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OPTIMAL INVENTORY MODEL WITH FUZZY PERFECTIVE RATE, DEMAND RATE, AND PURCHASING COST UNDER IMMEDIATE RETURN FOR DEFECTIVE ITEMS

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Abstract. *The purpose of this paper is to explore the inventory control problem with immediate return for defective items in fuzzy senses. First, the crisp case of the proposed model is constructed in terms of annual profit. Based on the features of the model and its practical applications, three parameters in the model: the perfective rate, demand rate and purchasing cost are fuzzified. The Yager's ranking method for fuzzy numbers is then utilized to determine the optimal order quantity. Due to the fact that triangular fuzzy numbers are used extensively, we also provide an expression of the optimal order quantity for the case that all of the three parameters are triangular fuzzy numbers. Finally, based on 125 combinations of triangular fuzzy numbers of the parameters, a numerical example is provided to illustrate the proposed model and assess the effects of fuzziness of the parameters on the optimal solution.*

Keywords: Inventory, Fuzzy, Defective items, Immediate return

1. **Introduction.** In the last few decades, inventory models have been widely applied in industries. However, one of the weaknesses of current inventory models is the unrealistic assumption that all items produced are of good quality [1]. In practice, due to imperfective production process, natural disasters or breakage in transit, the lot sizes produced/received may contain some defective items [2]. Hence, several researchers pay their attention to the inventory models with defective items [3]. Recently, Salameh and Jaber [4] and Cardenas-Barron [5] assumed that the defective items can be sold in a single batch by the end of the 100% screening process. The result indicates that the economic order quantity tends to increase as the average percentage of imperfect quality items increases. Based on the model of Salameh and Jaber [4], many new models for defective items were extended. For instance, Goyal and Cardenas-Barron [6] presented a simplified method to determine the optimal lot size in the model [4]. Chan et al. [7] proposed a non-shortage model similar to the model [4], wherein products can be classified as good quality, good quality after reworking, imperfect quality and scrap. Wee et al. [1] and Eroglu and Ozdemir [9] extended the model [4] to the case with shortage backordering. Chang [2] investigated the model [4] with fuzzy demand and perfective rates. Papachristos and Konstantaras [8] re-studied and developed the sufficient conditions for the models of [4,7]. However, in practice, in addition to the assumption of Salameh and Jaber [4], the imperfective items may be treated by other ways. One of the popular ways is that the defective items may be returned to suppliers directly. In practice, it is very popular for procurement operations of retailers. Hsu and Yu [10] have ever discussed the EOQ model with immediate return for imperfective items.

Another unrealistic assumption made for developing inventory models is the certainty assumptions on the parameters in the models, such as demand rate. The traditional inventory models that include these uncertainties are often based on the concept of randomness, and thus handled by probability theory. However, in practice, there may be a lack of historical data to estimate the probability distributions for the uncertain factors that are modeled by random variables. For such a situation, using linguistic terms such as high, low, or approximately equal to certain number to describe those parameters may be more appropriate. Thus, recently, inventory models have been widely discussed in a fuzzy sense, such as fuzzy demand rate (e.g., [11-13]), fuzzy storing cost (e.g., [14,15]), fuzzy perfective rate [2], fuzzy backorder rate (e.g., [16]), fuzzy shortage cost (e.g., [17,18]), and fuzzy elapsed time of production [19].

The purpose of this paper is to explore the inventory control problem with immediate return for defective items in fuzzy senses, including fuzzy perfective rate, fuzzy demand rate and fuzzy purchasing cost. In the relevant literature, most of studies focused on fuzzifying the first two parameters (perfective rate and fuzzy demand rate). However, due to the imbalance of supply and demand, exchange-rate or price negotiations, the purchasing price of items may also be fuzzy. Note that, due to computational difficulty and complexity, there are few articles exploring an inventory model with three fuzzy parameters.

In this paper, the crisp case of the proposed model is first constructed in terms of annual profit. Then, the crisp model is extended in fuzzy senses. For solving the proposed model, the Yager's ranking method [20] is utilized to rank the fuzzy annual profit and determine the optimal order quantity. Due to the fact that triangular fuzzy numbers are used extensively, we also provide an expression of the optimal order quantity for the case that all of the three parameters (perfective rate, demand rate and purchasing cost) are triangular fuzzy numbers. Finally, based on 125 combinations of triangular fuzzy numbers of those parameters, a numerical example is given to demonstrate the applications of the proposed model and assess the effects of fuzziness of the perfective rate, the demand rate, and the purchasing cost on the optimal solution.

2. Problem Statement and Preliminaries.

In this paper, the crisp inventory model with immediate return for imperfective items is first introduced [10]. The notation used in this paper is as follows:

- λ demand rate (unit/per year)
- s selling price per unit ($s > c$)
- c purchasing cost per unit
- b holding cost rate per unit/per unit time
- h holding cost per unit/per unit time, $h = bc$
- a ordering cost per order
- q perfective rate for each order
- p defective rate for each order ($p = 1 - q$)
- e screening rate (unit/per year)
- w screening cost per unit
- Q order size
- T cycle length
- C expected total cost per cycle
- R expected total revenue per cycle
- P_0 expected total profit per cycle
- P expected total profit per year

In the model, the primary assumptions are: (1) a lot size of Q is replenished instantaneously at the beginning of each inventory cycle. (2) The screening process and demand proceed simultaneously, and the screening rate is greater than the demand rate (i.e., $e > \lambda$). (3) Any of defective items found during the 100% screening process will be returned to the supplier immediately (or at least the retailer is not responsible for the safekeeping of the defectives any longer). The other justifications and assumptions are available in the traditional EOQ model.

The inventory-level behavior of the model can be illustrated as Figure 1, where $[t_1, t_2]$ denotes the time period of a cycle. In the model, it is assumed that (1) a lot size of Q is replenished instantaneously at the beginning of each inventory cycle, (2) a 100% screening process of the lot is started at time t_1 and finished at time t_e. The screening process and demand proceed simultaneously, and the screening rate is greater than the demand rate (i.e., $e > \lambda$), and (3) all imperfective items found during the 100% screening process will be returned to the supplier immediately. Hence, the demand rate during $t_1 \sim t_e$ can be regarded as $\lambda' = \lambda + e(1 - q)$. After the screening process is finished, i.e., at t_e, the demand rate will return to λ and the remaining non-defective items will be depleted at time t_2.

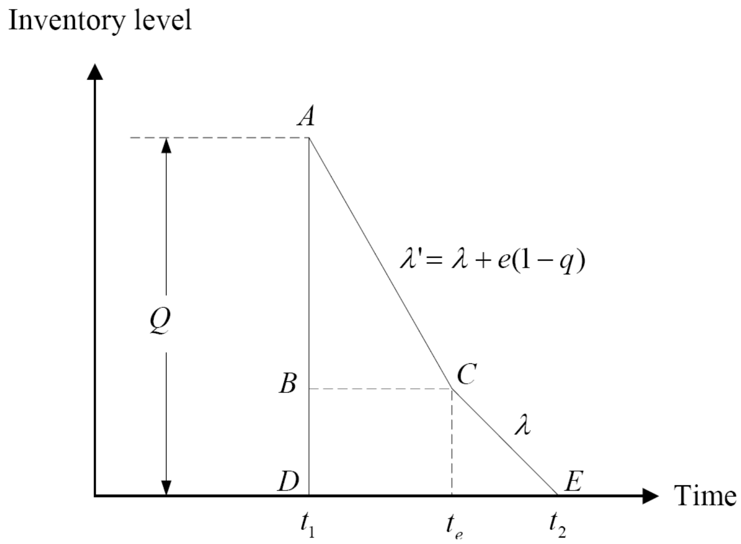

FIGURE 1. The inventory-level behavior of the model with immediate return for defective items

To avoid shortages, we have one more assumption that the number of good items is at least equal to the demand during the screening process. This leads to $e \geq \lambda/q$. The optimal Q is determined by maximizing the following expected total profit per year:

$$P(Q) = P_0(Q) * N,$$

where $P_0(Q)$ is the expected profit of a cycle and N is the number of orders per year.

For ease of exposition, $P_0(Q)$ can be expressed as follows:

$$P_0(Q) = R(Q) - C(Q), \tag{1}$$

where $R(Q)$ and $C(Q)$ denote the revenue and the total cost in each cycle, respectively. It is easily seen that

$$R(Q) = SR + RR, \tag{2}$$

where SR and RR denote the revenues for sale and for return in each cycle, respectively. It is clear that $SR = sQq$ and $RR = cQ(1 - q)$.

