International Journal of Innovative Computing, Information and Control
Volume **8**, Number **6**, June **2012**

ICIC International ©2012 ISSN 1349-4198
pp. 4133–4145

RELAXED STABILIZATION CONDITIONS FOR SWITCHING T-S FUZZY SYSTEMS WITH PRACTICAL CONSTRAINTS

Chung-Hsun Sun[1], Shih-Wei Lin[2] and Yin-Tien Wang[1]

[1]Department of Mechanical and Electro-mechanical Engineering
Tamkang University
No. 151, Yingzhuan Road, Tamsui District, New Taipei City 25137, Taiwan
{ chsun; ytwang }@mail.tku.edu.tw

[2]Nuclear Instrumentation Division
Institute of Nuclear Energy Research
Atomic Energy Council
No. 1000, Wenhua Rd., Jiaan Village, Longtan Township, Taoyuan 32546, Taiwan
wwlin@iner.gov.tw

Received January 2011; revised July 2011

Abstract. *Stability issues in Takagi-Sugeno (T-S) fuzzy control system have been widely investigated using the Lyapunov theorem. With respect to stability analysis, the group-fired rules and the maximal width of state steps are applied to deriving a more relaxed stability criterion for T-S fuzzy discrete models. For the stabilization problem, however, the controller synthesis and the analysis of the maximal width of state steps cannot be performed simultaneously. Evolutionary algorithms or the iterative linear matrix inequality (ILMI) methods can be employed to solve this problem, but the computational demand is high. In this study, the practical constraints of a control system are adopted in place of the iterative procedures of the ILMI method. The method reduces the computational burden of the previous stabilization design. Finally, two simulation results are carried out to demonstrate the effectiveness of the proposed controller design method.*
Keywords: Relaxed conditions, T-S fuzzy discrete system, Input constraint

1. **Introduction.** During the last decade, based on Takagi-Sugeno (T-S) fuzzy system and Lyapunov theorem, the fuzzy control issues have been extensively explored [1-13]. Tanaka and Sugeno employed a single quadratic Lyapunov function $V(k) = \mathbf{x}^T(k)\mathbf{P}\mathbf{x}(k)$ to verify the stability of a T-S fuzzy discrete system [1]. In [1], the stability criterion states that if a common positive definite matrix $\mathbf{P}$ can be found to satisfy all Lyapunov inequalities, then the stability of the T-S fuzzy discrete system is guaranteed. The common positive definite matrix $\mathbf{P}$ searching problem can be converted into a linear matrix inequality (LMI) problem, which can be solved efficiently by MATLAB LMI toolbox.

Other studies [14-19] have utilized a piecewise quadratic Lyapunov function $V(k) = \sum_j \delta_j \mathbf{x}^T(k)\mathbf{P}_j\mathbf{x}(k)$ to derive stability criteria for T-S fuzzy discrete systems. To derive these criteria using the piecewise quadratic Lyapunov function, the T-S fuzzy system should firstly be transformed into an equivalent switching T-S fuzzy system. Each switching region corresponds to a piecewise Lyapunov function $V(k) = \mathbf{x}^T(k)\mathbf{P}_j\mathbf{x}(k)$ and the stability criterion can be derived. In [16], the stability criterion is that if all the positive definite matrices $\mathbf{P}_j$ can be found to satisfy the corresponding Lyapunov inequalities, the stability of the T-S fuzzy system is guaranteed. Although the piecewise quadratic Lyapunov function yields more Lyapunov inequalities than a single quadratic Lyapunov function does, the piecewise quadratic Lyapunov function derives a more relaxed stability criterion. Also, the positive definite matrices $\mathbf{P}_j$ can be solved by LMI method.

According to the universal approximation theorem, the T-S fuzzy model can approximate any nonlinear system. In general, the more rules there are, the smaller modelling error is. Nevertheless, while a fuzzy system has large rules, the signal/piecewise Lyapunov functions may not be found even using the LMI tool. Therefore, large fuzzy rules induce conservative stability conditions. Hence, the relaxation of the stability conditions in T-S fuzzy systems with large fuzzy rules is proposed. Applying ideas of group-fired rules and the maximal width of state steps, enables the positive definite matrices $\mathbf{P}_j$ to satisfy fewer corresponding Lyapunov inequalities. Then, more relaxed stability criteria for discrete T-S fuzzy systems are proposed [14]. However, with respect to stabilization, the analysis of the maximal state step width and the design of the controller cannot be performed simultaneously. Presently, scholars solve this problem by computing the maximal width of state steps and control gains iteratively. Consequently, the problem of finding positive definite matrices $\mathbf{P}_j$ is transformed into an iterative linear matrix inequality (ILMI) problem [14,20,21], but the computational requirement is heavy. In this study, to reduce the computing demand of the aforementioned stabilization method, the iterative procedures will be replaced by a direct scheme. The concept of input constraint is considered in this direct scheme. In practical applications, a control signal always has a definite range, which is called the input constraint [2,3,23-27]. The constraint of a control input can be applied to estimating the maximal width of state steps. Then, according to the ideas of group-fired rules, one can find out the positive definite matrices $\mathbf{P}_j$ that satisfy corresponding Lyapunov inequalities. Additionally, all of the input constraints and Lyapunov inequalities can be represented in LMI form and solved simultaneously. Accordingly, no iterative computational scheme is required. Then, a novel methodology with a low computing demand and relaxed stabilization conditions is proposed. Finally, the effectiveness of the approach is demonstrated in two simulation results.

2. **Stability Conditions for Switching T-S Fuzzy Systems.** This section firstly reviews the stability criterion for a T-S fuzzy discrete system based on a piecewise quadratic Lyapunov function. Next, ideas of group-fired rules and the maximal width of state steps are applied to introducing a relaxed stability criterion.

Consider the following T-S fuzzy discrete system.

$$\begin{aligned} \text{Rule } l\text{: If } & x_1(k) \text{ is } M_{l1},\ x_2(k) \text{ is } M_{l2},\ \ldots,\ x_n(k) \text{ is } M_{ln}, \\ & \text{then } \mathbf{x}(k+1) = \mathbf{A}_l\mathbf{x}(k) + \mathbf{B}_l\mathbf{u}(k) \end{aligned} \tag{1}$$

where $l = 1, 2, \ldots, r$. r and n are the numbers of rules and state variables, respectively. $\mathbf{x}(k) = [x_1(k), x_2(k), \ldots, x_n(k)]^T$ is a state vector. $\mathbf{u}(k) = [u_1(k), u_2(k), \ldots, u_m(k)]^T$ is an input vector. M_{lj} is a fuzzy set. $\mathbf{A}_l$ and $\mathbf{B}_l$ are the system matrix and the input matrix, respectively. The final output of the above system is inferred as:

$$\mathbf{x}(k+1) = \sum_{l=1}^{r} w_l(k)[\mathbf{A}_l\mathbf{x}(k) + \mathbf{B}_l\mathbf{u}(k)] \tag{2}$$

where $w_l(k) = \prod_{h=1}^{n} M_{lh}(x_h(k)) / \sum_{l=1}^{r} \prod_{h=1}^{n} M_{lh}(x_h(k))$.

