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MODEL BASED FUZZY CONTROL WITH AFFINE T-S DELAYED MODELS APPLIED TO NONLINEAR SYSTEMS

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Abstract. *This paper mainly investigates fuzzy control with affine T-S delayed models, which can be applied to some nonlinear systems. Motivated from performing the linearization process associated with distinct operating points for nonlinear systems, an affine T-S fuzzy model with delayed state is addressed. The overall control then can be performed by the fuzzy inference mechanism, in which the consequent parts are represented by the locally linear affine subsystems with delayed state. Sufficient stability conditions for the unforced T-S affine models with delayed state are first derived. By involving the parallel distributed compensator (PDC), design conditions for the resulting closed-loop systems are further investigated. Since all the proposed criteria are formulated by the linear matrix inequalities (LMIs), we thus can perform the stability analysis or the PDC synthesis via current LMI solvers. A nonlinear numerical example and an applicable physical model with TCP/RED flowing control mechanism are given to demonstrate the validity and effectiveness of the proposed approach.*
Keywords: T-S fuzzy model, Delayed state, Affine systems, Linear matrix inequality (LMI)

1. **Introduction.** Model based fuzzy control has been successfully applied to miscellaneous systems, which are mathematically poorly modeled and where the knowledge and the experience of operators can achieve the control object well. It can approximately represent the states' behaviors of nonlinear systems or uncertain systems by appropriate transformation [1-4]. To date, Tanaka and Sugeno [5,6] first proposed a T-S fuzzy model and discussed its stability issue, where the consequent parts presented the locally linear models. By performing the fuzzy control, the model can be associated with the so-called "parallel distributed compensation (PDC)" [3]. Thereafter, the stability analysis and the controller synthesis for the T-S fuzzy model have attracted a great deal of research (e.g., [7-10] and the references therein). Recently, the linear matrix inequality (LMI) technique [11] was involved for deriving some explicit stability analysis and design criteria [12-16], and they could be readily evaluated by some commercial software [17].

In most physical and engineering systems, such as circuit systems, chemical processes, and long transmission lines in network systems, delay states cannot be neglected [18-21]. Since they in industrial plant are main sources of instability, oscillations, or degraded performances, the stability analysis and the PDC synthesis are extended to tackle the T-S fuzzy models with delayed state [22-30]. Moreover, for nonlinear system control, we can perform the linearization process on some distinct operating points and consequently obtain a set of the locally linear affine subsystems. Then, the overall control law can be implemented by the fuzzy inference system [31,32]. However, to the best of our knowledge, the stability analysis and the PDC synthesis for the T-S fuzzy affine model subjected to the delayed state seem not to be addressed in the past works, and the concerned control problems are still open.

In this work, we originally present an affine T-S fuzzy model with delayed state and discuss its stability analysis and PDC synthesis issues. Based on the Lyapunov stability theory, the stability and design criteria can be derived and objectively expressed in terms of LMIs. Then, we can readily verify the proposed conditions directly via LMI solver [17]. Demonstratively, the proposed results are illustrated by a numerical nonlinear system and a physical TCP/RED network flowing model [33]. The involved TCP/RED flows model indeed is a nonlinear system. By performing the linearization on some operating points, we can obtain a set of the locally affine linear models with delayed state; where it elaborately coincides with the proposed affine fuzzy models. Comparing with the past works, the main contributions of our method can be summarized as follows:

i) In the past, it seems that there are no results on the fuzzy T-S affine model with the time varying delayed-state. This work mainly focuses on the stability analysis and the PDC synthesis for the regarded affine systems subjected to the time varying delayed-state.
ii) Since all the proposed criteria can be formulated in terms of LMIs, the stability analyzing and the PDC synthesis for the considered systems are readily achieved by LMI solvers [17].
iii) The superiority and practicability of the proposed approach are demonstrated by a numerical nonlinear system and a physical TCP/RED network flowing model [33].

The rest of content is organized as follows. In Section 2, the concerned problem is formulated. Based on the Lyapunov-Krasovskii theorem, the stability analysis and the PDC controller synthesis are mainly discussed in Section 3. In Section 4, two nonlinear systems are given to verify the effectiveness and applicability of the proposed approach. Finally, concluding remarks are collected in Section 5.

2. Problem Formulation.

Consider an affine T-S fuzzy model with the time varying delayed-state in each rule described by

Rule i: If $x_1(t)$ is M_1^i and $x_2(t)$ is M_2^i and $\ldots$ $x_n(t)$ is M_n^i, then

$$\begin{aligned} &\dot{x}(t) = A_i x(t) + A_{di} x(t - d(t)) + B_i u(t) + \mu_i, \quad t > 0 \\ &x(s) = \Psi(s), \quad \bar{d} \leq s \leq 0 \quad i = 1, 2, \ldots, r, \end{aligned} \tag{1}$$

where M_j^i is a fuzzy set, $\mu_i \in R^n$ is a constant affine term, $x(t) \in R^n$ is the state vector, $u(t) \in R$ is a single control input, and $A_i \in R^{n\times n}$, $A_{di} \in R^{n\times n}$ are the system matrices. The input matrices $B_i \in R^n$ $\forall i$ are assumed to have the form $B_i = \begin{bmatrix} b_i & 0 \end{bmatrix}^T$ (or $B_i = \begin{bmatrix} 0 & b_i \end{bmatrix}^T$, respectively). The bounded time-delay function $d(t)$, satisfying $\dot{d}(t) < 1$, is continuous and nonnegative; there are some constants $\bar{d}$ and δ such that $0 \leq d(t) \leq \bar{d}$ and $\dot{d}(t) \leq \delta < 1$. The initial term $\Psi(s)$, $s \in [-\bar{d}, 0]$ is a continuous vector function. Thus, the overall fuzzy model is inferred as

$$\begin{aligned} \dot{x}(t) &= \frac{\sum_{i=1}^{r} \omega_i(x(t))[A_i x(t) + A_{di} x(t - d(t)) + B_i u(t) + \mu_i]}{\sum_{i=1}^{r} \omega_i(x(t))} \\ &= \sum_{i=1}^{r} h_i(x(t))[A_i x(t) + A_{di} x(t - d(t)) + B_i u(t) + \mu_i], \end{aligned} \tag{2}$$

where

$$\begin{cases} \omega_i(x(t)) = \prod_{j=1}^{p} M_j^i(x_j(t)) \geq 0 \\ \sum_{i=1}^{r} \omega_i(x(t)) > 0 \end{cases} \quad i = 1, 2, \ldots, r\,,$$

$$\begin{cases} h_i(x(t)) = \dfrac{\omega_i(x(t))}{\sum_{i=1}^{r} \omega_i(x(t))} \geq 0 \\ \sum_{i=1}^{r} h_i(x(t)) = 1 \end{cases} \quad i = 1, 2, \ldots, r\,,$$

and $M_j^i(x_j(t))$ is the grade of membership of $x_j(t)$ in M_j^i.

Based on the notion of PDC, the following control law is involved into the fuzzy system (2).

