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ROBUST RELIABLE H_∞ CONTROL FOR UNCERTAIN SYSTEMS WITH POLE CONSTRAINTS

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Abstract. *This paper investigates the robust reliable H_∞ control for uncertain systems. The system under consideration is subject to parameter uncertainties, external disturbances, actuator faults and pole constraints. A reliable output feedback controller is designed via a Lyapunov function approach in such a way that the closed-loop system will satisfy the system design requirements. The existence conditions for the admissible controller are given in terms of linear matrix inequalities (LMIs). The controller design is thus transformed into a convex optimization problem subject to LMI constraints. Two illustrative examples are provided to show the effectiveness of the proposed control design method.*

Keywords: Reliable H_∞ control, LMIs, Uncertain systems, Actuator faults, Pole constraints

1. **Introduction.** In reality, it is impossible to construct the exact model of a system. There are many phenomena that are not fully understood, and hence could not be modelled precisely. Furthermore, for all man-made systems, such as ship motion control systems, and spacecraft control systems, their components are subject to deterioration. Thus, in the past few decades, uncertain linear systems have attracted considerable attention (see [1, 2, 3]).

In the uncertain control system design, the system stability and performance are two fundamental requirements. A stable system must have a good dynamic performance such as fast response, small overshoot and effective load rejection. In the past, much attention has been focused on the study of H_∞ control problems where the objective is to design a controller such that the closed-loop system is stable and the H_∞-norm of the corresponding closed-loop transfer function is minimized. In this way, effects of the disturbance on the system are reduced. However, for both the standard H_∞ control and robust H_∞ control problems, concerns on the transient behavior of the closed-loop systems [4, 5, 6] have not been taken into proper consideration. In many practical applications, it is clear that the importance of their transient properties should not be overlooked. A practical approach to this problem is to place the poles of the closed-loop systems in a specified region. Therefore, in the past few decades, much attention has been devoted to the pole assignment problems. These include the works reported in [7], where the transient behaviors of the closed-loop systems are considered. However, little attention is given to their H_∞ performance. In practice, it is desirable to design a control system which possesses not only the required transient behaviors but also good H_∞ performance. Thus, it is required to place the poles of the control system in a specified region, while ensuring some desirable level of performance, such as the H_∞ performance, is achieved. The problem of H_∞ controller design with pole-placement constraints are studied extensively in [5, 6, 8], and some results are now available. The systems considered in [8] are time-invariant and have no parameter uncertainty. In [5], only systems with structured uncertainty are considered, where there are no H_∞ performance constraints. For [6], the problems of H_∞ control for uncertain discrete-time systems with circular pole constraints are considered, where the objective is to design state and output feedback controllers such that the resulting closed-loop system achieves not only robust pole locations, but also satisfies an H_∞-norm constraint for all admissible uncertainties. In all these papers, they address one, two or even multiple aspects of the design requirements. However, few attempts are made towards solving the spacecraft control problems with the multiple design requirements being taken into consideration simultaneously.

For the designs of the controllers addressed in these papers mentioned above, a common assumption is that the actuators can provide constant levels of signals. In practice, contingent faults of the actuators are, however, possible in a system. The faults may lead to the degradation of the H_∞ performance, causing the shift of the poles of the closed-loop system to outside the specified region. Thus, the closed-loop system may become unstable. In engineering applications, the reliability of the control system when encountering actuator failures is of great importance, as it takes time to repair the faulty actuators to proper working order. The so-called reliable control is referred to as the design of a controller such that the closed-loop system can retain the overall system stability with acceptable system performance when encountering abnormal operations of some control components. Some important results are now available. See, for example, [9, 10, 11, 12, 13] and the references cited therein, where several approaches are proposed. They include the algebraic Riccati equation based approach, the coprime factorization approach, the Hamilton-Jacobi (HJ)-based approach, the sliding-mode control (SMC)-based approach, and the LMI-based approach. Among the aforementioned reliable control approaches, the LMI-based design method has an advantage that it offers the flexibility for formulation of the multiple constraints in terms of LMI. In this way, the original design problem becomes a convex optimization problem. In addition, the robust control problems with parameter uncertainties can be formulated as equivalent matrix inequalities. These matrix inequalities are less conservative. Finally, this approach is simple and easy to implement.