Further, $C(Q)$ comprises the following four parts:

$$C(Q) = PC + AC + WC + HC,$$

where PC, AC, WC, and HC denote the procurement cost, ordering cost, screening cost, and holding cost in each cycle, respectively. Obviously, $PC = cQ$, $AC = a$, and $WC = wQ$. To compute HC, we need to calculate the total quantity of inventory in a cycle, which is equal to the sum of the areas of △ABC and □BCDE in Figure 1. By direct algebraic manipulations, we have:

$$V = \frac{Q^2}{2}\left(\frac{q^2}{\lambda} + \frac{(1-q)}{e}\right).$$

Hence,

$$HC = h * V = cb * \frac{Q^2}{2}\left(\frac{q^2}{\lambda} + \frac{(1-q)}{e}\right).$$

The total annual profit, $P(Q)$, thus can be derived as:

$$\begin{aligned} P(Q) &= P_0(Q) \cdot N = [(SR + RR) - (PC + AC + WC + HC)] \cdot N \\ &= \left\{[sQq + cQ(1-q)] - \left[cQ + a + wQ + cb\frac{Q^2}{2}\left(\frac{q^2}{\lambda} + \frac{(1-q)}{e}\right)\right]\right\}\frac{\lambda}{Qq} \\ &= s\lambda - c\lambda + \frac{bQ}{2e} \cdot c\lambda - \frac{bQ}{2} \cdot cq - \left(\frac{a}{Q} + w\right) \cdot \lambda/q - \left(\frac{bQ}{2e}\right) \cdot c\lambda/q. \end{aligned} \tag{3}$$

Taking the first derivative of $P(Q)$ with respect to Q and setting the result to zero, we have:

$$P'(Q) = \frac{bc\lambda}{2e} - \frac{bcq}{2} + \frac{a\lambda}{Q^2 q} - \frac{bc\lambda}{2eq} = 0.$$

By solving the above equation, the optimal solution, termed as Q_c^*, can be found as:

$$Q_c^* = \sqrt{\frac{2a\lambda e}{cb[eq^2 + (1-q)\lambda]}}. \tag{4}$$

It is easy to find the second derivative of $P(Q)$ with respect to Q is:

$$P''(Q) = -\frac{2a\lambda}{Q^3 q} < 0, \text{ for } Q > 0.$$

Thus, the Q_c^* is the global maximum solution of $P(Q)$. Substituting Equation (4) into Equation (3), we can obtain the corresponding total profit $P_c^*(= P(Q_c^*))$. It is easily seen that if the defective rate is zero (i.e., $q = 1$), then Equation (4) will reduce to $Q^* = \sqrt{2a\lambda/cb}$, the traditional EOQ formula.

3. **The Fuzzy Model.**

Suppose the perfective rate, demand rate and purchasing cost are LR-type fuzzy numbers with parameters: $\tilde{q} = [l_q, m_q, u_q]$, $\tilde{\lambda} = [l_\lambda, m_\lambda, u_\lambda]$ and $\tilde{c} = [l_c, m_c, u_c]$, where, $l_q > 0$, $l_\lambda > 0$ and $l_c > 0$. That is, their membership functions can be described as follows:

$$\mu_{\tilde{q}}(x_q) = \begin{cases} L_q(x_q), & l_q \le x_q \le m_q \\ R_q(x_q), & m_q \le x_q \le u_q \\ 0, & \text{otherwise} \end{cases},$$

$$\mu_{\tilde{\lambda}}(x_\lambda) = \begin{cases} L_\lambda(x_\lambda), & l_\lambda \le x_\lambda \le m_\lambda \\ R_\lambda(x_\lambda), & m_\lambda \le x_\lambda \le u_\lambda \\ 0, & \text{otherwise} \end{cases}$$

and

$$\mu_{\tilde{c}}(x_c) = \begin{cases} L_c(x_c), & l_c \le x_c \le m_c \\ R_c(x_c), & m_c \le x_c \le u_c \\ 0, & \text{otherwise} \end{cases},$$

where $L_q(x_q)$ and $R_q(x_q)$ are the left-shape and right-shape functions of $\tilde{q}$; $L_\lambda(x_\lambda)$ and $R_\lambda(x_\lambda)$ are the left-shape and right-shape functions of $\tilde{\lambda}$; and $L_c(x_c)$ and $R_c(x_c)$ are the left-shape and right-shape functions of $\tilde{c}$. Those three membership functions can be depicted by α-level cuts as:

$$\tilde{q}(\alpha) = \left[\min .\mu_{\tilde{q}}^{-1}(\alpha), \max .\mu_{\tilde{q}}^{-1}(\alpha)\right] = \left[L_q^{-1}(\alpha), R_q^{-1}(\alpha)\right], \quad 0 \le \alpha \le 1, \tag{5}$$

$$\tilde{\lambda}(\alpha) = \left[\min .\mu_{\tilde{\lambda}}^{-1}(\alpha), \max .\mu_{\tilde{\lambda}}^{-1}(\alpha)\right] = \left[L_\lambda^{-1}(\alpha), R_\lambda^{-1}(\alpha)\right], \quad 0 \le \alpha \le 1, \tag{6}$$

$$\tilde{c}(\alpha) = \left[\min .\mu_{\tilde{c}}^{-1}(\alpha), \max .\mu_{\tilde{c}}^{-1}(\alpha)\right] = \left[L_c^{-1}(\alpha), R_c^{-1}(\alpha)\right], \quad 0 \le \alpha \le 1. \tag{7}$$

Based on the concept of α-level cut, for $L_q^{-1}(\alpha) > 0$, $L_\lambda^{-1}(\alpha) > 0$, and $L_c^{-1}(\alpha) > 0$ (i.e., $l_q > 0$, $l_\lambda > 0$, $l_c > 0$), several arithmetic operations about the three parameters ($\tilde{q}$, $\tilde{\lambda}$, $\tilde{c}$) can be defined as follows [21]:

(i) $(\tilde{\lambda} + \tilde{q})(\alpha) = [L_{(\lambda+q)}^{-1}(\alpha), R_{(\lambda+q)}^{-1}(\alpha)] = [L_\lambda^{-1}(\alpha) + L_q^{-1}(\alpha), R_\lambda^{-1}(\alpha) + R_q^{-1}(\alpha)]$.

(ii) $(\tilde{\lambda} - \tilde{q})(\alpha) = [L_{(\lambda-q)}^{-1}(\alpha), R_{(\lambda-q)}^{-1}(\alpha)] = [L_\lambda^{-1}(\alpha) - R_q^{-1}(\alpha), R_\lambda^{-1}(\alpha) - L_q^{-1}(\alpha)]$.

(iii) $k \cdot \tilde{\lambda}(\alpha) = [L_{k\lambda}^{-1}(\alpha), R_{k\lambda}^{-1}] = \begin{cases} \left[kL_\lambda^{-1}(\alpha), kR_\lambda^{-1}(\alpha)\right], & k > 0 \\ \left[kR_\lambda^{-1}(\alpha), kL_\lambda^{-1}(\alpha)\right], & k < 0 \end{cases}$, where k is a constant.

(iv) $(\tilde{c} \times \tilde{\lambda})(\alpha) = [L_{(c\times\lambda)}^{-1}(\alpha), R_{(c\times\lambda)}^{-1}(\alpha)] = [L_c^{-1}(\alpha) \cdot L_\lambda^{-1}(\alpha), R_c^{-1}(\alpha) \cdot R_\lambda^{-1}(\alpha)]$.

(v) $(\tilde{\lambda}/\tilde{q})(\alpha) = [L_{(\lambda/q)}^{-1}(\alpha), R_{(\lambda/q)}^{-1}(\alpha)] = [L_\lambda^{-1}(\alpha)/R_q^{-1}(\alpha), R_\lambda^{-1}(\alpha)/L_q^{-1}(\alpha)]$.