Control inputs:

Rule i: If $x_1(t)$ is M_1^i and $x_2(t)$ is M_2^i and $\ldots$ $x_p(t)$ is M_p^i, then

$$u(t) = K_i x(t) + K_{di} x(t - d(t)) + a_i, \quad i = 1, 2, \ldots, r,$$

where M_j^i $\forall i, j$ are the same fuzzy sets as in (2). The overall fuzzy control can be integrated by

$$\begin{aligned} u(t) &= \frac{\sum_{i=1}^{r} \omega_i(x(t))\left[K_i x(t) + K_{di} x(t - d(t)) + a_i\right]}{\sum_{i=1}^{r} \omega_i(x(t))} \\ &= \sum_{i=1}^{r} h_i(x(t))\left[K_i x(t) + K_{di} x(t - d(t)) + a_i\right]. \end{aligned} \tag{3}$$

Substituting (3) into (2), the resulting closed-loop fuzzy system thus can be represented by

$$\dot{x}(t) = \sum_{i,j=1}^{r} h_i(x(t)) h_j(x(t))[(A_i + B_i K_j)x(t) + (A_{di} + B_i K_{dj})x(t - d(t)) + \mu_i + a_j B_i]. \tag{4}$$

Assumption 2.1. *Let I_0 be the set of indexes for the fuzzy rules that contain the origin point $x = 0$ and satisfy $h_o(0) \neq 0$. The affine terms μ_i in (2) or (4) are assumed to be 0 when $i \in I_0$, and the corresponding bias input a_i in (3) or (4) will be set to 0.*

3. **Stability Analysis and PDC Controller Synthesis.**

A sufficient condition for the unforced fuzzy system (4), i.e., $u(t) \equiv 0$ in (4), is first derived in the following.

Theorem 3.1. *The equilibrium of the unforced fuzzy system (4) is asymptotically stable in the large, if there exist positive definite symmetric matrices P, Q and scalars $\tau_{il} \geq 0$ $\forall i, l$ such that*

$$\begin{bmatrix} A_i^T P + P A_i + Q & P A_{di} \\ A_{di}^T P & -(1 - \delta) Q \end{bmatrix} < 0 \quad i \in I_o, \tag{5}$$

$$\begin{bmatrix} A_i^T P + P A_i + Q - \sum_l \tau_{il} T_{il} & P A_{di} & P \mu_i - \sum_l \tau_{il} v_{il} \\ A_{di}^T P & -(1 - \delta) Q & 0 \\ \mu_i^T P - \sum_l \tau_{il} v_{il}^T & 0 & -\sum_l \tau_{il} r_{il} \end{bmatrix} < 0 \quad i \notin I_0, \tag{6}$$

where parameters T_{il}, v_{il}, and r_{il} satisfy $F_{il}(x) = x^T T_{il} x + 2 v_{il}^T x + r_{il} \leq 0$, $l = 1, 2, \ldots, n$.

Proof: Assume that there exist positive definite symmetric matrices P, Q and scalars $\tau_{il} \geq 0$ satisfying (5) and (6). Choose a candidate quadratic Lyapunov function as

$$V(x(t),t) = x^T(t)Px(t) + \int_{t-d(t)}^{t} x^T(\delta)Qx(\delta)d\delta.$$

The time derivative of $V(x(t),t)$ along the trajectories of the unforced fuzzy system (4) is

$$\begin{aligned}
&\dot{V}(x(t),t)\\
&= \sum_{i=1}^{r} h_i(x(t))\Big[x^T(t)\left(A_i^TP + PA_i\right)x(t) + 2x^T(t)PA_{di}x(t-d(t)) + 2\mu_i^TPx(t)\Big]\\
&\quad + x^T(t)Qx(t) - (1-\dot{d}(t))x^T(t-d(t))Qx(t-d(t))\\
&\leq \sum_{i=1}^{r} h_i(x(t))\Big[x^T(t)\left(A_i^TP + PA_i\right)x(t) + 2x^T(t)PA_{di}x(t-d(t)) + 2\mu_i^TPx(t)\Big]\\
&\quad + x^T(t)Qx(t) - (1-\delta)x^T(t-d(t))Qx(t-d(t))\\
&\leq \sum_{i=1}^{r} h_i(x(t))\Big[x^T(t)(A_i^TP + PA_i + Q - \sum_l \tau_{il}T_{il})x(t) + 2x^T(t)PA_{di}x(t-d(t))\\
&\quad + 2\left(\mu_i^TP - \sum_l \tau_{il}v_{il}\right)x(t) - (1-\delta)x^T(t-d(t))Qx(t-d(t)) - \sum_l \tau_{il}r_{il}\Big]\\
&= \sum_{i\in I_0} h_i(x(t))\begin{bmatrix} x(t) \\ x(t-d(t)) \end{bmatrix}^T \begin{bmatrix} A_i^TP + PA_i + Q & PA_{di} \\ A_{di}^TP & -(1-\delta)Q \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-d(t)) \end{bmatrix}\\
&\quad + \sum_{i\notin I_0} h_i(x(t))\begin{bmatrix} x(t) \\ x(t-d(t)) \\ 1 \end{bmatrix}^T\\
&\quad \begin{bmatrix} A_i^TP + PA_i + Q - \sum_l \tau_{il}T_{il} & PA_{di} & P\mu_i - \sum_l \tau_{il}v_{il} \\ A_{di}^TP & -(1-\delta)Q & 0 \\ \mu_i^TP - \sum_l \tau_{il}v_{il}^T & 0 & -\sum_l \tau_{il}r_{il} \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-d(t)) \\ 1 \end{bmatrix}
\end{aligned}$$

By (5) and (6), the above equation implies $\dot{V}(x(t),t) < 0$. Thus, the unforced fuzzy system (4) is asserted to be asymptotically stable in the large according to the Lyapunov-Krasovskii theorem.

The parameters T_{il}, v_{il}, and r_{il} in (6) can be initially determined in the following.

Remark 3.1. *[32] Assume that the region in which the inferred fuzzy rule $h_i(x) \neq 0$. The range of the individual entices x_l of x can be divided by three scalar regions with two corresponding bounds α_{il} and β_{il}, $\alpha_{il} \leq \beta_{il}$ as*

for x_1, $x_1 \leq \alpha_{i1}$ or $x_1 \geq \beta_{i1}$ or $\alpha_{i1} \leq x_1 \leq \beta_{i1}$,

for x_2, $x_2 \leq \alpha_{i2}$ or $x_2 \geq \beta_{i2}$ or $\alpha_{i2} \leq x_2 \leq \beta_{i2}$,

$\vdots$

for x_n, $x_n \leq \alpha_{in}$ or $x_n \geq \beta_{in}$ or $\alpha_{in} \leq x_n \leq \beta_{in}$.

Thus, for x_l, the T_{il}, v_{il}, and r_{il} satisfying $F_{il}(x) = x^TT_{il}x + 2v_{il}^Tx + r_{il} \leq 0$ are evaluated as

Case $x_l \leq \alpha_{il}$:

$$\begin{aligned}
T_{il} &= 0_{n\times n},\\
v_{il} &= \begin{bmatrix} 0 & 0 & \cdots & -1/2 & \cdots & 0 \end{bmatrix}^T, \text{ } l\text{th element},\\
r_{il} &= -\alpha_{il}.
\end{aligned}$$

Case $x_l \geq \beta_{il}$:

$$\begin{aligned} &T_{il} = 0_{n\times n}, \\ &v_{il} = \begin{bmatrix} 0 & 0 & \cdots & -1/2 & \cdots & 0 \end{bmatrix}^T, \ \textit{lth element}, \\ &r_{il} = \beta_{il}. \end{aligned}$$

Case $\alpha_{il} \leq x_l \leq \beta_{il}$:

$$\begin{aligned} &T_{il} = diag\begin{pmatrix} 0 & \cdots & 1 & \cdots & 0 \end{pmatrix}, \ \textit{lth element}, \\ &v_{il} = \begin{bmatrix} 0 & 0 & \cdots & -1/2(\alpha_{il}+\beta_{il}) & \cdots & 0 \end{bmatrix}^T, \textit{lth element}, \\ &r_{il} = \alpha_{il}\beta_{il}. \end{aligned}$$

Based on Theorem 3.1, the PDC synthesis for the resulting closed-loop fuzzy system (4) is mainly deduced in the sequel.

