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A ROBUST DOA/BEAMFORMING ALGORITHM USING THE CONSTANT MODULUS FEATURE

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Abstract. *In the conventional Space-Division Multiple-Access (SDMA) scheme, Direction-of-Arrival (DoA) estimation and beamforming are usually performed sequentially and independently. Such a cascade structure may transmit the error or deviation from the DoA stage to the beamforming stage. Two of the main reasons for such error or deviation are the effect of imprecise array manifold and the effect of finite sample size. The deviation, especially error may subsequently lead a beamformer to produce a beam pointing in wrong DoA that can severely degrade the performance of the beamformer. To address this problem, we propose a robust algorithm both for DoA estimation and for beamforming using the constant modulus (CM) feature. In the algorithm, the DoA estimation and the beamforming are no longer independent but work in an inter cooperative way. One can regard them as two parallel outputs of the algorithm. In this way, the robustness of DoA estimation and beamforming can be improved simultaneously. Finally, numerical experiments show the effectiveness of the proposed method.*
Keywords: Direction-of-Arrival, Beamformer, Constant modulus

1. **Introduction.** In the past decades, several high-resolution algorithms, such as Maximum Likelihood method, MUSIC method and ESPRIT method, were developed for estimating DoA [1]. However, the performance of these methods is known to be degraded by the effects of imprecise array manifold and/or finite sample size of signal [2-7]. Due to the changes of weather, surrounding environment, antenna locations, etc., the response of the array may differ from the one obtained during the last calibration. Furthermore, the calibration itself may not be ideal [2]. Therefore, in practical applications, the exact array manifold is unavailable; only an approximation one can be obtained, which may worsen the DoA estimation dramatically. Besides, these conventional methods require a long sample size. If the sample size is short, their performance will degrade. Therefore, many methods have been proposed to improve the robustness of DoA estimation. These methods include the weighted-based algorithms, beamspace-based algorithms, self-calibration algorithms, EM algorithms, etc. A good overview of these can be found in [1]. The methods mentioned above are general and do not rely on the special property of signal.

On the other hand, some other methods have been proposed based on certain special properties of the signals. In [8], the authors presented an algorithm in the case of the signals with known waveforms. The properties of the signals such as nonGaussianity or cyclostationarity are also exploited in some methods [9-11]. Moreover, the CM property is also invoked in [12]. If all sources impinging on the array are CM sources, the method proposed in [12] can work well to estimate the DoAs. However, in practical communication systems, though the signals transmitting from digital communication users satisfy the CM property, the interference sources might be analog (non-CM). In this case, this method will become invalid.

In some digital communication applications, the purpose of DoA estimation is usually for providing the beamformers with steering vectors to recover the signals of communication users. However, the performance of beamformers will degrade dramatically if the knowledge about the complex steering vector for the desired user is imprecise and/or if the sample size is finite. Therefore, in the last decades, several robust beamformers have been developed for mitigating the effects of steering vector mismatch and finite sample size, e.g., the diagonal loading-based (DL) beamformers [13,14], the subspace-based (SSB) beamformers [15] and the beamformers based on some optimization methods [16,17]. Moreover, the method so-called blind beamformer is also proposed based on CM property for digital communications [18]. A good overview of these can be found in [19].

By reviewing the past researches, we can find that the robust DoA estimation and beamformers are usually discussed separately, which means the relation between them is neglected. However, in practical applications, since the steering vector utilized in beamformers is generally provided by DoA estimation, the beamformers should be interrelated with DoA estimation methods. The main difference between the proposed method and the conventional methods is that the proposed method considers the DoA estimation and the beamformer jointly and in a mutual and cooperative way. Though DoA estimation and beamforming are widely used in radar, sonar and many other applications, in this paper, we consider them used only for digital communication applications. Invoking the CM property, we propose a robust algorithm that can implement both DoA estimation and beamforming. In the algorithm, the DoA estimation and the beamformer are performed in an inter cooperative way, i.e., fused as a single algorithm, so that the robustness of DoA estimation and beamformer can be improved together. We shall investigate the features of the algorithm regarding both DoA estimation and beamforming.

1.1. **The contributions of the proposed method for DoA estimation.** In the proposed method, a beamformer is employed for directing to any angle. This beamformer may be the DL beamformer, the beamformer based on the optimization method, or others. In this paper, we choose the SSB beamformer as an example to present our approach. We choose the SSB beamformer rather than the others just because this beamformer is popular and is well known to most readers. Of course, one can choose other advanced and latest robust beamformer for providing a better performance of DoA estimation.

In this paper, we use the beam produced by the SSB beamformer to scan the region of DoA space. If, in a definite angle at which one of the CM sources is impinging, the SSB beamformer with this angle will cancel the interference efficiently. In this case, the outputs of SSB beamformer should also satisfy the CM property more exactly than that with other angles. Therefore, minimizing CM errors (MCME) can act as a natural and reasonable criterion for determining the DoA of the CM source. Since the CM property is a strong and natural condition, the proposed method can significantly improve the robustness about DoA estimation against the effects of imprecise array manifold and/or finite sample size without any additional assumption.

