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DISCRETE-TIME CONSTRAINED PREDICTIVE CONTROL FOR DISTILLATION COLUMNS

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Abstract. *In this paper, the basic concepts for the design of a discrete-time constrained predictive controller for a nonlinear process are presented in a didactic way. The controller uses a standard predictive algorithm, whose solution is obtained by solving a well-known convex quadratic programming (QP) problem. This controller combines the simplicity of the linear models with the essential nonlinearities of the process using an on-line linearised model of the process for the output prediction. The controller has been developed for a distillation column, presenting as main advantages the easy of tuning and its adaptation to different operating points without need of readjusting the controller parameters. Simulation results for diverse studied cases are presented along with experimental tests carried out on a pilot distillation plant for the validation of the mathematical model.*
Keywords: Predictive control, Nonlinear systems, Distillation column

1. **Introduction.** Distillation is probably one of the most employed operations in the chemical, pharmaceutics and oil industry to separate different components from a flow or for product purification. This process is very energy-intensive [21], so designing a high performance control system for a distillation column is therefore important for improving product quality and energy saving.

A modern control algorithm that has had success in industry is predictive control (MPC). The original industrial algorithms [5,13] use a finite impulse or step response model to predict the process behaviour, i.e., they can only capture local (linear) and open-loop stable dynamics. However, distillation processes feature complicated multivariable nonlinear dynamics, for which it is expensive and time-consuming to develop simplified models that can be used for MPC. Moreover, a nonlinear model of the process is a better approach to address nonlinearities, particulary if the column must operate at different purity levels, and to develop soft-sensors (observers) for estimating product purity accurately [6,15].

The use of a nonlinear model for the output prediction involves computational issues related to the cost and the ability to solve the optimisation problem (nonlinear programming), where convergence to a globally optimal solution cannot be guaranteed having implications for closed-loop stability. For this problem, different solutions in the literature have been proposed, mainly finding feasible local solutions using different algorithms to solve the nonlinear optimisation problem, e.g., [17,20]. Nevertheless, the elevated complexity of these algorithms introduce computational delays, having in consequence long processing times and difficulties for real-time implementation.

On the other hand, there are several papers about distillation control but mainly in simulation [3,10]. Most of MPC applied to a distillation process use linear models lowering the computational burden but restricting the operation region [1,7]. Another approach is to work with different local models of de plant and scheduler coordinates the local models according to the actual operation point of the plant to enlarge the operation region [4], but the principal drawback of the paper is that it uses a DMC algorithm which only allows open-loop stable models for prediction.

The goal of this paper is to develop an improved discrete-time MPC for a binary distillation column. The controller uses a linearised model for prediction in each sampling time as an approximate alternative to the constrained nonlinear MPC. The use of a lineralised model in each sample time provides low computational requirements (compared with nonlinear programming) and guarantees less sensitivity to set point changes (compared with lineal MPC). Another contribution is to establish a methodology for the implementation of MPC to a nonlinear process, in particular to binary distillations columns. Furthermore, the paper is presented as a practical guide for implementing MPC algorithms to nonlinear processes, and it allows to visualise the ease of the technique, to both, process engineers or students of control systems.

2. Model-Based Predictive Control.

The methodology of the controllers belonging to the MPC family is characterised by the following steps [14,19]: (i) at each sampling time k, a finite horizon optimal control problem is solved over a prediction horizon n_p, using the current state of the process as the initial state; (ii) the on-line optimisation problem takes account of system dynamics, constraints and control objectives, the optimisation yields an optimal control sequence, and only the control action for the current time is applied while the rest of the calculated sequence is discarded; (iii) at the next time instant the horizon is shifted one sample and the optimisation is restarted with the information of the new measurements, using the concept of receding horizon.

2.1. State-space formulation.

Consider the following linear time-invariant deterministic discrete-time system represented by use of a state-space model:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k; \quad \mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{d}_k, \tag{1}$$

where $\mathbf{x}_k \in \mathbb{R}^{n_x}$, $\mathbf{y}_k \in \mathbb{R}^{n_y}$ and $\mathbf{u}_k \in \mathbb{R}^{n_u}$ are the state vector, the measured output and the process input respectively. By iterating the model, the output prediction with a prediction horizon n_p and a control horizon n_c is given by ($n_p \gg n_c$):

$$\underrightarrow{\mathbf{y}}_k = \mathbf{F} + \mathbf{G}\underrightarrow{\Delta\mathbf{u}}_k, \tag{2}$$