To prove the stability criterion using a piecewise quadratic Lyapunov function, the T-S fuzzy system should be transformed into an equivalent switching T-S fuzzy system in advance. The switching T-S fuzzy system is constructed from crisp region rules and local fuzzy rules. In each region rule (say sub-region), the state vector $\mathbf{x}(k)$ fires the same local fuzzy rules meaning that the state space $\mathbf{X}$ is divided into some non-overlapping sub-regions. For example, in Figure 1, the universal set of the premise variable x_h is divided into several intervals. Figure 2 presents an example of membership functions and the associated rules table for $\mathbf{x}(k) \in \mathbf{R}^2$ [17]. For example, the states that are in the

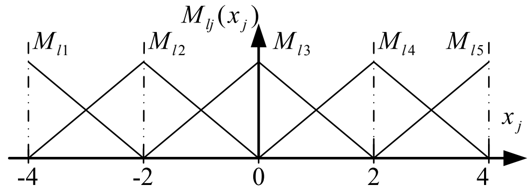

FIGURE 1. The partition of membership functions

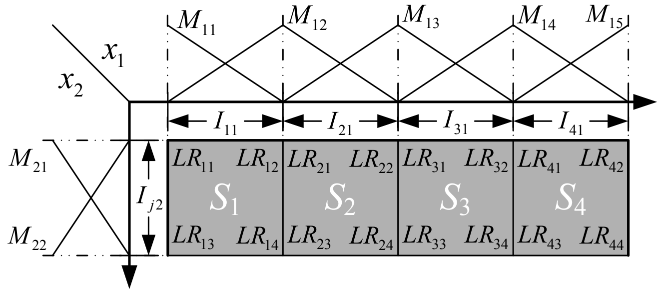

FIGURE 2. The geometry of a switching T-S fuzzy system

sub-region $\mathbf{S}_1$, fire the same local rules $\{\mathbf{LR}_{11}, \mathbf{LR}_{12}, \mathbf{LR}_{13}, \mathbf{LR}_{14}\}$. The system (1) can be rewritten as an equivalent switching T-S fuzzy system in the following form [3,17,20].

Region rule j: If $x(k) \in \mathbf{S}_j$, then

Local Plant rule $\mathbf{LR}_{jl}$: If $x_1(k)$ is M_{jl1}, $x_2(k)$ is M_{jl2}, $\ldots$, $x_n(k)$ is M_{jln},

$$\text{then } \mathbf{x}(k+1) = \mathbf{A}_{jl}\mathbf{x}(k) + \mathbf{B}_{jl}\mathbf{u}(k) \tag{3}$$

where $l = 1, 2, \ldots, \beta_j$; $j = 1, 2, \ldots, s$. $\mathbf{S}_j$ represents the j-th sub-region; s is the number of partitioned sub-regions; $\mathbf{LR}_{jl}$ denotes the l-th rule in $\mathbf{S}_j$, and β_j is the number of local fuzzy rules in the sub-region $\mathbf{S}_j$. The state space $\mathbf{X}$ is divided into s non-overlapping partitions. Accordingly, $\bigcup_{j=1}^{s} \mathbf{S}_j = \mathbf{X}$, $\mathbf{S}_i \bigcap \mathbf{S}_j = \phi$ for $i, j = 1, 2, \ldots, s$. The final output of the switching T-S fuzzy system is inferred as

$$\mathbf{x}(k+1) = \sum_{l=1}^{\beta_j} w_{jl}(k)[\mathbf{A}_{jl}\mathbf{x}(k) + \mathbf{B}_{jl}\mathbf{u}(k)], \text{ for } \mathbf{x}(k) \in \mathbf{S}_j \tag{4}$$

where $w_{jl}(k) = \prod_{h=1}^{n} M_{lh}(x_h(k)) / \sum_{l=1}^{\beta_j} \prod_{h=1}^{n} M_{lh}(x_h(k))$. Any state $\mathbf{x}(k)$ in the sub-region $\mathbf{S}_j$ can be represented as

$$\mathbf{x}(k) = \sum_{l=1}^{\beta_j} w_{jl}(k)\mathbf{x}_{jl}^{v}, \text{ for } \mathbf{x}(k) \in \mathbf{S}_j \tag{5}$$

where $\mathbf{x}_{jl}^{v}$ denotes the vertex of the sub-region $\mathbf{S}_j$. Consider, for example, the geometry of a 2-D state space. Figure 3 displays the membership functions and vertices of the sub-region $\mathbf{S}_j$, and the any state $\mathbf{x}(k) \in \mathbf{S}_j$ can be represented as follows [17].

$$\mathbf{x}(k) = \sum_{l=1}^{4} w_{jl}(k)\mathbf{x}_{jl}^{v}, \text{ for } \mathbf{x}(k) \in \mathbf{S}_j \tag{6}$$

Then, the piecewise quadratic Lyapunov function is employed to prove the stability. The

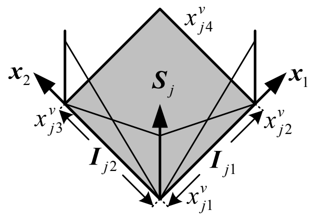

FIGURE 3. The geometry of 2-D sub-region

piecewise quadratic Lyapunov function candidate for each sub-region $\mathbf{S}_j$ is of the form

$$V_j(k) = \mathbf{x}(k)^T\mathbf{P}_j\mathbf{x}, \text{ for } \mathbf{x}(k) \in \mathbf{S}_j \tag{7}$$

The candidate Lyapunov function for the overall switching T-S fuzzy system is

$$V(k) = \sum_{j=1}^{s} g_j(k)V_j(k) = \sum_{j=1}^{s} g_j(k)\mathbf{x}(k)^T\mathbf{P}_j\mathbf{x} \tag{8}$$

where $g_j(k) = \begin{cases} 1, & \mathbf{x}(k) \in \mathbf{S}_j \\ 0, & \text{otherwise} \end{cases}$. Therefore, the stability criterion has been obtained based on piecewise quadratic Lyapunov function.

Theorem 2.1. *[16-18] Consider the switching T-S fuzzy discrete system (4). If positive definite matrices $\mathbf{P}_j$ exist such that the LMIs*

$$\mathbf{A}_{jl}^T\mathbf{P}_j\mathbf{A}_{jl} - \mathbf{P}_j < 0, \quad i, j = 1, 2, \ldots, s;\ l = 1, 2, \ldots, \beta_j. \tag{9}$$

hold, then the switching T-S fuzzy discrete system (4) is asymptotically stable.

Notably, that the matrix $\mathbf{P}_j$ in (9) is associated with the piecewise quadratic Lyapunov function for $\mathbf{x}(k) \in \mathbf{S}_j$. And the $\mathbf{P}_i$ in (9) is associated with the piecewise quadratic Lyapunov function for $\mathbf{x}(k+1) \in \mathbf{S}_i$. Define a set Ω to denote the possible state transition from $\mathbf{S}_j$ to $\mathbf{S}_i$ [18].

$$\Omega \equiv \{\langle j, i\rangle | \mathbf{x}(k) \in \mathbf{S}_j,\ \mathbf{x}(k+1) \in \mathbf{S}_i\} \tag{10}$$

In case $j = i$, it means that $\mathbf{x}(k)$ and $\mathbf{x}(k+1)$ are in the same sub-region. In case $j \neq i$, it means that state $\mathbf{x}(k)$ will go into $\mathbf{S}_i$ from $\mathbf{S}_j$. In Theorem 2.1, $\Omega = \{\langle j, i\rangle | i, j = 1, 2, \ldots, s\}$ meaning that any two consecutive states, $\mathbf{x}(k)$ and $\mathbf{x}(k+1)$, may locate anywhere of the state space $\mathbf{X}$. If the relative positions of two consecutive states, $\mathbf{x}(k)$ and $\mathbf{x}(k+1)$, could be estimated, fewer $\langle j, i\rangle$ pairs in set Ω are associated with Theorem 2.1. It implies that the number of LMIs (9) can be reduced. Next, the maximal width of state steps is utilized to estimate relative positions of $\mathbf{x}(k)$ and $\mathbf{x}(k+1)$.