Theorem 3.2. *Assume* $B_i \equiv \begin{bmatrix} b_i & 0 \end{bmatrix}^T$ *in (4). The equilibrium of the closed-loop fuzzy system (4) is asymptotically stable in the large, if there exist positive definite symmetric matrices* $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_3 \end{bmatrix}$, Q *with appreciate dimensions and scalars* $\tau_{il} \geq 0 \ \forall i, l$ *such that*

$$\begin{bmatrix} A_i^T P + PA_i + b_j \begin{bmatrix} F_i^T & 0 \end{bmatrix} + b_j \begin{bmatrix} F_i \\ 0 \end{bmatrix} + Q & PA_{di} + b_j \begin{bmatrix} F_{di} \\ 0 \end{bmatrix} \\ A_{di}^T P + b_j \begin{bmatrix} F_{di}^T & 0 \end{bmatrix} & -(1-\delta)Q \end{bmatrix} < 0, \quad i \in I_o, \quad j = 1, 2, \tag{7}$$

$$\begin{bmatrix} A_i^T P + PA_i + b_j \begin{bmatrix} F_i^T & 0 \end{bmatrix} + b_j \begin{bmatrix} F_i \\ 0 \end{bmatrix} + Q - \sum_l \tau_{il} T_{il} & PA_{di} + b_j \begin{bmatrix} F_{di} \\ 0 \end{bmatrix} & P\mu_i + \alpha_i \begin{bmatrix} b_j \\ 0 \end{bmatrix} - \sum_l \tau_{il} v_{il} \\ A_{di}^T P + b_j \begin{bmatrix} F_{di}^T & 0 \end{bmatrix} & -(1-\delta)Q & 0 \\ \mu_i^T P + \alpha_i \begin{bmatrix} b_j & 0 \end{bmatrix} - \sum_l \tau_{il} v_{il}^T & 0 & -\sum_l \tau_{il} r_{il} \end{bmatrix} < 0, \quad i \notin I_0, \quad j = 1, 2, \tag{8}$$

where $\begin{bmatrix} b_1 & 0 \end{bmatrix}^T \equiv \min_i \begin{bmatrix} b_i & 0 \end{bmatrix}^T$ *and* $\begin{bmatrix} b_2 & 0 \end{bmatrix}^T \equiv \max_i \begin{bmatrix} b_i & 0 \end{bmatrix}^T$, *parameters* T_{il}, v_{il}, *and* r_{il} *satisfy* $F_{il}(x) = x^T T_{il} x + 2v_{il}^T x + r_{il} \leq 0$, $l = 1, 2, \ldots, n$. *Then, the equilibrium of the system (4) with the control gains* $K_i = F_i P_1^{-1}$, $K_{di} = F_{di} P_1^{-1}$, $a_i = \alpha_i P_1^{-1}$ $\forall i$ *is asymptotically stable in the large.*

Proof: Assume that there exist positive definite symmetric matrices $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_3 \end{bmatrix}$, Q and parameters $\tau_{il} \geq 0$ satisfying (7) and (8).

Define $\underline{B} \equiv \min_i \begin{bmatrix} b_i & 0 \end{bmatrix}^T$, $\overline{B} \equiv \max_i \begin{bmatrix} b_i & 0 \end{bmatrix}^T$, and a polytypic set

$$\Omega_B \equiv \left\{ \tilde{B} : \tilde{B} = \beta \underline{B} + (1-\beta)\overline{B}, \beta \in [0, 1] \right\}, \tag{9}$$

where there exist $B_i \in \Omega_B \ \forall i$, and choose a candidate quadratic Lyapunov function as

$$V(x(t),\ t) = x^T(t)Px(t) + \int_{t-d(t)}^{t} x^T(\delta)Qx(\delta)d\delta.$$

By (9), the time derivative of $V(x(t),t)$ along the trajectories of the closed-loop fuzzy system (4) is

$$
\begin{aligned}
&\dot V(x(t),t)\\
=\ & \sum_{i=1}^{r} h_i(x(t))\Big[x^T(t)(A_i^TP+PA_i)x(t)+2x^T(t)PA_{di}x(t-d(t))+2\mu_i^TPx(t)\Big]\\
&+\sum_{j=1}^{r} h_j(x(t))\Big[2x^T(t)P\sum_{i=1}^{r} h_i(x(t))B_i(K_jx(t)+K_{dj}x(t-d(t))+a_j)\Big]\\
&+x^T(t)Qx(t)-\sum_{i=1}^{r}(1-\dot h(t))x^T(t-d(t))Qx(t-d(t))\\
\le\ & \sum_{i=1}^{r} h_i(x(t))\Big[x^T(t)(A_i^TP+PA_i)x(t)+2x^T(t)PA_{di}x(t-d(t))+2\mu_i^TPx(t)\Big]\\
&+\sum_{i=1}^{r} h_i(x(t))\Big[2x^T(t)P\tilde B(K_ix(t)+K_ix(t-d(t))+a_i)\Big]\\
&+x^T(t)Qx(t)-(1-\delta)x^T(t-d(t))Qx(t-d(t))\\
\le\ & \sum_{i=1}^{r} h_i(x(t))\Big[x^T(t)(A_i^TP+PA_i+K_i^T\tilde B^TP+P\tilde BK_i)x(t)+2x^T(t)(PA_{di}+P\tilde BK_{di})x(t-d(t))+2\mu_i^TPx(t)\\
&+2a_i\tilde BPx(t)\Big]+x^T(t)Qx(t)-(1-\delta)x^T(t-d(t))Qx(t-d(t))\\
\le\ & \sum_{i=1}^{r} h_i(x(t))\Big[x^T(t)(A_i^TP+PA_i+K_i^T\tilde B^TP+P\tilde BK_i+Q-\sum_l\tau_{il}T_{il})x(t)+2x^T(t)(PA_{di}+P\tilde BK_{di})x(t-d(t))\\
&+2\left(\mu_i^TP+a_i\tilde BP-\sum_l\tau_{il}v_{il}\right)x(t)-(1-\delta)x^T(t-h(t))Qx(t-d(t))-\sum_l\tau_{il}r_{il}\Big]\\
=\ & \sum_{i\in I_0} h_i(x(t))\begin{bmatrix} x(t)\\ x(t-d)\end{bmatrix}^T\begin{bmatrix} A_i^TP+PA_i+K_i^T\tilde B^TP+P\tilde BK_i+Q & PA_{di}+P\tilde BK_{di}\\ A_{di}^TP+K_{di}^T\tilde B^TP & -(1-\delta)Q\end{bmatrix}\begin{bmatrix} x(t)\\ x(t-d)\end{bmatrix}\\
&+\sum_{i\notin I_0} h_i(x(t))\begin{bmatrix} x(t)\\ x(t-d)\\ 1\end{bmatrix}^T\\
&\times\begin{bmatrix} A_i^TP+PA_i+K_i^T\tilde B^TP+P\tilde BK_i+Q-\sum_l\tau_{il}T_{il} & PA_{di}+P\tilde BK_{di} & P\mu_i+a_i\tilde BP-\sum_l\tau_{il}v_{il}\\ A_{di}^TP+K_{di}^T\tilde B^TP & -(1-\delta)Q & 0\\ \mu_i^TP+a_iP\tilde B^T-\sum_l\tau_{il}v_{il}^T & 0 & -\sum_l\tau_{il}r_{il}\end{bmatrix}\begin{bmatrix} x(t)\\ x(t-d)\\ 1\end{bmatrix}
\end{aligned}
$$