where $\underrightarrow{\mathbf{y}}_k$, $\underrightarrow{\Delta\mathbf{u}}_k$, $\mathbf{F}$ and $\mathbf{G}$ are defined as follows:

$$\underrightarrow{\mathbf{y}}_k = \begin{bmatrix} \mathbf{y}_{k+1|k} & \dots & \mathbf{y}_{k+n_p|k} \end{bmatrix}^T; \quad \underrightarrow{\Delta\mathbf{u}}_k = \begin{bmatrix} \Delta\mathbf{u}_{k|k} & \dots & \Delta\mathbf{u}_{k+n_c|k} \end{bmatrix}^T;$$

$$\mathbf{F} = \begin{bmatrix} \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{n_c} \\ \mathbf{CA}^{n_c+1} \\ \vdots \\ \mathbf{CA}^{n_p} \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} \mathbf{CB} \\ \vdots \\ \sum_{i=0}^{n_c-1} \mathbf{CA}^i\mathbf{B} \\ \sum_{i=0}^{n_c} \mathbf{CA}^i\mathbf{B} \\ \vdots \\ \sum_{i=0}^{n_p-1} \mathbf{CA}^i\mathbf{B} \end{bmatrix} \mathbf{u}_{k-1};$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{CB} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{n_c-1} \mathbf{CA}^i\mathbf{B} & \dots & \mathbf{CB} \\ \sum_{i=0}^{n_c} \mathbf{CA}^i\mathbf{B} & \dots & \mathbf{CAB} + \mathbf{CB} \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{n_p-1} \mathbf{CA}^i\mathbf{B} & \dots & \sum_{i=0}^{n_p-n_c} \mathbf{CA}^i\mathbf{B} \end{bmatrix}.$$

2.2. **Constraints.** The constraints considered are: limits in the control signal ($\mathbf{U}_{\min}$, $\mathbf{U}_{\max}$), limits in the slew rate of the actuator ($\Delta\mathbf{u}_{\min}$, $\Delta\mathbf{u}_{\max}$) and limits in the output signals ($\mathbf{y}_{\min}$, $\mathbf{y}_{\max}$). Then, for a process with n_u inputs, n_y outputs, the constraints along the horizons n_p and n_c can be expressed as:

$$\mathbf{R}\underrightarrow{\Delta\mathbf{u}}_k \leq \mathbf{c}, \tag{3}$$

where

$$\mathbf{R} = \begin{bmatrix} \mathbf{I}_{n_c\times n_c} \\ -\mathbf{I}_{n_c\times n_c} \\ \mathbf{T} \\ -\mathbf{T} \\ \mathbf{G} \\ -\mathbf{G} \end{bmatrix}; \quad \mathbf{c} = \begin{bmatrix} \mathbf{1}_{n_u}\Delta\mathbf{u}_{\max} \\ \mathbf{1}_{n_u}\Delta\mathbf{u}_{\min} \\ \mathbf{1}_{n_u}\mathbf{U}_{\max} - \mathbf{1}_{n_u}\mathbf{u}_{k-1} \\ -\mathbf{1}_{n_u}\mathbf{U}_{\min} + \mathbf{1}_{n_u}\mathbf{u}_{k-1} \\ \mathbf{1}_{n_y}\mathbf{y}_{\max} - \mathbf{F} \\ -\mathbf{1}_{n_y}\mathbf{y}_{\min} + \mathbf{F} \end{bmatrix}.$$

$\mathbf{1}_{n_u}$ is an $(n_p\times n_u)\times n_u$ matrix formed by n_c $n_u\times n_u$ identity matrices; $\mathbf{1}_{n_y}$ is an $(n_p\times n_y)\times n_y$ matrix formed by n_p $n_y\times n_y$ identity matrices; and $\mathbf{T}$ its a lower triangular block matrix whose nonnull block entries are $n_c\times n_c$ identity matrices.

2.3. **Control signal.** The set of control increments $\underrightarrow{\Delta\mathbf{u}}_k$ is calculated by minimising an objective function over the prediction horizon. In the context of predictive control, it is common to take the following quadratic performance index as the objective to be minimised at each sample [14]:

$$J = \sum_{j=1}^{n_p} \left(\mathbf{w}_{k+j|k} - \mathbf{y}_{k+j|k}\right)^2 + \sum_{j=0}^{n_c-1} \lambda \left(\Delta\mathbf{u}_{k+j|k}\right)^2, \tag{4}$$

where $\mathbf{w}_k$ is the reference trajectory, the second term in Equation (4) is the control effort and λ is the weighting sequence factor. $\mathbf{w}_k$, is the desired output in the closed-loop of the system, and is given by:

$$\mathbf{w}_{k+i|k} = \mathbf{s}_{k+i} - \alpha^i\left(\mathbf{s}_k - \mathbf{y}_k\right); \quad 1 \leq i \leq n_p, \tag{5}$$

where $\mathbf{s}_k$ is the set point and $0 < \alpha < 1$ determines the smoothness of the approach from $\mathbf{y}_k$ to $\mathbf{s}_k$. The parameter α will have a substantial impact, but is rarely discussed except in PFC approaches [18].

