

## MODELING AND SIMULATION OF FUZZY LOGIC CONTROLLER-BASED MODEL REFERENCE ADAPTIVE CONTROLLER

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**ABSTRACT.** *The aim of this paper is to design a Fuzzy Logic Controller-based Model Reference Adaptive Controller. It consists of Fuzzy Logic Controller (FLC) along with a conventional Model Reference Adaptive Control (MRAC) scheme. The idea is to control the plant by conventional MRAC with a suitable single reference model, and at the same time control the plant by FLC. In the conventional MRAC scheme, the controller is designed to realize plant output converges to reference model output based on the plant which is linear. This scheme is for controlling linear plant effectively with unknown parameters. However, using MRAC scheme to control the nonlinear system at real time is difficult. In this paper, it is proposed to incorporate an FLC in MRAC scheme to overcome the problem. The control input is given by the sum of the output of conventional MRAC and the output of FLC. The rules for the FLC are obtained from the conventional PI Controller. The effectiveness of the proposed control scheme is demonstrated by simulations. The proposed Fuzzy Logic Controller-based Model Reference Adaptive Controller (FLC-MRAC) can significantly improve the system's behavior and force the system to follow the reference model and minimize the error between the model and plant output.*

**Keywords:** Model reference adaptive control (MRAC), Fuzzy logic controller (FLC), Proportional-integral (PI) controller

**1. Introduction.** Model Reference Adaptive Control (MRAC) is one of the main schemes used in adaptive system. Recently, MRAC has received considerable attention, and many new approaches have been applied to practical processes [1,2]. In the MRAC scheme, the controller is designed to realize plant output converges to reference model output based on the assumption that plant can be linearized. Therefore, this scheme is effective for controlling linear plants with unknown parameters. However, it may not assure for controlling nonlinear plants with unknown structure. It is well known that fuzzy technique has been widely used in many physical and engineering systems, especially for systems with incomplete plant information [3-8]. In addition to fuzzy logic, it has been widely applied to controller designs for nonlinear systems [9-13]. A novel fuzzy model reference based controller for controlling nonlinear plants can be found in [14]. H. Han [15] proposed an adaptive FLC for a class of nonlinear system with disturbance. A problem of Fuzzy-Approximation-Based adaptive control for a class of nonlinear time-delay systems with unknown nonlinearities and strict-feedback structure is discussed in [16]. C.-W. Chen et

al. [17] discussed a proposed a method of stability analysis for a GA-Based reference ANNC which is capable of handling problems in a nonlinear system.

FL technique has been selected to replace PI controllers in different error minimization applications [18,19]. Various applications of FL have shown a fast growth in the past few years. FLC has become popular in the field of industrial control applications for solving control, estimation and optimization problems [20]. An adaptive control approach for time-varying permanent-magnet synchronous motor (PMSM) systems with chaotic behavior is discussed in [21]. Observer-based model reference output feedback tracking control design for switched linear systems with time delay is investigated in [22]. A learning approach of combining MRAC with the use of fuzzy systems as reference models and controllers for control dynamical systems can be found in [23]. A hybrid approach by combining FLC and neural networks for learning-based control is proposed in [24].

The adaptive controller which is used in various practical applications has attracted much attention in the field of control engineering. This is due to its promising capability of tackling the presence of unknown parameters or unknown variations in plant parameters better than that of the one based on constant gain feedback control scheme. In general, the external load disturbances always exist although they are bounded. So, the controller cannot stabilize the closed-loop control system without considering the disturbances and nonlinearities existing in the system. A solution to this problem is to incorporate dead-zone technique in the adaptive controller (Peterson et al. 1982, Sastry 1984). With this approach, the controller will stop updating the control parameters when the identifier error is smaller than some fixed threshold. Thus, it can prevent the estimated parameters from being infinity. However, the regulation error of the system will only be asymptotically bounded if large threshold is used, resulting in undesirable closed-loop performance. All control techniques have their individual characteristics. Every control technique has its own individual characteristics. Hence, by combining the merits of the adaptive control scheme with those of the FL, a new stabilizing controller can effectively be designed to have better performance than the one based on the concept of control theory.

PI controllers are widely used in industrial control systems applications. They have a simple structure and can offer a satisfactory performance over a wide range of operation. Therefore, the majority of adaptation schemes described in the literature for MRAS speed observer employ a simple fixed-gain linear PI controller to generate the estimated output. However, due to the continuous variations in the system parameters and the operating conditions, in addition to the nonlinearities present in the system, PI-MRAC scheme may not be able to provide the required standard performance. Not much attention has been devoted to study of other types of adaptation mechanisms used to minimize the error to obtain the estimated output. In this paper, the FPI-MRAC scheme is designed to replace the classical PI controller used in PI-MRAC scheme by a Fuzzy Logic Controller (FLC).

The FLC-MRAC scheme is proposed as a nonlinear optimizer to ensure that the plant output trajectory tracks the reference model output trajectory and the tracking error becomes zero with a possible minimum time. The performance of the new and conventional scheme is compared based on simulation tests with two examples. The rules and membership function of FLC are formed from the input and output waveforms of PI controller of designed PI-MRAC scheme. The FLC is connected in parallel with an MRAC and its output is added and then given to the plant input. The FLC is used to compensate the nonlinearity of the plant and it is not taken into consideration in the conventional MRAC. The role of MRAC is to perform the model matching for the uncertain linearized system to a given reference model. Finally, to confirm the effectiveness of proposed method, it is compared with the simulation results of the conventional MRAC.

**2. Structure of an MRAC Design.** The MRAC is one of the major approaches in adaptive control. The desired performance is expressed as a reference model, which gives the wished response to an input signal. The adjustment mechanism changes the parameters of the regulator by minimizing the error between the system output and the reference model.

**2.1. The plant model and reference model system.** To consider a Single Input and Single Output (SISO), Linear Time Invariant (LTI) plant with strictly proper transfer function

$$G(s) = \frac{y_p(s)}{u_p(s)} = K_p \frac{Z_p(s)}{R_p(s)} \tag{1}$$

where  $u_p$  is the plant input and  $y_p$  is the plant output. Also, the reference model is given by

$$G_m(s) = \frac{y_m(s)}{r(s)} = K_m \frac{Z_m(s)}{R_m(s)} \tag{2}$$

where  $r$  and  $y_m$  are the model's input and output. Define the output error as

$$e = y_p - y_m \tag{3}$$

Now the objective is to design the control input  $U_{mr}$  such that the output error,  $e$  goes to zero asymptotically for arbitrary initial condition, where the reference signal  $r(t)$  is piecewise continuous and uniformly bounded. The plant and reference model satisfy the following assumptions:

Assumptions:

1.  $Z_p(s)$  is a monic Hurwitz polynomial of degree  $m_p$ .
2. An upper bound  $n$  of degree  $n_p$  of  $R_p(S)$ .
3. The relative degree  $n^* = n_p - m_p$  of  $G(s)$ .
4. The sign of the high frequency gain  $K_p$  are known.
5.  $Z_m(s), R_m(s)$  are monic Hurwitz polynomials of degree  $q_m, p_m$  respectively, where  $p_m \leq n$ .
6. The relative degree  $n_m^* = p_m - q_m$  of  $G_m(s)$  is the same as that of  $G(S)$ , i.e.,  $n_m^* = n^*$ .

**2.2. Relative degree  $n = 1$ .** As in [1] the following input and output filters are used,

$$\dot{\omega}_1 = F\omega_1 + gu_p, \quad \dot{\omega}_2 = F\omega_2 + gy_p \tag{4}$$

where  $F$  is an  $(n-1) \times (n-1)$  stable matrix such that  $\det(SI - F)$  is a Hurwitz polynomial whose roots include the zeros of the reference model and that  $(F, g)$  is a controllable pair. It is defined as the "regressor" vector

$$\omega = [\omega_1^T, \omega_2^T, y_p, r]^T \tag{5}$$

In the standard adaptive control scheme, the control  $U_{mr}$  is structured as

$$U_{mr} = \theta^T \omega \tag{6}$$

where  $\theta = [\theta_1, \theta_2, \theta_3, C_0]^T$  is a vector of adjustable parameters, and is considered as an estimate of a vector of unknown system parameters  $\theta^*$ .

The dynamic of tracking error

$$e = G_m(s)\rho^* \tilde{\theta}^T \omega \tag{7}$$

where  $\rho^* = \frac{K_p}{K_m}$  and  $\tilde{\theta} = \theta(t) - \theta^*$  represents parameter error. Now in this case, since the transfer function between the parameter error  $\tilde{\theta}$  and the tracking error  $e$  is strictly

positive real (SPR) [1], the adaptation rule for the controller gain  $\theta$  is given by

$$\dot{\theta} = -\Gamma e_1 \omega \operatorname{sgn}(\rho^*) \quad (8)$$

where  $\Gamma$  is a positive gain.