$$\begin{aligned} \|\Delta\mathbf{x}(k)\|_\infty &= \|\mathbf{x}(k+1) - \mathbf{x}(k)\|_\infty \\ &= \left\| \left[\sum_{l=1}^{\beta_j} w_{jl}(k)\mathbf{A}_{jl} - \mathbf{I} \right] \mathbf{x}(k) \right\|_\infty, \text{ for } \mathbf{x}(k) \in \mathbf{S}_j \end{aligned} \tag{11}$$

where $\mathbf{I}$ is an identity matrix with appropriate dimensions. According to (5), and $0 \leq w_{jl} \leq 1$, (11) can be rewritten as

$$\begin{aligned} \|\Delta\mathbf{x}(k)\|_\infty &\leq \left\| \left[\sum_{l=1}^{\beta_j} w_{jl}(k)\mathbf{A}_{jl} - \mathbf{I} \right] \left[\sum_{l=1}^{\beta_j} w_{jl}(k)\mathbf{x}_{jl}^v \right] \right\|_\infty \\ &\leq \max_l \left\| (\mathbf{A}_{jl} - \mathbf{I})\,\mathbf{x}_{jl}^v \right\|_\infty, \text{ for } \mathbf{x}(k) \in \mathbf{S}_j \end{aligned} \tag{12}$$

Equation (12) yields the maximal width of state steps. Therefore, the possible state transitions from $\mathbf{S}_j$ to $\mathbf{S}_i$, i.e., Ω, can be obtained according to the following relation.

$$\|\Delta\mathbf{x}(k)\|_\infty \le \delta_j \cdot \min_l(\mathbf{I}_{jl}), \text{ for } \mathbf{x}(k) \in \mathbf{S}_j \tag{13}$$

where $\mathbf{I}_{jl}$ is the width of sub-region $\mathbf{S}_j$, as shown in Figure 2 and Figure 3. δ_j is the minimum positive integer that satisfies (13). Consider, for example, a switching fuzzy system with four sub-regions $\mathbf{S}_j$. Figure 4 shows the possible transitions of the two consecutive states $\mathbf{x}(k)$ and $\mathbf{x}(k+1)$, that is $\Omega = \{\langle j,i\rangle | \mathbf{x}(k) \in \mathbf{S}_j,\ \mathbf{x}(k+1) \in \mathbf{S}_i\}$. If all $\delta_j = 1$, then $\Omega = \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 3,2\rangle, \langle 3,3\rangle, \langle 3,4\rangle, \langle 4,3\rangle, \langle 4,4\rangle\}$. If all $\delta_j = 2$, then $\Omega = \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 1,3\rangle, \langle 2,1\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 2,4\rangle, \langle 3,1\rangle, \langle 3,2\rangle, \langle 3,3\rangle, \langle 3,4\rangle, \langle 4,2\rangle, \langle 4,3\rangle, \langle 4,4\rangle\}$. If all $\delta_j = 3$, the set Ω is as Theorem 2.1, $\Omega = \{\langle j,i\rangle | i,j = 1,2,\ldots,s\}$. Now, the set Ω associated with the maximal width of state steps is defined as follows.

$$\Omega \equiv \{\langle j,i\rangle | j = 1,2,\ldots,s;\ i = j(\pm 1,2,\ldots,\delta_j);\ i > 0\} \tag{14}$$

According to the information on possible state transitions, i.e., (14), then a relaxed stability criterion based on a piecewise quadratic Lyapunov function is obtained.

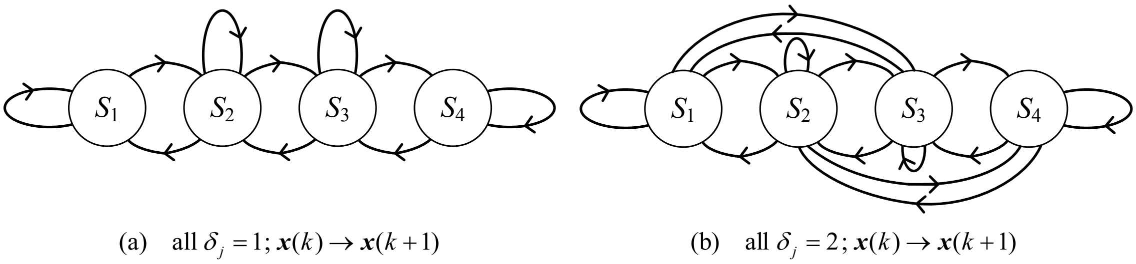

FIGURE 4. Possible transitions of two consecutive states

Theorem 2.2. *[14,17] Consider the switching T-S fuzzy discrete system (4). If there exist positive definite matrices* $\mathbf{P}_j$ *for* $j = 1,2,\ldots,s$, *such that the LMIs*

$$\mathbf{A}_{jl}^T\mathbf{P}_j\mathbf{A}_{jl} - \mathbf{P}_j < 0, \quad \langle i,j\rangle \in \Omega;\ l = 1,2,\ldots,\beta_j. \tag{15}$$

hold, then the switching T-S fuzzy discrete system (4) is asymptotically stable. Notably, the set $\Omega = \{\langle j,i\rangle | j = 1,2,\ldots,s;\ i = j \pm (1,2,\ldots,\delta_j);\ i > 0\}$ *and* δ_j *are obtained from (12) and (13).*

Remark 2.1. *In Theorem 2.1, all possible* $\langle j,i\rangle$ *pairs, for* $i,j = 1,2,\ldots,s$, *should be considered in checking the stability of a T-S fuzzy switching system. However, in Theorem 2.2 only the* $\langle j,i\rangle$ *pairs, for* $j = 1,2,\ldots,s$; $i = j \pm (1,2,\ldots,\delta_j)$; $i > 0$, *have to be checked. Theorem 2.2 yields a more relaxed result than Theorem 2.1.*

3. **Controller Synthesis for Switching T-S Fuzzy Systems.** The aim of this section is to design a controller for the switching T-S fuzzy system based on the above relaxed stability concept. The well-known parallel distributed compensation (PDC) is employed to stabilize the switching T-S fuzzy system. To apply above relaxed stability concept to controller synthesis, the PDC fuzzy controller should also be rewritten as a switching fuzzy controller:

Region rule j: If $x(k) \in \mathbf{S}_j$,

Local Plant rule $\mathbf{LR}_{jl}$: If $x_1(k)$ is M_{jl1}, $x_2(k)$ is M_{jl2}, …, $x_n(k)$ is M_{jln},

then $\mathbf{u}(k) = -\mathbf{F}_{jl}\mathbf{x}(k)$, (16)

$l = 1, 2, \ldots, \beta_j$; $j = 1, 2, \ldots, s$. The final output of the switching fuzzy controller is inferred as:

$$\mathbf{u}(k) = -\sum_{l=1}^{\beta_j} w_{jl}(k)\mathbf{F}_{jl}\mathbf{x}(k), \quad \mathbf{x}(k) \in \mathbf{S}_j \tag{17}$$

By (4) and (17), the final output of the closed-loop switching fuzzy system is

$$\mathbf{x}(k+1) = \sum_{p=1}^{\beta_j}\sum_{q=1}^{\beta_j} w_{jp}(k)w_{jq}(k)[\mathbf{A}_{jp} - \mathbf{B}_{jp}\mathbf{F}_{jq}]\mathbf{x}(k), \text{ for } \mathbf{x}(k) \in \mathbf{S}_j \tag{18}$$

In the relaxed stabilization problem, the maximal width of state steps is

$$\begin{aligned} \|\Delta\mathbf{x}(k)\|_\infty &= \|\mathbf{x}(k+1) - \mathbf{x}(k)\|_\infty \\ &\le \left\| \left\{ \sum_{p=1}^{\beta_j}\sum_{q=1}^{\beta_j} w_{jp}(k)w_{jq}(k)\left[\mathbf{A}_{jp} - \mathbf{B}_{jp}\mathbf{F}_{jq}\right] - \mathbf{I} \right\} \left\{ \sum_{p=1}^{\beta_j} w_{jp}(k)\mathbf{x}_{jp}^v \right\} \right\|_\infty \\ &\le \max_{p,q,l} \left\| \left(\mathbf{A}_{jp} - \mathbf{B}_{jp}\mathbf{F}_{jq} - \mathbf{I}\right)\mathbf{x}_{jl}^v \right\|_\infty, \text{ for } \mathbf{x}(k) \in \mathbf{S}_j \end{aligned} \tag{19}$$

To obtain the possible state transitions from $\mathbf{S}_j$ to $\mathbf{S}_i$, i.e., Ω, we try to find the minimum positive integer δ_j as (13) to satisfy

$$\|\Delta\mathbf{x}(k)\|_\infty \le \max_{p,q,l} \left\| \left(\mathbf{A}_{jp} - \mathbf{B}_{jp}\mathbf{F}_{jq} - \mathbf{I}\right)\mathbf{x}_{jl}^v \right\|_\infty \le \delta_j \cdot \min_p(\mathbf{I}_{jp}), \text{ for } \mathbf{x}(k) \in \mathbf{S}_j \tag{20}$$

where $\mathbf{I}_{jp}$ is the width of sub-region $\mathbf{S}_j$, as shown in Figure 2 and Figure 3, Consequently, a corollary of Theorem 2.2 applies.