The reliable controller design methods mentioned above are all based on a basic assumption that the control component faults are modelled as outages, i.e., when a fault occurs, the signal (in the case of sensors) or the control action (in the case of actuators) simply becomes zero. The outage model is the simplest case of control component faults. Different from the outage model, a more general fault model is adopted for sensor and actuator faults in [14], where a continuous scaling factor to the signal or to the control action is to be measured. The fault model is called continuous gain fault model. In this paper, we extend the fault model of outage to the continuous gain fault model, and propose a robust reliable H_∞ control design method for a class of linear systems subject to parameter uncertainties, external perturbations, and pole constraints. A reliable output feedback controller is designed via a Lyapunov function approach. It ensures that the closed-loop system will satisfy the system design requirements. The existence conditions for admissible controllers are given in terms of linear matrix inequalities (LMIs). The controller design is thus transformed into a convex optimization problem subject to LMI constraints. Two illustrative examples are provided to show the effectiveness of the proposed control design method.

The rest of this paper is organized as follows. Section 2 presents the dynamic model of uncertain systems, and the formulation of the robust control design problem. In Section 3, the output-feedback controller design method is proposed. Then, two examples are given to illustrate the applicability of the approach proposed in Section 4. Finally, Section 5 draws the conclusions.

2. **Problem Formulation.** For real symmetric matrices P and Q, $P > Q$ ($P < Q$) means that the matrix $P - Q$ is positive (negative) definite, and $P \geq Q$ ($P \leq Q$) means that the matrix $P - Q$ is positive (negative) semi-definite. $\mathcal{H}_e\{A\} = A + A^T$. A^T and A^{-1} represent, respectively, the transpose and the inverse of the matrix A, and $||\cdot||_2$ denotes either the Euclidean vector norm or its induced matrix 2-norm. diag$\{\cdot\}$ stands for a block-diagonal matrix. I and 0 denote the identity matrix and zero matrix with appropriate dimensions. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

Consider the system model

$$\dot{x} = (A_0 + \Delta A)x + Bu + B_\omega \omega \tag{1}$$

$$y = Cx \tag{2}$$

where x is the state vector, u is the control input vector, y is the control output vector, ω is the external disturbance, (A_0, B, B_ω, C) are known real constant matrices with appropriate dimensions, and C is of full row rank.

The admissible uncertainty matrix ΔA is assumed to be of the form:

$$\Delta A = MH(t)N, \tag{3}$$

where (M, N) are known real constant matrices, and $H(t)$ is a random matrix-valued function satisfying $H^T(t)H(t) \leq I$.

Instead of the actuator outage model, a more general actuator fault model is adopted in this paper. Let u be a normal signal output feedback controller, which is of the form: $u = Ky$. Then, the actuator fault model is described as

$$u^f = Fu, \tag{4}$$

F is the actuator fault matrix of the form:

$$F = \text{diag}\{f_1, f_2, \cdots, f_m\}, \quad 0 \leq F_l \leq F \leq F_u,$$

where $F_l \geq 0$ and $F_u \geq I$ are diagonal matrices.

For the actuator fault matrix F, $f_i = 0$ means the outage of the ith actuator control signal; $f_i = 1$ means the normal operation of the ith actuator control signal; $f_i \neq 0, 1$ indicates partial fault of the ith actuator control signal. Therefore, the actuator fault matrix can be described by the matrix inequality of the form:

$$0 \leq F_l \leq F \leq F_u.$$

Based on (1), (2) and (4), the closed-loop system can be written as follows:

$$\dot{x} = A_{cl}x + B_\omega \omega \tag{5}$$

$$y = Cx \tag{6}$$

where $A_{cl} = A_0 + \Delta A + BFKC$.