(vi) $(\tilde{\lambda} \times \tilde{c}/\tilde{q})(\alpha) = [L_{((\lambda\times c)/q)}^{-1}(\alpha), R_{((\lambda\times c)/q)}^{-1}(\alpha)]$

$$= [L_\lambda^{-1}(\alpha) \cdot L_c^{-1}(\alpha)/R_q^{-1}(\alpha), R_\lambda^{-1}(\alpha) \cdot R_c^{-1}(\alpha)/L_q^{-1}(\alpha)].$$

Referring to Equation (3), since the perfective rate, the demand rate, and the purchasing cost have been fuzzified, the corresponding total profit (denoted by $\tilde{P}(Q)$) is fuzzy as well. By extension principle [11] and Equation (3), the $\tilde{P}(Q)$ can be derived as:

$$\tilde{P}(Q) = s\tilde{\lambda} - (\tilde{c} \times \tilde{\lambda}) + \frac{bQ}{2e} \cdot (\tilde{c} \times \tilde{\lambda}) - \frac{bQ}{2} \cdot (\tilde{c} \times \tilde{q}) - \left(\frac{a}{Q} + w\right)(\tilde{\lambda}/\tilde{q}) - \frac{bQ}{2e}(\tilde{c} \times \tilde{\lambda}/\tilde{q}).$$

Note that the expression for $\tilde{P}(Q)$ consists of five fuzzy numbers: $\tilde{\lambda}$, $(\tilde{c} \times \tilde{\lambda})$, $(\tilde{c} \times \tilde{q})$, $(\tilde{\lambda}/\tilde{q})$, and $(\tilde{c} \times \tilde{\lambda}/\tilde{q})$. Further, the membership function of $\tilde{P}(Q)$ can also be depicted by α-level cut as:

$$\tilde{P}(\alpha) = \left[L_P^{-1}(\alpha), R_P^{-1}(\alpha)\right], \tag{8}$$

where, by Equations (5)$\sim$(7) and the arithmetic operations (i)$\sim$(vi), the $L_p^{-1}(\alpha)$ and $R_p^{-1}(\alpha)$ can be derived as:

$$\begin{cases} L_P^{-1}(\alpha) = sL_\lambda^{-1}(\alpha) - \left[R_c^{-1}(\alpha) \times R_\lambda^{-1}(\alpha)\right] + \dfrac{bQ}{2e}\left[L_c^{-1}(\alpha) \times L_\lambda^{-1}(\alpha)\right] - \dfrac{bQ}{2} \cdot \left[R_c^{-1}(\alpha)\right. \\ \qquad \left. \times R_q^{-1}(\alpha)\right] - \left(\dfrac{a}{Q} + w\right)\left[R_\lambda^{-1}(\alpha)/L_q^{-1}(\alpha)\right] - \dfrac{bQ}{2e}\left[R_c^{-1}(\alpha) \cdot R_\lambda^{-1}(\alpha)/L_q^{-1}(\alpha)\right] \\ R_P^{-1}(\alpha) = sR_\lambda^{-1}(\alpha) - \left[L_c^{-1}(\alpha) \times L_\lambda^{-1}(\alpha)\right] + \dfrac{bQ}{2e}\left[R_c^{-1}(\alpha) \times R_\lambda^{-1}(\alpha)\right] - \dfrac{bQ}{2} \cdot \left[L_c^{-1}(\alpha)\right. \\ \qquad \left. \times L_q^{-1}(\alpha)\right] - \left(\dfrac{a}{Q} + w\right)\left[L_\lambda^{-1}(\alpha)/R_q^{-1}(\alpha)\right] - \dfrac{bQ}{2e}\left[L_c^{-1}(\alpha) \cdot L_\lambda^{-1}(\alpha)/R_q^{-1}(\alpha)\right] \end{cases}. \tag{9}$$

Since different Q results in different $\tilde{P}(Q)$, we can find the maximum $\tilde{P}(Q)$ by ranking the $\tilde{P}(Q)$ with respect to Q. In the literature, there are a lot of ranking methods which have been proposed. One popular method which lends itself to this purpose is Yager's ranking method [20]. This method has an advantage of not requiring the knowledge of the explicit form of the membership functions of the fuzzy numbers, and further, it is very simple to apply [11]. Hence, the Yager's ranking method is employed to find the optimal order quantity in this paper.

The Yager's ranking method is to calculate a ranking index for a fuzzy number from its α-level cut form. For example, for the fuzzy number of total profit $\tilde{P}(=\tilde{P}(Q))$, its Yager's index $I(\tilde{P})$ is defined as:

$$I(\tilde{P}) = \frac{1}{2}\int_0^1 \left[L_P^{-1}(\alpha) + R_P^{-1}(\alpha)\right] d\alpha. \tag{10}$$

Thus, from Equations (8) and (9), the Yager's index of $\tilde{P}$ then can be derived as:

$$\begin{aligned} I(\tilde{P}) = s \cdot I(\tilde{\lambda}) - I(\tilde{c}\times\tilde{\lambda}) + \frac{bQ}{2e} \cdot I(\tilde{c}\times\tilde{\lambda}) - \frac{bQ}{2} \cdot I(\tilde{c}\times\tilde{q}) \\ - \left(\frac{a}{Q} + w\right) \cdot I(\tilde{\lambda}/\tilde{q}) - \frac{bQ}{2e} \cdot I(\tilde{c}\times\tilde{\lambda}/\tilde{q}), \end{aligned}$$

where

$$\begin{cases} I(\tilde{\lambda}) = \frac{1}{2} \cdot \left[\int_0^1 L_\lambda^{-1}(\alpha)d\alpha + \int_0^1 R_\lambda^{-1}(\alpha)d\alpha\right] \\ I(\tilde{c}\times\tilde{\lambda}) = \frac{1}{2} \cdot \left[\int_0^1 L_c^{-1}(\alpha)L_\lambda^{-1}(\alpha)d\alpha + \int_0^1 R_c^{-1}(\alpha)R_\lambda^{-1}(\alpha)d\alpha\right] \\ I(\tilde{c}\times\tilde{q}) = \frac{1}{2}\left[\int_0^1 L_c^{-1}(\alpha)L_q^{-1}(\alpha)d\alpha + \int_0^1 R_c^{-1}(\alpha)R_q^{-1}(\alpha)d\alpha\right] \\ I(\tilde{\lambda}/\tilde{q}) = \frac{1}{2} \cdot \left[\int_0^1 \frac{L_\lambda^{-1}(\alpha)}{R_q^{-1}(\alpha)}d\alpha + \int_0^1 \frac{R_\lambda^{-1}(\alpha)}{L_q^{-1}(\alpha)}d\alpha\right] \\ I(\tilde{c}\times\tilde{\lambda}/\tilde{q}) = \frac{1}{2} \cdot \left[\int_0^1 \frac{L_c^{-1}(\alpha)L_\lambda^{-1}(\alpha)}{R_q^{-1}(\alpha)}d\alpha + \int_0^1 \frac{R_c^{-1}(\alpha)R_\lambda^{-1}(\alpha)}{L_q^{-1}(\alpha)}d\alpha\right] \end{cases}.$$

Taking the first derivative of $I(\tilde{P})$ with respect to Q and setting the result to zero, we have:

$$\frac{dI(\tilde{P})}{dQ} = \frac{b}{2e} \cdot I(\tilde{c}\times\tilde{\lambda}) - \frac{b}{2} \cdot I(\tilde{c}\times\tilde{q}) + \frac{a}{Q^2} \cdot I(\tilde{\lambda}/\tilde{q}) - \frac{b}{2e} \cdot I(\tilde{c}\times\tilde{\lambda}/\tilde{q}) = 0.$$

By solving the above equation, the optimal solution Q^* can then be obtained as follows:

$$Q^* = \left(\frac{2aeI(\tilde{\lambda}/\tilde{q})}{b\left[I(\tilde{c}\times\tilde{\lambda}/\tilde{q}) + eI(\tilde{c}\times\tilde{q}) - I(\tilde{c}\times\tilde{\lambda})\right]}\right)^{\frac{1}{2}}. \tag{11}$$

The global optimality and the uniqueness of Q^* reside in the fact that $I(\tilde{P})$ is concave, which is implied by the following second derivative of $I(\tilde{P})$ with respect to Q:

$$I_Q''(\tilde{P}) = -\frac{2a}{Q^3} \cdot I(\tilde{\lambda}/\tilde{q}) < 0, \text{ for } Q > 0.$$

Let $\tilde{P}^*(=\tilde{P}(Q^*))$ denote the optimal total profit associated with Q^*. Then, from Equations (8), (9), and (11), the $\tilde{P}^*$ can be expressed as:

$$\tilde{P}^*(\alpha) = \left[(L_P^{-1}(\alpha)|Q=Q^*), (R_P^{-1}(\alpha)|Q=Q^*)\right]. \tag{12}$$