Corollary 3.1. *Consider the switching T-S fuzzy discrete system (4). If there exist positive definite matrices $\mathbf{P}_j$ for $j = 1, 2, \ldots, s$, such that the LMIs*

$$(\mathbf{A}_{jp} - \mathbf{B}_{jp}\mathbf{F}_{jq})^T\mathbf{P}_j(\mathbf{A}_{jp} - \mathbf{B}_{jp}\mathbf{F}_{jq}) - \mathbf{P}_i < 0, \quad \langle i, j\rangle \in \Omega;\ l = 1, 2, \ldots, \beta_j \tag{21}$$

hold, then the switching T-S fuzzy discrete system (4) is asymptotically stable. Notably, $\Omega = \{\langle j, i\rangle | j = 1, 2, \ldots, s;\ i = j \pm (1, 2, \ldots, \delta_j);\ i > 0\}$, where δ_j is obtained from (20).

Remark 3.1. *To estimate δ_j in (20), the feedback control gain $\mathbf{F}_{jl}$ must be determined in advance. Then, δ_j can be obtained and the stability is checked according to Corollary 3.1. If the stability conditions of the closed-loop system are not met, the above process must be repeated. This design procedure involves an iterative process and heavy computational requirement. Most recently developed stabilization theorems for the T-S fuzzy system, however, can obtain the control gains and ensure the stability of the closed-loop system simultaneously. A more effective stabilization scheme is presented below.*

In practical applications, the limited power, driver circuit devices, mechanism, approximation error and other limitations cause the outputs of a controller to be always with definite ranges that are defined as the input constraints [2,3,23-27]. Accordingly, the input constraint (say $\|\mathbf{u}(k)\|_2 < \mu$) is employed firstly to estimate δ_j. Then, controller synthesis and stability checking can be achieved simultaneously.

According to (4), $\|\Delta\mathbf{x}(k)\|_\infty$ for $\mathbf{x}(k)\in\mathbf{S}_j$ is rewritten as

$$\begin{aligned}\|\Delta\mathbf{x}(k)\|_\infty &= \|\mathbf{x}(k+1)-\mathbf{x}(k)\|_\infty\\ &= \left\|\sum_{l=1}^{\beta_j} w_{jl}(k)\big[\mathbf{A}_{jl}+\mathbf{B}_{jl}\mathbf{u}(k)\big]-\mathbf{x}(k)\right\|_\infty\\ &= \left\|\left[\sum_{l=1}^{\beta_j} w_{jl}(k)\mathbf{A}_{jl}-\mathbf{I}\right]\mathbf{x}(k)+\sum_{l=1}^{\beta_j} w_{jl}(k)\mathbf{B}_{jl}\mathbf{u}(k)\right\|_\infty \\ &\le \left\|\left[\sum_{l=1}^{\beta_j} w_{jl}(k)\mathbf{A}_{jl}-\mathbf{I}\right]\mathbf{x}(k)\,\right\|_\infty+\left\|\sum_{l=1}^{\beta_j} w_{jl}(k)\mathbf{B}_{jl}\mathbf{u}(k)\right\|_\infty \quad \text{for } \mathbf{x}(k)\in\mathbf{S}_j\end{aligned} \tag{22}$$

According to (5) and assume $\|\mathbf{u}(k)\|_\infty\le\|\mathbf{u}(k)\|_2<\mu$, then

$$\begin{aligned}\|\Delta\mathbf{x}(k)\|_\infty &\le \left\|\left[\sum_{l=1}^{\beta_j} w_{jl}(k)\mathbf{A}_{jl}-\mathbf{I}\right]\left[\sum_{l=1}^{\beta_j} w_{jl}(k)\mathbf{x}_{jl}^v\right]\right\|_\infty+\left\|\sum_{l=1}^{\beta_j} w_{jl}(k)\mathbf{B}_{jl}\right\|_\infty\left\|\mathbf{u}(k)\right\|_\infty\\ &\le \max_l\left\|(\mathbf{A}_{jl}-\mathbf{I})\,\mathbf{x}_{jl}^v\right\|_\infty+\max_l\left\|\mathbf{B}_{jl}\right\|_\infty\cdot\mu\\ &\le \delta_j\cdot\min_p(\mathbf{I}_{jp}),\ \text{ for } \mathbf{x}(k)\in\mathbf{S}_j\end{aligned} \tag{23}$$

where $\mathbf{I}$ is an identity matrix with appropriate dimensions; $\mathbf{I}_{jp}$ is the width of sub-region $\mathbf{S}_j$, as displayed in Figure 2 and Figure 3, and δ_j is the minimum positive integer satisfying (23). Consequently, the set $\Omega=\{\langle j,i\rangle|j=1,2,\ldots,s;\ i=j\pm(1,2,\ldots,\delta_j);\ i>0\}$ as (14) is obtained. Furthermore, by the following lemma, the input constraints $\|\mathbf{u}(k)\|_2<\mu$ can be represented in LMI form and solved simultaneously with stabilization problem.

Lemma 3.1. *For the system (2), consider the single quadratic Lyapunov function $V(k)=\mathbf{x}(k)^T\mathbf{P}\mathbf{x}(k)$. Assume that the unknown initial condition $\mathbf{x}(0)$ is bounded, i.e., $\|\mathbf{x}(0)\|<\phi$. The constraint $\|\mathbf{u}(k)\|_2<\mu$ is enforced at all time $k\ge 0$ if the LMIs*

$$\mathbf{Q}>\phi^2\mathbf{I} \tag{24}$$

$$\begin{bmatrix}\mathbf{Q} & \mathbf{K}_l^T\\ \mathbf{K}_l & \mu^2\mathbf{I}\end{bmatrix}>0 \tag{25}$$

hold, where $\mathbf{Q}=\mathbf{P}^{-1}$, $\mathbf{K}_j=\mathbf{F}_l\mathbf{Q}$ and $j=1,2,\ldots,r$.

Therefore, we have the following main result.

Theorem 3.1. *Consider the switching T-S fuzzy discrete system (4). The switching T-S fuzzy discrete system is asymptotically stable, if there exist positive definite matrices $\mathbf{P}_j$ for $j=1,2,\ldots,s$, such that the LMIs*

$$\mathbf{Q}_j>\phi^2\mathbf{I} \tag{26}$$

$$\begin{bmatrix}\mathbf{Q}_j & *\\ \mathbf{K}_{jq} & \mu^2\mathbf{I}\end{bmatrix}>0 \tag{27}$$

$$\begin{bmatrix}4\mathbf{Q}_j & *\\ \mathbf{A}_{jp}\mathbf{Q}_j-\mathbf{B}_{jp}\mathbf{K}_{jq}+\mathbf{A}_{jq}\mathbf{Q}_j-\mathbf{B}_{jq}\mathbf{K}_{jp} & \mathbf{Q}_i\end{bmatrix}>0,\quad \langle j,i\rangle\in\Omega;\ 1\le p\le q\le\beta_j \tag{28}$$

hold, where $\mathbf{Q}_j=\mathbf{P}^{-1}$, $\mathbf{K}_{jq}=\mathbf{F}_{jq}\mathbf{Q}_j$, and ϕ is the upper bound of the unknown initial condition $\mathbf{x}(0)$, i.e., $\|\mathbf{x}(0)\|<\phi$. The asterisk indicates the transposed element for symmetric positions. The set $\Omega=\{\langle j,i\rangle|j=1,2,\ldots,s;\ i=j\pm(1,2,\ldots,\delta_j);\ i>0\}$ and δ_j are obtained from (23).