When substituting the matrices $P=\begin{bmatrix} P_1 & 0\\ 0 & P_3\end{bmatrix}$, $F_i\equiv K_iP_1$, $F_{di}\equiv K_{di}P_1$, and $\alpha_i\equiv a_iP_1$ into the above equation, we obtain

$$
\begin{aligned}
\dot V(x(t),t)=\ & \sum_{i\in I_0} h_i(x(t))\begin{bmatrix} x(t)\\ x(t-d)\end{bmatrix}^T\begin{bmatrix} A_i^TP+PA_i+\tilde b\begin{bmatrix} F_i^T & 0\end{bmatrix}+\tilde b\begin{bmatrix} F_i\\ 0\end{bmatrix}+Q & PA_{di}+\tilde b\begin{bmatrix} F_{di}\\ 0\end{bmatrix}\\ A_{di}^TP+\tilde b\begin{bmatrix} F_{di}^T & 0\end{bmatrix} & -(1-\delta)Q\end{bmatrix}\begin{bmatrix} x(t)\\ x(t-d)\end{bmatrix}\\
&+\sum_{i\notin I_0} h_i(x(t))\begin{bmatrix} x(t)\\ x(t-d)\\ 1\end{bmatrix}^T\\
\times&\begin{bmatrix} A_i^TP+PA_i+\tilde b\begin{bmatrix} F_i^T & 0\end{bmatrix}+\tilde b\begin{bmatrix} F_i\\ 0\end{bmatrix}+Q-\sum_l\tau_{il}T_{il} & PA_{di}+\tilde b\begin{bmatrix} F_{di}\\ 0\end{bmatrix} & P\mu_i+\alpha_i\begin{bmatrix} \tilde b\\ 0\end{bmatrix}-\sum_l\tau_{il}v_{il}\\ A_{di}^T+\tilde b\begin{bmatrix} F_{di}^T & 0\end{bmatrix} & -(1-\delta)Q & 0\\ \mu_i^TP+\alpha_i\begin{bmatrix} \tilde b & 0\end{bmatrix}-\sum_l\tau_{il}v_{il}^T & 0 & -\sum_l\tau_{il}r_{il}\end{bmatrix}\begin{bmatrix} x(t)\\ x(t-d)\\ 1\end{bmatrix}
\end{aligned}
$$

By (7) and (8), it implies $\dot V(x(t),\ t)<0$. Thus, the resulting closed-loop fuzzy system (4) with the control gains $K_i=F_iP_1^{-1}$, $K_{di}=F_{di}P_1^{-1}$, and $a_i=\alpha_iP_1^{-1}$ $\forall i$ is asserted to be asymptotically stable in the large according to the Lyapunov-Krasovskii theorem.

Corollary 3.1. *Assume* $B_i\equiv\begin{bmatrix} 0 & b_i\end{bmatrix}^T$ *in (4). The equilibrium of the closed-loop fuzzy system (4) is asymptotically stable in the large, if there exist positive definite symmetric* *matrices* $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_3 \end{bmatrix}$, Q *with appreciate dimensions and scalars* $\tau_{il} \geq 0 \ \forall i, l$ *such that*

$$\begin{bmatrix} A_i^T P + PA_i + b_j \begin{bmatrix} 0 & F_i^T \end{bmatrix} + b_j \begin{bmatrix} 0 \\ F_i \end{bmatrix} + Q & PA_{di} + b_j \begin{bmatrix} 0 \\ F_{di} \end{bmatrix} \\ A_{di}^T P + b_j \begin{bmatrix} 0 & F_{di}^T \end{bmatrix} & -(1-\delta)Q \end{bmatrix} < 0, \quad i \in I_o, \quad j = 1, 2,$$

$$\begin{bmatrix} A_i^T P + PA_i + b_j \begin{bmatrix} 0 & F_i^T \end{bmatrix} + b_j \begin{bmatrix} 0 \\ F_i \end{bmatrix} + Q - \sum_l \tau_{il} T_{il} & PA_{di} + b_j \begin{bmatrix} 0 \\ F_{di} \end{bmatrix} & P\mu_i + \alpha_i \begin{bmatrix} 0 \\ b_j \end{bmatrix} - \sum_l \tau_{il} v_{il} \\ A_{di}^T P + b_j \begin{bmatrix} 0 & F_{di}^T \end{bmatrix} & -(1-\delta)Q & 0 \\ \mu_i^T P + \alpha_i \begin{bmatrix} 0 & b_j \end{bmatrix} - \sum_l \tau_{il} v_{il}^T & 0 & -\sum_l \tau_{il} r_{il} \end{bmatrix} < 0,$$

$$i \notin I_0, \quad j = 1, 2,$$

where $\begin{bmatrix} 0 & b_1 \end{bmatrix}^T \equiv \min_i \begin{bmatrix} 0 & b_i \end{bmatrix}^T$ *and* $\begin{bmatrix} 0 & b_2 \end{bmatrix}^T \equiv \max_i \begin{bmatrix} 0 & b_i \end{bmatrix}^T$, *parameters* T_{il}, v_{il}, *and* r_{il} *satisfy* $F_{il}(x) = x^T T_{il} x + 2v_{il}^T x + r_{il} \leq 0$, $l = 1, 2, \ldots, n$. *Then, the equilibrium of the system (4) with the control gains* $K_i = F_i P_3^{-1}$, $K_{di} = F_{di} P_3^{-1}$, $a_i = \alpha_i P_3^{-1}$ $\forall i$ *is asymptotically stable in the large.*

Proof: Following the same line of Theorem 3.2, the proof can be similarly attained.

4. Illustrating Examples via Applicable Nonlinear Systems.

Example 4.1. *Consider a nonlinear system with delayed state described as*

$$\dot{x}_1(t) = 2\beta \sin x_1(t) + \alpha x_2(t) + (\beta + 1)x_1(t - d(t)) + \tilde{b}u(t),$$

$$\dot{x}_2(t) = \alpha\beta \sin x_1(t) + \beta x_2(t) + (\alpha - 2)x_1(t - d(t)),$$

where $x_i(t) \in R^n$ *is the state variables,* $d(t) \equiv 0.5(1 + \cos t)$ *is a delayed function,* α *and* β *are two constant parameters, and* $\tilde{b} \in [2, 3]$ *is an uncertain input parameter.*

When performing the linearization for the nonlinear term $\sin x_1(t)$ on three divided intervals: $\begin{bmatrix} -3\pi/2 & -\pi/2 \end{bmatrix}$, $\begin{bmatrix} -\pi/2 & \pi/2 \end{bmatrix}$, $\begin{bmatrix} \pi/2 & 3\pi/2 \end{bmatrix}$, the considered system can be approximately represented as a three rules with second order affine T-S fuzzy model

Rule 1: IF $x_1(t)$ is M_1
Then $\dot{x}(t) = A_1 x(t) + A_{d1}(t - d(t)) + B_1 u(t) + \mu_1$,
Rule 2: IF $x_1(t)$ is M_2
Then $\dot{x}(t) = A_2 x(t) + A_{d2}(t - d(t)) + B_2 u(t)$,
Rule 3: IF $x_1(t)$ is M_3
Then $\dot{x}(t) = A_3 x(t) + A_{d3}(t - d(t)) + B_3 u(t) + \mu_3$,