Upon the CM, as is well known, several kinds of CM cost functions have been proposed in the field of blind equalizations to evaluate the CM errors. However, these functions cannot be directly used in DoA estimation for our purpose. In [20], the authors have proposed a modified CM cost function that can address this problem. Recently, we proposed a DL beamformer based on the CM property [20]. A key point of the DL beamformer is how to determine the diagonal loading factor (DLF). In [20], we have proposed a modified CM cost function based on the minimizing CM errors (MCME) criterion to determine the one-dimensional parameter, i.e., DLF. In this paper, by generalizing the cost function in [20], we propose a cost function to estimate the one-dimensional variable, i.e., the DoAs for CM sources. Since the cost function in the proposed method depends only on one parameter, DoA, the optimization can be solved easily by one-dimensional search algorithm no matter the cost function is convex or not.

The CM property is also exploited in [12]; however, all sources are CM which is required in that. As a comparison, the proposed method is quite different from that of the method presented in [12] so that the condition that all sources are CM sources is no longer required. However, if there exist both CM sources and non-CM sources impinging on the array, the proposed method can only estimate the DoAs for the CM sources (More details see Section 3). Though this method cannot estimate the DoAs about the non-CM sources, if the purpose of DoA estimation is only for providing the beamformers with steering vectors to recover the communication users signals, the method is enough for this purpose. The reason is that, in digital communications, a non-CM source is definitely an interference source.

1.2. **The contributions of the proposed method for beamformer.** In the proposed method, since the DoA is optimized by the MCME criterion, the SSB beamformer with this optimal DoA will eliminate the interferences effectively so that its outputs should be more closer to CM signals than those of the SSB beamformer with the other angles are. Therefore, using the MCME criterion can improve the DoA estimation and sequentially improve the performance of SSB beamformer.

Remark 1.1. *In most digital communication applications, the ultimate objective of array signal processing is to recover the desired users signals effectively. However, due to the effects of imprecise array manifold and/or finite sample size, the best performance of a beamformer should be achieved at a certain DoA within a near region of the true one that may be different from the true DoA of CM source itself. The goal of the proposed method is to estimate DoA for a better performance of SSB beamformer. Therefore, it can search a DoA so that the SSB beamformer can achieve a good performance that is even better than that by true DoA (See the numerical experiment 4.3 in Section 4).*

The rest of this paper is organized as follows. In Section 2, we present the mathematical models of DoA estimation and beamforming. In Section 3, we propose a robust DoA/Beamforming algorithm. In Section 4, we show some numerical experimental results. Conclusions and discussions are given in Section 5.

2. The Mathematical Models of DoA Estimation and Beamforming.

2.1. **The mathematical description of DoA estimation.** Assume a uniform linear array with M sensors and L narrow-band, far-field signal sources. The array manifold is calibrated beforehand and is represented as $\Omega = \{\boldsymbol{a}(\theta) : \theta \in \Theta\}$, where Θ denotes the region of DoA space. The array output $\boldsymbol{x}(k) \in \mathbb{C}^{M\times 1}$ can be written as [8]

$$\boldsymbol{x}(k) = \sum_{i=1}^{L} \boldsymbol{a}(\theta_i) c_i s_i(k) + \boldsymbol{n}(k), \tag{1}$$

where k denotes the time index; $\boldsymbol{a}(\theta_i) \in \mathbb{C}^{M\times 1}$ is the complex array response, i.e., the steering vector for the ith source with direction of arrival θ_i; $\boldsymbol{n}(k) \in \mathbb{C}^{M\times 1}$ represents additive noise; c_i and $s_i(k)$ is the channel attenuation and the normalized signal for the ith source, respectively. In this paper, $s_i(k)$ may or may not be digital signals. For the cases of analog signal sources, $s_i(k) \in \mathbb{C}^1$ and $\frac{1}{N}\sum_{k=1}^{N} \|s_i(k)\|^2{=}1$, where N is the number of snapshots (sample size); in the cases of CM sources, $s_i(k) \in \{+1, -1\}$ [12].

If the array manifold Ω is known and the sample size is infinite, one can use DoA estimation methods to extract the users' DoAs efficiently. In practical applications, however, the sample size is usually finite. Moreover, the exact array manifold is unavailable; only the approximation $\widetilde{\Omega}$ can be obtained

$$\widetilde{\Omega} = \{\widetilde{\boldsymbol{a}}(\theta) : \widetilde{\boldsymbol{a}}(\theta) = \boldsymbol{a}(\theta) + \Delta(\theta), \theta \in \Theta\} \tag{2}$$

where $\Delta(\theta)$ is the deviation vector between $\widetilde{\boldsymbol{a}}(\theta)$ and $\boldsymbol{a}(\theta)$. In these cases, the performance of the DoA estimation methods will degrade dramatically.

2.2. **The mathematical description of SSB beamformer.** A beamformer is a spatial filter that operates on the observations of an array of M sensors in order to enhance the desired signal relative to directional interference and background noise. That is, its output, for approaching the desired source, is given by

$$y(k) = \boldsymbol{w}^H \boldsymbol{x}(k) \tag{3}$$

where $\boldsymbol{w} \in \mathbb{C}^{M\times 1}$ denotes the beamformer weights, which need be determined by the beamformer.