Proof: Firstly, for the convenience of presentation, (18) is rewritten as

$$\mathbf{x}(k+1) = \sum_{p=1}^{\beta_j}\sum_{q=1}^{\beta_j} w_{jp}(k)w_{jq}(k)\mathbf{A}^c_{jpq}\mathbf{x}(k), \text{ for } \mathbf{x}(k) \in \mathbf{S}_j \tag{29}$$

where $\mathbf{A}^c_{jpq} = \mathbf{A}_{jp} - \mathbf{B}_{jp}\mathbf{F}_{jq}$. Considering the piecewise Lyapunov function (4) and taking the difference between $V(k+1)$ and $V(k)$, we have

$$\Delta V(k) = V(k+1) - V(k) = \sum_{i=1}^{s} g_i(k+1)\mathbf{x}(k+1)^T\mathbf{P}_i\mathbf{x}(k+1) - \sum_{j=1}^{s} g_j(k)\mathbf{x}(k)^T\mathbf{P}_i\mathbf{x}(k) \tag{30}$$

According to $g_j(k) = \begin{cases} 1, & \mathbf{x}(k) \in \mathbf{S}_j \\ 0, & \text{otherwise} \end{cases}$ and $\Omega = \{\langle j,i\rangle | \mathbf{x}(k) \in \mathbf{S}_j,\ \mathbf{x}(k+1) \in \mathbf{S}_i\}$, then

$$\begin{aligned}
\Delta V(k) &= \mathbf{x}(k+1)^T\mathbf{P}_i x(k+1) - \mathbf{x}(k)^T\mathbf{P}_j\mathbf{x}(k) \\
&= \sum_{p=1}^{\beta_j}\sum_{q=1}^{\beta_j}\sum_{u=1}^{\beta_j}\sum_{v=1}^{\beta_j} w_{jp}(k)w_{jq}(k)w_{ju}(k)w_{jv}(k)\mathbf{x}(k)^T\left[\left(\mathbf{A}^c_{jpq}\right)^T\mathbf{P}_i\mathbf{A}^c_{jpq} - \mathbf{P}_j\right]\mathbf{x}(k) \\
&\le \frac{1}{4}\sum_{p=1}^{\beta_j}\sum_{q=1}^{\beta_j} w_{jp}(k)w_{jq}(k)\mathbf{x}(k)^T\left[\left(\mathbf{A}^c_{jpq} + \mathbf{A}^c_{jqp}\right)^T\mathbf{P}_i\left(\mathbf{A}^c_{jpq} + \mathbf{A}^c_{jqp}\right) - 4\mathbf{P}_j\right]\mathbf{x}(k) \\
&= \frac{1}{4}\sum_{p=1}^{\beta_j}(w_{jp}(k))^2\mathbf{x}(k)^T\left[\left(\mathbf{A}^c_{jpp} + \mathbf{A}^c_{jpp}\right)^T\mathbf{P}_i\left(\mathbf{A}^c_{jpp} + \mathbf{A}^c_{jpp}\right) - 4\mathbf{P}_j\right]\mathbf{x}(k) \\
&\quad + \frac{2}{4}\sum_{p=1}^{\beta_j}\sum_{q>p} w_{jp}(k)w_{jq}(k)\mathbf{x}(k)^T\left[\left(\mathbf{A}^c_{jpq}+\mathbf{A}^c_{jqp}\right)^T\mathbf{P}_i\left(\mathbf{A}^c_{jpq}+\mathbf{A}^c_{jqp}\right) - 4\mathbf{P}_j\right]\mathbf{x}(k)
\end{aligned} \tag{31}$$

Then $\Delta V(k) < 0$, if the following LMIs hold:

$$\left(\mathbf{A}^c_{jpp} + \mathbf{A}^c_{jpp}\right)^T\mathbf{P}_i\left(\mathbf{A}^c_{jpp} + \mathbf{A}^c_{jpp}\right) - 4\mathbf{P}_j < 0; \quad \langle j,i\rangle \in \Omega;\ p = 1,2,\ldots,\beta_j. \tag{32}$$

$$\left(\mathbf{A}^c_{jpq} + \mathbf{A}^c_{jqp}\right)^T\mathbf{P}_i\left(\mathbf{A}^c_{jpq} + \mathbf{A}^c_{jqp}\right) - 4\mathbf{P}_j < 0; \quad \langle j,i\rangle \in \Omega;\ 1 \le p < q \le \beta_j. \tag{33}$$

Combining (32) with (33) and using Schur complements, then we have the following equivalent LMIs.

$$\begin{bmatrix} 4\mathbf{P}_j & * \\ \mathbf{A}_{jp} - \mathbf{B}_{jp}\mathbf{F}_{jq} + \mathbf{A}_{jq} - \mathbf{B}_{jq}\mathbf{F}_{jp} & \mathbf{P}_i^{-1} \end{bmatrix} > 0, \quad \langle j,i\rangle \in \Omega;\ 1 \le p \le q \le \beta_j. \tag{34}$$

Multiplying (34) on the right and left by *block-diag*$[\mathbf{P}^{-1} \quad \mathbf{I}]$ and defining $\mathbf{Q}_j = \mathbf{P}_j^{-1}$ and $\mathbf{K}_{jq} = \mathbf{F}_{jq}\mathbf{Q}_j$, then we can obtain (28).

Moreover, according to the concept of the maximal width of state steps and input constraints, i.e., $\|\mathbf{u}(k)\|_\infty \le \|\mathbf{u}(k)\|_2 < \mu$, then δ_j can be obtained from (23). Consequently, $\Omega = \{\langle j,i\rangle | j = 1,2,\ldots,s;\ i = j \pm (1,2,\ldots,\delta_j);\ i > 0\}$ summarizes the relation of any two consecutive states $\mathbf{x}(k) \in \mathbf{S}_j$ and $\mathbf{x}(k+1) \in \mathbf{S}_i$. Equations (26) and (27) are corollaries of Lemma 3.1. The proof is completed.

4. **Examples.** This section presents two examples to prove the effectiveness of the proposed idea.

Example 4.1. *Consider a nonlinear T-S fuzzy system with eight rules as follows:*

$$\textit{Rule } l\textit{: If } x_1(k) \textit{ is } M_{l1}\textit{, then } \mathbf{x}(k+1) = \mathbf{A}_l\mathbf{x}(k) + \mathbf{B}_l\tan(\mathbf{u}(k)) \tag{35}$$

where

$$\mathbf{A}_1 = \begin{bmatrix} 0.8148 & 0.2689 \\ 0.0689 & 0.7308 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 0.8103 & 0.1771 \\ 0.0708 & 0.8103 \end{bmatrix}, \quad \mathbf{A}_3 = \begin{bmatrix} 0.8041 & 0.0715 \\ 0.0357 & 0.8219 \end{bmatrix},$$
$$\mathbf{A}_4 = \begin{bmatrix} 0.8033 & 0.0714 \\ 0.0178 & 0.8212 \end{bmatrix}, \quad \mathbf{A}_5 = \begin{bmatrix} 0.8087 & 0.1416 \\ 0.0708 & 0.8087 \end{bmatrix}, \quad \mathbf{A}_6 = \begin{bmatrix} 0.8056 & 0.0707 \\ 0.0707 & 0.8056 \end{bmatrix},$$
$$\mathbf{A}_7 = \begin{bmatrix} 0.8072 & 0.0711 \\ 0.1067 & 0.8161 \end{bmatrix}, \quad \mathbf{A}_8 = \begin{bmatrix} 0.8107 & 0.0657 \\ 0.1806 & 0.6957 \end{bmatrix}.$$