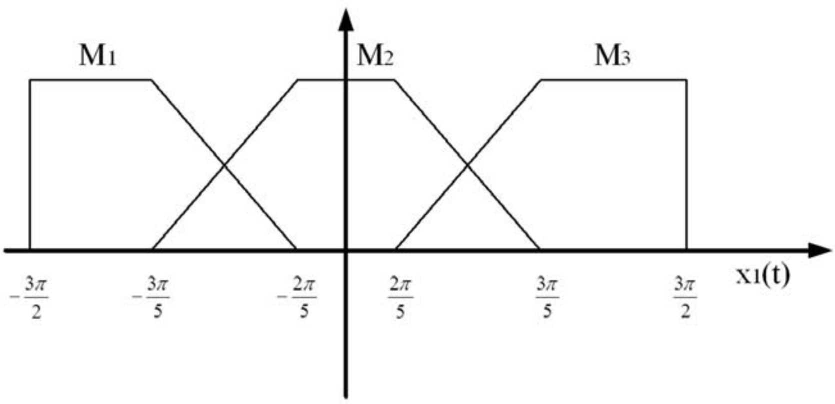

FIGURE 1. Membership functions of M_1, M_2, M_3

where the fuzzy sets M_1, M_2, and M_3 are given in Figure 1. The systems' matrices are then formed by

$$A_1 = \begin{bmatrix} \frac{-4\beta}{\pi} & \alpha \\ \frac{-2\alpha\beta}{\pi} & \beta \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} \beta+1 & 0 \\ \alpha-2 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad \mu_1 = \begin{bmatrix} -4\beta \\ -2\alpha\beta \end{bmatrix},$$

$$A_2 = \begin{bmatrix} \frac{4\beta}{\pi} & \alpha \\ \frac{2\alpha\beta}{\pi} & \beta \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} \beta+1 & 0 \\ \alpha-2 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2.5 \\ 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} \frac{-4\beta}{\pi} & \alpha \\ \frac{-2\alpha\beta}{\pi} & \beta \end{bmatrix}, \quad A_{d3} = \begin{bmatrix} \beta+1 & 0 \\ \alpha-2 & 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \quad \mu_3 = \begin{bmatrix} 4\beta \\ 2\alpha\beta \end{bmatrix}.$$

Denoting the parameters $\alpha = 3$, $\beta = -1$ and the initial condition $x(0) = [\ 3 \quad 1\ ]^T$, $\Psi(s) = 0$, $-1 \leq s \leq 0$ in the system, the simulated result with the unforced input $u(t) = 0$ is first plotted in Figure 2. By observation, this unforced system has unstable states, and a stabilizing control law needs to be involved. But, by the previous result [32] and other previous works, they all cannot cope with the PDC synthesis for the delayed T-S model with the affine terms.

Based on Theorem 3.2 associated with M_1, M_2, and M_3 depicted in Figure 1, we can previously determine $\dot{d}(t) \leq \delta = 0.5$ and

$$T_{11} = T_{31} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad v_{11} = \begin{bmatrix} \frac{19\pi}{20} \\ 0 \end{bmatrix}, \quad v_{31} = \begin{bmatrix} \frac{-19\pi}{20} \\ 0 \end{bmatrix}, \quad r_{11} = r_{31} = \frac{3\pi^2}{5}.$$

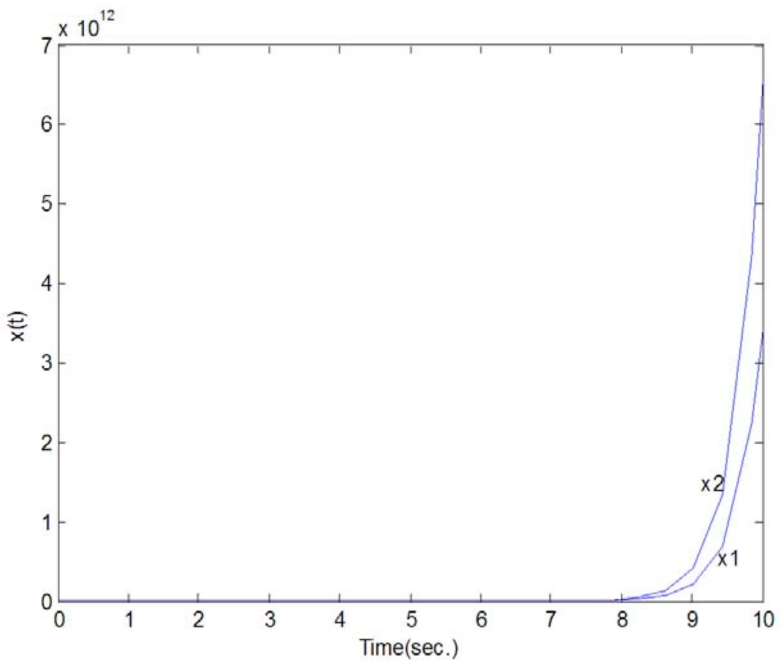

FIGURE 2. Simulated results for $x(t)$ with the unforced input

Thus, design conditions for the PDC synthesis can be constructed from (7) and (8). By the LMI solver [17], we thus obtain a set of feasible solutions as

$$P = \begin{bmatrix} 14.3105 & 0 \\ 0 & 5.5779 \end{bmatrix} > 0, \quad Q = \begin{bmatrix} 20.1747 & -0.4192 \\ -0.4192 & 1.2418 \end{bmatrix} > 0,$$
$$F_1 = \begin{bmatrix} -9.7593 & -18.2872 \end{bmatrix}, \quad F_2 = \begin{bmatrix} -266.1028 & -12.4131 \end{bmatrix},$$
$$F_3 = \begin{bmatrix} -320.0335 & -15.3828 \end{bmatrix}, \quad F_{d1} = \begin{bmatrix} -3.8319 & -0.2044 \end{bmatrix} \times 10^{-3},$$
$$F_{d2} = \begin{bmatrix} -18.45 & 0.0001 \end{bmatrix} \times 10^{-2}, \quad F_{d3} = \begin{bmatrix} -33.4843 & 0.0343 \end{bmatrix} \times 10^{-3},$$
$$\alpha_1 = 9.4121, \quad \alpha_2 = -9.7593, \quad t_{11} = 66.9702, \quad t_{31} = 1762.9639.$$

The stabilizing PDC gains then be evaluated as

$$K_1 = F_1 P_1^{-1} = \begin{bmatrix} -0.6820 & -1.2779 \end{bmatrix}, K_{d1} = F_{d1} P_1^{-1} = \begin{bmatrix} -0.2678 & -0.0143 \end{bmatrix} \times 10^{-3},$$
$$K_2 = F_2 P_1^{-1} = \begin{bmatrix} -18.5950 & -0.8674 \end{bmatrix}, K_{d2} = F_{d2} P_1^{-1} = \begin{bmatrix} -12.8933 & 0.0001 \end{bmatrix} \times 10^{-3},$$
$$K_3 = F_3 P_1 - 1 = \begin{bmatrix} -22.3636 & -1.0749 \end{bmatrix}, K_{d3} = F_{d3} P_1^{-1} = \begin{bmatrix} -2.3398 & 0.0024 \end{bmatrix} \times 10^{-3},$$
$$a_1 = 0.6577, a_3 = -4.8797.$$

By the given initial condition $x(0) = \begin{bmatrix} 3 & 1 \end{bmatrix}^T$, $\Psi(s) = 0$, $-1 \leq s \leq 0$, the simulated results for $x(t)$ and $u(t)$ are depicted in Figure 3 and Figure 4, respectively. It is shown that the PDC controller with the obtained gains can stabilize the regarded nonlinear system indeed.

In the following example, the proposed approach will be demonstrated by a physical nonlinear model, an AQM-Based TCP flows dynamic model [33].