The SSB beamformer is a popular one which can improve robustness against the mismatch of the array response and the effect of finite sample size. The covariance matrix of the observations vector $\boldsymbol{x}(k)$ can be estimated by

$$\boldsymbol{R} = \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{x}(n)\boldsymbol{x}^H(n). \tag{4}$$

The weight vector of the SSB beamformer for the ith source is given by [15]

$$\boldsymbol{w}_{\mathrm{SSB}} = \alpha\, \boldsymbol{U}_s \Sigma_s^{-1} \boldsymbol{U}_s^H \widetilde{\boldsymbol{a}}(\hat{\theta}_i) \tag{5}$$

where $\alpha = \left(\widetilde{\boldsymbol{a}}^H(\hat{\theta}_i)\, \boldsymbol{U}_s \Sigma_s^{-1} \boldsymbol{U}_s^H \widetilde{\boldsymbol{a}}(\hat{\theta}_i)\right)^{-1}$. Here, $\boldsymbol{U}_s \in \mathbb{C}^{M\times L}$ is stacked up by the L eigenvectors of signal subspace of $\boldsymbol{R}$; $\Sigma_s \in \mathbb{R}^{L\times L}$ is a diagonal matrix in which the elements are the eigenvalues of signal subspace. $\hat{\theta}_i$ denotes the DoA of the ith source obtained from the DoA estimation methods.

Remark 2.1. *In the conventional beamformer, the DoA of desired user is required that usually is estimated in the former DoA stage. This is different from the proposed algorithm, as shown in the next section, in which DoA is also a unknown parameter to be determined.*

3. **The Proposed DoA/Beamforming Algorithm.** In the beginning of DoA estimation stage, the DoAs are unknown and $\boldsymbol{w}_{\text{SSB}}$ cannot be determined by (5). We propose a weight vector of SSB beamformer which depends on θ. That is,

$$\boldsymbol{w}_{\text{SSB}}(\theta) = \alpha_\theta \boldsymbol{U}_s \Sigma_s^{-1} \boldsymbol{U}_s^H \widetilde{\boldsymbol{a}}(\theta) \tag{6}$$

where $\alpha_\theta = \left(\widetilde{\boldsymbol{a}}^H(\theta) \boldsymbol{U}_s \Sigma_s^{-1} \boldsymbol{U}_s^H \widetilde{\boldsymbol{a}}(\theta)\right)^{-1}$.

For more easier understanding, we would like to give an illustrative example. In this example, there are one CM source and $L-1$ analog signal sources impinging on the array. Assume the first source is the desired CM source, i.e., $s_1(k) \in \{+1, -1\}$; the others are analog signal sources. From (3) and (6), the output of the SSB beamformer with an arbitrary angle θ can be written as

$$y(k) = \boldsymbol{w}_{\text{SSB}}^H(\theta)\boldsymbol{x}(k) = \gamma s_1(k) + \boldsymbol{w}_{\text{SSB}}^H(\theta)\left(\sum_{i=2}^{L} \boldsymbol{a}(\theta_i) c_i s_i(k) + \boldsymbol{n}(k)\right) \tag{7}$$

where $\gamma = c_1 \boldsymbol{w}_{\text{SSB}}^H(\theta)\boldsymbol{a}(\theta_1)$. Define $\boldsymbol{x}_{\text{in}}(k) = \sum_{i=2}^{L} \boldsymbol{a}(\theta_i) c_i s_i(k) + \boldsymbol{n}(k)$. (7) can be rewritten as

$$\frac{y(k)}{\gamma} = s_1(k) + \frac{\boldsymbol{w}_{\text{SSB}}^H(\theta)\boldsymbol{x}_{\text{in}}(k)}{\gamma} \tag{8}$$

Though $s_1(k)$ is unknown in (8), it satisfies CM property, i.e., $|s_1(k)|^2 = 1$. If θ is closer to the DoA of the first source, the SSB beamformer shown in (6) can cancel the interference term $\boldsymbol{x}_{\text{in}}(k)$ more efficiently. In this case, $y(k)/\gamma$ will approximate to $s_1(k)$ more exactly. Therefore, if γ is known, we can use CM cost functions, such as Godard cost function $\mathrm{G}(\theta)$ to determine the DoA of desired user.

$$\mathrm{G}(\theta) = \mathrm{E}\left\{\left(\left|\frac{y(k)}{\gamma}\right|^2 - 1\right)^2\right\} = \mathrm{E}\left\{\left(\left|s_1(k) + \frac{\boldsymbol{w}_{\text{SSB}}^H(\theta)\boldsymbol{x}_{\text{in}}(k)}{\gamma}\right|^2 - 1\right)^2\right\} \tag{9}$$

