 & 0.1 \\ 0.1 & -0.8 & 0.0 \\ 0.1 & 0.0 & -0.8 \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.0 \\ 0.1 & 0.0 & 0.2 \end{bmatrix},$$

and $d_1 = 1$, $d_2 = 2$, $\beta = 0.8$, $\tau = 0.6$. We consider the average dwell time approach proposed in this paper, and set $\mu = 1.5 > 1$, thus $T_a > T_a^* = \text{ceil}\left(-\frac{\ln \mu}{\ln \beta}\right) = 2$. Solving LMI (5) with (7), we can obtain a feasible solution of $(P(1), P(2), Q(1), Q(2), R(1), R(2), Z(1), Z(2), S_1(1), S_1(2), S_2(1), S_2(2), L(1), L(2), M(1), M(2), N(1), N(2))$. Therefore, we can conclude that the above discrete-time switched system is exponentially stable.

In addition, for $d_1 = 1$, $\mu = 1.5$ and $\tau = 0.6$, considering different β, the upper bound of d_2 for different cases are listed in Table 1.

TABLE 1. Upper bound of d_2 (denoted by $\hat{d}_2$) for different β

β	0.5	0.6	0.7	0.8	0.9
$\hat{d}_2$	1.3344	1.7345	2.2247	2.9278	4.1299

Example 4.2 (**Stabilization**). *Consider the system in (1) with $N = 2$, and its parameters are given as follows:*
Subsystem 1.

$$\begin{aligned} A(1) &= \begin{bmatrix} -0.9 & 0.2 & -0.2 \\ 0.2 & -0.1 & 0.3 \\ -0.3 & 0.1 & 0.3 \end{bmatrix}, \quad A_d(1) = \begin{bmatrix} 0.2 & 0 & 0.1 \\ 0.1 & 0.3 & 0.1 \\ 0.3 & 0.1 & 0.2 \end{bmatrix}, \\ A_\tau(1) &= \begin{bmatrix} -0.2 & 0.0 & 0.1 \\ 0.2 & 0.1 & 0.1 \\ 0.0 & -0.2 & -0.1 \end{bmatrix} \end{aligned} \tag{36}$$

Subsystem 2.

$$\begin{aligned} A(2) &= \begin{bmatrix} -0.8 & -0.1 & -0.2 \\ 0.2 & -0.1 & 0.3 \\ 0.2 & -0.1 & 0.2 \end{bmatrix}, \quad A_d(2) = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.3 \end{bmatrix}, \\ A_\tau(2) &= \begin{bmatrix} -0.2 & 0.0 & 0.1 \\ 0.2 & 0.1 & 0.1 \\ 0.0 & -0.2 & -0.1 \end{bmatrix}, \\ B_1 &= B_2 = B = \begin{bmatrix} 0 & 0 & 2 \end{bmatrix}^T \end{aligned} \tag{37}$$

with $d_1(k) = d_2(k) = 2.5 + (-1)^k/2$ and $f(x)$ defined in Example 4.1, and $d_1 = 1$, $d_2 = 3$, $\mu = 1.5$, $\beta = 0.9$, $\tau = 0.5$, $x(\theta) = [-0.3\ 1\ -0.8]^T$, $\theta \in [-3, 0]$.

The switching signal is given in Figure 1 (which is generated randomly, here, '1' and '2' represent the first and the second subsystem, respectively). The states trajectories of the open-loop system are shown in Figure 2, from which we can see that the open-loop system is not stable. In this situation, we will design a state feedback stabilization controller such that the closed-loop system is stable. To this end, by solving the LMI conditions in Theorem 3.2, we obtain

$$K_1 = \begin{bmatrix} 0.1480 & -0.0307 & -0.1965 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.2096 & 0.0475 & -0.1501 \end{bmatrix}.$$

The states trajectories of the closed-loop system are displayed in Figure 3, which demonstrates the effectiveness of the proposed method in this paper.