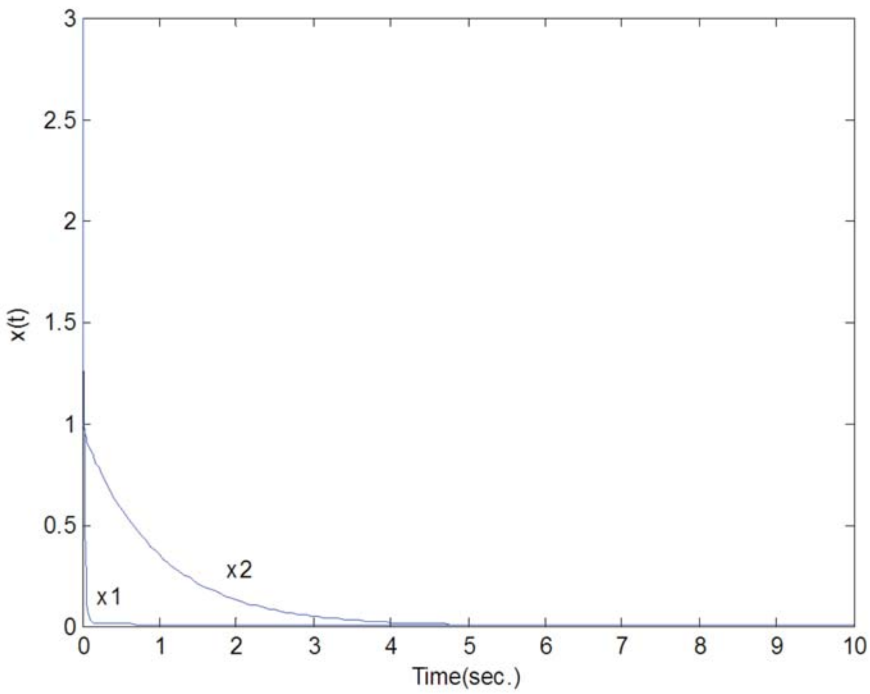

FIGURE 3. Simulated results for $x(t)$ with the PDC control

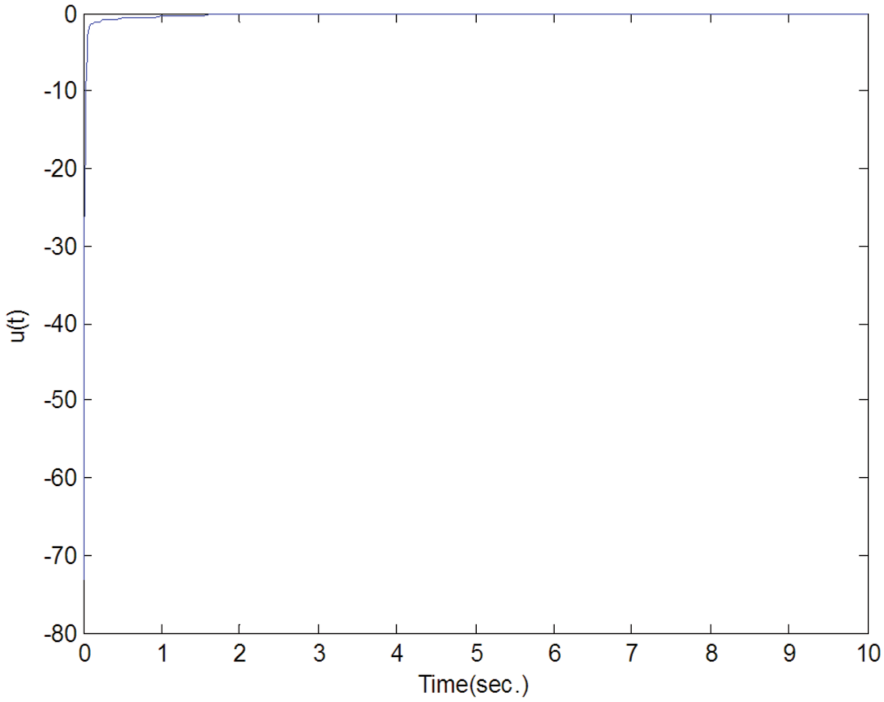

FIGURE 4. Simulated result for $u(t)$

Example 4.2. *Consider a feedback mechanism of AQM behavior by a time-delayed dynamic system described by*

$$f_1(t) \equiv \dot{W}(t) = \frac{1}{\dfrac{q(t)}{C} + T_p} - \frac{W(t)}{2} \frac{W(t-R_0)}{\dfrac{q(t-R_0)}{C} + T_p} p(t-R_0) \tag{10}$$

$$f_2(t) \equiv \dot{q}(t) = \begin{cases} -C + \dfrac{N(t)}{R(t)} W(t), & q > 0 \\ \max\left\{0,\ -C + \dfrac{N(t)}{R(t)} W(t)\right\}, & q = 0 \end{cases} \tag{11}$$

where $W(t)$ is the average TCP window size (packets), q is the average queue length (packets), $R(t) = q(t)/C + T_p$ is the round-trip time (secs), C is the link capacity (packets/sec), T_P is the propagation delay (secs), N is the number of TCP sessions, and $p(t)$ is the probability of dropping/marking.

By letting $\dot{W}(t) = 0$, $\dot{q}(t) = 0$, an equivalent point (W_0, q_0, p_0) can be obtained as

$$p_0 = \frac{2}{W_0^2}, \quad W_0 = \frac{R_0 C}{N}, \quad R_0 = \frac{q_0}{C} + T_p. \tag{12}$$

For conveniently analyzing, we modify the dynamic system (10) and (11) by involving the biased variables $\delta W \equiv W - W_0$, $\delta q \equiv q - q_0$, $\delta p \equiv p - p_0$; where there is favorably associated with the system with the new equivalent point at $(0,0,0)$. Then, a linearization dynamic model can be formed as

$$\delta\dot{W}(t) = -\frac{N}{R_0^2 C}(\delta W(t) + \delta W(t-R_0)) - \frac{1}{R_0^2 C}(\delta q(t) - \delta q(t-R_0)) - \frac{R_0 C^2}{2N^2}\delta p(t-R_0) \tag{13}$$

$$\delta\dot{q}(t) = \frac{N}{R_0}\delta W(t) - \frac{1}{R_0}\delta q(t). \tag{14}$$

Define the state-space variables as $x^T(t) = [\ x_1(t) \quad x_2(t)]^T \equiv [\ \delta W(t) \quad \delta q(t)]^T$ and the controlled input as $u(t) \equiv \delta p(t)$. The state equations for the original point (0, 0, 0) and other operator points $(W_i,\ q_i,\ p_i)$ can be represented by

$$\dot{x}(t) = \begin{bmatrix} -\dfrac{N}{R_0^2 C} & -\dfrac{1}{R_0^2 C} \\ \dfrac{N}{R_0} & -\dfrac{1}{R_0} \end{bmatrix} x(t) + \begin{bmatrix} -\dfrac{N}{R_0^2 C} & \dfrac{1}{R_0^2 C} \\ 0 & 0 \end{bmatrix} x(t-R_0) + \begin{bmatrix} -\dfrac{R_0 C^2}{2N^2} \\ 0 \end{bmatrix} u(t-R_0) \tag{15}$$

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + A_{id} x(t-R_i) + B_i u(t-R_i) + \mu_i \\ &= \begin{bmatrix} -\dfrac{N_i}{R_i^2 C_i} & -\dfrac{1}{R_i^2 C} \\ \dfrac{N_i}{R_i} & -\dfrac{1}{R_i} \end{bmatrix} x(t) + \begin{bmatrix} -\dfrac{N_i}{R_i^2 C_i} & \dfrac{1}{R_i^2 C_i} \\ 0 & 0 \end{bmatrix} x(t-R_i) + \begin{bmatrix} -\dfrac{R_i C_i^2}{2N_i^2} \\ 0 \end{bmatrix} u(t-R_i) + \mu_i \end{aligned} \tag{16}$$

where $\mu_i = \begin{bmatrix} f_1 & f_2 \end{bmatrix}^T \Big|_{\substack{W=W_i \\ q=q_i \\ p=p_i}} - A_i \begin{bmatrix} W_i \\ q_i \end{bmatrix} - A_{id} \begin{bmatrix} W_i \\ q_i \end{bmatrix} - B_i p_i$.