$\mathbf{B}_1 = \mathbf{B}_2 = \cdots = \mathbf{B}_8 = [0.1\ 0.4]^T$. *Moreover, membership functions of this system are shown as Figure 5.*

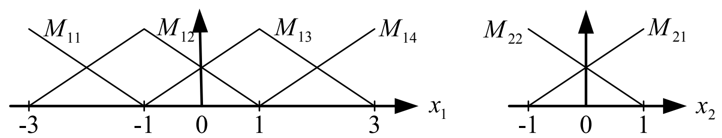

FIGURE 5. Membership functions of Example 4.1

Firstly, convert the nonlinear T-S fuzzy system (35) system to a switching T-S fuzzy system as (3), where $s = 3$ and $\beta_j = 4$. Additionally, vertices $\mathbf{x}^v_{jl}$ of each sub-region $\mathbf{S}_j$ are as follows:

$$\begin{array}{lllll} \mathbf{x}^v_{11} = [-3\ 1]^T, & \mathbf{x}^v_{12} = [-1\ 1]^T, & \mathbf{x}^v_{13} = [-3\ \ -1]^T, & \mathbf{x}^v_{14} = [-1\ \ -1]^T, & \text{for } \mathbf{S}_1 \\ \mathbf{x}^v_{21} = [-1\ 1]^T, & \mathbf{x}^v_{22} = [1\ 1]^T, & \mathbf{x}^v_{23} = [-1\ \ -1]^T, & \mathbf{x}^v_{24} = [1\ \ -1]^T, & \text{for } \mathbf{S}_2 \\ \mathbf{x}^v_{31} = [1\ 1]^T, & \mathbf{x}^v_{32} = [3\ 1]^T, & \mathbf{x}^v_{33} = [1\ \ -1]^T, & \mathbf{x}^v_{34} = [3\ \ -1]^T, & \text{for } \mathbf{S}_3 \end{array}$$

Therefore, we have $\phi = \max_{i,j} \mathbf{x}^v_{jl} = 3.1623$. Also, if the control input is small enough, this nonlinear function $\tan(\mathbf{u}(k))$ approximates to $\mathbf{u}(k)$. In the approximation of tangent function ($\tan(\mathbf{u}(k))$), $\|\mathbf{u}(k)\|_2 < 0.5$ is reasonable. Therefore, we determine the input constraint $\mu = 0.5$ in this example. Then, the nonlinear T-S fuzzy system (35) system is transformed into a switching T-S fuzzy system as

Region rule j: If $x(k) \in \mathbf{S}_j$, then
Local Plant rule $\mathbf{LR}_{jl}$: If $x_1(k)$ is M_{jl1}, $x_2(k)$ is M_{jl2},
then $\mathbf{x}(k+1) = \mathbf{A}_{jl}\mathbf{x}(k) + \mathbf{B}_{jl}\mathbf{u}(k)$ (36)

where $l = 1, 2, 3, 4$ and $j = 1, 2, 3$. $\mathbf{A}_{11} = \mathbf{A}_1$, $\mathbf{A}_{12} = \mathbf{A}_2$, $\mathbf{A}_{13} = \mathbf{A}_5$, $\mathbf{A}_{14} = \mathbf{A}_6$, $\mathbf{A}_{21} = \mathbf{A}_2$, $\mathbf{A}_{22} = \mathbf{A}_3$, $\mathbf{A}_{23} = \mathbf{A}_6$, $\mathbf{A}_{24} = \mathbf{A}_7$, $\mathbf{A}_{31} = \mathbf{A}_3$, $\mathbf{A}_{32} = \mathbf{A}_4$, $\mathbf{A}_{33} = \mathbf{A}_7$, $\mathbf{A}_{34} = \mathbf{A}_8$ and $\mathbf{B}_{jl} = [0.1\ 0.4]^T$ for $l = 1, 2, 3, 4$ and $j = 1, 2, 3$. Notably, feasible solutions to the stability analysis problem cannot be found for Theorem 2.1 using Matlab LMI toolbox. However, the stability can be guaranteed using Theorem 2.2. This example also demonstrates the effectiveness of the ideas of group-fired rules and the maximal width of state steps. According to $\mu = 0.5$ and (23), $\delta_1 = \delta_2 = \delta_3 = 1$ can be obtained by using (23). Additionally, $\Omega = \{\langle 1, 1\rangle, \langle 1, 2\rangle, \langle 2, 1\rangle, \langle 2, 2\rangle, \langle 2, 3\rangle, \langle 3, 2\rangle, \langle 3, 3\rangle\}$. Then, the controller $\mathbf{F}_{11} - \mathbf{F}_{34}$ can be solved based on Theorem 3.1.

$$\begin{array}{lll} \mathbf{F}_{11} = [0.0601\ 5.721], & \mathbf{F}_{21} = [0.0638\ 0.0746], & \mathbf{F}_{31} = [0.0212\ 0.0445], \\ \mathbf{F}_{12} = [0.0486\ 0.0600], & \mathbf{F}_{22} = [0.0278\ 0.0581], & \mathbf{F}_{32} = [0.0174\ 0.0424], \\ \mathbf{F}_{13} = [0.0400\ 0.0527], & \mathbf{F}_{23} = [0.0362\ 0.0562], & \mathbf{F}_{33} = [0.0375\ 0.0516], \\ \mathbf{F}_{14} = [0.0254\ 0.0412], & \mathbf{F}_{24} = [0.0479\ 0.0642], & \mathbf{F}_{34} = [0.0405\ 0.0326]. \end{array}$$

Figure 6 plots the simulation results in eight different initial conditions, which proves that Theorem 3.1 can provide an asymptotically stable control.

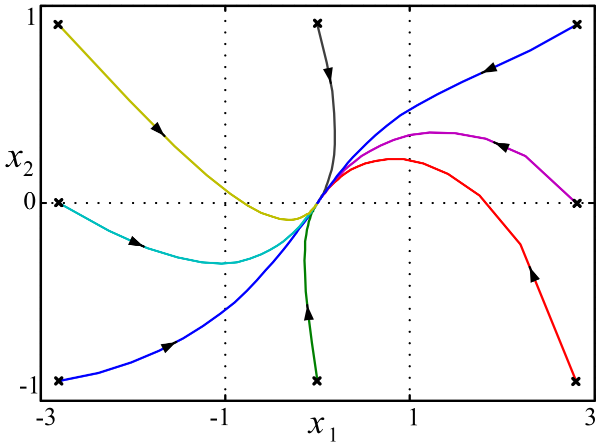

FIGURE 6. Simulation results of eight different initial conditions (Example 4.1)

Remark 4.1. *Based on Theorem 3.1, only seven* $\langle j,i\rangle \in \Omega$ *pairs should be considered for Example 4.1. However, all possible* $\langle j,i\rangle$ *pairs for* $i,j = 1,2,3$, *i.e., nine pairs, should be considered based on Theorem 3 of [16] for Example 4.1. Therefore, this work proposes a more relaxed stabilization criterion.*

Remark 4.2. *Based on Theorem 1 of [17], the control gains* $\mathbf{F}_{jq}$ *and* $\langle j,i\rangle \in \Omega$ *cannot be solved simultaneously. That is, in [17] some* δ_j *should be assumed in advance to obtain* $\Omega = \{\langle j,i\rangle | j = 1,2,\ldots,s;\ i = j \pm (1,2,\ldots,\delta_j);\ i > 0\}$ *and then solve Lyapunov inequalities to obtain* $\mathbf{F}_{jq}$ *and* $\mathbf{P}_j$. *Next, a real* δ_j *is derived from the feedback control gains* $\mathbf{F}_{jq}$. *If the predetermined* δ_j *and real* δ_j *are contradictory, another* δ_j *should be assumed and repeat control design. However, in this work, the real* δ_j *is derived from input constraint (say* $\|\mathbf{u}(k)\|_2 < \mu$*) and then* $\mathbf{F}_{jq}$ *and* $\mathbf{P}_j$ *are obtained based on Theorem 3.1. Consequently, no iterative computational scheme is required. Then, a novel methodology with a low computing demand and relaxed stabilization conditions is proposed.*