Also, the Yager's index of $\tilde{P}^*(Q)$can be found as:

$$I(\tilde{P}^*) = s \cdot I(\tilde{\lambda}) - I(\tilde{c} \times \tilde{\lambda}) + \frac{bQ^*}{2e} \cdot I(\tilde{c} \times \tilde{\lambda}) \\ - \frac{bQ^*}{2} \cdot I(\tilde{c} \times \tilde{q}) - \left(\frac{a}{Q^*} + w\right) \cdot I(\tilde{\lambda}/\tilde{q}) - \frac{bQ^*}{2e} \cdot I(\tilde{c} \times \tilde{\lambda}/\tilde{q}). \tag{13}$$

4. **The Fuzzy Model with Triangle Fuzzy Numbers.** Let perfective rate, demand rate and purchasing cost be triangular fuzzy numbers with respective parameters: $\tilde{q}_t = [q - \Delta_1, q, q + \Delta_2]$, $\tilde{\lambda}_t = [\lambda - \Delta_3, \lambda, \lambda + \Delta_4]$ and $\tilde{c}_t = [c - \Delta_5, c, c + \Delta_6]$, where $\Delta_1 \sim \Delta_6$ are positive numbers such that $q - \Delta_1 > 0$, $\lambda - \Delta_3 > 0$, and $c - \Delta_5 > 0$. By α-level cuts, those fuzzy numbers can be described as:

$$\begin{cases} \tilde{q}_t(\alpha) = [(q - \Delta_1) + \Delta_1\alpha,\ (q + \Delta_2) - \Delta_2\alpha],\ 0 \le \alpha \le 1 \\ \tilde{\lambda}_t(\alpha) = [(\lambda - \Delta_3) + \Delta_3\alpha,\ (\lambda + \Delta_4) - \Delta_4\alpha],\ 0 \le \alpha \le 1 \\ \tilde{c}_t(\alpha) = [(c - \Delta_5) + \Delta_5\alpha,\ (c + \Delta_6) - \Delta_6\alpha],\ 0 \le \alpha \le 1. \end{cases}$$

According to Equation (11), for finding the optimal Q^*, we need to calculate the following Yager's indexes:

1. $I(\tilde{\lambda}_t)$

Since

$$\int_0^1 L_{\lambda_t}^{-1}(\alpha)d\alpha = \int_0^1 [(\lambda - \Delta_3) + \Delta_3\alpha]d\alpha = \lambda - \Delta_3/2$$

and

$$\int_0^1 R_{\lambda_t}^{-1}(\alpha)d\alpha = \int_0^1 [(\lambda + \Delta_4) - \Delta_4\alpha]d\alpha = \lambda + \Delta_4/2,$$

from Equation (10), the Yager's index of $\tilde{\lambda}_t$ can be found as:

$$I(\tilde{\lambda}_t) = \frac{1}{2} \cdot \left[\int_0^1 L_{\lambda_t}^{-1}(\alpha)d\alpha + \int_0^1 R_{\lambda_t}^{-1}(\alpha)d\alpha\right] = \lambda + \frac{1}{4}(\Delta_4 - \Delta_3). \tag{14}$$

2. $I(\tilde{\lambda}_t/\tilde{q}_t)$

From the arithmetic operation (v) and Equation (10), we can derive the $I_t(\tilde{\lambda}/\tilde{q})$ as follows:

$$\begin{aligned} I(\lambda_t/q_t) &= \frac{1}{2} \cdot \left[\int_0^1 \frac{L_{\lambda_t}^{-1}(\alpha)}{R_{q_t}^{-1}(\alpha)}d\alpha + \int_0^1 \frac{R_{\lambda_t}^{-1}(\alpha)}{L_{q_t}^{-1}(\alpha)}d\alpha\right] \\ &= \frac{1}{2} \cdot \int_0^1 \left[\frac{(\lambda - \Delta_3) + \Delta_3\alpha}{(q + \Delta_2) - \Delta_2\alpha} + \frac{(\lambda + \Delta_4) - \Delta_4\alpha}{(q - \Delta_1) + \Delta_1\alpha}\right]d\alpha \\ &= \frac{1}{2} \cdot \left[\frac{(q\Delta_3 + \lambda\Delta_2)}{\Delta_2^2} \ln \frac{(q + \Delta_2)}{q} - \frac{\Delta_3}{\Delta_2} + \frac{(q\Delta_4 + \lambda\Delta_1)}{\Delta_1^2} \ln \frac{q}{(q - \Delta_1)} - \frac{\Delta_4}{\Delta_1}\right]. \end{aligned} \tag{15}$$

3. $I(\tilde{c}_t \times \tilde{\lambda}_t)$ and $I(\tilde{c}_t \times \tilde{q}_t)$

From the arithmetic operation (iv) and Equation (10), we can derive the $I_t(\tilde{c}_t \times \tilde{\lambda}_t)$ as:

$$\begin{aligned} I(\tilde{c}_t \times \tilde{\lambda}_t) &= \frac{1}{2}\left[\int_0^1 L_{c_t}^{-1}(\alpha)L_{\lambda_t}^{-1}(\alpha)d\alpha + \int_0^1 R_{c_t}^{-1}(\alpha)R_{\lambda_t}^{-1}(\alpha)d\alpha\right] \\ &= \frac{1}{2}\left[(c - \Delta_5)(\lambda - \Delta_3) + \frac{(c - \Delta_5)\Delta_3}{2} + \frac{(\lambda - \Delta_3)\Delta_5}{2} + \frac{\Delta_5\Delta_3}{3}\right. \\ &\quad \left. +(c + \Delta_6)(\lambda + \Delta_4) - \frac{(c + \Delta_6)\Delta_4}{2} - \frac{(\lambda + \Delta_4)\Delta_6}{2} + \frac{\Delta_6\Delta_4}{3}\right]. \end{aligned} \tag{16}$$

Similarly, we have:

$$\begin{aligned} I(\tilde{c}_t \times \tilde{q}_t) =& \frac{1}{2}\left[\int_0^1 L_{c_t}^{-1}(\alpha)L_{q_t}^{-1}(\alpha)d\alpha + \int_0^1 R_{c_t}^{-1}(\alpha)R_{q_t}^{-1}(\alpha)d\alpha\right] \\ =& \frac{1}{2}\Bigg[(c-\Delta_5)(q-\Delta_1) + \frac{(c-\Delta_5)\Delta_1}{2} + \frac{(q-\Delta_1)\Delta_5}{2} + \frac{\Delta_5\Delta_1}{3} \\ & +(c+\Delta_6)(q+\Delta_2) - \frac{(c+\Delta_6)\Delta_2}{2} - \frac{(q+\Delta_2)\Delta_6}{2} + \frac{\Delta_6\Delta_2}{3}\Bigg]. \end{aligned} \tag{17}$$

4. $I(\tilde{c}_t \times \tilde{\lambda}_t/\tilde{q}_t)$

From the arithmetic operation (vi) and Equation (10), we can derive the $I(\tilde{c}_t \times \tilde{\lambda}_t/\tilde{q}_t)$ as:

$$\begin{aligned} & I(\tilde{c}_t \times \tilde{\lambda}_t/\tilde{q}_t) \\ =& \frac{1}{2}\cdot\left[\int_0^1 \frac{L_{c_t}^{-1}(\alpha)L_{\lambda_t}^{-1}(\alpha)}{R_{q_t}^{-1}(\alpha)}d\alpha + \int_0^1 \frac{R_{c_t}^{-1}(\alpha)R_{\lambda_t}^{-1}(\alpha)}{L_{q_t}^{-1}(\alpha)}d\alpha\right] \\ =& \frac{1}{2}\Bigg[\frac{(c-\Delta_5)(\lambda-\Delta_3)\Delta_2^2 + (q+\Delta_2)\left[(c-\Delta_5)\Delta_3\Delta_2 + (\lambda-\Delta_3)\Delta_5\Delta_2 + (q+\Delta_2)\Delta_5\Delta_3\right]}{(-\Delta_2^3)} \\ & \left(\ln\frac{q}{q+\Delta_2}\right) - \frac{\Delta_5\Delta_3}{2\Delta_2} - \frac{(c-\Delta_5)\Delta_3\Delta_2 + (\lambda-\Delta_3)\Delta_5\Delta_2 + (q+\Delta_2)\Delta_5\Delta_3}{\Delta_2^2} \\ & + \frac{(c+\Delta_6)(\lambda+\Delta_4)\Delta_1^2 + (q-\Delta_1)\left[(c+\Delta_6)\Delta_4\Delta_1 + (\lambda+\Delta_4)\Delta_6\Delta_1 + (q-\Delta_1)\Delta_6\Delta_4\right]}{\Delta_1^3} \\ & \left(\ln\frac{q}{q-\Delta_1}\right) + \frac{\Delta_6\Delta_4}{2\Delta_1} - \frac{(c+\Delta_6)\Delta_4\Delta_1 + (\lambda+\Delta_4)\Delta_6\Delta_1 + (q-\Delta_1)\Delta_6\Delta_4}{\Delta_1^2}\Bigg]. \end{aligned} \tag{18}$$