In this paper, we consider the system (1)-(2) with possible actuator faults. Our goal is to design a reliable output feedback controller such that the closed-loop system (5) is robustly stable and the H_∞ performance index $\|G(s)\|_\infty < \gamma$ is guaranteed subject to the parameter uncertainties, external disturbances and pole constraints, where γ is a known real constant number.

Remark 2.1. *In the literature, many papers consider only the state-feedback control problems, under the assumption that the real-time state signals can be transmitted accurately. Compared with the state-feedback control problem, the output-feedback control problem is more important in real applications, because it is often that only output information is available for measurement (see* [15]*). However, output-feedback control problems are more difficult to deal with than state-feedback control problems.*

3. **Main Results.** Before we prove our main result (i.e., Theorem 3.1), we first introduce some essential preliminary lemmas.

Lemma 3.1. [16] *(Schur complement lemma) For a given constant real symmetric matrix Ξ, the following statements are equivalent.*

1: $\Xi \triangleq \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{12}^T & \Xi_{22} \end{bmatrix} > 0$

2: $\Xi_{11} > 0$ *and* $\Xi_{22} - \Xi_{12}^T \Xi_{11}^{-1} \Xi_{12} > 0$

3: $\Xi_{22} > 0$ *and* $\Xi_{22} - \Xi_{12} \Xi_{22}^{-1} \Xi_{12}^T > 0$

Lemma 3.2. [17] *Let U, F, W and M be real matrices of appropriate dimensions with M satisfying $M = M^T$. Then,*

$$M + UFW + W^T (F)^T U^T < 0$$

for any diagonal matrix $|F| \leq \Lambda$, where Λ is a given diagonal matrix of appropriate dimension, if and only if there exists a scalar $\varepsilon > 0$ such that

$$M + U\Lambda U^T + W^T \Lambda W < 0.$$

Lemma 3.3. [18] *Consider a continuous-time transfer function $G(s)$ of (not necessarily minimal) realization, $G(s) = D + C(sI - A)^{-1}B$. The following statements are equivalent:*

(i): $\|G(s)\|_\infty < \gamma$ *and A is stable in the continuous-time sense ($Re(\lambda_i(A)) < 0$).*

(ii): *There exists a symmetric positive definite solution X to the LMI:*

$$\begin{bmatrix} A^T X + XA & XB & C^T \\ B^T X & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0. \tag{7}$$

Lemma 3.4. [19] *Let $A_{cl} \in R^{n\times n}$ be a given matrix. The eigenvalues of A_{cl} belong to the disk region $D(\alpha, r)$ (centered in α with radius r in the complex plane), if and only if there exists a symmetric matrix $Q \in R^{n\times n}$ such that*

$$\begin{bmatrix} -Q & \frac{1}{r}\left(\bar{A} - \alpha I\right) Q \\ \frac{1}{r} Q \left(\bar{A} - \alpha I\right)^T & -Q \end{bmatrix} < 0. \tag{8}$$

We now present our main result in the following theorem.

Theorem 3.1. *Consider the uncertain linear system (5)-(6), let γ be a given real scalar. If there exist scalars ε_i, γ_i $(i = 1, 2)$ and matrices $S, W, X > 0$ satisfying*

$$CX = SC \tag{9}$$

$$\begin{bmatrix} \Omega_{11} & B_\omega & XC^T & XN^T & (J^{1/2}WC)^T \\ B_\omega^T & -\gamma I & 0 & 0 & 0 \\ CX & 0 & -\gamma I & 0 & 0 \\ NX & 0 & 0 & -\gamma_1 I & 0 \\ J^{1/2}WC & 0 & 0 & 0 & -\gamma_2 I \end{bmatrix} < 0 \tag{10}$$

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & 0 & 0 \\ \Theta_{12}^T & \Theta_{22} & (NX)^T & \Theta_{24} \\ 0 & NX & -\varepsilon_1 I & 0 \\ 0 & \Theta_{24}^T & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{11}$$