Example 4.2. *Consider an autonomous vehicle system [28] as shown in Figure 7.*

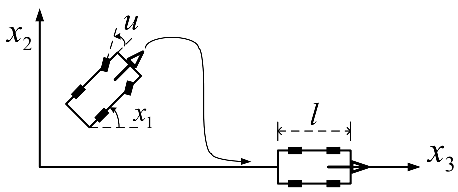

FIGURE 7. Autonomous vehicle system

The kinematic equations of the autonomous vehicle system are presented as follows:

$$x_1(k+1) = x_1(k) + \frac{vt}{l}\tan(\mathbf{u}(k)) \tag{37}$$

$$x_2(k+1) = x_2(k) + vt\sin(x_1(k)) \tag{38}$$

$$x_3(k+1) = x_3(k) + vt\cos(x_1(k)) \tag{39}$$

where $x_1(k)$ is the angle of the vehicle. $x_2(k)$ and $x_3(k)$ are the vertical and horizontal positions of the center of the vehicle, respectively. l is the length of the vehicle, v is the forward speed of the vehicle, $\mathbf{u}(k)$ is the steering angle, and t is the sampling time. In this study, $l = 0.5(m)$, $v = 1(m/s)$ and $t = 0.5(s)$. Furthermore, the control purpose is $\lim_{k\to\infty} x_1(k) = 0$ and $\lim_{k\to\infty} x_2(k) = 0$. As Example 4.1, if $\|\mathbf{u}(k)\|_2 < 0.5$, the nonlinear function $\tan(\mathbf{u}(k))$ approximates to $\mathbf{u}(k)$. Therefore, the control object is described as

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} x_1(k) \\ x_2(k) + vt\sin(x_1(k)) \end{bmatrix} + \begin{bmatrix} vt/l \\ 0 \end{bmatrix} \mathbf{u}(k) \tag{40}$$

By using local approximation method [29] to replace the nonlinear term $\sin(x_1(k))$ in (40), then we have the switching T-S fuzzy system for the autonomous vehicle as:

$$\begin{aligned} &\textbf{Region Rule } j\text{: If } x_1(k) \in \mathbf{S}_j, \\ &\quad \textbf{Local Fuzzy rule LR}_{jl}\text{: If } x_1(k) \text{ is } M_{jl}, \text{ then} \\ &\qquad\qquad \text{then } \mathbf{x}(k+1) = \mathbf{A}_{jl}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \end{aligned} \tag{41}$$

where $l = 1, 2$; $j = 1, 2, \ldots, 8$. $\mathbf{A}_{11} = \mathbf{A}_{82} = \begin{bmatrix} 1 & 0 \\ gvt & 1 \end{bmatrix}$, $\mathbf{A}_{42} = \mathbf{A}_{51} = \begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix}$, $\mathbf{A}_{12} = \mathbf{A}_{21} = \mathbf{A}_{72} = \mathbf{A}_{81} = \begin{bmatrix} 1 & 0 \\ 0.1501 & 1 \end{bmatrix}$, $\mathbf{A}_{22} = \mathbf{A}_{31} = \mathbf{A}_{62} = \mathbf{A}_{71} = \begin{bmatrix} 1 & 0 \\ 0.3183 & 1 \end{bmatrix}$, $\mathbf{A}_{32} = \mathbf{A}_{41} = \mathbf{A}_{52} = \mathbf{A}_{61} = \begin{bmatrix} 1 & 0 \\ 0.4502 & 1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} vt/l \\ 0 \end{bmatrix}$ and $g = 10^{-2}/\pi$ for avoid uncontrollable situation [28]. Moreover, membership functions and switching regions of this system are shown as Figure 8. Since $\mu = 0.5$, then $\delta_1 = \delta_2 = \cdots = \delta_8 = 1$ can be obtained by using

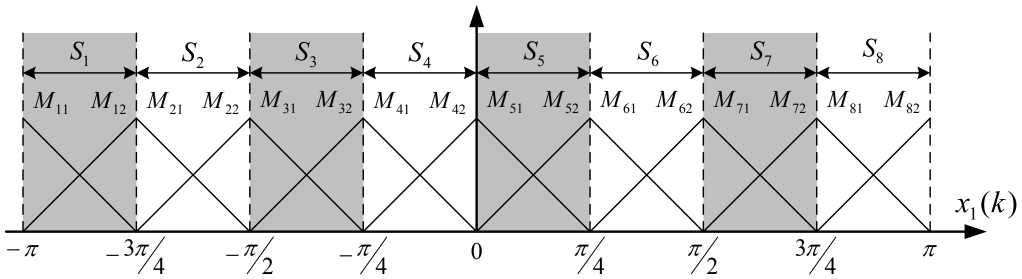

FIGURE 8. Membership functions and switching regions of Example 4.2

(23). Additionally, $\Omega = \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 3,2\rangle, \langle 3,3\rangle, \langle 3,4\rangle, \langle 4,3\rangle, \langle 4,4\rangle, \langle 4,5\rangle, \langle 5,4\rangle, \langle 5,5\rangle, \langle 5,6\rangle, \langle 6,5\rangle, \langle 6,6\rangle, \langle 6,7\rangle, \langle 7,6\rangle, \langle 7,7\rangle, \langle 7,8\rangle, \langle 8,7\rangle, \langle 8,8\rangle\}$. Next, the autonomous vehicle system can be stabilized according to Theorem 3.1. Figure 9 summarizes the simulation results with four different initial states (x_2), $\mathbf{x}(0) = [0,\ 10,\ 0]^T$, $\mathbf{x}(0) = [0,\ 5,\ 0]^T$, $\mathbf{x}(0) = [0,\ -5,\ 0]^T$ and $\mathbf{x}(0) = [0,\ -10,\ 0]^T$, respectively. Also, Figure 10 shows the trajectories of the vehicle with four different initial states (angles/x_1), $\mathbf{x}(0) = [\pi/2,\ 10,\ 0]^T$, $\mathbf{x}(0) = [-\pi/2,\ 10,\ 10]^T$, $\mathbf{x}(0) = [0; -10,\ 0]^T$ and $\mathbf{x}(0) = [-0.99\pi,\ -10,\ 10]^T$, respectively. In Figure 9 and Figure 10, the block denotes the instant attitude of the vehicle. Notably, to avoid the complexity, only partial instant attitudes of vehicle show in the following figures. Simulation results indicate that the control design based on Theorem 3.1 can stabilize the autonomous vehicle system.

5. **Conclusions.** This paper addressed the relaxed stabilization problem for a T-S fuzzy discrete system. A switching PDC fuzzy controller is designed under some sufficient conditions in Theorem 3.1, such that the whole closed-loop systems are asymptotically

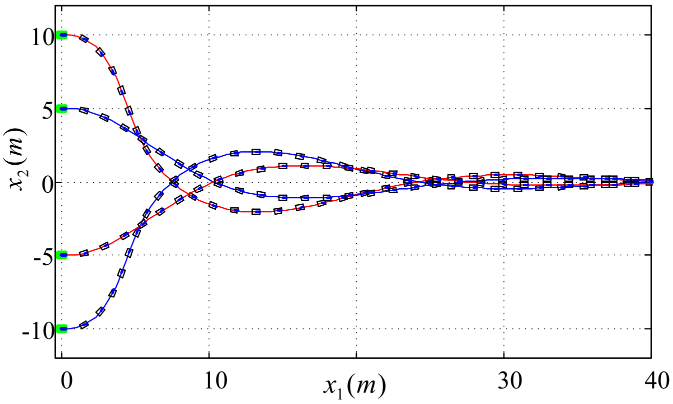

FIGURE 9. Simulation of four different initial conditions (Example 4.2)

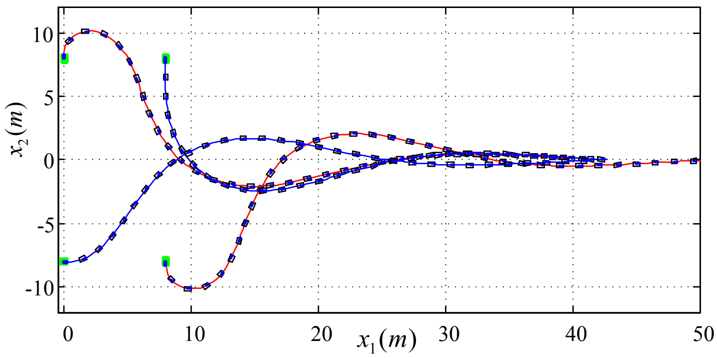

FIGURE 10. Simulation of four different initial conditions (Example 4.2)

stable. The ideas of group-fired rules and the maximal width of the state steps are considered. The practical input constraint is also adopted to reduce computational effort required by the stabilization design. The stable control law can be solved using the Matlab LMI toolbox. Finally, a numerical example and a practical application are presented to demonstrate the effectiveness of the proposed method.