Substituting the results in Equations (14)-(18) into Equation (11), we can find the optimal solution Q_t^* for the case with triangular fuzzy numbers as follows:

$$Q_t^* = \left(\frac{2aeI(\tilde{\lambda}_t/\tilde{q}_t)}{b\left[I(\tilde{c}_t \times \tilde{\lambda}_t/\tilde{q}_t) + e\cdot I(\tilde{c}_t \times \tilde{q}_t) - I(\tilde{c}_t \times \tilde{\lambda}_t)\right]}\right)^{\frac{1}{2}}. \tag{19}$$

Furthermore, from Equations (12) and (19), the total profit $\tilde{P}_t^*(=\tilde{P}(Q_t^*))$ corresponding to Q_t^* can be expressed as:

$$\tilde{P}_t^*(\alpha) = [(L_P^{-1}(\alpha)|Q = Q_t^*), (R_P^{-1}(\alpha)|Q = Q_t^*)].$$

Note that if $(\tilde{\lambda}_t, \tilde{q}_t, \tilde{c}_t)$ degenerates to (λ, q, c) (i.e., $\Delta_3 = \Delta_4 = 0$, $\Delta_1 = \Delta_2 = 0$ and $\Delta_5 = \Delta_6 = 0$), then it can be shown that Q_t^* in Equation (19) will reduce to the Q_c^* in Equation (4). The result is summarized as follows (see Appendix):

Proposition 4.1. *If* $(\tilde{\lambda}_t, \tilde{q}_t, \tilde{c}_t)$ *degenerates to* (λ, q, c)*, then*

$$Q_t^* = Q_c^* = \sqrt{\frac{2ae\cdot\lambda}{cb[eq^2 + (1-q)\lambda]}}.$$

5. **Numerical Example.** To illustrate the proposed model, we set

$$(a, b, c, q, w, s, \lambda) = (100, 0.2, 25, 0.98, 0.5, 175200, 50000).$$

By Equations (3) and (4), we have the optimal solution of the crisp model as:

$$(Q_c^*, P_c^*) = (1438.806, 1{,}217{,}397).$$

To assess the effects of fuzziness of the perfective rate, the demand rate, and the purchasing cost on the optimal solution, we compute and compare the optimal order quantities and the corresponding Yager's indexes associated with 125 combinations of $\tilde{q}_t$, $\tilde{\lambda}_t$, and $\tilde{c}_t$ with $(q, \lambda, c) = (0.98, 50000, 25)$. The 125 combinations consist of five triangular fuzzy numbers of $\tilde{q}_t$, five triangular fuzzy numbers of $\tilde{\lambda}_t$, and five triangular fuzzy numbers of $\tilde{c}_t$. For $\tilde{q}_t$, $\tilde{\lambda}_t$, and $\tilde{c}_t$, the five triangular fuzzy numbers are respectively set as follows:
(Δ_1, Δ_2): $(0.00^+, 1.00)$, $(0.25, 0.75)$, $(0.50, 0.50)$, $(0.75, 0.25)$, $(1.00, 0.00^+)$,
(Δ_3, Δ_4): $(1000, 0.00^+)$, $(750, 250)$, $(500, 500)$, $(250, 750)$, $(0.00^+, 1000)$,
(Δ_5, Δ_6): $(0.00^+, 5.00)$, $(1.25, 3.75)$, $(2.50, 2.50)$, $(3.75, 1.25)$, $(5.00, 0.00^+)$.

For ease of exposition, the following index (called the skewness rate) is defined to measure the extent to which $\tilde{q}_t$, $\tilde{\lambda}_t$, and $\tilde{c}_t$ skew:

$$\begin{cases} SR_{\tilde{q}} = (\Delta_2 - \Delta_1)/(\Delta_1 + \Delta_2) \\ SR_{\tilde{\lambda}} = (\Delta_4 - \Delta_3)/(\Delta_3 + \Delta_4) \\ SR_{\tilde{c}} = (\Delta_6 - \Delta_5)/(\Delta_5 + \Delta_6). \end{cases} \tag{20}$$

Conceptually, such settings for $\tilde{q}_t$, $\tilde{\lambda}_t$, and $\tilde{c}_t$ represent the skewness from the most right to the most left. Specifically, the values of $SR_{\tilde{q}}$, $SR_{\tilde{\lambda}}$, and $SR_{\tilde{c}}$ are all 1.0 (skew to the most righ), 0.5, 0.0 (symmetric), -0.5, and -1.0 (skew to the most left). For $\tilde{q}_t$, when $\Delta_2 > \Delta_1$ $(\Delta_2 < \Delta_1)$, $SR_{\tilde{q}} > 0 (SR_{\tilde{q}} < 0)$, implying that $\tilde{q}_t$ is right (left) skew or $\tilde{q}_t > q$ $(\tilde{q}_t < q)$; when $SR_{\tilde{q}} = 1$ $(SR_{\tilde{q}} = -1)$, $\tilde{q}_t$ is the most right (left) skew; when $SR_{\tilde{q}} = 0$, $\tilde{q}_t$ is symmetric. The above results are also available for $\tilde{\lambda}_t$ with $SR_{\tilde{\lambda}}$ and $\tilde{c}_t$ with $SR_{\tilde{c}}$.

Similarly, associated with the skewness rates (*SR*) of $\tilde{q}_t$, $\tilde{\lambda}_t$, and $\tilde{c}_t$, the following index (called the variation rate) is defined to measure the extent to which the corresponding Q_t^* and $I(\tilde{P}_t^*)$ variate:

$$\begin{cases} VR_{Q^*} = [(Q_t^* - Q_c^*)/Q_c^*] \times 100\% \\ VR_{I(P^*)} = [(I(P_t^*) - I(P_c^*))/I(P_c^*)] \times 100\%. \end{cases} \tag{21}$$

It is also clear that for Q_t^*, $VR_{Q^*} > 0$ $(VR_{Q^*} < 0)$ implies $Q_t^* > Q_c^*$ $(Q_t^* < Q_c^*)$; $VR_{Q^*} = 0$ implies $Q_t^* = Q_c^*$. The result is also available for $I(\tilde{P}_t^*)$ with $VR_{I(P^*)}$. The 125 combinations and their corresponding results are shown as Tables 1∼5.

From the results of Table 1∼Table 5, it is seen that for fixed λ and q, a decreasing $SR_{\tilde{c}}$ results in increasing VR_{Q^*} and $VR_{I(\tilde{P}^*)}$. For example, for fixed $(SR_{\tilde{q}}, SR_{\tilde{\lambda}}) = (1.0, -1.0)$, from the data of No. 1∼No. 5 in the last three fields of Table 1, we have that a decreasing $SR_{\tilde{c}}$ (from 1.0 to -1.0) results in increasing VR_{Q^*} (from -2.858 to 2.130) and $VR_{I(\tilde{P}^*)}$ (from -5.644 to 4.585). This result implies that a decreasing $\tilde{c}_t$ results in increasing Q_t^* and $\tilde{P}_t^*$. Likewise, for fixed λ and c, a decreasing $SR_{\tilde{q}}$ results in an increasing VR_{Q^*} and a decreasing $VR_{I(\tilde{P}^*)}$. For example, for fixed $(SR_{\tilde{\lambda}}, SR_{\tilde{c}}) = (-1.0, 1.0)$, from the data of No. 1, No. 6, No. 11, No. 16 and No. 21 in Table 1, we have that a decreasing $SR_{\tilde{q}}$ (from 1.0 to -1.0) results in an increasing VR_{Q^*} (from -2.858 to -2.434) and a decreasing $VR_{I(\tilde{P}^*)}$(from -5.644 to -5.655). This result indicates that a decreasing $\tilde{q}_t$ results in an increasing Q_t^* and a decreasing $\tilde{P}_t^*$. Furthermore, for fixed q and c, an increasing $SR_{\tilde{\lambda}}$ results in increasing VR_{Q^*} and $VR_{I(\tilde{P}^*)}$, For example, for fixed $(SR_{\tilde{q}}, SR_{\tilde{c}}) = (1.0, 1.0)$, from the data of No. 1 in Table 1∼Table 5, we have that a increasing $SR_{\tilde{\lambda}}$ (from -1.0 to 1.0) results in increasing VR_{Q^*} (from -2.858 to -2.375) and $VR_{I(\tilde{P}^*)}$ (from -5.644 to -4.710). This implies that an increasing λ also results in increasing Q_t^* and $\tilde{P}_t^*$.