The vector of future increments can be calculated as follows:

$$\begin{aligned} \underrightarrow{\Delta \mathbf{u}}^{*}_{k-1} &= \arg\min_{\underrightarrow{\Delta \mathbf{u}}_{k-1}} \frac{1}{2} \underrightarrow{\Delta \mathbf{u}}^{T}_{k-1} \mathbf{H} \underrightarrow{\Delta \mathbf{u}}_{\mathbf{k-1}} + \mathbf{f^{T}} \underrightarrow{\Delta \mathbf{u}}_{\mathbf{k-1}} + \mathbf{b} \\ &\quad \text{subject to} \quad \mathbf{R} \underrightarrow{\Delta \mathbf{u}}_{\mathbf{k-1}} \leq \mathbf{c}; \\ \mathbf{H} = 2\left(\mathbf{G}^{T}\mathbf{G} + \lambda \mathbf{I}\right); &\quad \mathbf{f}^{T} = 2\left(\mathbf{F} - \mathbf{W}\right)^{T} \mathbf{G}; \quad \mathbf{b} = \left(\mathbf{F} - \mathbf{W}\right)^{T} \left(\mathbf{F} - \mathbf{W}\right). \end{aligned} \tag{6}$$

Therefore, the solution takes the form of a standard Quadratic Programming (QP) formulation. The objective function in Equation (6) is convex since it is quadratic with positive definite Hessian ($\mathbf{H} > 0$) and linear constraints. Given these conditions, it is a well known result that a local minimum, if exists, is also a global minimum (see, for example, *Theorem 4.3.8* in [2]).

3. Mathematical Model of a Distillation Column.

The dynamical model of a distillation column consists in a set of differential equations derived from mass and energy balances in each stage of the column, and a set of algebraic equations used to describe the chemical and thermodynamical properties. The mathematical model presented here takes into account the following assumptions: (**A1**) the mixture is binary and ideal; (**A2**) the purity of the components is 100%; (**A3**) constant pressure along the column; (**A4**) constant molar flows; (**A5**) the molar vapour holdup is negligible compared with the molar liquid holdup; (**A6**) the volumetric mass and liquid hold up in each tray are constant; (**A7**) the feed temperature is at bubble point; (**A8**) the boiler is a theoretical tray; (**A9**) there are no losses of heat; (**A10**) the condenser is total.

3.1. Equilibrium relationship.

When both vapour and liquid are in intimate contact for a long period of time, equilibrium is attained between the two phases. If vapour-liquid equilibrium exists then the vapour composition y_i and the liquid composition x_i can be computed by correlating equations of the form $(y_i^{equ}, x_i^{equ}) = K_i(T_i, P_T)$, where T_i is the temperature, P_T is the total pressure, y_i^{equ} and x_i^{equ} are the vapour and liquid composition respectively. The equilibrium constant K_i depends of the thermodynamical properties of the mixture. For an ideal mixture:

$$K_i = \frac{y_i^{equ}}{x_i^{equ}} = \frac{P_i T_i}{P_T}, \tag{7}$$

where P_i is the vapour pressure of the light component.

In order to deal with mass transfer effects, the Murphree's efficiencies are introduced. The Murphree stage efficiency in the stripping (rectifying) section E_i (e_i) is the ratio between the actual change in vapour (liquid) composition between two stages and the change that would occur if the vapour (liquid) was in equilibrium with the liquid (vapour) leaving the stage. In mathematical form:

$$E_i = \frac{y_i - y_{i+1}}{y_i^{equ} - y_{i+1}}; \quad e_i = \frac{x_i - x_{i-1}}{x_i^{equ} - x_{i-1}}. \tag{8}$$