Then, the closed-loop system (5) is robustly asymptotically stable with disturbance attenuation level γ, where

$$\Omega_{11} = \mathcal{H}_e\{(A_0X + BF_0WC)\} + \gamma_1 MM^T + \gamma_2 BF_0JF_0B^T$$
$$\Theta_{11} = -Q + \varepsilon_1 MM^T + \varepsilon_2 BF_0JF_0B^T$$
$$\Theta_{12} = \eta_0(A_0X + BF_0WC - \alpha X)$$
$$\Theta_{22} = \eta_0^2 r^2 (Q - 2X)$$
$$\Theta_{24} = (J^{1/2}WC)^T$$

Proof: From Lemma 3.3, the closed-loop system (5) is asymptotically stable and $\|G(s)\|_\infty < \gamma$ if and only if the following matrix inequality in variables P and K admit a solution.

$$\begin{bmatrix} A_{cl}^T P + PA_{cl} & PB_\omega & C^T \\ (PB_\omega)^T & -\gamma I & 0 \\ C & 0 & -\gamma I \end{bmatrix} < 0 \tag{12}$$

From Lemma 3.1 and Lemma 3.2, it holds that for any the admissible uncertainty, (12) is satisfied if and only if there exists a positive scalar γ_1 such that the following inequality holds:

$$\begin{bmatrix} \Delta_{11} & PB_\omega & C^T & N^T \\ (PB_\omega)^T & -\gamma I & 0 & 0 \\ C & 0 & -\gamma I & 0 \\ N & 0 & 0 & -\gamma_1 I \end{bmatrix} < 0, \tag{13}$$

where $\Delta_{11} = \mathcal{H}_e\{P(A_0 + BFKC)\} + \gamma_1 PMM^TP$.

Let $J = \frac{1}{2}(F_u - F_l)(F_u + F_l)^{-1}$, $F_0 = \frac{1}{2}(F_l + F_u)$, $L = (F - F_0)F_0^{-1}$. Then, $F = F_0(I + L)$. Substituting $F = F_0(I + L)$ into (13), we obtain

$$\begin{bmatrix} \tilde{\Delta}_{11} & PB_\omega & C^T & N^T \\ (PB_\omega)^T & -\gamma I & 0 & 0 \\ C & 0 & -\gamma I & 0 \\ N & 0 & 0 & -\gamma_1 I \end{bmatrix} + \mathcal{H}_e\left\{\begin{bmatrix} PBF_0 \\ 0 \end{bmatrix} L \begin{bmatrix} KC & 0 \end{bmatrix}\right\} < 0, \tag{14}$$

where $\tilde{\Delta}_{11} = \mathcal{H}_e\{P(A_0 + BF_0KC)\} + \gamma_1 PMM^TP$.

Applying Lemma 3.1 and Lemma 3.2 to (14), it follows that (14) is satisfied if and only if the following matrix inequality holds

$$\begin{bmatrix} \Lambda_{11} & PB_\omega & C^T & N^T & (J^{1/2}KC)^T \\ (PB_\omega)^T & -\gamma I & 0 & 0 & 0 \\ C & 0 & -\gamma I & 0 & 0 \\ N & 0 & 0 & -\varepsilon_1 I & 0 \\ J^{1/2}KC & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix} < 0, \tag{15}$$

where $\Lambda_{11} = \tilde{\Delta}_{11} + \varepsilon_2 PBF_0JF_0B^TP$.

Multiplying (15) on the left and right sides by $\mathrm{diag}\{P^{-1}, I\}$ and its transpose respectively, we obtain

$$\begin{bmatrix} P^{-1}\Lambda_{11}P^{-1} & B_\omega & P^{-1}C^T & P^{-1}N^T & P^{-1}(J^{1/2}KC)^T \\ B_\omega^T & -\gamma I & 0 & 0 & 0 \\ CP^{-1} & 0 & -\gamma I & 0 & 0 \\ NP^{-1} & 0 & 0 & -\gamma_1 I & 0 \\ J^{1/2}KCP^{-1} & 0 & 0 & 0 & -\gamma_2 I \end{bmatrix} < 0 \tag{16}$$

Let $X = P^{-1}$ and $KS = W$. Then, it follows from (9) and (16) that (10) holds.