By the linearization process, the nonlinear AQM-Based TCP dynamic model (10) and (11) can be approximately represented as a set of linear affine models in (15) and (16) corresponding to the different operating points. Thus, we can apply the proposed fuzzy model-based control to this system.

Numerical illustration. *Consider a TCP flows model in (15) and (16) with the given network parameters: link capacity* $C = 3750$ *packets/sec., propagation delay* $T_P = 0.2$ *seconds, and three different TCP session and queue pairs:* $N_1 = 80$, $q_1 = 50$, $N_2 = 70$, $q_2 = 100$, *and* $N_3 = 60$, $q_3 = 150$. *From (12) we then correspondingly denote three operating points as* $(W_1, q_1, p_1) = (10, 50, 0.0200)$, $(W_2, q_2, p_2) = (12.14, 100, 0.0136)$, $(W_3, q_3, p_3) = (15, 150, 0.0089)$.

Let the state-space variables as $x^T(t) = [\ x_1(t) \quad x_2(t)]^T \equiv [\ W(t) - W_2 \quad q(t) - q_2]^T$ and the controlled input as $u(t) \equiv p(t) - p_2$. The affine linear models with three operating points can be presented as

(W_1, q_1, p_1):

$$\begin{aligned} \dot{x}(t) &= A_1 x(t) + A_{d1} x(t-0.2133) + B_1 u(t-0.2133) + \mu_1 \\ &= \begin{bmatrix} -0.4688 & -0.0059 \\ 375 & -4.6875 \end{bmatrix} x(t) + \begin{bmatrix} -0.4688 & 0.0059 \\ 0 & 0 \end{bmatrix} x(t-0.2133) \\ &\quad + \begin{bmatrix} -234.3750 \\ 0 \end{bmatrix} u(t-0.2133) + \begin{bmatrix} -0.5005 \\ 569.1964 \end{bmatrix} \end{aligned}$$

(W_2, q_2, p_2):

$$\begin{aligned} \dot{x}(t) &= A_2 x(t) + A_{d2} x(t-0.2267) + B_2 u(t-0.2267) \\ &= \begin{bmatrix} -0.3633 & -0.0052 \\ 308.8235 & -4.4118 \end{bmatrix} x(t) + \begin{bmatrix} -0.3633 & 0.0052 \\ 0 & 0 \end{bmatrix} x(t-0.2267) \\ &\quad + \begin{bmatrix} -325.2551 \\ 0 \end{bmatrix} u(t-0.2267) \end{aligned}$$

(W_3, q_3, p_3):

$$\begin{aligned} \dot{x}(t) &= A_3 x(t) + A_{d3} x(t-0.24) + B_3 u(t-0.24) + \mu_3 \\ &= \begin{bmatrix} -0.2778 & -0.0046 \\ 250 & -4.1667 \end{bmatrix} x(t) + \begin{bmatrix} -0.2778 & 0.0046 \\ 0 & 0 \end{bmatrix} x(t-0.24) \\ &\quad + \begin{bmatrix} -468.75 \\ 0 \end{bmatrix} u(t-0.24) + \begin{bmatrix} -0.6042 \\ -505.9524 \end{bmatrix} \end{aligned}$$

By surveying the past works, we cannot find an applicable result on the stability issues of this system. However, by the proposed fuzzy affine model (1) associated with Theorem 3.2, a three rules and second order fuzzy system with delayed state is formed as

Rule 1: IF $x_2(t)$ is M_4
Then $\dot{x}(t) = A_1x(t) + A_{d1}(t - d(t)) + B_1u(t - d(t)) + \mu_1$,
Rule 2: IF $x_2(t)$ is M_5
Then $\dot{x}(t) = A_2x(t) + A_{d2}(t - d(t)) + B_2u(t - d(t))$,
Rule 3: IF $x_2(t)$ is M_6
Then $\dot{x}(t) = A_3x(t) + A_{d3}(t - d(t)) + B_3u(t - d(t)) + \mu_3$,

where $d(t) \equiv 0.22 + 0.02\sin(t)$, M_4, M_5, and M_6 are given in Figure 5.

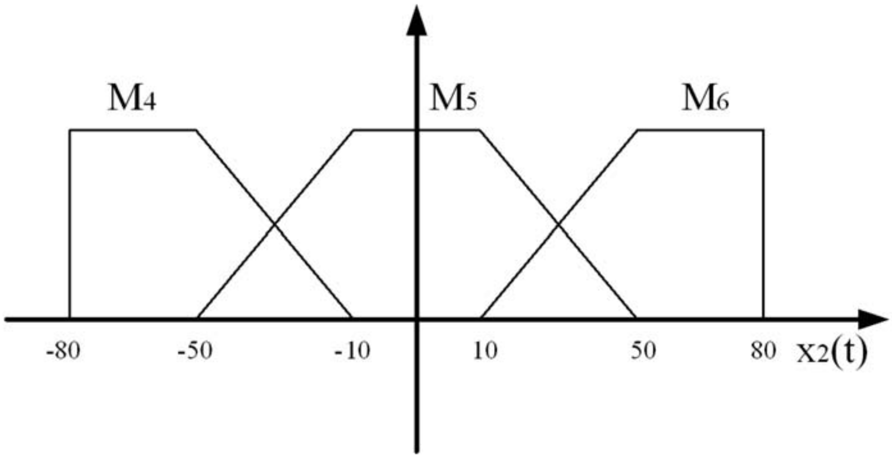

FIGURE 5. Membership functions of M_4, M_5, M_6

From Remark 3.1 with M_4, M_5, and M_6, we can first determine the parameters

$$T_{12} = T_{32} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad v_{12} = \begin{bmatrix} 0 \\ 45 \end{bmatrix}, \quad v_{32} = \begin{bmatrix} 0 \\ -45 \end{bmatrix}, \quad r_{12} = r_{32} = 800,$$

and $\dot{d}(t) \leq \delta = 0.02$. Thus, a set of LMI conditions for the PDC synthesis can be constructed from (7) and (8). By the LMI solver, we then obtain a set of feasible solutions as

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_3 \end{bmatrix} = \begin{bmatrix} 25.5014 & 0 \\ 0 & 0.0014 \end{bmatrix} > 0,$$
$$Q = \begin{bmatrix} 7.8548 & -0.1291 \\ -0.1291 & 0.0091 \end{bmatrix} > 0,$$
$$F_{d1} = \begin{bmatrix} -36.8332 & 0.4834 \end{bmatrix} \times 10^{-3},$$
$$F_{d2} = \begin{bmatrix} -24.2141 & 0.3447 \end{bmatrix} \times 10^{-3},$$
$$F_{d3} = \begin{bmatrix} -19.8474 & 0.3302 \end{bmatrix} \times 10^{-3},$$
$$\alpha_1 = -0.0311, \quad \alpha_3 = -0.0368, \quad t_{12} = 0.0175, \quad t_{32} = 0.0189.$$