**2.3. Relative degree  $n = 2$ .** In the standard adaptive control scheme, the control  $U_{mr}$  is structured as

$$U_{mr} = \theta^T \omega + \dot{\theta}^T \Phi = \theta^T \omega - \theta^T \Gamma \phi e_1 \operatorname{sgn}(K_p/K_m) \quad (9)$$

where  $\theta = [\theta_1, \theta_2, \theta_3, C_0]^T$  is a vector of adjustable parameters, and is considered as an estimate of a vector of unknown system parameters  $\theta^*$ .

The dynamic of tracking error is

$$e = G_m(s)(s + p_0)\rho^* \tilde{\theta}^T \phi \quad (10)$$

where  $\rho^* = \frac{K_p}{K_m}$  and  $\tilde{\theta} = \theta(t) - \theta^*$  represent the parameter error.  $G_m(s)(s + p_0)$  is strictly proper and Strictly Positive Real (SPR). Now in this case, since the transfer function between the parameter error  $\tilde{\theta}$  and the tracking error  $e$  is SPR, [1] and the adaptation rule for the controller gain  $\theta$  is given

$$\dot{\theta} = \Gamma \phi e_1 \operatorname{sgn}(K_p/K_m) \quad (11)$$

where  $e_1 = y_p - y_m$  and  $\Gamma$  is a positive gain.

The developed adaptive laws and control schemes are based on a plant model that is free of disturbances, noise and unmodeled dynamics. These schemes are to be implemented on actual plants that most likely deviate from the plant models on which their design is based. An actual plant may be infinite dimensional, nonlinear and its measured input and output may be corrupted by noise and external disturbances. It is shown that an adaptive scheme designed for a disturbance free plant model may go unstable under small disturbances.

**3. PI Controller-based Model Reference Adaptive Control.** The PI algorithm remains the most popular approach for industrial process control despite of continual advances in control theory. This is because the PI algorithm has a simple structure which is conceptually easy to understand and implement in practice but also the algorithm provides adequate performance in the vast majority of applications. A PI controller may be considered as an extreme form of a phase lag compensator. The transfer function of PI controller is generally written in the ‘‘Parallel form’’ given by (12)

$$G_{PI}(S) = \frac{U_{pi}(S)}{E(S)} = K_p + \frac{K_i}{S} \quad (12)$$

where  $U_{pi}(s)$  is the control signal acting on the error signal  $E(s)$ ,  $K_p$  is the proportional gain,  $K_i$  is the integral gain and  $T_i$  is the integral time constant. The ‘‘two term’’ functionalities are highlighted by the following.

- The proportional term-providing an overall control action proportional to the error signal through the all pass gain factor.
- The integral term-reducing steady state errors through low frequency compensation by an integrator.

The disturbances and nonlinear component are fed to the plant input of the conventional MRAC, in this case, the tracking error does not reach to zero and the plant output is not tracked with the reference model plant output. The conventional MRAC fails completely under the action of the external disturbances and nonlinearities, where degradation in the performance due to overshoot, is observed. To improve the system performance, the

PI-MRAC scheme is designed. In this scheme, the controller is designed by using parallel combination of the conventional MRAC system and PI controller. The Block diagram of PI-MRAC scheme is shown in Figure 1. The control input  $U_{mp}$  of the plant is given by the following equation,

$$U_{mp} = U_{mr} + U_{pi} \tag{13}$$

where  $U_{mr}$  is the output of the adaptive controller and  $U_{pi}$  is the output of the PI controller.

$$U_{mr} = \theta^T \omega$$

where  $\theta$  is the update law vector and  $\omega$  is the parameter vector.

The input to the PI controller is the error which is the difference between the plant output  $y_p(t)$  and the reference model output  $y_m(t)$ . In this case also, the disturbance (random noise signal) and nonlinear component is fed to the input of the plant. The PI-MRAC improves the system performance compared to the conventional MRAC scheme. However, due to the continuous variations in the system parameters and the operating conditions, in addition to the nonlinearities present in the system, PI-MRAC scheme may not be able to provide the required standard performance.

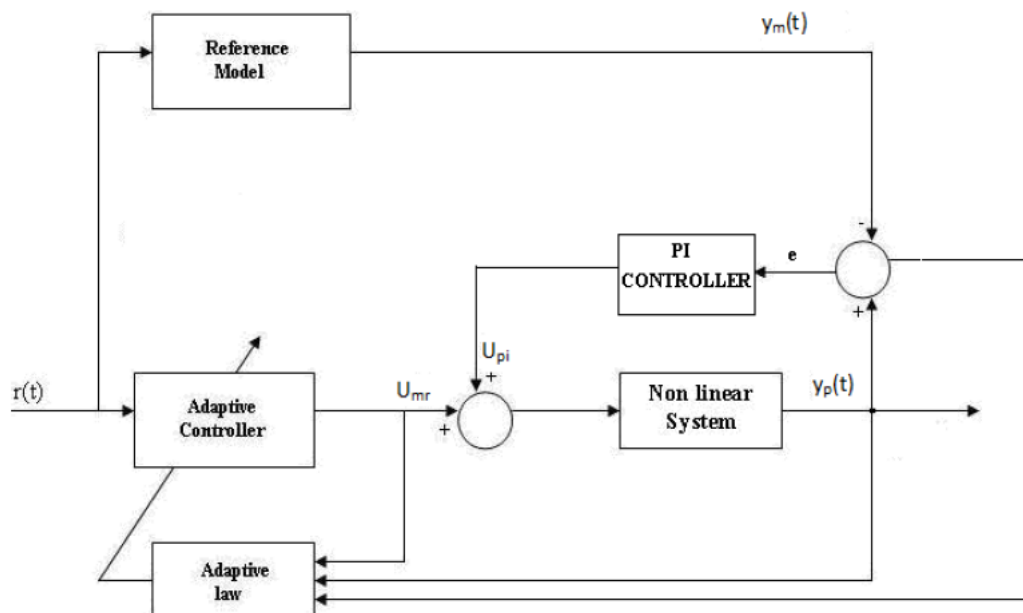


FIGURE 1. Block diagram of PI-MRAC scheme

**4. Fuzzy Logic Controller-based Model Reference Adaptive Controller.** Various applications of Fuzzy Logic (FL) have shown a fast growth for the past few years. The FLC has become popular in the field of industrial control applications for solving control, estimation, and optimization problems. In this section the FPI-MRAC scheme is proposed to replace the classical PI controller used in PI-MRAC scheme by a Fuzzy Logic Controller (FLC) and it is used for error minimization. In the PI-MRAC scheme, the PI controller generates a quantity so as to minimize a specified error. Therefore, the FLC can replace the conventional PI controller to solve the optimization problem. A Fuzzy Logic Controller-based Model Reference Adaptive Control (FLC-MRAC) scheme is proposed to improve the system performance. The proposed controller structure of the FPI-MRAC is shown in Figure 2 which consists of a parallel combination of MRAC and FLC. While the MRAC forces the plant output to closely follow the output of the model which represents

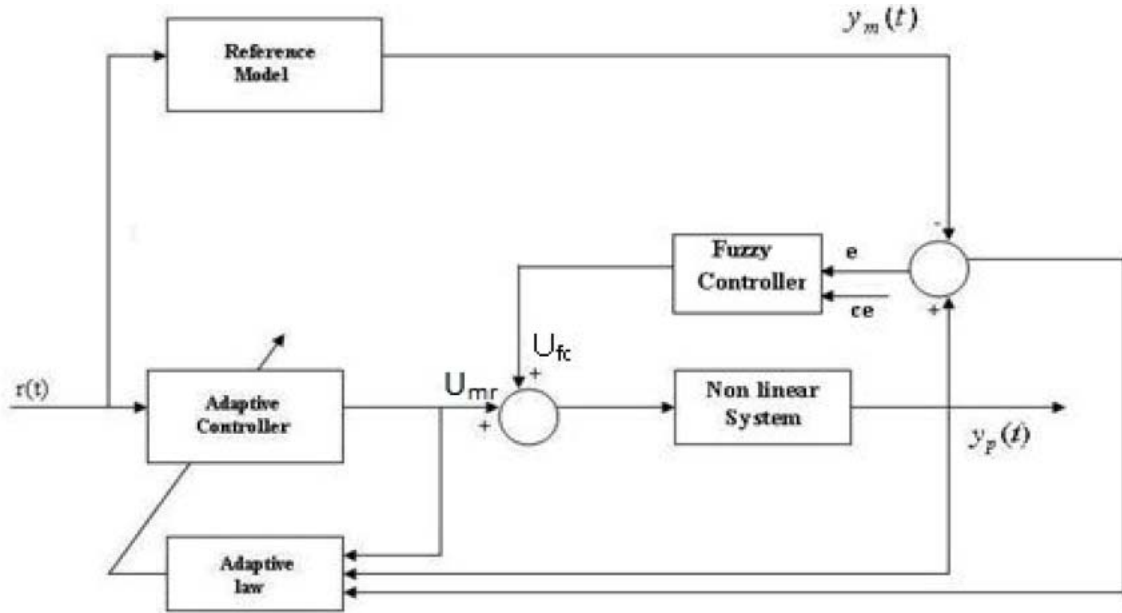


FIGURE 2. Block diagram of proposed FPI-MRAC scheme

the desired closed loop behavior, and the FLC is used in various operating conditions, the objective of the FL control is to determine the control signal for controlling nonlinear processes. The error and the change in error are given input to the FLC. The rules and membership function of FLC are formed from the input and output waveforms of PI controller of designed PI-MRAC scheme. This scheme is restricted to a case of Single Input Single Output (SISO) control, noting that the extension to Multiple Input Multiple Output (MIMO) is possible. To keep the plant output  $y_p$  in track of the reference model output  $y_m$ , it is synthesized to control input  $U_{mf}$  by the following equation

$$U_{mf} = U_{mr} + U_{fc} \quad (14)$$

where  $U_{mr}$  is the output of the adaptive controller and  $U_{fc}$  is the output of the fuzzy logic controller

$$U_{mr} = \theta^T \omega \quad (15)$$

$$\theta = [\theta_1, \theta_2, \theta_3, C_0]^T, \quad \omega = [\omega_1, \omega_2, y_p, r]^T$$

where  $\theta$  is the update law vector, and  $\omega$  is the parameter vector.