TABLE 1. The results for $(\Delta_3, \Delta_4) = (1000, 0^+)$ with $SR_{\tilde{\lambda}} = -1.0$

No.	$(\Delta_1, \Delta_2)(\%)$	(Δ_5, Δ_6)	Q^*	$I(\tilde{P}^*)$	$SR_{\tilde{q}}$	$SR_{\tilde{c}}$	$VR_{Q^*}(\%)$	$VR_{I(\tilde{P}^*)}(\%)$
1	$(0.00^+, 1.00)$	$(0.00^+, 5.00)$	1,397.682	1,148,685	1.0	1.0	−2.858	−5.644
2		(1.25, 3.75)	1,414.636	1,179,814		0.5	−1.680	−3.087
3		(2.50, 2.50)	1,432.222	1,210,943		0.0	−0.458	−0.530
4		(3.75, 1.25)	1,450.481	1,242,074		−0.5	0.811	2.027
5		$(5.00, 0.00^+)$	1,469.456	1,273,206		−1.0	2.130	4.584
6	(0.25, 0.75)	$(0.00^+, 5.00)$	1,399.202	1,148,652	0.5	1.0	−2.753	−5.647
7		(1.25, 3.75)	1,416.169	1,179,780		0.5	−1.573	−3.090
8		(2.50, 2.50)	1,433.769	1,210,910		0.0	−0.350	−0.533
9		(3.75, 1.25)	1,452.042	1,242,040		−0.5	0.920	2.024
10		$(5.00, 0.00^+)$	1,471.032	1,273,172		−1.0	2.240	4.581
11	(0.50, 0.50)	$(0.00^+, 5.00)$	1,400.728	1,148,618	0.0	1.0	−2.646	−5.650
12		(1.25, 3.75)	1,417.709	1,179,747		0.5	−1.466	−3.093
13		(2.50, 2.50)	1,435.323	1,210,876		0.0	−0.242	−0.536
14		(3.75, 1.25)	1,453.610	1,242,007		−0.5	1.029	2.022
15		$(5.00, 0.00^+)$	1,472.614	1,273,138		−1.0	2.350	4.579
16	(0.75, 0.25)	$(0.00^+, 5.00)$	1,402.257	1,148,585	−0.5	1.0	−2.540	−5.652
17		(1.25, 3.75)	1,419.252	1,179,713		0.5	−1.359	−3.095
18		(2.50, 2.50)	1,436.879	1,210,842		0.0	−0.134	−0.538
19		(3.75, 1.25)	1,455.181	1,241,973		−0.5	1.138	2.019
20		$(5.00, 0.00^+)$	1,474.200	1,273,105		−1.0	2.460	4.576
21	$(1.00, 0.00^+)$	$(0.00^+, 5.00)$	1,403.787	1,148,551	−1.0	1.0	−2.434	−5.655
22		(1.25, 3.75)	1,420.799	1,179,679		0.5	−1.252	−3.098
23		(2.50, 2.50)	1,438.445	1,210,809		0.0	−0.025	−0.541
24		(3.75, 1.25)	1,456.765	1,241,939		−0.5	1.248	2.016
25		$(5.00, 0.00^+)$	1,475.804	1,273,071		−1.0	2.571	4.573

Note: $0^+ = 10^{-5}$

Furthermore, as the above explanations, we also have that the more $\tilde{\lambda}_t$ skews to the right (i.e., increasing), or the more $\tilde{q}_t$ and $\tilde{c}_t$ skew to the left (i.e., decreasing), the larger Q_t^* and $\tilde{P}_t^*$. Further, when $\tilde{\lambda}_t$ is the most right skew ($SR_{\tilde{\lambda}} = 1.0$) and ($\tilde{q}_t$, $\tilde{c}_t$) is the most left skew ($SR_{\tilde{c}} - 1.0$ and $SR_{\tilde{c}} = -1.0$) (see the row No. 25 in Table 5), $(VR_{Q^*}, VR_{I(\tilde{P}^*)}) = (3.081, 5.645)$ is the maximum one with $(Q_t^*, I(\tilde{P}_t^*)) = (1483.138, 1, 286, 113)$. On the contrary, the more $\tilde{\lambda}_t$ skews to the left (i.e., decreasing), or the more $\tilde{q}_t$ and $\tilde{c}_t$ skew to the right (i.e., increasing), the smaller Q_t^* and $\tilde{P}_t^*$. When $\tilde{\lambda}_t$ is the most left skew ($SR_{\tilde{\lambda}} = -1.0$), ($\tilde{q}_t$, $\tilde{c}$) is the most right skew ($SR_{\tilde{c}} = 1.0$ and $SR_{\tilde{c}} = 1.0$) (see the row No. 1 in Table 1), $(VR_{Q^*}, VR_{I(\tilde{P}^*)}) = (-2.858, -5.644)$ is the minimum one with $(Q_t^*, I(\tilde{P}_t^*)) = (11, 397.682, 1, 148, 685)$. Furthermore, when all of the $\tilde{q}_t$, $\tilde{c}_t$ and $\tilde{\lambda}_t$ are symmetric (see the row No. 13 in Table 3), i.e., $SR_{\tilde{\lambda}} = SR_{\tilde{q}} = SR_{\tilde{c}} = 0$, $(Q_t^*, I(\tilde{P}_t^*)) = (1438.093, 1, 216.981) \approx (Q_c^*, P(Q_c^*)) = (1438.806, 1, 217, 397)$, which implies that the fuzzy case is close to the crisp case.

TABLE 2. The results for $(\Delta_3, \Delta_4) = (750, 250)$ with $SR_{\tilde{\lambda}} = -0.5$

No.	$(\Delta_1, \Delta_2)(\%)$	(Δ_5, Δ_6)	Q^*	$I(\tilde{P}^*)$	$SR_{\tilde{q}}$	$SR_{\tilde{c}}$	$VR_{Q^*}(\%)$	$VR_{I(\tilde{P}^*)}(\%)$
1	$(0.00^+, 1.00)$	$(0.00^+, 5.00)$	1,399.423	1,151,529	1.0	1.0	−2.737	−5.411
2		(1.25, 3.75)	1,416.399	1,182,762		0.5	−1.557	−2.845
3		(2.50, 2.50)	1,434.008	1,213,996		0.0	−0.333	−0.279
4		(3.75, 1.25)	1,452.291	1,245,231		−0.5	0.937	2.286
5		$(5.00, 0.00^+)$	1,471.291	1,276,467		−1.0	2.258	4.852
6	(0.25, 0.75)	$(0.00^+, 5.00)$	1,400.948	1,151,496	0.5	1	−2.631	−5.413
7		(1.25, 3.75)	1,417.937	1,182,728		0.5	−1.450	−2.848
8		(2.50, 2.50)	1,435.559	1,213,962		0.0	−0.226	−0.282
9		(3.75, 1.25)	1,453.855	1,245,197		−0.5	1.046	2.284
10		$(5.00, 0.00^+)$	1,472.868	1,276,433		−1.0	2.367	4.849
11	(0.50, 0.50)	$(0.00^+, 5.00)$	1,402.476	1,151,462	0.0	1.0	−2.525	−5.416
12		(1.25, 3.75)	1,419.478	1,182,695		0.5	−1.343	−2.851
13		(2.50, 2.50)	1,437.114	1,213,928		0.0	−0.118	−0.285
14		(3.75, 1.25)	1,455.424	1,245,163		−0.5	1.155	2.281
15		$(5.00, 0.00^+)$	1,474.452	1,276,399		−1.0	2.477	4.847
16	(0.75, 0.25)	$(0.00^+, 5.00)$	1,404.007	1,151,428	−0.5	1.0	−2.419	−5.419
17		(1.25, 3.75)	1,421.023	1,182,661		0.5	−1.236	−2.853
18		(2.50, 2.50)	1,438.673	1,213,895		0.0	−0.009	−0.288
19		(3.75, 1.25)	1,456.997	1,245,130		−0.5	1.264	2.278
20		$(5.00, 0.00^+)$	1,476.040	1,276,365		−1.0	2.588	4.844
21	$(1.00, 0.00^+)$	$(0.00^+, 5.00)$	1,405.538	1,151,394	−1.0	1.0	−2.312	−5.422
22		(1.25, 3.75)	1,422.570	1,182,627		0.5	−1.128	−2.856
23		(2.50, 2.50)	1,440.238	1,213,861		0.0	0.100	−0.290
24		(3.75, 1.25)	1,458.580	1,245,096		−0.5	1.374	2.275
25		$(5.00, 0.00^+)$	1,477.641	1,276,332		−1.0	2.699	4.841

Note: $0^+ = 10^{-5}$

Synthesizing the above results, we conclude that the more the fuzzy number of demand rate skews to the right, or the more the fuzzy numbers of perfective rate and purchasing cost skew to the left, the larger are the optimal order quantity and its optimal profit.