However, this cost function also includes $\boldsymbol{a}(\theta_1)$ and c_1 (both are include in γ), which are also unknown. We cannot employ Godard function directly. Fortunately, this situation will be changed if we use the modified Godard cost function that has been proposed in [20]. The modified Godard cost function can be expressed as

$$\mathrm{G}_{\text{MCME}}(\theta) = \mathrm{E}\left\{\left(\frac{|y(k)|^2}{\mathrm{E}\left\{|y(k)|^2\right\}} - 1\right)^2\right\} = \mathrm{E}\left\{\left(\frac{\left|\widetilde{\boldsymbol{a}}^H(\theta) \boldsymbol{U}_s \Sigma_s^{-1} \boldsymbol{U}_s^H \boldsymbol{x}(k)\right|^2}{\mathrm{E}\left\{\left|\widetilde{\boldsymbol{a}}^H(\theta) \boldsymbol{U}_s \Sigma_s^{-1} \boldsymbol{U}_s^H \boldsymbol{x}(k)\right|^2\right\}} - 1\right)^2\right\} \tag{10}$$

A notable difference between the Godard function $\mathrm{G}(\theta)$ and $\mathrm{G}_{\text{MCME}}(\theta)$ in (10) is that $\mathrm{G}_{\text{MCME}}(\theta)$ does not depend on γ but $\mathrm{G}(\theta)$ does. Let us analyze the reason why $\mathrm{G}_{\text{MCME}}(\theta)$ can replace the Godard function. (10) can be rewritten as

$$\mathrm{G}_{\text{MCME}}(\theta) = \mathrm{E}\left\{\left(\frac{\left|\gamma s_1(k) + \boldsymbol{w}_{\text{SSB}}^H(\theta)\boldsymbol{x}_{\text{in}}(k)\right|^2}{\mathrm{E}\left\{\left|\gamma s_1(k) + \boldsymbol{w}_{\text{SSB}}^H(\theta)\boldsymbol{x}_{\text{in}}(k)\right|^2\right\}} - 1\right)^2\right\} \tag{11}$$

As is well known, the aim of beamforming is to remove interference. Obviously, if θ is closer to the DoA of the first source, the interference $\boldsymbol{x}_{\text{in}}(k)$ should be eliminated more efficiently; in this case, $y(k)$ should be closer to $\gamma s_1(k)$ and $\boldsymbol{w}_{\text{SSB}}^H(\theta)\boldsymbol{x}_{\text{in}}(k)$ should be closer to 0, for every k. If so, $\mathrm{G}_{\text{MCME}}(\theta)$ will more exactly approach its minimum, $\mathrm{E}\left\{\left(|s_1(k)|^2 - 1\right)^2\right\}$, which is also just the minimum of the Godard cost function.

Remark 3.1. *If θ is closer to the DoA of the first source, the SSB beamformer shown in (6) can cancel the interference term $\mathbf{x}_{in}(k)$ more efficiently. In this case, the function (10) will approach its minimum more exactly. However, for the other analog sources, since their signals do not satisfy the CM property, the outputs of the SSB beamformer should be non-CM even though θ is close to one of the DoAs of analog sources. Therefore, the function (10) will show one valley point corresponding to the DoA of CM source, rather than* L *valley points. In other words, our method can only estimate the DoA for the CM source.*

In practical applications, since the number of snapshots is finite, (10) should be rewritten as

$$\hat{\mathrm{G}}_{\mathrm{MCME}}(\theta) = \frac{1}{N}\sum_{k=1}^{N}\left(\frac{N\cdot|y(k)|^2}{\sum_{i=1}^{N}|y(i)|^2}-1\right)^2 = \frac{1}{N}\sum_{k=1}^{N}\left(\frac{N\cdot\left|\widetilde{\boldsymbol{a}}^H(\theta)\,\boldsymbol{U}_s\Sigma_s^{-1}\boldsymbol{U}_s^H\boldsymbol{x}(k)\right|^2}{\sum_{i=1}^{N}\left|\widetilde{\boldsymbol{a}}^H(\theta)\,\boldsymbol{U}_s\Sigma_s^{-1}\boldsymbol{U}_s^H\boldsymbol{x}(i)\right|^2}-1\right)^2 \tag{12}$$

Therefore, the DoA of the first source θ_1 can be determined by solving the optimization problem

$$\min_{\theta}\hat{\mathrm{G}}_{\mathrm{MCME}}(\theta) \tag{13}$$

Obviously, if there are several CM sources impinging on the array, the cost function (12) will show several valley points corresponding to the DoAs of CM sources. Therefore, the cost function (12) can be employed to estimate the DoAs of multi-CM sources.

Since the function (12) is complicated, its gradient is also complicated. A more serious problem is that the function (12) is not convex and has local optimal points. Fortunately, (12) is a one-dimensional problem that permits us to apply a direct search algorithm for the global optimization. The similar search algorithm is also used in some conventional DoA estimation methods such as MUSIC method.