Stability of the system and adaptability are then achieved by an adaptive control law  $U_{mr}$  tracking the system output to a suitable reference model output,  $e = y_p - y_m = 0$  is obtained asymptotically. The FLC provides an adaptive control for better system performance and solution for controlling nonlinear processes. The proposed FLC is a Mamdani-type rule base where the inputs are the error ( $e$ ) and error change ( $ce$ ) which can be defined as

$$e(k) = y_m(k) - y_p(k), \quad ce(k) = e(k) - e(k-1)$$

where  $y_m(k)$  is the response of the reference model at  $k$ th sampling interval,  $y_p(k)$  is the response of the plant output at  $k$ th sampling interval,  $e(k)$  is the error signal at  $k$ th sampling interval,  $ce(k)$  is the error change signal at  $k$ th sampling interval.

FLC consists of three stages: fuzzification, rule execution, and defuzzification. In the first stage, the crisp variables  $e(kT)$  and  $ce(kT)$  are converted into fuzzy variables error ( $e$ ) and change in error ( $ce$ ) using the triangular membership functions. Each fuzzy variable is

a member of the subsets with a degree of membership varying between '0' (non-member) and '1' (full member). In the second stage of the FLC, the fuzzy variables error ( $e$ ) and change in error ( $ce$ ) are processed by an inference engine that executes a set of control rules containing in a rule base. In this paper the control rules are formulated using the input and output waveforms of the PI controller of designed PI-MRAC system behavior and the experience of control engineers. The reverse of fuzzification is called defuzzification. The FLC produces the required output in a linguistic variable (fuzzy number). According to real-world requirements, the linguistic variables have to be transformed to crisp output. As the centroid method is considered to be the best well-known defuzzification method, it is utilized in the proposed method. The feature of the proposed scheme is that the FLC can compensate for the nonlinearity of the system to linearize the dynamics from the output of the adaptive controller to the system output, while the role of the adaptive controller is to perform the model-matching for the linearized system.

TABLE 1. Linguistic rule base

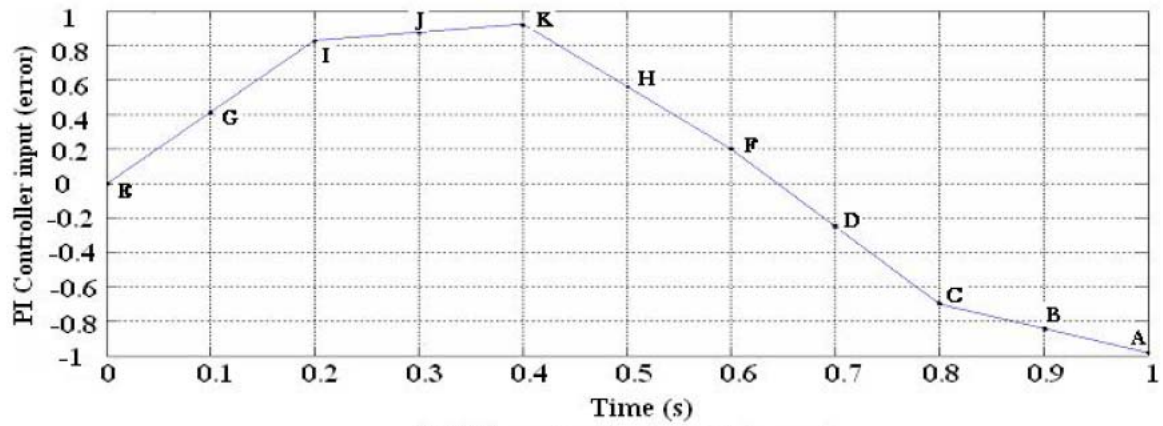
1.	If error is 'A' and change in error is 'A' then the output is 'D'
2.	If error is 'B' and change in error is 'B' then the output is 'F'
3.	If error is 'C' and change in error is 'D' then the output is 'H'
4.	If error is 'D' and change in error is 'F' then the output is 'J'
5.	If error is 'E' and change in error is 'C' then the output is 'A'
6.	If error is 'F' and change in error is 'I' then the output is 'K'
7.	If error is 'G' and change in error is 'C' then the output is B
8.	If error is 'H' and change in error is 'H' then the output is 'I'
9.	If error is 'I' and change in error is 'C' then the output is 'C'
10.	If error is 'J' and change in error is 'E' then the output is 'E'
11.	If error is 'K' and change in error is 'G' then the output is 'G'

**4.1. Construction of fuzzy rules.** In this paper, the fuzzy rules are formulated by using the input and output waveforms of the PI controller of designed PI-MRAC scheme behavior and the experience of control engineers. Let us consider an example of a PI controller input (error), change in error and PI controller output waveforms are given by Figure 3. Fuzzy rules and membership for error ( $e$ ) and change in error ( $ce$ ) and output ( $U_{fc}$ ) can be developed by using PI-controller input and output waveforms are shown in Figure 3. The developed fuzzy rules are given in Table 1. The membership functions for fuzzy variable error ( $e$ ), change in error ( $ce$ ) and output ( $U_{fc}$ ) are shown in Figure 4.

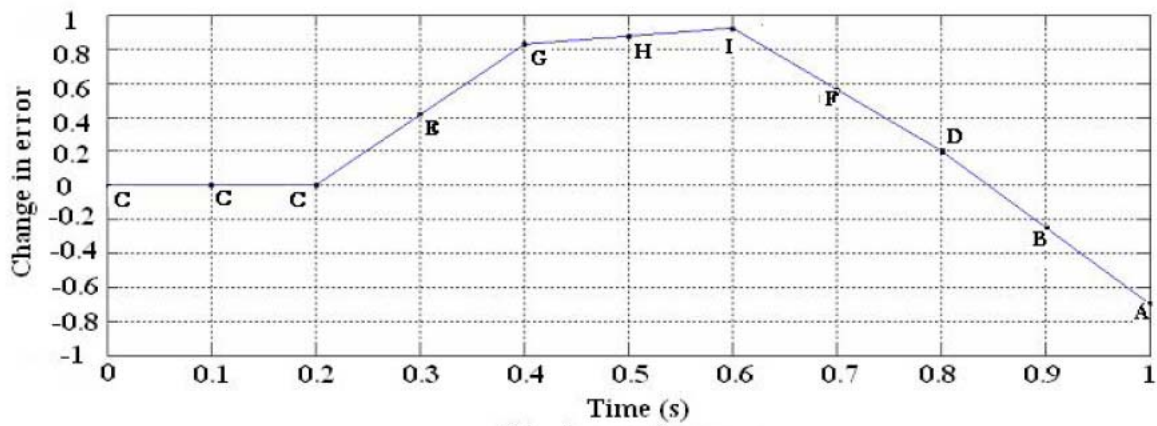
**5. Results and Discussion.** In this section, the results of computer simulation are obtained by using MATLAB environment for the conventional MRAC, PI-MRAC and FLC-MRAC scheme is evaluated by applying inputs of varying magnitude under nonlinearities and disturbances in the plant. The results show the effectiveness of the proposed FLC-MRAC scheme and reveal its performance superiority to the conventional MRAC technique. A detailed simulation comparison between the three schemes has been carried out using with two examples.

**5.1. Example 1.** In this example, backlash nonlinearities and disturbances (random noise signal) in the input of plant are shown in Figure 5.