Finally, from the absolute values of $SR_{\tilde{\lambda}}$, $SR_{\tilde{q}}$ and $SR_{\tilde{c}}$, we can observe that as all of the $|SR_{\tilde{\lambda}}|$, $|SR_{\tilde{q}}|$ and $|SR_{\tilde{c}}|$ decrease, both of the $|VR_{Q^*}|$ and $|VR_{I(\tilde{P}^*)}|$ decrease. This result implies that the less uncertain$\tilde{\lambda}_t$, $\tilde{q}_t$, and $\tilde{c}_t$ are, the closer between $(Q_t^*, \tilde{P}_t^*)$ and $(Q_c^*, I(P_c^*))$. In other words, the less uncertain the three parameters (demand rate, perfective rate and purchasing cost) are, the closer between the fuzzy inventory policy and the crisp inventory policy.

6. **Conclusions.** The purpose of the paper is to explore the inventory control problem with immediate return for defective items. Practically, this case is very popular for procurement operations of retailers. Further, for enhancing the practical applications of the model, this paper also extends the model in fuzzy senses, including the fuzziness of

TABLE 3. The results for $(\Delta_3, \Delta_4) = (500,\ 500)$ with $SR_{\tilde{\lambda}} = 0.0$

No.	$(\Delta_1, \Delta_2)(\%)$	(Δ_5, Δ_6)	Q^*	$I(\tilde{P}^*)$	$SR_{\tilde{q}}$	$SR_{\tilde{c}}$	$VR_{Q^*}(\%)$	$VR_{I(\tilde{P}^*)}(\%)$
1	$(0.00^+,\ 1.00)$	$(0.00^+,\ 5.00)$	1,401.162	1,154,373	1.0	1.0	−2.616	−5.177
2		(1.25, 3.75)	1,418.160	1,185,710		0.5	−1.435	−2.603
3		(2.50, 2.50)	1,435.792	1,217,048		0.0	−0.209	−0.029
4		(3.75, 1.25)	1,454.098	1,248,387		−0.5	1.063	2.546
5		$(5.00,\ 0.00^+)$	1,473.123	1,279,728		−1.0	2.385	5.120
6	(0.25, 0.75)	$(0.00^+,\ 5.00)$	1,402.692	1,154,340	0.5	1.0	−2.510	−5.180
7		(1.25, 3.75)	1,419.702	1,185,676		0.5	−1.328	−2.606
8		(2.50, 2.50)	1,437.346	1,217,015		0.0	−0.101	−0.031
9		(3.75, 1.25)	1,455.665	1,248,354		−0.5	1.172	2.543
10		$(5.00,\ 0.00^+)$	1,474.702	1,279,694		−1.0	2.495	5.117
11	(0.50, 0.50)	$(0.00^+,\ 5.00)$	1,404.222	1,154,306	0.0	1.0	-2.404	-5.182
12		(1.25, 3.75)	1,421.245	1,185,643		0.5	−1.221	−2.608
13		(2.50, 2.50)	1,438.903	1,216,981		0.0	0.007	−0.034
14		(3.75, 1.25)	1,457.236	1,248,320		−0.5	1.281	2.540
15		$(5.00,\ 0.00^+)$	1,476.288	1,279,660		−1.0	2.605	5.114
16	(0.75, 0.25)	$(0.00^+,\ 5.00)$	1,405.754	1,154,272	−0.5	1.0	−2.297	−5.185
17		(1.25, 3.75)	1,422.791	1,185,609		0.5	−1.113	−2.611
18		(2.50, 2.50)	1,440.463	1,216,947		0.0	0.115	−0.037
19		(3.75, 1.25)	1,458.811	1,248,286		−0.5	1.390	2.537
20		$(5.00,\ 0.00^+)$	1,477.877	1,279,626		−1.0	2.716	5.112
21	$(1.00,\ 0.00^+)$	$(0.00^+,\ 5.00)$	1,407.287	1,154,238	−1.0	1.0	−2.191	−5.188
22		(1.25, 3.75)	1,424.340	1,185,575		0.5	−1.005	−2.614
23		(2.50, 2.50)	1,442.028	1,216,913		0.0	0.224	−0.040
24		(3.75, 1.25)	1,460.392	1,248,252		−0.5	1.500	2.535
25		$(5.00,\ 0.00^+)$	1,479.476	1,279,592		−1.0	2.827	5.109

Note: $0^+ = 10^{-5}$

defective rate, demand rate and purchasing cost. In the previous studies, due to computational difficulty and complexity, there are few articles exploring an inventory model with three fuzzy parameters. The results improve the practical applications of fuzzy theory significantly.

Due to the fact that triangular fuzzy numbers are used extensively, this paper also expresses the optimal order quantity for the case that all of the three parameters are triangular fuzzy numbers. Based on 125 combinations of triangular fuzzy numbers of the parameters, a numerical example is provided to illustrate the model and to assess the effects of fuzziness of the parameters on the optimal solution. The results indicate that (1) as the three parameters reduce to their crisp values, the fuzzy model degenerates to the crisp model, (2) the more the fuzzy number of demand rate skews to the right, or the more the fuzzy numbers of perfective rate and purchasing cost skew to the left, the larger are the optimal order quantity and its optimal profit, and (3) the less uncertain the three

TABLE 4. The results for $(\Delta_3, \Delta_4) = (250,\ 750)$ with $SR_{\tilde{\lambda}} = 0.5$

No.	$(\Delta_1, \Delta_2)(\%)$	(Δ_5, Δ_6)	Q^*	$I(\tilde{P}^*)$	$SR_{\tilde{q}}$	$SR_{\tilde{c}}$	$VR_{Q^*}(\%)$	$VR_{I(\tilde{P}^*)}(\%)$
1	$(0.00^+,\ 1.00)$	$(0.00^+,\ 5.00)$	1,402.899	1,157,217	1.0	1.0	−2.496	−4.943
2		(1.25, 3.75)	1,419.918	1,188,658		0.5	−1.313	−2.361
3		(2.50, 2.50)	1,437.573	1,220,101		0.0	−0.086	0.222
4		(3.75, 1.25)	1,455.903	1,251,544		−0.5	1.188	2.805
5		$(5.00,\ 0.00^+)$	1,474.952	1,282,989		−1.0	2.512	5.388
6	(0.25, 0.75)	$(0.00^+,\ 5.00)$	1,404.434	1,157,183	0.5	1	−2.389	−4.946
7		(1.25, 3.75)	1,421.465	1,188,625		0.5	−1.205	−2.363
8		(2.50, 2.50)	1,439.131	1,220,067		0.0	0.023	0.219
9		(3.75, 1.25)	1,457.472	1,251,511		−0.5	1.297	2.802
10		$(5.00,\ 0.00^+)$	1,476.533	1,282,955		−1.0	2.622	5.385
11	(0.50, 0.50)	$(0.00^+,\ 5.00)$	1,405.965	1,157,149	0.0	1.0	−2.283	−4.949
12		(1.25, 3.75)	1,423.010	1,188,591		0.5	−1.098	−2.366
13		(2.50, 2.50)	1,440.690	1,220,033		0.0	0.131	0.217
14		(3.75, 1.25)	1,459.046	1,251,477		−0.5	1.407	2.799
15		$(5.00,\ 0.00^+)$	1,478.121	1,282,921		−1.0	2.732	5.382
16	(0.75, 0.25)	$(0.00^+,\ 5.00)$	1,407.499	1,157,115	−0.5	1.0	−2.176	−4.952
17		(1.25, 3.75)	1,424.558	1,188,557		0.5	−0.990	−2.369
18		(2.50, 2.50)	1,442.252	1,219,999		0.0	0.240	0.214
19		(3.75, 1.25)	1,460.622	1,251,442		−0.5	1.516	2.797
20		$(5.00,\ 0.00^+)$	1,479.712	1,282,887		−1.0	2.843	5.380
21	$(1.00,\ 0.00^+)$	$(0.00^+,\ 5.00)$	1,409.034	1,157,081	−1.0	1.0	−2.069	−4.955
22		(1.25, 3.75)	1,426.107	1,188,522		0.5	−0.883	−2.372
23		(2.50, 2.50)	1,443.816	1,219,965		0.0	0.348	0.211
24		(3.75, 1.25)	1,462.202	1,251,408		−0.5	1.626	2.794
25		$(5.00,\ 0.00^+)$	1,481.308	1,282,853		−1.0	2.954	5.377

Note: $0^+ = 10^{-5}$

parameters are, the closer between the fuzzy inventory policy and the crisp inventory policy is.