Acknowledgment. This work was supported by the National Science Council of Taiwan under the Grant NSC99-2221-E-032-065 and NSC100-2221-E-032-008. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

[1] G. Feng, A survey on analysis and design of model-based fuzzy control systems, *IEEE Trans. on Fuzzy Syst.*, vol.14, pp.676-697, 2006.
[2] K. Tanaka and H. O. Wang, *Fuzzy Control System Design and Analysis: A Linear Matrix Inequality Approach*, Wiley, New York, 2001.
[3] K. Tanaka, M. Iwasaki and H. O. Wang, Switching control of an R/C hovercraft: Stabilization and smooth switching, *IEEE Trans. on Syst., Man, Cybern. B*, vol.31, pp.853-863, 2001.
[4] D. J. Choi and P. G. Park, H_∞ state-feedback controller design for discrete-time fuzzy systems using fuzzy weighting-dependent Lyapunov functions, *IEEE Trans. on Fuzzy Syst.*, vol.11, pp.271-278, 2003.
[5] R. J. Wang, W. W. Lin and W. J. Wang, Stabilizability of linear quadratic state feedback for uncertain fuzzy time-delay systems, *IEEE Trans. on Syst., Man, Cybern. B*, vol.34, pp.1288-1292, 2004.

[6] C. S. Tseng, B. K. Lee and Y. F. Li, Robust fuzzy filter design for a class of nonlinear discrete-time uncertain systems, *Int. J. Fuzzy Syst.*, vol.8, pp.113-118, 2006.
[7] C. W. Park, B. S. Kim and J. B. Lee, Digital stabilization of fuzzy systems with time-delay and its application to backing up control of a truck-trailer, *Int. J. Fuzzy Syst.*, vol.9, pp.14-21, 2007.
[8] H. Chung, S. Wu and W. Chang, Regional fuzzy control for nonlinear ship steering systems, *International Journal of Innovative Computing, Information and Control*, vol.4, no.7, pp.1635-1646, 2008.
[9] J. S. Chiou et al., Stability analysis and controller design of the nonlinear switched systems via T-S discrete-time fuzzy model, *Int. J. Fuzzy Syst.*, vol.11, pp.213-224, 2009.
[10] L. Luoh, Control design of T-S fuzzy large-scale systems, *International Journal of Innovative Computing, Information and Control*, vol.5, no.9, pp.2869-2880, 2009.
[11] Z. Gao, B. Jiang, P. Shi and Y. Xu, Fault accommodation for near space vehicle attitude dynamics via T-S fuzzy models, *International Journal of Innovative Computing, Information and Control*, vol.6, no.11, pp.4843-4856, 2010.
[12] X.-H. Chang and G.-H. Yang, Relaxed results on stabilization and state feedback H_∞ control conditions for T-S fuzzy systems, *International Journal of Innovative Computing, Information and Control*, vol.7, no.4, pp.1753-1764, 2011.
[13] Y. Xu, B. Jiang, G. Tao and Z. Gao, Fault accommodation for near space hypersonic vehicle with actuator fault, *International Journal of Innovative Computing, Information and Control*, vol.7, no.5(A), pp.2187-2200, 2011.
[14] W. J. Wang and C. H. Sun, Relaxed stability and stabilization conditions for a T-S fuzzy discrete system, *Fuzzy Sets and Syst.*, vol.156, pp.208-225, 2005.
[15] M. Johansson, A. Rantzer and K. E. Årzén, Piecewise quadratic stability of fuzzy systems, *IEEE Trans. on Fuzzy Syst.*, vol.7, pp.713-722, 1999.
[16] L. Wang and G. Feng, Piecewise H_∞ controller design of discrete time fuzzy systems, *IEEE Trans. on Syst., Man, Cybern. B*, vol.34, pp.682-686, 2004.
[17] W. J. Wang, Y.-J. Chen and C. H. Sun, Relaxed stabilization criteria for discrete-time T-S fuzzy systems based on switching fuzzy model and piecewise Lyapunov function, *IEEE Trans. on Syst., Man, Cybern. B*, vol.37, pp.551-559, 2009.
[18] D. J. Choi and P. G. Park, Guaranteed cost controller design for discrete-time switching fuzzy system, *IEEE Trans. on Syst., Man, Cybern. B*, vol.34, pp.110-119, 2004.
[19] I. Zamani and M. H. Zarif, An approach for stability analysis of T-S fuzzy systems via piecewise quadratic stability, *International Journal of Innovative Computing, Information and Control*, vol.6, no.9, pp.4041-4054, 2010.
[20] H. Y. Chung and S. M. Wu, Hybrid approaches for regional Takagi-Sugeno static output feedback fuzzy controller design, *Expert Syst. Appl.*, vol.36, pp.1720-1730, 2009.
[21] G. Feng, C. L. Chen, D. Sun and Y. Zhu, Controller synthesis of fuzzy dynamic systems based on piecewise Lyapunov functions and bilinear matrix inequalities, *IEEE Trans. on Fuzzy Syst.*, vol.13, pp.94-105, 2005.
[22] K. Tanaka, T. Hori and H. O. Wang, A multiple Lyapunov function approach to stabilization of fuzzy control systems, *IEEE Trans. on Fuzzy Syst.*, vol.11, pp.582-589, 2003.
[23] T. Zhang, G. Feng and J. H. Lu, Fuzzy constrained min-max model predictive control based on piecewise Lyapunov functions, *IEEE Trans. on Fuzzy Syst.*, vol.15, pp.686-698, 2007.
[24] Y. J. Chen, W. J. Wang and C. L. Chang, Guaranteed cost control for an overhead crane with practical constraints: Fuzzy descriptor system approach, *Eng. Appl. Artif. Intell.*, vol.22, pp.639-645, 2009.
[25] H. P. Du and N. Zhang, Fuzzy control for nonlinear uncertain electrohydraulic active suspensions with input constraint, *IEEE Trans. on Fuzzy Syst.*, vol.17, pp.343-356, 2009.
[26] S. H. Kim, C. H. Lee and P. G. Park, State-feedback control for fuzzy systems with input saturation via fuzzy weighting-dependent Lyapunov functions, *Comput. Math. Appl.*, vol.57, pp.981-990, 2009.
[27] M. Khairy, A. L. Elshafei and H. M. Emara, LMI based design of constrained fuzzy predictive control, *Fuzzy Sets and Syst.*, vol.161, pp.893-918, 2010.
[28] K. Tanaka and M. Sano, Trajectory stabilization of a model car via fuzzy control, *Fuzzy Sets and Syst.*, vol.70, pp.155-170, 1995.
[29] M. C. M. Teixeira and S. H. Żak, Stabilizing controller design for uncertain nonlinear systems using fuzzy models, *IEEE Trans. on Fuzzy Syst.*, vol.7, pp.133-142, 1999.