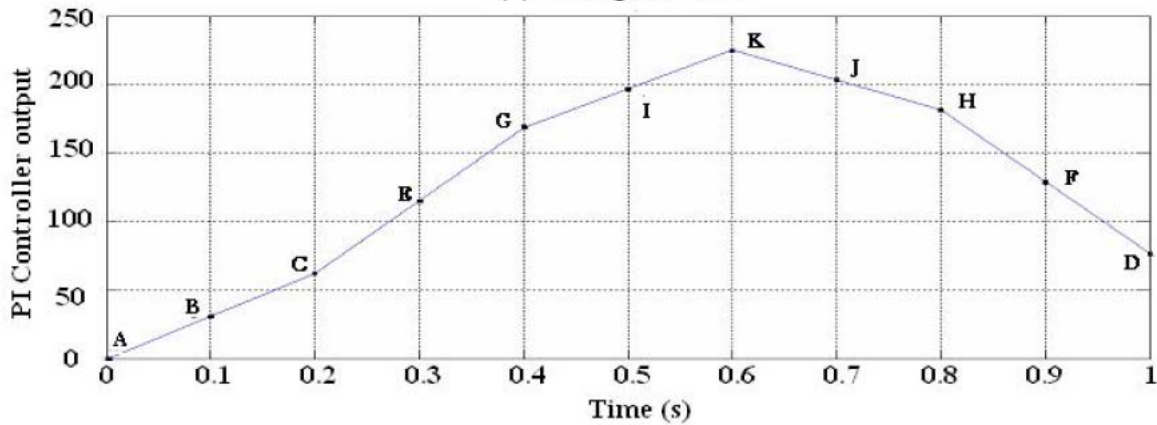
The simulation was carried out for the conventional MRAC, PI-MRAC and FLC-MRAC schemes with MATLAB for time duration  $t = [0, 50]s$ .



(a) PI controller input (error)



(b) change in error



(c) PI controller output (U<sub>pi</sub>)

FIGURE 3. PI controller input error (e), change in error (ce) and output (U<sub>pi</sub>)

Let us consider a linear part of the controlled system and the reference model are given by,

$$G(S) = \frac{2S + 5}{S^3 + 6S^2 + 7S - 4}, \quad G_M(S) = \frac{S + 2.5}{S^3 + 6S^2 + 11S + 6}$$

which have relative degree  $n^* = 2$ .

The input to the reference model is chosen as  $r(t) = 15 \sin 4.9t$ .



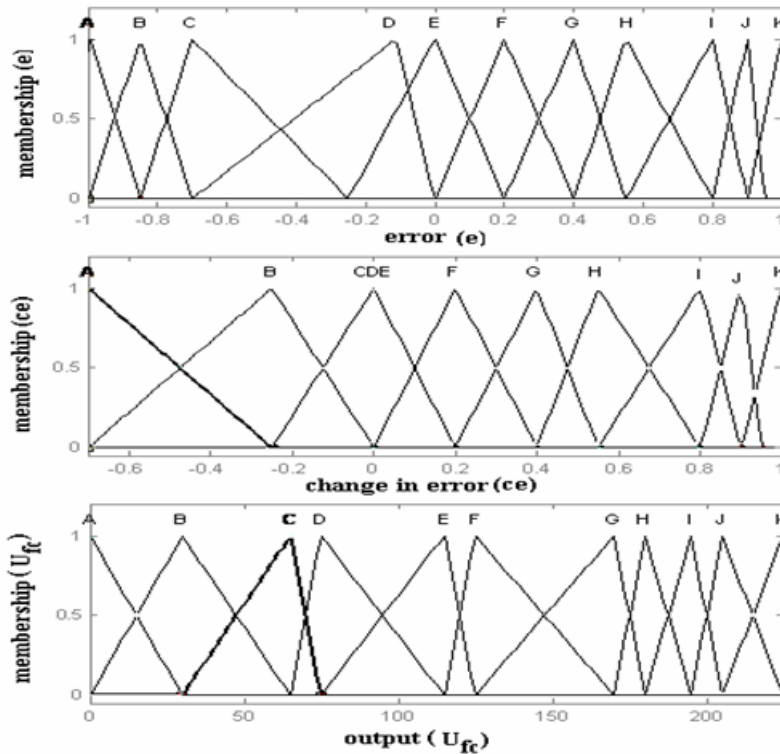


FIGURE 4. Membership functions for fuzzy variable error (e), change in error (ce) and output ( $U_{fc}$ )

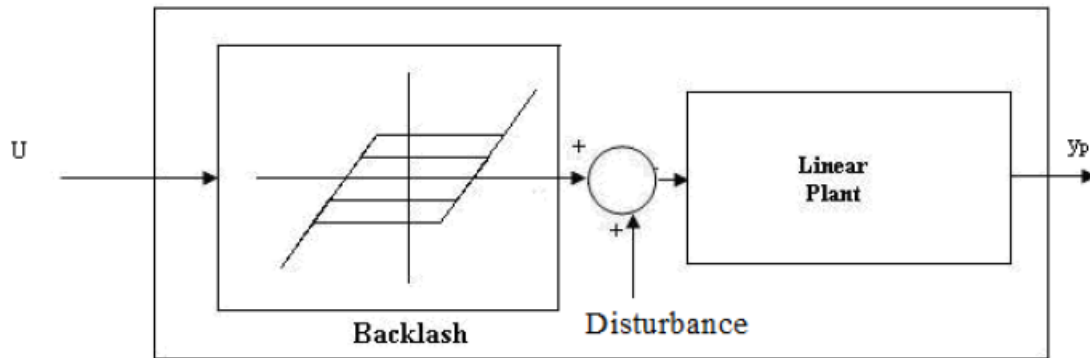


FIGURE 5. Nonlinear system

The output error is  $e = y_p - y_m$  and the  $U_{mr}$  is the control input of the plant for conventional MRAC denoted by

$$U_{mr} = \theta^T \omega + \dot{\theta}^T \Phi = \theta^T \omega - \theta^T \Gamma \phi e_1 \text{sgn}(K_p/K_m)$$

where  $\theta = [\theta_1, \theta_2, \theta_3, C_0]^T$  is the update law vector,  $\omega = [\omega_1, \omega_2, y_p, r]^T$  is the regressor vector and  $\dot{\omega}_1 = F\omega_1 + gu_p$ ,  $\dot{\omega}_2 = F\omega_2 + gy_p$  where  $F$  is an  $(n - 1) * (n - 1)$  stable matrix such that  $\det(SI - F)$  is a Hurwitz polynomial whose roots include the zeros of the reference model and that  $(F, g)$  is a controllable pair.

The initial value of the conventional MRAC scheme controller parameters are chosen as  $\theta(0) = [0.5, 0, 0, 0]^T$ .

The PI controller gains can be selected as high as possible but are limited by the noise. In the PI-MRAC scheme, the value of the PI controller gains  $K_p = 10$  and  $K_i = 75$ , are

shown to provide a better performance for the PI-MRAC scheme. The  $U_{mp}$  is the control input of the plant for the PI-MRAC scheme

$$U_{mp} = U_{mr} + U_{pi}$$

The simulink model of the PI-MRAC scheme developed is given in Figure 6. To obtain optimal performance compared to PI-MRAC scheme, the proposed FLC-MRAC scheme is employed. In FLC-MRAC scheme, the fuzzy variables ‘e’ and ‘ce’ are processed by an inference engine that executes a set of control rules containing in  $(6 \times 6)$  rule base as shown in Table 2.

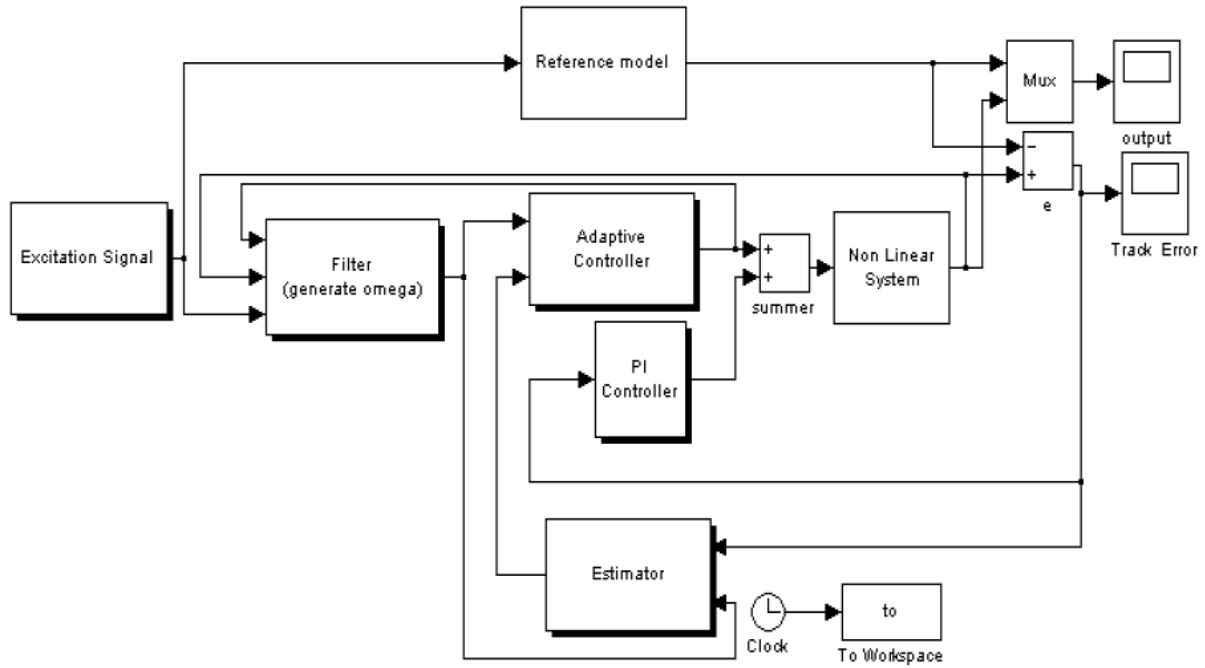


FIGURE 6. Simulink of the PI-MRAC scheme

TABLE 2. Linguistic rule base

e \ ce	NH	NL	ZE	PS	PM	PH
NH	NH	ZE	ZE	PS	NL	PH
NL	NH	PS	ZE	PS	NH	NL
ZE	PS	ZE	ZE	PS	PS	PM
PS	NH	PS	ZE	PS	PS	NH
PM	NH	ZE	ZE	PS	PM	PH
PH	NL	ZE	ZE	NL	NH	PH