Next, we will analyze the computational complexity of the proposed method comparing with that of the MUSIC method. The cost function of the MUSIC method can be expressed as [1]

$$\max_{\theta} f_{\mathrm{MUSIC}}(\theta) = \frac{\widetilde{\boldsymbol{a}}^H(\theta)\widetilde{\boldsymbol{a}}(\theta)}{\widetilde{\boldsymbol{a}}^H(\theta)\,\boldsymbol{U}_n\boldsymbol{U}_n^H\widetilde{\boldsymbol{a}}(\theta)} \tag{14}$$

where $\boldsymbol{U}_n$ is stacked up by the $M-L$ eigenvectors of noise subspace of $\boldsymbol{R}$.

Assume there are P points to search, and assume the search region is bounded by $[\theta_{\min},\theta_{\max}]$. In this case, the search interval Δ is $(\theta_{\max}-\theta_{\min})/(P-1)$. Therefore, in the lth search, $\theta^{(l)}$ is equal to $\theta_{\min}+(l-1)\Delta$ $(l=1,2,\ldots,P)$. For each search point, the main cost of MUSIC method is to calculate $\widetilde{\boldsymbol{a}}^H(\theta^{(l)})\,\boldsymbol{U}_n\boldsymbol{U}_n^H\widetilde{\boldsymbol{a}}(\theta^{(l)})$, the complexity is $O(M(M$-$L))$, $\forall l=1,2,\ldots,P$. Finally, one can determine an optimal value of DoA from P search points, corresponding to the maximum of the MUSIC cost function (14). Therefore, the total computational complexity of MUSIC method is $O(PM(M-L))$.

Let's consider the proposed method. Since the term $\boldsymbol{U}_s\Sigma_s^{-1}\boldsymbol{U}_s^H\boldsymbol{x}(k)$ is independent from the variable θ and search points in (12), one can compute this term before searching. The complexity for computing this term is $O(NLM)$. Once the terms $\boldsymbol{U}_s\Sigma_s^{-1}\boldsymbol{U}_s^H\boldsymbol{x}(k)$, $\forall k=1,2,\ldots,N$ have been calculated, we sequentially calculate the value of (12) in each search point. For the lth search, the main cost of the proposed algorithm is to calculate $\widetilde{\boldsymbol{a}}^H(\theta^{(l)})\,\boldsymbol{U}_s\Sigma_s^{-1}\boldsymbol{U}_s^H\boldsymbol{x}(k)$, $\forall k=1,2,\ldots,N$ with the computation complexity $O(NM)$. Finally, one can determine an optimal value of DoA from P search points, corresponding to the minimum of the proposed cost function (12). Therefore, the total computation complexity of the proposed method is

$$O(NLM) + O(PNM) \tag{15}$$

Since the sample size N is usually greater than the number of sources L, the computation complexity of the proposed method is usually greater than that of the MUSIC method, especially when N is large. Though the proposed algorithm has larger computational cost, it can achieve better performance. Moreover, if the channels are time invariant or varied slowly in realistic applications, the computational complexity of the proposed algorithm can be reduced tremendously. In this case, updating our algorithm is only required to be performed completely from the beginning for time invariant channels and to be performed at a low periodicity for slow time-varying ones. After updating, one can just use a conventional beamformer to estimate the source signals in real time, taking advantage of the DoA information estimated previously by the proposed algorithm.

Once the DoAs of CM sources are solved by optimization of (13), the weights of SSB beamformer (6) can be sequentially determined. One can employ these weight vectors to recover the CM signals.

4. **Numerical Experiments and Results.** Assume a uniform linear array with $M = 4$ omni-directional sensors spaced at half-wavelength intervals. In this case, the presumed array manifold can be characterized analytically. That is,

$$\widetilde{\Omega} = \{\widetilde{\boldsymbol{a}}(\theta) : \widetilde{\boldsymbol{a}}(\theta) = (1, \ldots, \exp\{-j\pi(M-1)\sin\theta\})^H, \theta \in \Theta\}. \tag{16}$$

Assume $\Delta(\theta)$ in (2) is a white noise vector with covariance matrix $\sigma_{na}^2\boldsymbol{I}$. The uncertainty ratio (UR) is defined as [13]

$$UR = 10\log\sigma_{na}^2 \tag{17}$$

In all experiments, the signal-to-noise ratio (SNR) is fixed to 15 dB.

4.1. **The effectivity of DOA estimation for the proposed method.** In this setting, we assigned one CM signal source and two interference sources (Non-CM with the white Gaussian distribution) with plane wavefronts; their DoAs are 30° (CM), 10° (user 2) and 50° (user 3), respectively. In order to present the algorithm performance both for high-power and low-power interference cases (simulate a near-far effect), the channel attenuation coefficients are assigned to $c_1 = 1$; $c_2 = 10$; $c_3 = 1$.

Figure 1 shows the typical results obtained from the proposed method and the MUSIC method in scenarios of $UR = -40$dB, $N = 500$ and $UR = -20$dB, $N = 100$, respectively. In fact, many other well known DoA estimation methods, such as Maximum Likelihood method, ESPRIT method and some other robust methods [4,6], etc., had also been evaluated in our experiments. However, since the typical results of them are similar or worse than those of the MUSIC method under the given experimental environment, in Figure 1, we only compare the typical results of our method with that of MUSIC algorithm. Since there are one CM source (30°) and two analog sources (10° and 50°) impinging on the array, the plots of MUSIC method should appear three peaks near 10°, 30° and 50°. Whereas, the proposed method can only estimate the DoA for the CM source, the plots of the proposed method should appear one valley near 30°.