And, the stabilizing PDC gains are thus evaluated by

$$K_{d1} = F_{d1}P_1^{-1} = \begin{bmatrix} -14.4436 & 0.1896 \end{bmatrix} \times 10^{-4}, \quad a_1 = \alpha_1 P_1^{-1} = -12.1980 \times 10^{-4},$$
$$K_{d2} = F_{d2}P_1^{-1} = \begin{bmatrix} -9.4952 & 0.1352 \end{bmatrix} \times 10^{-4},$$
$$K_{d3} = F_{d3}P_1^{-1} = \begin{bmatrix} -7.7829 & 0.1295 \end{bmatrix} \times 10^{-4}, \quad a_3 = \alpha_3 P_1^{-1} = -14.4436 \times 10^{-4}.$$

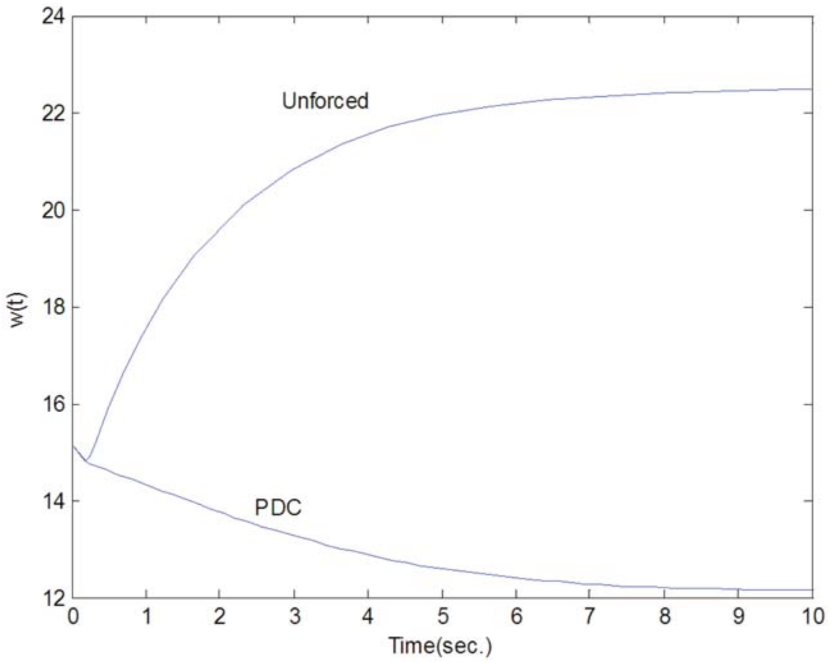

Figure 6. Simulated results for $W(t)$

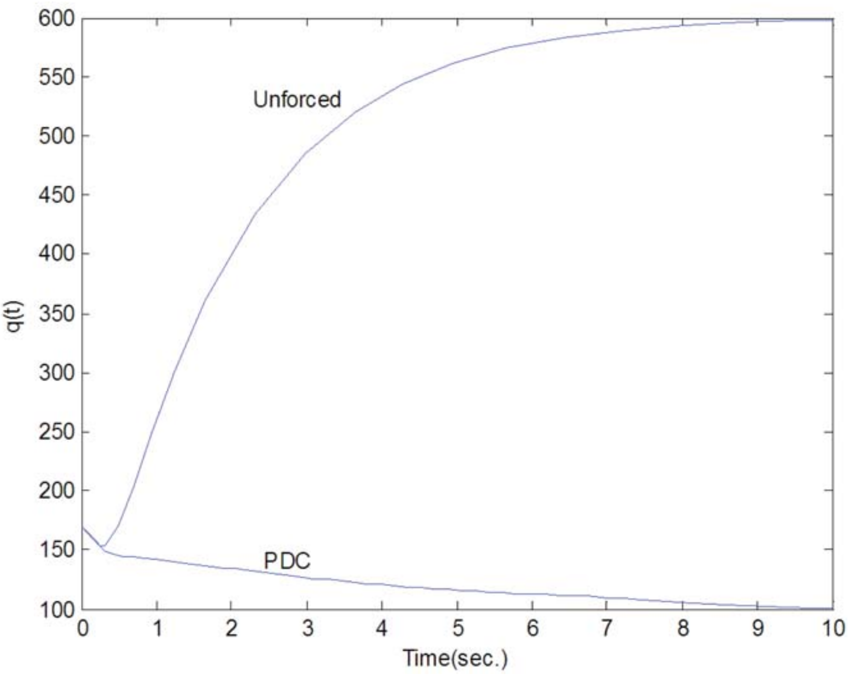

Figure 7. Simulated results for $q(t)$

For comparison, by the initially condition $x(0) = [\ 3 \quad 70\ ]^T$ and $\Psi(s) = 0$, $-0.24 \leq s \leq 0$, the systems equipped with the zero input and the delayed PDC controller $u(t-h(t)) = (h_1K_{d1} + h_2K_{d2} + h_3K_{d3})x(t-h(t)) + h_1a_1 + h_3a_3$ are both simulated. And, the results for $w(t)$, $q(t)$, and $p(t)$ are depicted in Figures 6-8, respectively. By observing Figure 6 and Figure 7, the unforced results tend to unexpected parameters' tent while the results equipped the PDC are all operating in the allowable network circumstances and well convergent to the desired points.

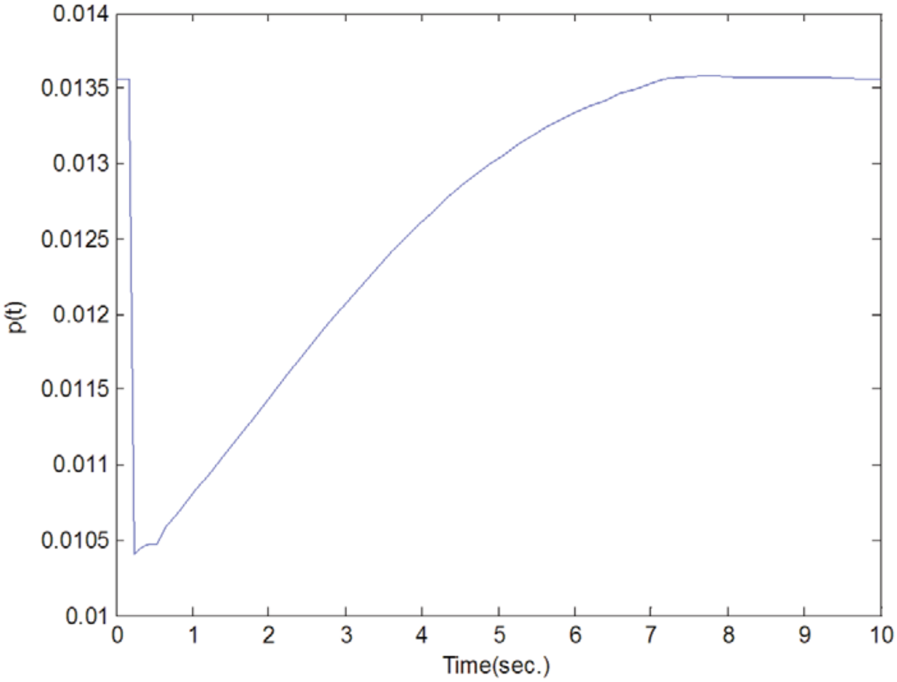

Figure 8. Simulated results for $p(t)$ with the PDC control

5. **Conclusions.** Motivated from the linearization process for some nonlinear systems on different operating points, the stability analysis and PDC synthesis for the affine T-S delayed model had been investigated in this work. Based on the Lyapunov-Krasovskii theorem, the stability condition for the unforced system was first derived. By involving the PDC synthesis, we further proposed the PDC synthesis criteria for the resulting closed-loop system. Since all the proposed criteria were expressed in terms of LMIs, we could readily perform the stability analyzing and the PDC design via the existing LMI solver. Finally, the given nonlinear numerical system and the physical TCP/RED flowing model demonstrated the superiority and applicability of the proposed approach.

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