The fuzzy rules and membership functions are formulated using the input and output waveforms of the PI controller of designed PI-MRAC scheme and the experience of control engineers. Each variable of the FLC has six membership functions. The fuzzy sets are used as NH (Negative High), NL (Negative Large), ZE (Zero), PS (Positive Small), PM (Positive Medium) and PH (Positive High).

The  $U_{mf}$  is the control input of the plant for the FLC-MRAC scheme.

$$U_{mf} = U_{mr} + U_{fc}$$

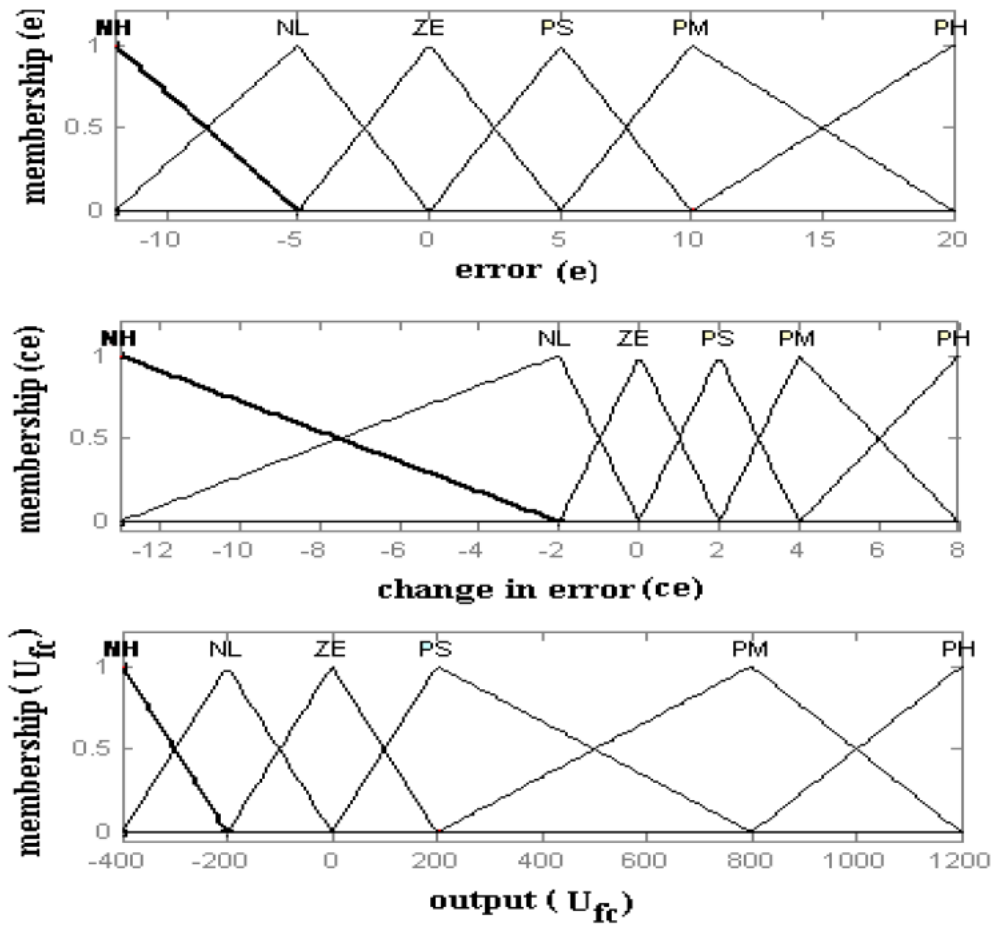


FIGURE 7. Membership functions for fuzzy variable error ( $e$ ), change in error ( $ce$ ) and output ( $U_{fc}$ )

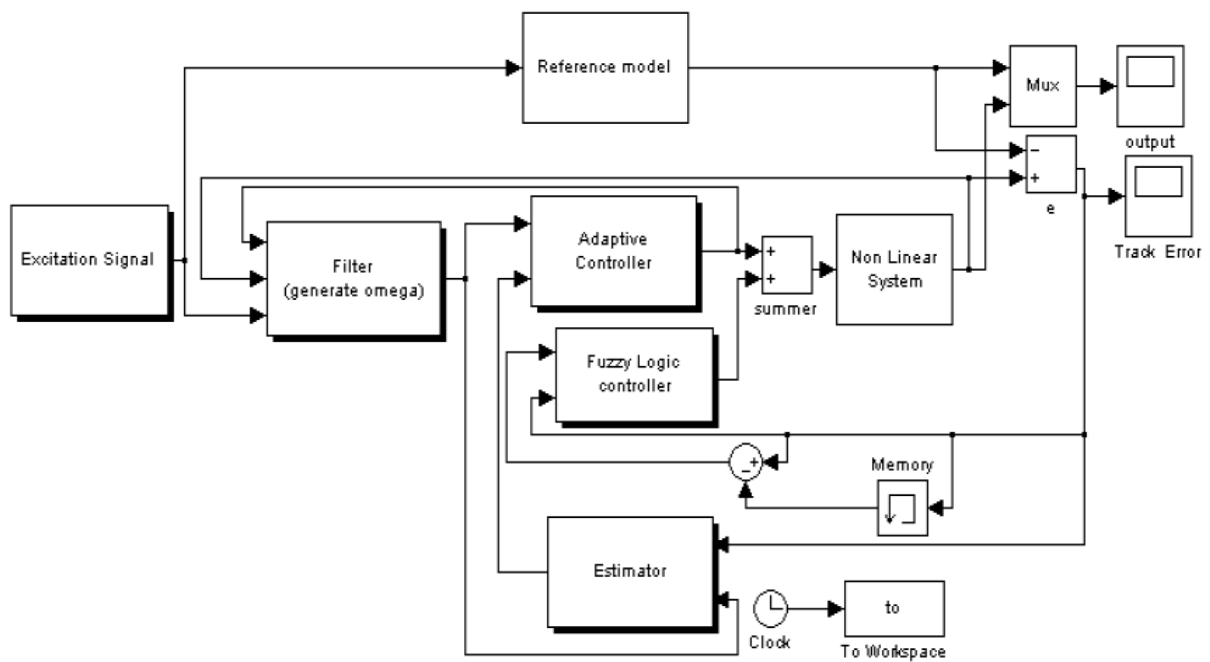


FIGURE 8. Simulink model of the FLC-MRAC scheme

The membership functions for fuzzy variable error ( $e$ ), change in error ( $ce$ ) and output ( $U_{fc}$ ) are shown in Figure 7. The simulink model of the FLC-MRAC scheme is given in Figure 8. Figures 9-11 show the performance of the MRAC, PI-MRAC and FLC-MRAC scheme for Example 1 with input  $r(t) = 15 \sin 4.9t$  under nonlinearities and disturbance in the plant.