In this paper, the Yager's ranking method [20] is used to rank the proposed model and find the optimal solution. In the literature, there have been a lot of ranking methods developed. Certainly, different ranking methods may lead to different results. Thus, for improving the validation of the result, a comparison with the results obtained by the other ranking methods could be considered in the future research. Return is one of the popular ways for imperfective items in current procurements. Uncertainty is another common problem for the procurements. The result of this paper provides a solution for those two problems. The results are very practical and applicable for procurements in real world.

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Table 5. The results for $(\Delta_3, \Delta_4) = (0^+, 1000)$ with $SR_{\tilde{\lambda}} = 1.0$

No.	$(\Delta_1, \Delta_2)(\%)$	(Δ_5, Δ_6)	Q^*	$I(\tilde{P}^*)$	$SR_{\tilde{q}}$	$SR_{\tilde{c}}$	$VR_{Q^*}(\%)$	$VR_{I(\tilde{P}^*)}(\%)$
1	$(0.00^+, 1.00)$	$(0.00^+, 5.00)$	1,404.633	1,160,061	1.0	1.0	−2.375	−4.710
2		(1.25, 3.75)	1,421.675	1,191,607		0.5	−1.191	−2.118
3		(2.50, 2.50)	1,439.352	1,223,153		0.0	0.038	0.473
4		(3.75, 1.25)	1,457.706	1,254,701		−0.5	1.314	3.064
5		$(5.00, 0.00^+)$	1,476.780	1,286,250		−1.0	2.639	5.656
6	(0.25, 0.75)	$(0.00^+, 5.00)$	1,406.173	1,160,027	0.5	1	−2.268	−4.713
7		(1.25, 3.75)	1,423.226	1,191,573		0.5	−1.083	−2.121
8		(2.50, 2.50)	1,440.914	1,223,119		0.0	0.147	0.470
9		(3.75, 1.25)	1,459.278	1,254,667		−0.5	1.423	3.061
10		$(5.00, 0.00^+)$	1,478.363	1,286,216		−1.0	2.749	5.653
11	(0.50, 0.50)	$(0.00^+, 5.00)$	1,407.706	1,159,993	0.0	1.0	−2.162	−4.715
12		(1.25, 3.75)	1,424.772	1,191,539		0.5	−0.975	−2.124
13		(2.50, 2.50)	1,442.474	1,223,085		0.0	0.255	0.467
14		(3.75, 1.25)	1,460.853	1,254,633		−0.5	1.532	3.059
15		$(5.00, 0.00^+)$	1,479.952	1,286,182		−1.0	2.860	5.650
16	(0.75, 0.25)	$(0.00^+, 5.00)$	1,409.242	1,159,959	−0.5	1.0	−2.055	−4.718
17		(1.25, 3.75)	1,426.322	1,191,504		0.5	−0.868	−2.127
18		(2.50, 2.50)	1,444.038	1,223,051		0.0	0.364	0.464
19		(3.75, 1.25)	1,462.431	1,254,599		−0.5	1.642	3.056
20		$(5.00, 0.00^+)$	1,481.545	1,286,148		−1.0	2.970	5.647
21	$(1.00, 0.00^+)$	$(0.00^+, 5.00)$	1,410.779	1,159,925	−1.0	1.0	−1.948	−4.721
22		(1.25, 3.75)	1,427.872	1,191,470		0.5	−0.760	−2.130
23		(2.50, 2.50)	1,445.602	1,223,017		0.0	0.472	0.462
24		(3.75, 1.25)	1,464.010	1,254,565		−0.5	1.752	3.053
25		$(5.00, 0.00^+)$	1,483.138	1,286,113		−1.0	3.081	5.645

Note: $0^+ = 10^{-5}$

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Appendix. According to Equation (19), in order to prove Proposition 4.1, what we have to do is to evaluate $I(\tilde{\lambda}_t)$, $I(\tilde{\lambda}_t/\tilde{q}_t)$, $I(\tilde{c}_t \times \tilde{\lambda}_t)$, $I(\tilde{c}_t \times \tilde{q}_t)$, and $I(\tilde{c}_t \times \tilde{\lambda}_t/\tilde{q}_t)$.

1. $I(\tilde{\lambda}_t)$:

Obviously, if $\tilde{\lambda}_t$ degenerate to λ, then $\Delta_3 = \Delta_4 = 0$. Thus, according to Equation (14), it is easily seen that $I(\tilde{\lambda}_t) = \lambda$.

2. $I(\tilde{\lambda}_t/\tilde{q}_t)$:

If $\tilde{\lambda}_t$ degenerates to λ, then $\Delta_3 = \Delta_4 = 0$. Thus, Equation (15) will reduce to

$$I(\tilde{\lambda}_t/\tilde{q}_t) = \frac{1}{2}\lambda\left[\frac{1}{\Delta_2}\ln\frac{(q+\Delta_2)}{q} + \frac{1}{\Delta_1}\ln\frac{q}{(q-\Delta_1)}\right]. \tag{A-1}$$

Further, for Equation (A-1), if $\tilde{q}_t$ also degenerates to q (i.e., $\Delta_1 = \Delta_2 \to 0$), then by the L'Hospital's rule, we have $I(\tilde{\lambda}_t/\tilde{q}_t) = \lambda/q$.

3. $I(\tilde{c}_t \times \tilde{\lambda}_t)$:

If $(\tilde{c}_t, \tilde{\lambda}_t)$ degenerates to (c, λ), we have $\Delta_5 = \Delta_6 = 0$ and $\Delta_3 = \Delta_4 = 0$. Thus, according to Equation (16), we can easily obtain $I(\tilde{c}_t \times \tilde{\lambda}_t) = c \cdot \lambda$.

4. $I(\tilde{c}_t \times \tilde{q}_t)$:

Similarly, according to Equation (17), if $(\tilde{c}_t, \tilde{q}_t)$ degenerates to (c, q) (i.e., $\Delta_5 = \Delta_6 = 0$ and $\Delta_1 = \Delta_2 = 0$), then we easily obtain $I(\tilde{c}_t \times \tilde{q}_t) = c \cdot q$.

5. $I(\tilde{c}_t \times \tilde{\lambda}_t/\tilde{q}_t)$:

According to Equation (18), if $\tilde{c}_t$ reduces to c (i.e., $\Delta_5 = \Delta_6 = 0$), then we have

$$I(\tilde{c}_t \times \tilde{\lambda}_t/\tilde{q}_t) = c \cdot \frac{1}{2}\left[\frac{(q\Delta_3 + \lambda\Delta_2)}{\Delta_2^2}\left(\ln\frac{q+\Delta_2}{q}\right) - \frac{\Delta_3}{\Delta_2} + \frac{(\lambda\Delta_4 + q\Delta_1)}{\Delta_1^2}\left(\ln\frac{q}{q-\Delta_1}\right) - \frac{\Delta_4}{\Delta_1}\right] \tag{A-2}$$

Comparing Equation (A-2) with Equation (15), it is clear that if $\tilde{c}_t$ reduces to c then

$$I(\tilde{c}_t \times \tilde{\lambda}_t/\tilde{q}_t) = c \cdot I(\tilde{\lambda}_t/\tilde{q}_t). \tag{A-3}$$

From Equation (A-3) and the result in (A-2), we have that if $(\tilde{c}_t, \tilde{\lambda}_t, \tilde{q}_t)$ reduces to (c, λ, q), then $I(\tilde{c}_t \times \tilde{\lambda}_t/\tilde{q}_t) = c\lambda/q$.

Substituting the above results 1$\sim$5 into Equation (19), the result of Proposition 4.1 follows immediately.