Figure 1 shows that, when the uncertainty ratio is small and the sample size is long ($UR = -40$dB, $N = 500$), the MUSIC algorithm can estimate the DoAs effectively. However, if the uncertainty ratio is great and sample size is too short ($UR = -20$dB, $N = 100$), the MUSIC algorithm becomes invalid, the peak in 30° disappears. In contrast to this, the proposed method can obtain the DoA of CM source that is very close to the true value, 30°, even if the UR is great ($UR = -20$dB) and/or the sample size is very short ($N = 100$).

To evaluate the performance of our method versus uncertainty ratio, we conduct 1,000 blocks data and the length of each block is equal to the sample size $N = 100$. The uncertainty ratio varies from -25dB to -15dB. The DoA estimation absolute error for each block is defined as $|\theta_1 - \hat{\theta}_1|$. We show the average estimation absolute errors about the CM source's DoA in Table 1, which shows that our method can efficiently estimate the DoA of the CM source even UR is increased to -15dB.

From the experimental results, we can conclude that, if there exist both CM sources and non-CM sources impinging on the array, the proposed method can estimate the DoA of the CM source effectively even in the case that the conventional DoA estimation methods become invalid. Though this method cannot estimate the DoAs of the non-CM sources, if the purpose of DoA estimation is only for providing the beamformers with steering vectors to recover the communication users signals, the method is enough for this purpose. Since, in digital communications, a non-CM source is definitely an interference source.

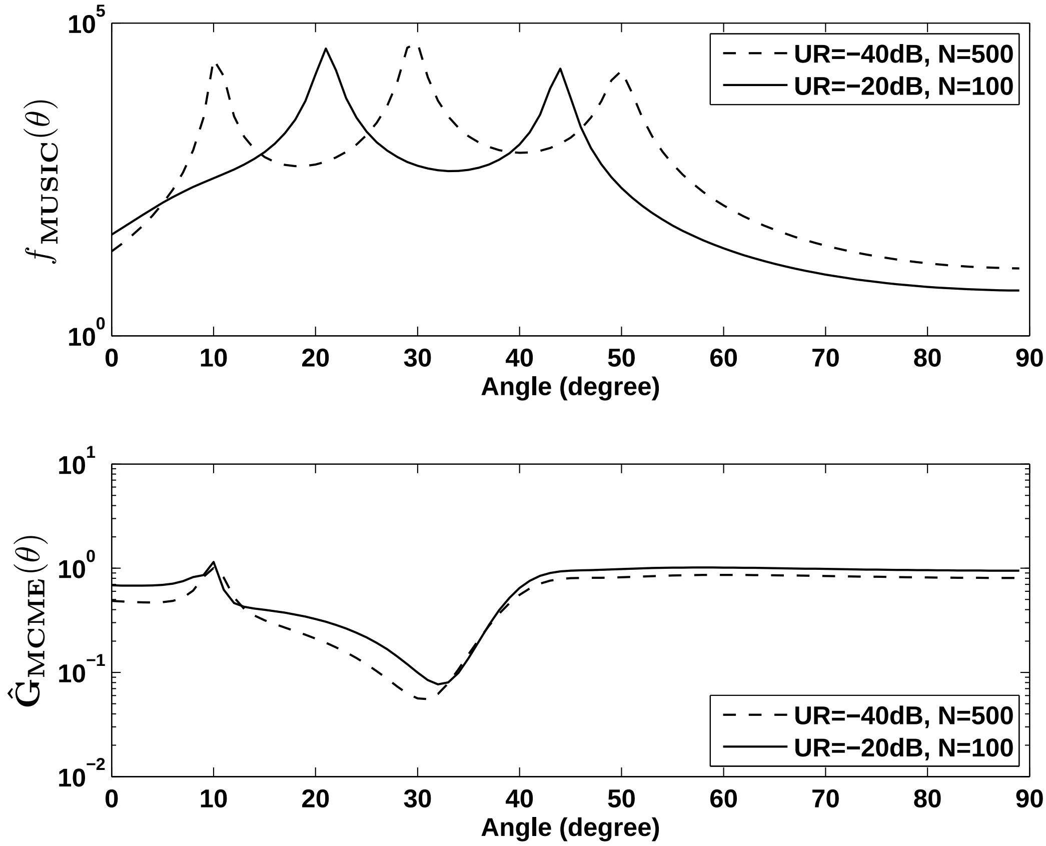

Figure 1. The effectivity of DOA estimation for the proposed method compared with the MUSIC method

Table 1. The average estimation absolute errors (AEAE) versus UR

UR (dB)	-25	-23	-21	-19	-17	-15
AEAE (Degree)	1.575	1.611	1.764	1.999	2.257	2.658

4.2. **The resolution of DOA estimation for the proposed method.** To evaluate the resolution of the proposed method, we assigned two CM signal sources with the same channel attenuation coefficients. One DoA of CM source (CM1) is fixed to 30°; another source's (CM2) DoA is varied. The UR is fixed to -20dB.