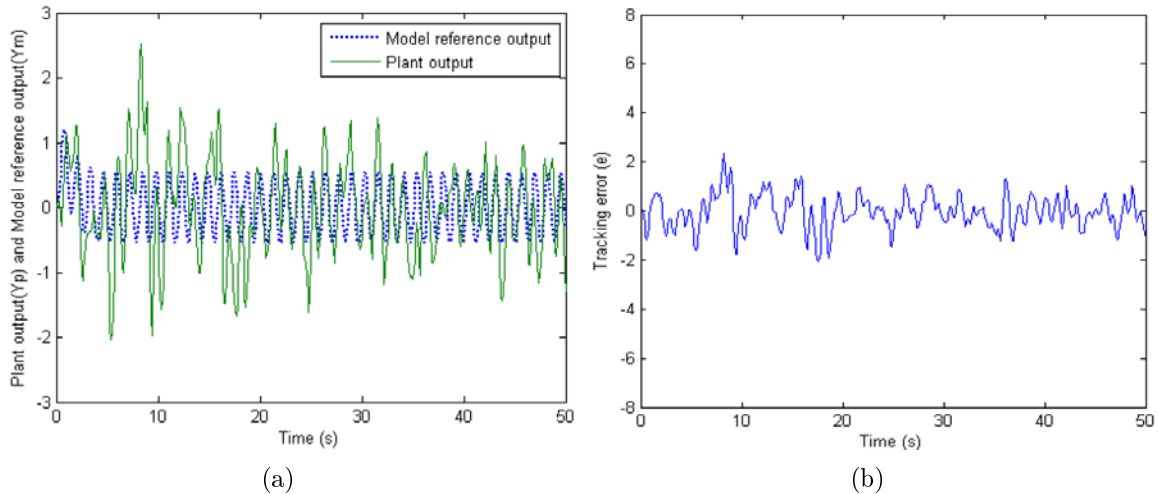


FIGURE 9. Response of the MRAC scheme of Example 1: (a) plant and model reference response, (b) tracking error

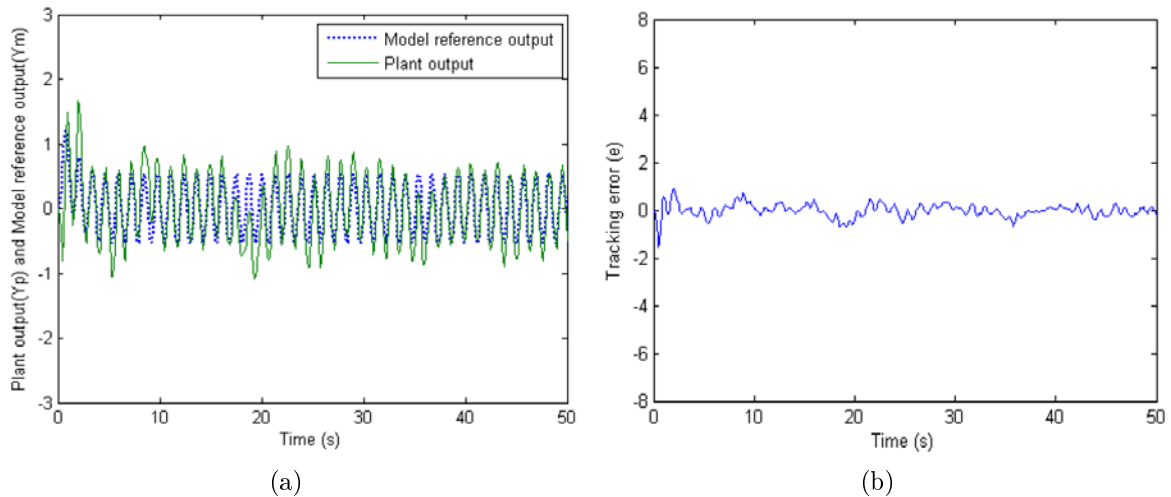


FIGURE 10. Response of the PI-MRAC scheme of Example 1: (a) plant and model reference response, (b) tracking error

**5.2. Example 2.** In this example, dead zone nonlinearities and disturbances (random noise signal) in the input of plant.

Let us consider a linear part of the controlled system and the reference model are given by,

$$G(S) = \frac{S + 1}{S^2 + 3S - 10}, \quad G_M(S) = \frac{1}{S + 1}$$

which have relative degree  $n^* = 1$ .

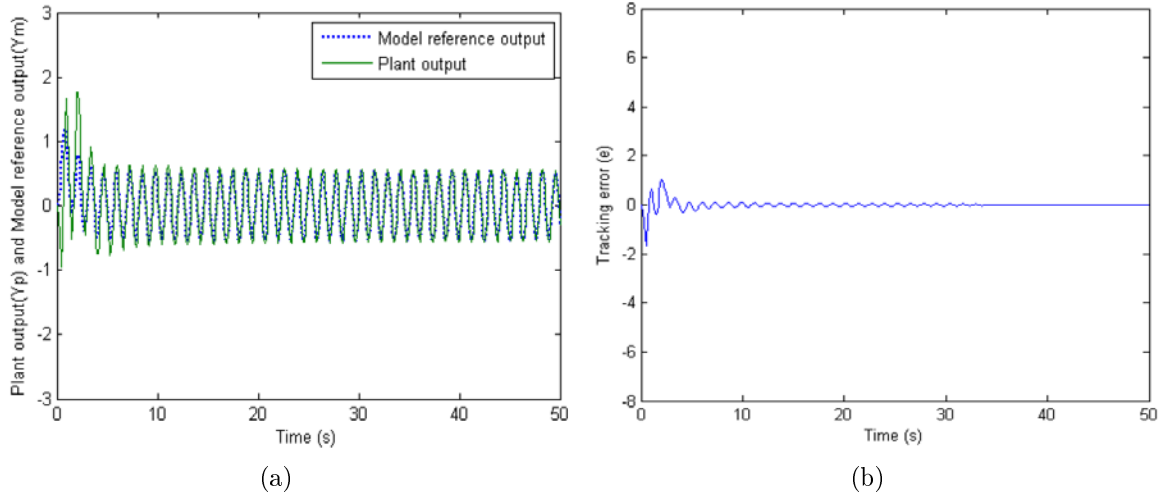


FIGURE 11. Responses of the FLC-MRAC scheme of Example 1: (a) plant and model reference response, (b) tracking error

The input to the reference model is chosen as  $r(t) = 15 + 12 \sin 0.7t + 35 \cos 5.9t$ .

The initial value of conventional MRAC scheme the controller parameters are chosen as  $\theta(0) = [3, 18, -8, 3]^T$ .  $U_{mr}$  is the control input of the plant for conventional MRAC denoted by  $U_{mr} = \theta^T \omega$ .

In the PI-MRAC scheme, the value for the PI controller gains  $K_p$  and  $K_i$  are equal to 12 and 95 respectively. In the FLC-MRAC scheme, seven linguistic variables are used for the input variable error and change in error. They are Extremely Negative (EN), High Negative (HN), Medium Negative (MN), Small Negative (SN), Zero (ZE), Medium Positive (MP) and High Positive (HP). The seven linguistic variables are used for the output variable are Very Low (VL), Low (L), Nearly Low (NL), Medium (M), Medium high (MH), High (H) and Extremely Positive (EP). Table 3 shows the fuzzy rule base with 49 rules. The input and output membership functions are as shown in Figure 12.

TABLE 3. Linguistic rule base

$e$ \ $ce$	EN	HN	MN	SN	ZE	MP	HP
EN	L	H	M	MH	MH	L	H
HN	H	M	M	MH	MH	H	H
MN	M	M	MH	MH	MH	M	H
SN	M	MH	MH	MH	MH	MH	L
ZE	M	MH	MH	MH	VL	M	EH
MP	H	H	M	MH	MH	H	L
HP	L	L	M	MH	MH	L	NL

Figures 13-15 show the performance of the MRAC, PI-MRAC and FLC-MRAC scheme for Example 2 with input  $r(t) = 15 + 12 \sin 0.7t + 35 \cos 5.9t$  under nonlinearities and disturbance in the plant. The results show the effectiveness of the FLC-MRAC scheme to force the plant to follow the model, under uncertainties. Extensive simulation tests were carried out to compare the three adaptation schemes: conventional MRAC, PI-MRAC scheme and FLC-MRAC scheme. In the simulation results of conventional MRAC, PI-MRAC and FLC-MRAC schemes, the dotted line and solid line represents the model reference trajectory and plant trajectory respectively. In conventional MRAC scheme,