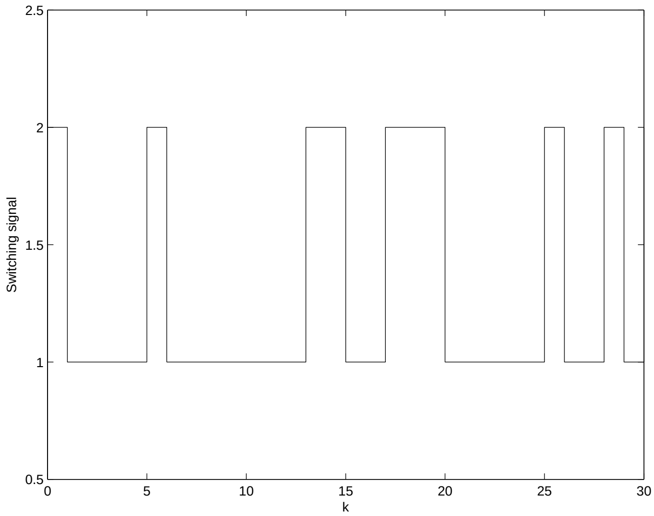

FIGURE 1. Switching signal

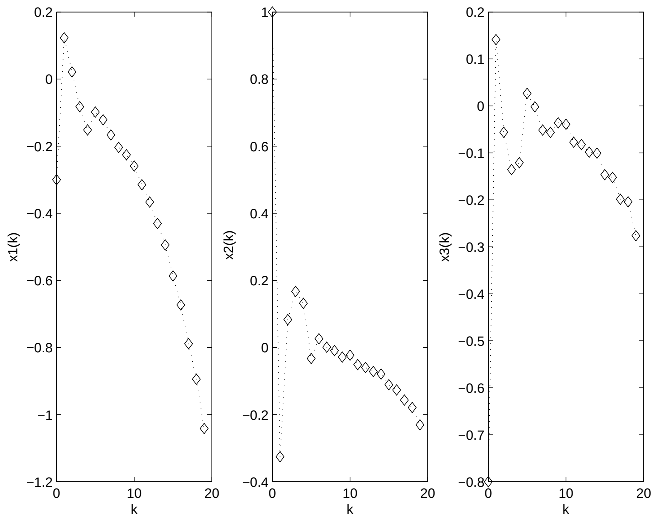

FIGURE 2. States of the open-loop system

5. **Conclusion.** The stability analysis and stabilization problems have been investigated for discrete-time switched nonlinear systems with time delays. By using the average dwell time approach and the piecewise Lyapunov function technique, a delay-dependent sufficient condition has been proposed to guarantee the exponential stability of the considered system. To reduce the conservatism of the delay-dependent condition, we has introduced some slack matrix variables to seek the relationship between the Newton-Leibniz formula, instead of applying model transformation. A strict LMI stabilization controller design approach has developed. An explicit expression for the desired state feedback control law has also been given. Finally, two numerical examples have been provided to illustrate the effectiveness of the proposed theory.

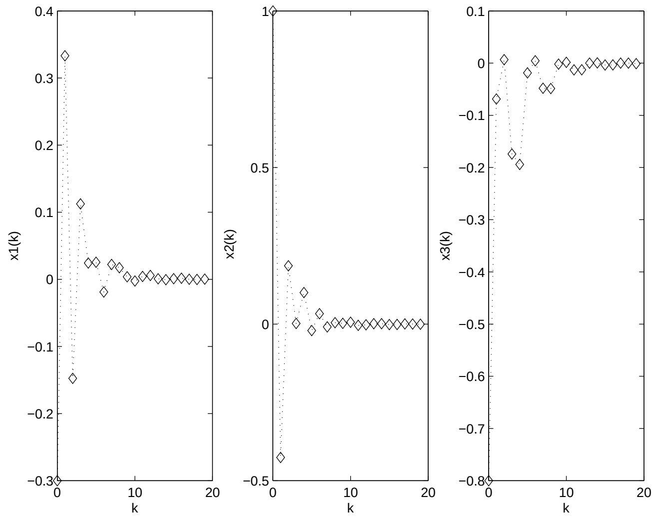

FIGURE 3. States of the closed-loop system

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