Figure 2 shows the typical results obtained from the proposed method and the MUSIC method in scenarios of DoA of CM2 are 18° and 26°, respectively. Figure 2 shows that, due to the effects of imprecise knowledge about the array manifold and finite sample size, the MUSIC algorithm cannot discriminate the DoAs located in a near region. However, the proposed method can distinguish the DoAs even if the DoA of CM2 is 26°.

We also conduct 1,000 blocks data and the length of each block is equal to the sample size $N = 100$. The uncertainty ratio is fixed to -20dB. We show the AEAE about two CM source's DoAs in Table 2. Table 2 shows that our method can efficiently estimate the DoAs of the CM sources even though their positions are near.

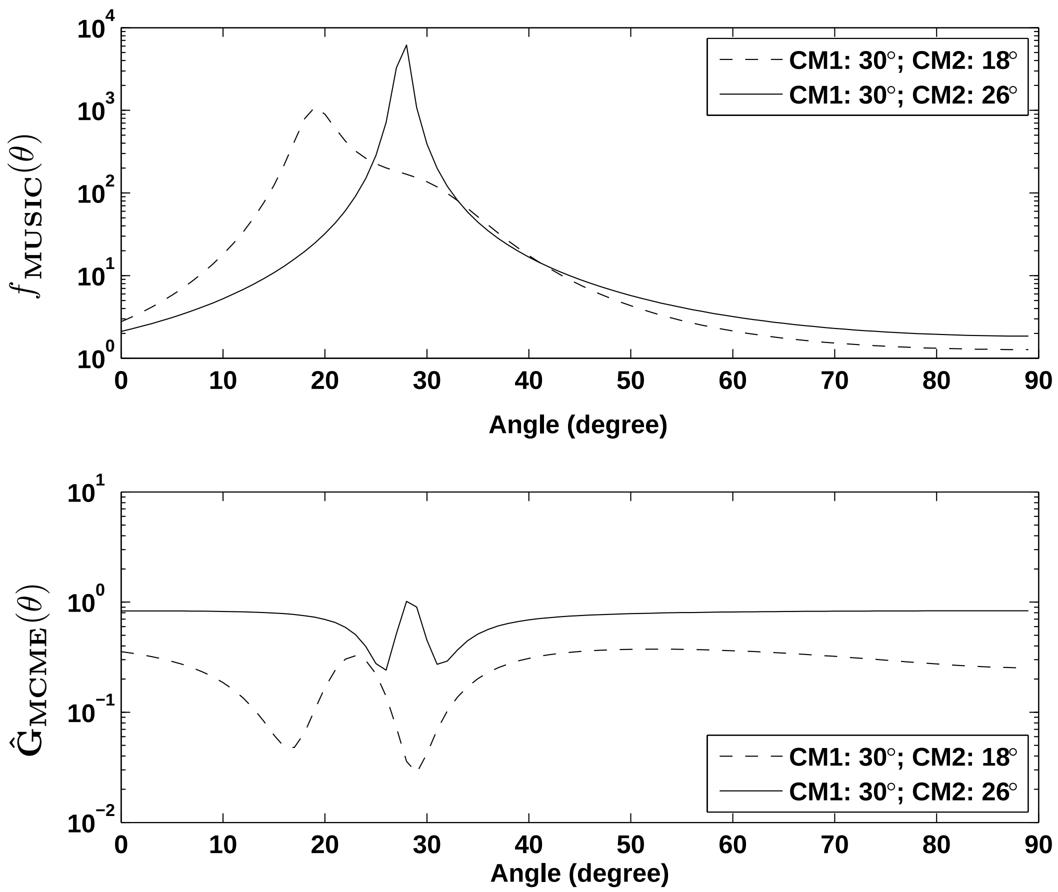

FIGURE 2. The resolution of DOA estimation for the proposed method compared with the MUSIC method

TABLE 2. The AEAE versus DoA of CM2

DoA of CM2 (Degree)	18	20	22	24	26
AEAE (Degree)	1.848	1.449	1.105	0.973	1.033

4.3. **The performance of beamformer for the proposed method.** In this setting, the experiment data are same as those of experiment 4.1. That is, 100,000 samples are divided into 1000 blocks. The uncertainty ratio varies from −25dB to −15dB. The average estimation absolute errors about the CM source's DoA has been shown in Table 1. We use the mean output Signal to Interference plus Noise Ratio (SINR) to evaluate the performance of SSB beamformer [20]. The output SINR for each block is defined as:

$$\mathrm{SINR} = \frac{\hat{\sigma}_1^2 |c_1|^2 |\boldsymbol{w}_{\mathrm{SSB}}^H(\hat{\theta}_1)\boldsymbol{a}(\theta_1)|^2}{\boldsymbol{w}_{\mathrm{SSB}}^H(\hat{\theta}_1)\hat{\boldsymbol{R}}_{\mathrm{in}}\boldsymbol{w}_{\mathrm{SSB}}(\hat{\theta}_1)} \tag{18}$$

where $\hat{\sigma}_1^2 = \frac{1}{N}\sum_{k=1}^{N}|s_1(k)|^2$; $\hat{\theta}_1$ is the estimation of θ_1 that is obtained from the proposed DoA estimation method; $\hat{\boldsymbol{R}}_{\mathrm{in}} = \frac{1}{N}\sum_{k=1}^{N}\boldsymbol{x}_{\mathrm{in}}(k)\boldsymbol{x}_{\mathrm{in}}^H(k)$. In each scenario, the results of 1,000 simulation runs have been averaged to obtain the mean output of SINR.