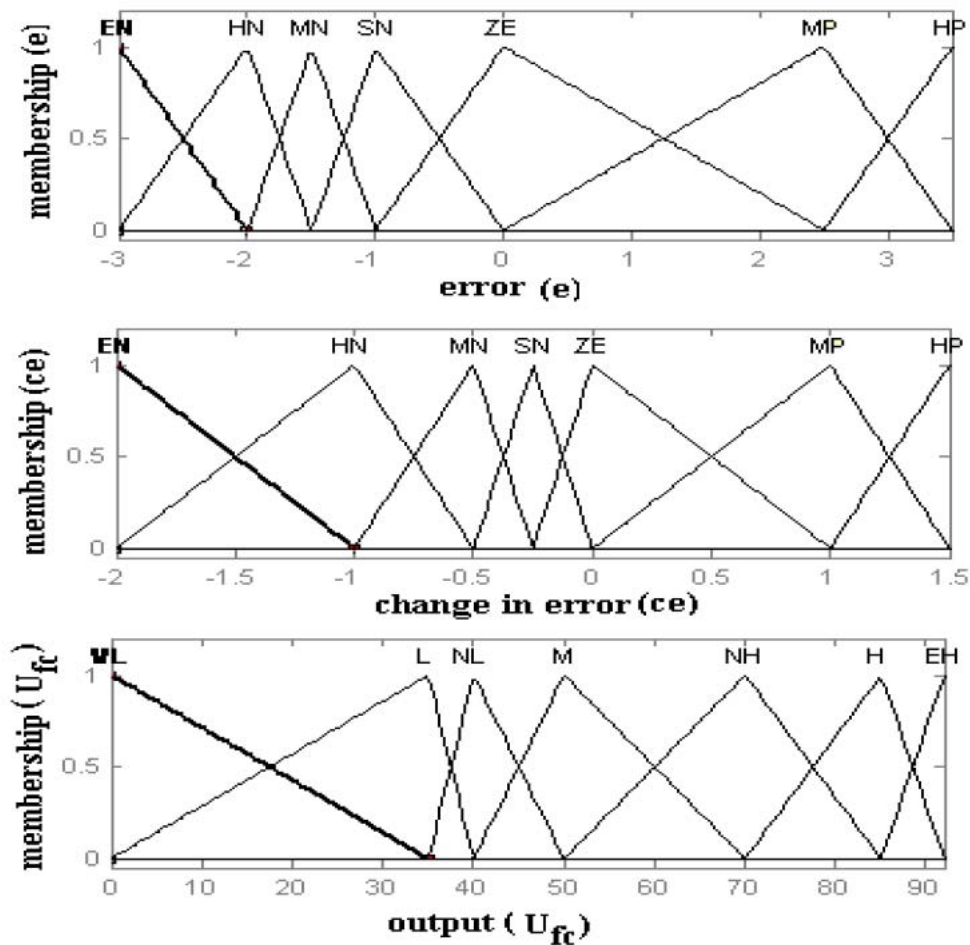


FIGURE 12. Membership functions for fuzzy variable error ( $e$ ), change in error ( $ce$ ) and output ( $U_{fc}$ )

the plant output is poor with large overshoots and oscillations as shown in Figures 9 and 13. Figures 10 and 14 show the response of the PI-MRAC scheme. In this case, the overshoots and the oscillations are much smaller, yielding a much better performance than the conventional MRAC scheme.

However, due to the continuous variation in the system parameters and the operating conditions, in addition to the nonlinearities present in the system, fixed-gain PI-MRAC scheme may not be able to provide the required performance. In the proposed FLC-based MRAC scheme, the plant output has tracked with the reference model output and the tracking error becomes zero within few seconds with less control effort as shown in Figures 11 and 15. In conventional MRAC, the plant output does not track the reference model output. Therefore, conventional MRAC fails completely under the action of the external disturbances and nonlinearities, where degradation in the performance is observed due to high peak overshoot.

The responses performed by the control algorithm only with the MRAC scheme are observed to be inferior to that of the control algorithm both with FLC and the MRAC scheme. Also, the response of the control algorithm only with the MRAC scheme shows large overshoot and oscillation. Further, the response of the output performed by the control algorithm both with the FLC and the MRAC scheme shows more satisfactory results for the bounded disturbances with unknown as well as time-varying characteristics

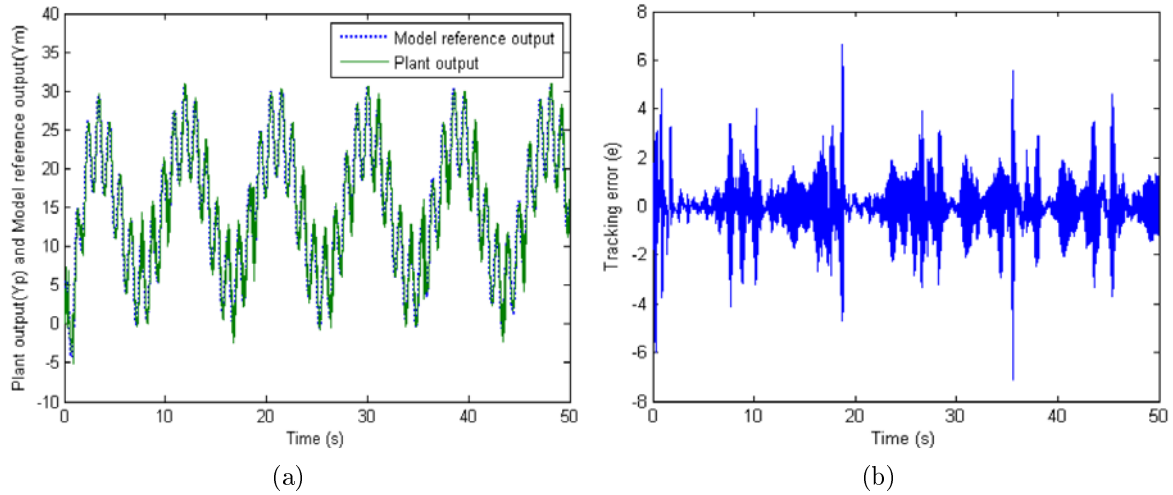


FIGURE 13. Response of the MRAC scheme of Example 2: (a) plant and model reference response, (b) tracking error

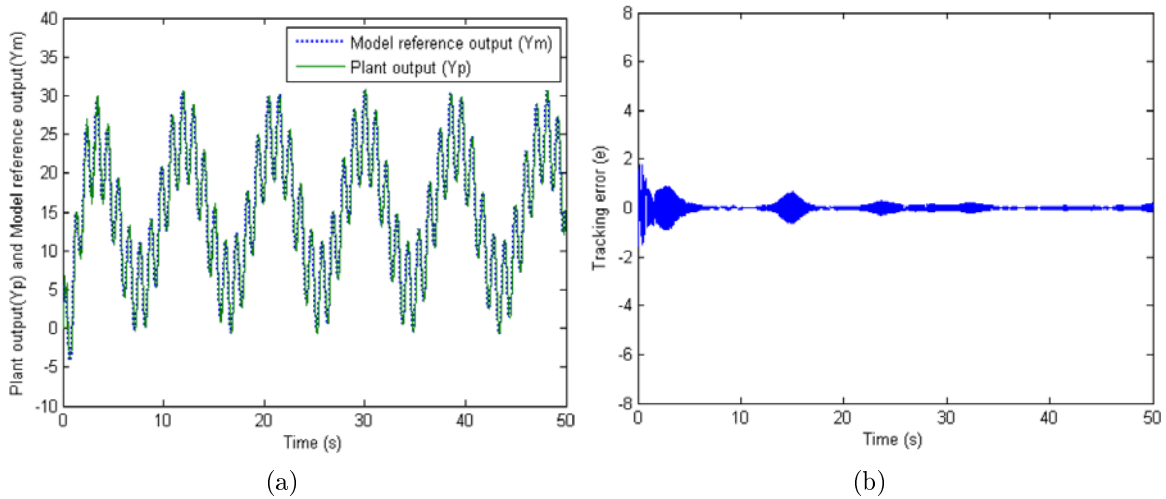


FIGURE 14. Response of the PI-MRAC scheme of Example 2: (a) plant and model reference response, (b) tracking error

than that of the control algorithm only with the MRAC scheme. From the simulation results, because of the existing nonlinearities and bounded disturbances, the controlled system using the control algorithm only using the MRAC scheme will be unstable. But when using the FLC and the MRAC scheme in coordination in which the control law is used to cope with nonlinearities and bounded disturbances, the controlled system can be robustly stabilized all the time. From the above discussions, the proposed control algorithm both with the FLC and the conventional MRAC scheme can be a promising way to tackle the problem of controlling the nonlinear systems and bounded time-varying disturbances. From the above simulations, it is shown that the control algorithm using only the MRAC scheme will not stabilize the nonlinear controlled systems with disturbances. From Figures 11 and 15, it is seen that the control algorithm both with the FLC and the MRAC scheme working in coordination can cope up with the uncertain dynamic system and bounded disturbances, but the control algorithm without the fuzzy compensating control cannot. Compared with the conventional MRAC, PI-MRAC scheme and

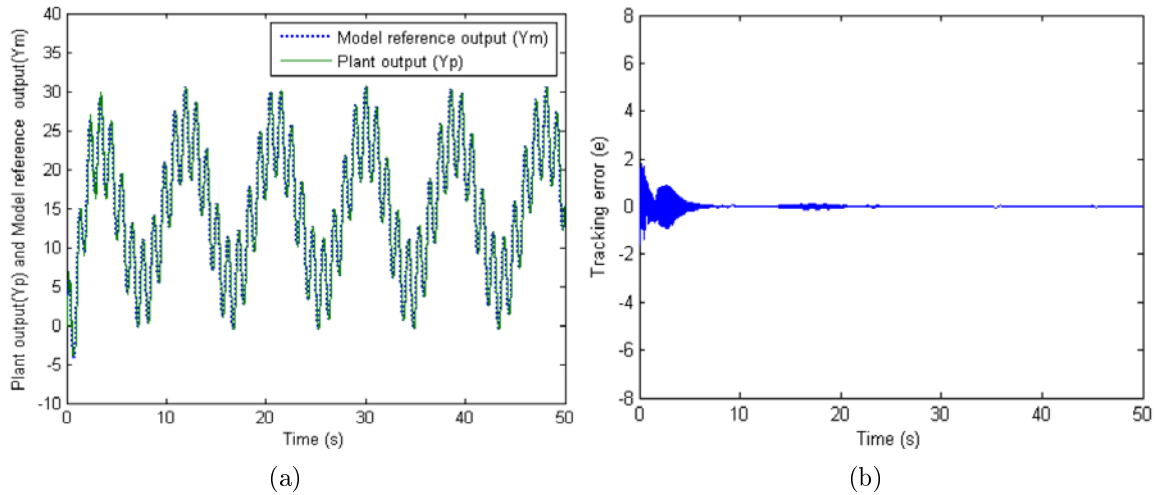


FIGURE 15. Response of the FLC-MRAC scheme of Example 2: (a) plant and model reference response, (b) tracking error

FLC-MRAC scheme still show a faster response during transients. Moreover, the FLC-MRAC scheme shows faster response compared to the PI-MRAC scheme. An optimal response was obtained for the FLC-MRAC scheme compared with the PI MRAC scheme. The proposed FLC-MRAC scheme shows better control results compared to those by the conventional MRAC and PI-MRAC system.