From Lemma 3.1 and Lemma 3.2, we see that (11) is equivalent to the following matrix inequality

$$\begin{bmatrix} -Q & \Theta_{12} \\ \Theta_{12}^T & \Theta_{22} \end{bmatrix} + \mathcal{H}_e\left\{\begin{bmatrix} M \\ 0 \end{bmatrix} H \begin{bmatrix} 0 & NX \end{bmatrix}\right\} + \mathcal{H}_e\left\{\begin{bmatrix} BF_0 \\ 0 \end{bmatrix} L \begin{bmatrix} 0 & KCX \end{bmatrix}\right\} < 0, \tag{17}$$

where

$$\Theta_{12} = \eta_0(A_0 + BF_0KC - \alpha I)X$$
$$\Theta_{22} = \eta_0^2 r^2 (Q - 2X).$$

Substitute $\Delta A = MHN$ and $F = F_0(I + L)$ into (17). It gives rise to

$$\begin{bmatrix} -Q & \eta_0(A_{cl} - \alpha I)X \\ \eta_0 X(A_{cl} - \alpha I)^T & r^2\Theta_{22} \end{bmatrix} < 0. \tag{18}$$

For $(Q - X)^TQ^{-1}(Q - X) \geq 0$, we have $-XQ^{-1}X \leq Q - 2X$. This, together with (18), implies that

$$\begin{bmatrix} -Q & \eta_0(A_{cl} - \alpha I)X \\ \eta_0 X(A_{cl} - \alpha I)^T & -\eta_0^2 r^2 XQ^{-1}X \end{bmatrix} < 0. \tag{19}$$

Multiplying (19) on the left and right sides by $\mathrm{diag}\{I, \eta_0^{-1}QX^{-1}\}$ and its transpose respectively, we have

$$\begin{bmatrix} -Q & (A_{cl} - \alpha I)Q \\ Q(A_{cl} - \alpha I)^T & -r^2 Q \end{bmatrix} < 0. \tag{20}$$

From Lemma 3.4, we see that (20) holds if and only if that the eigenvalues of A_{cl} are in the disk region $D(\alpha, r)$ (centered in α with radius r in the complex plane). This completes the proof.

Remark 3.1. *In view of (2), it is seen that the system output matrix C is of full row rank. Thus, the matrix S in (9) is nonsingular for $X > 0$. Here, our goal is to satisfy conditions of Theorem 3.1. It is easy to solve the matrix inequalities (10) and (11) by using the Matlab LMI toolbox, but a difficulty is added because of equality constraint (9). However, it is much harder if (9), (10) and (11) are to be solved simultaneously. In* [20]*, it is shown that the equality constraint (9) of Theorem 3.1 can be transformed into an optimization problem: Minimize η subject to (10) and*

$$\begin{bmatrix} -\eta I & CX - SC \\ (CX - SC)^T & -\eta I \end{bmatrix} < 0. \tag{21}$$

For CX to approach SC with satisfactory precision, a sufficiently small positive scalar η should be selected in advance in (21).

Remark 3.2. *In Theorem 3.1, we select a sufficiently small positive scalar η. The scalar γ is regarded to be decision a variable in the optimization of the H_∞ disturbance attention level bound. Then, the minimum H_∞ disturbance attention level bound in terms of the feasibility of admissible controllers can be readily found by solving the following convex optimization problem:*

$$\textit{Minimize } \gamma \textit{ subject to the LMIs } (10), (11) \textit{ and } (21) \tag{22}$$

4. Two Simulation Examples.

4.1. **Example 1.** Consider a version of the pitch axis model for the AFTI/F-16 flying at 3000 ft and Mach 0.6 [21]. The equations of motion in the state-space form are given by

$$\dot{x} = (A_0 + \Delta A)x + Bu^F + B_\omega \omega, \tag{23}$$
$$y = Cx, \tag{24}$$

where

$$A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.87 & 43.22 \\ 0 & 0.99 & -1.34 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ -17.25 & -1.58 \\ -0.17 & -0.25 \end{bmatrix},$$
$$B_\omega = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

and the structured uncertain matrices are described by

$$\Delta A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & r_1 & r_2 \\ 0 & r_3 & r_4 \end{bmatrix}$$

with $|r_1| < 0.2$, $|r_2| < 10$, $|r_3| < 0.2$ and $|r_4| < 0.3$.