Figure 3 shows that, due to the effects of imprecise array manifold and/or finite sample size, the highest mean output SINR of SSB beamformer is usually achieved at a certain DoA within a near region of the true one that may be different from the true one of CM source itself. Since the proposed method utilized the CM property to estimate DoA, it can really search the DoA to make the SSB beamformer achieving higher mean output SINR even than the estimation by using the true value of DoA.

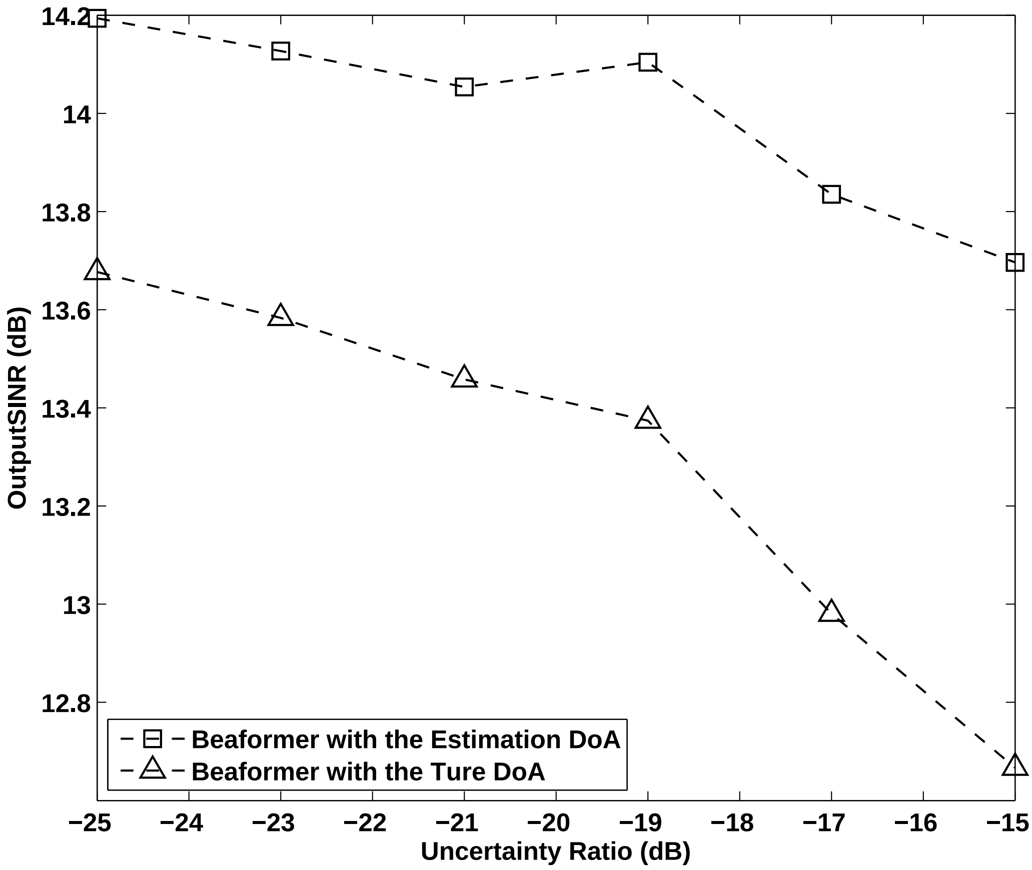

FIGURE 3. The output SINRs of SSB beamformer with the estimation DoA and with the true one

5. **Conclusions and Discussions.** In this paper, invoking the CM feature, we proposed a robust DoA estimation method inter-cooperative with a SSB beamformer. In this algorithm, the DoA estimation and beamforming can be done simultaneously. In this way, the algorithm can (i) effectively estimate the DoAs of CM sources in the case of great uncertainty ratio with a short sample size; (ii) by using these DoA estimations, the SSB beamformer can achieve a good performance that is even better than that by using the true DoAs.

In [20], the authors have presented a modified Godard cost function for multilevel modulation systems. Therefore, in principle, it is no doubt to employ the proposed method for the multilevel modulation systems, such as M-ary PAM and M-ary QAM systems. However, the CM-based methods require much more samples if the modulation systems are multilevel. As the formula (15) shows, the computational complexity of the proposed method is in direct proportion to the sample size N. If we apply the method to the multilevel modulation systems, the computational cost might be unacceptable in practical applications. Therefore, how to reduce the computational complexity of the proposed method is a valuable problem and worth researching in the future.

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