From these simulation results it is observe that:

1. In conventional MRAC scheme, the plant output does not track the reference model output. Therefore conventional MRAC scheme fails completely under the action of the external disturbances and nonlinearities, where degradation in the performance is observed due to high peak overshoot.
2. The PI-MRAC scheme reduces the overshoots and the oscillations compared to those of the conventional MRAC scheme. In the PI-MRAC scheme, the plant output nearly tracks the reference model output. However, it will not be able to provide the required performance in terms of steady state and transient performances.
3. The proposed FPI-MRAC scheme design approach can keep the plant output in track with the reference model and tracking error becomes zero in 6 seconds. The proposed FPI-MRAC scheme gives better performances in terms of steady state error, settling time and overshoot.

**5.3. Implementation issue.** The proposed method can be widely used in most of the industrial nonlinear and complex applications such as machine tools, industrial robot control, position control, and other engineering practices. The proposed FLC-MRAC scheme is relatively simple and does not require complex mathematical operations. It can be readily implemented using conventional microprocessors or microcontrollers. The execution speed of the FLC-MRAC scheme can be improved by using advanced processors such as reduced instruction set computing (RISC) processors or digital signal processors (DSP's) or fuzzy logic ASIC's (application specific integrated circuits).

**6. Conclusion.** In this paper, an FLC-MRAC scheme is proposed to replace the fixed-gain PI controller of PI-MRAC scheme by a FL strategy. In FLC-MRAC the fuzzy rules and membership functions are formed from the input and output waveforms of PI controller of PI- MRAC scheme. A detailed simulation comparison between the conventional



MRAC, PI-MRAC and FLC-MRAC schemes has been carried out using the two examples. The FPI-MRAC scheme improves both the steady state and transient performances compared to other existing schemes. Hence it can be concluded that the proposed FPI-MRAC scheme has more robust performance than the other schemes do. In proposed FPI-MRAC scheme, the system output tracks very closely the reference model in spite of the disturbances and nonlinearities. Thus the FPI-MRAC controller is found to be extremely effective, efficient and useful. Due to its simple operation, the proposed FPI-MRAC scheme can be readily implemented using conventional microprocessors.

## REFERENCES

- [1] K. J. Astrom and B. Wittenmark, *Adaptive Control*, 2nd Edition, Addison-Wesley, 1995.
- [2] P. A. Ioannou and J. sun, *Robust Adaptive Control*, Prentice-Hall, Upper Saddle River, NJ, 1996.
- [3] J. Dong, Y. Wang and G.-H. Yang, Control synthesis of continuous time T-S fuzzy systems with local nonlinear models, *IEEE Trans. Fuzzy Syst.*, vol.39, no.5, pp.1245-1258, 2009.
- [4] J.-H. Park, G.-T. Park, S.-H. Huh, S.-H. Kim and C.-J. Moon, Direct adaptive self-structuring fuzzy controller for nonaffine nonlinear system, *Fuzzy Sets and Systems*, vol.153, no.3, pp.429-445, 2005.
- [5] N. Al-Holou, T. Lahdhiri, D. S. Joo, J. Weaver and F. Al-Abbas, Sliding mode neural network inference fuzzy logic control for active suspension systems, *IEEE Trans. Fuzzy Syst.*, vol.10, pp.234-246, 2002.
- [6] R.-J. Wai, M.-A. Kuo and J.-D. Lee, Cascade direct adaptive fuzzy control design for a nonlinear two-axis inverted-pendulum servomechanism, *IEEE Trans. Syst., Man, Cybern., Part B*, vol.38, no.2, pp.439-454, 2008.
- [7] T.-H. S. Li, S.-J. Chang and W. Tong, Fuzzy target tracking control of autonomous mobile robots by using infrared sensors, *IEEE Trans. Fuzzy Systems*, vol.12, no.4, pp.491-501, 2004.
- [8] K. Tanaka and M. Sano, A robust stabilization problem of fuzzy control systems and its application to backing up control of a truck trailer, *IEEE Trans. Fuzzy Syst.*, vol.2, no.1, pp.119-134, 1994.
- [9] S. Labiod and T. M. Guerra, Adaptive fuzzy control of a class of SISO nonaffine nonlinear systems, *Fuzzy Sets and Systems*, vol.158, no.10, pp.1126-1137, 2007.
- [10] G. Feng, A survey on analysis and design of model-based fuzzy control systems, *IEEE Trans. Fuzzy Syst.*, vol.14, no.5, pp.676-697, 2006.
- [11] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, Wiley, New York, NY, 2001.
- [12] H. O. Wang, K. Tanaka and M. Griffin, An approach to fuzzy control of nonlinear systems: Stability and design issues, *IEEE Trans. Fuzzy Syst.*, vol.4, no.1, pp.14-23, 1996.
- [13] K. Y. Lian and J. J. Liou, Output tracking control for fuzzy systems via output feedback design, *IEEE Trans. Fuzzy Syst.*, vol.14, no.5, pp.628-639, 2006.
- [14] M. A. Khanesar and M. Teshnehlab, Direct fuzzy model reference controller for SISO nonlinear plants using observer, *International Journal of Innovative Computing, Information and Control*, vol.6, no.1, pp.297-306, 2010.
- [15] H. Han, Adaptive fuzzy controller for a class of nonlinear systems, *International Journal of Innovative Computing, Information and Control*, vol.1, no.4, pp.727-742, 2005.
- [16] B. Chen, X. Liu, K. Liu and C. Lin, Fuzzy-approximation-based adaptive control of strict-feedback nonlinear systems with time delays, *IEEE Trans. Fuzzy Syst.*, vol.18, no.5, pp.883-892, 2010.
- [17] C.-W. Chen and P.-C. Chen, GA-based adaptive neural network controllers for nonlinear systems, *International Journal of Innovative Computing, Information and Control*, vol.6, no.4, pp.1793-1803, 2010.
- [18] S. Mir, M. E. Elbuluk and D. S. Zinger, PI and fuzzy estimators for tuning the stator resistance in direct torque control of induction machines, *IEEE Trans. Power Electron.*, vol.13, no.2, pp.279-287, 1998.
- [19] B. Karanayil, M. F. Rahman and C. Grantham, PI and fuzzy estimators for on-line tracking of rotor resistance of indirect vector controlled induction motor drive, *Proc. of IEEE Int. Electr. Mach. Drives Conf.*, pp.820-825, 2001.
- [20] P. Vas, *Artificial-Intelligence-Based Electrical Machines and Drives-Application of Fuzzy, Neural, Fuzzy-Neural and Genetic Algorithm Based Techniques*, Oxford Univ. Press, New York, NY, 1999.

- [21] H. C. Cho, K. S. Lee and M. S. Fadali, Adaptive control of PMSM systems with chaotic nature using lyapunov stability based feedback linearization, *International Journal of Innovative Computing, Information and Control*, vol.5, no.2, pp.479-488, 2009.
- [22] Q.-K. Li, X.-J. Liu, J. Zhao and X.-M. Sun, Observer based model reference output feedback tracking control for switched linear systems with time delay: Constant delay case, *International Journal of Innovative Computing, Information and Control*, vol.6, no.11, pp.5047-5059, 2010.
- [23] J. R. Layne and K. M. Passino, Fuzzy model reference learning control for cargo ship steering, *IEEE Contr. Syst. Mag.*, vol.13, no.12, pp.23-34, 1993.
- [24] J. T. Spooner and K. Passino, Stable adaptive control using fuzzy systems and neural networks, *IEEE Trans. Fuzzy Syst.*, vol.4, pp.339-359, 1996.
- [25] B. B. Peterson and K. S. Narendra, Bounded error adaptive control, *IEEE Trans. Automat Contr.*, vol.27, pp.1161-1168, 1982.
- [26] S. S. Sastry, Model reference adaptive control: Stability, parameter convergence and robustness, *I.M.A.J. Contr. Inform.*, vol.1, pp.27-66, 1984.