Note that the structured uncertain matrices can be expressed as $\Delta A = MHN$, where

$$M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad H(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.25r_1 & 0.0625r_2 \\ 0 & r_3 & 0.25r_4 \end{bmatrix}, \quad N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$

Then, $H^T(t)H(t) \le I$. Let $F_l = \begin{bmatrix} 0.7 & 0 \\ 0 & 1 \end{bmatrix}$ and $F_u = \begin{bmatrix} 1.2 & 0 \\ 0 & 1 \end{bmatrix}$. By solving the convex optimization problem (22), the eigenvalues of the nominal system are

$$\{-1.6541, -5.5859, -7.2835\}.$$

These closed-loop poles can be calculated with the gain matrix

$$K = \begin{bmatrix} 0.3880 & -0.2042 & -0.0172 \\ 23.5327 & -19.4920 & 4.2011 \end{bmatrix}.$$

The minimum H_∞ disturbance attention level bound obtained is $\gamma_{\min} = 0.9802$.

From the results obtained above, we observe that even if there exists a partial fault of the actuator, the system, under external perturbations, is still stable with fast response performance.

4.2. **Example 2.** Consider the dynamics of a helicopter in a vertical plane for an airspeed range of 60-170 knots [22]. There are four state variables, which are x_1 = horizontal velocity (knot/sec), x_2 = vertical velocity (knot/sec), x_3 = pitch rate (deg/sec) and x_4 = pitch angle (deg). The two control variables are u_1 = collective pitch control and u_2 = longitudinal cyclic pitch control. In the airspeed range of 60 knots to 170 knots, significant changes occur only in the components a_{32} and a_{34}. For this range of operating conditions, we have

$$A_0 = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.2855 & -0.707 & 1.3229 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \Delta A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & r_{32} & 0 & r_{34} \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.0447 & -7.5922 \\ -5.52 & 4.99 \\ 0 & 0 \end{bmatrix}, \quad B_\omega = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0.6 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with $|r_{32}| < 0.2192$ and $|r_{34}| < 1.2031$.

Note that the structured uncertain matrices can be expressed as $\Delta A = MHN$, where

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad H(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.5r_{32} & 0 & 0.5r_{34} \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad N = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then, $H^T(t)H(t) \le I$. Let $F_l = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}$ and $F_u = \begin{bmatrix} 1 & 0 \\ 0 & 1.2 \end{bmatrix}$. By solving the convex optimization problem (22), the eigenvalues of the nominal system obtained are

$$\{-7.9998, -0.1684, -1.7716, -4.0539\}.$$

These closed-loop poles can be calculated with the gain matrix

$$K = \begin{bmatrix} -0.7948 & 5.1617 & 2.8603 & 2.6940 \\ 0.4665 & 5.2357 & -0.1654 & -1.7560 \end{bmatrix}.$$

The minimum H_∞ disturbance attention level bound obtained is $\gamma_{\min} = 2.8772$.

From the results obtained above, we observe that even if there exist the partial fault of all the actuators, the system, under external perturbations, remains also stable with fast response performance.

5. Conclusions.

This paper proposed an output feedback robust H_∞ controller design method for linear system subject to parameter uncertainties, external perturbations and pole constraints. By using the robust control and LMI techniques, the uncertain control system problem is transformed into a convex optimization problem with linear matrix inequality constraints. Two examples are solved, showing the effectiveness of the proposed controller design method.

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