3.2. Flow rates, masses and input parameters.

The distillation column has internal and externals flows of vapour and liquid. Considering the assumptions **A1** to **A10**, they can be computed by the following equations [16] (see appendix for nomenclature):

$$V_R = V_S + (1 - q) F; \tag{9}$$

$$L_R = (1 - R) V_R; \tag{10}$$

$$V_S = \frac{Q_B}{\Delta H_1^{vap} x_{1n} + \Delta H_2^{vap} (1 - x_{1n})}; \tag{11}$$

$$L_S = L_R + qF; \tag{12}$$

$$F = F_V \left[\rho_1 W t_1 + \rho_2 (1 - W t_1)\right] \left[\frac{x_{f1}}{M_{W1}} + \frac{1 - x_{f1}}{M_{W2}}\right]; \tag{13}$$

$$D = (1 - R) V_R; \tag{14}$$

$$B = (L_S - V_S) V_B. \tag{15}$$

The molar mass liquid of the light component can be obtained with [16]:

$$M_{1i} = V_{1i} \left[\left(\frac{\rho_1}{M_{W1}} - \frac{\rho_2}{M_{W2}}\right) W t_1 + \frac{\rho_2}{M_{W2}}\right]. \tag{16}$$

The quality of the input molar flow rate is described by the following equation:

$$q = 1 + \frac{Cp_c (T_{bc} - T_f)}{\Delta H_c^{vap}}. \tag{17}$$

The molar composition of the light component in the feed can be determined by its weight fraction:

$$x_{ci} = \frac{M_{W2} W t_1}{(M_{W2} - M_{W1}) W t_1 + M_{W1}}. \tag{18}$$

3.3. **Mass balances.** Equations (19) and (20) describe the principle of the matter conservation for an ideal mixture. For the light component:

$$\begin{cases} \dot{M}_1 = V_2 - L_1 - D; \\ \dot{M}_i = V_{i+1} - L_i - V_i + L_{i-1} + \pi(i) F; \;\; i = 1, \ldots, n-1 \\ \dot{M}_n = L_{n-1} - V_n - B; \end{cases} \tag{19}$$

$$\begin{cases} \dot{M}_1 \dot{x}_1 = V_2 y_2 - L_1 x_1 - D x_1; \\ \dot{M}_i \dot{x}_i = V_{i+1} y_{i+1} - L_i x_i - V_i y_i + L_{i-1} x_{i-1} + \pi(i) F x_f; \;\; i = 1, \ldots, n-1 \\ \dot{M}_n \dot{x}_n = L_{n-1} x_{n-1} - V_n y_n - B x_n, \end{cases} \tag{20}$$

where $\pi(i) = 0$ when $i \neq f$ and $\pi(i) = 1$ when $i = f$.

Considering assumptions **A1** to **A10**, the model given in Equation (20) (expressed in terms of the light component) can be reduced to:

$$\begin{cases} M_1 \dot{x}_1 = V_R (y_2 - x_1); \\ M_i \dot{x}_i = V_R (y_{i+1} - y_i) + L_R (x_{i-1} - x_i); \;\; i = 2, \ldots, f-1 \\ M_f \dot{x}_f = V_S y_{f+1} - V_R y_f + L_R x_{f-1} - L_S x_f + F x_f; \\ M_i \dot{x}_i = V_S (y_{i+1} - y_i) + L_S (x_{i-1} - x_i); \;\; i = f+1, \ldots, n-1 \\ M_n \dot{x}_n = V_S (x_n - y_n) + L_S (x_{n-1} - x_n). \end{cases} \tag{21}$$

Finally, the assumption of a constant vapour flow through the distillation column, makes negligible the energy balance, then, it can be discarded from the formulation of the model [16].

Summarising, the dynamical model of the distillation column considered for the MPC control design, is given by Equations (9)-(18) and the set of differential-equations (21).

3.4. **Experimental validation of the model.** In order to validate the mathematical model several experiments were made on a distillation pilot-plant, e.g., Figure 1; one of these experiments is presented in this section. The pilot plant for the experiments has $n = 12$ stages, 10 trays, the feed tray is located in the 7th stage.

FIGURE 1. Distillation pilot-plant

The considered mixture was Methanol-Ethanol, whose thermodynamic and chemical properties (T_b, ρ_c, ΔH_c^{vap}, M_W, Cp_c) can be found in [11]. The purity of the components was 96%. Nevertheless, they were both assumed as pure components to exclude a third element (water) in the equilibrium relationship.

From the temperature measurements of the condenser, the boiler and the trays 4, 6, 7, 9; the molar compositions were obtained using the equilibrium relationship (7), with $P_T = 101.3$ kPa.

The input values for the process (Q_B, F_V and R) were used to calculate the molar flows (V_R, L_R, V_S, L_S, F, D and B) using Equations (9)-(15). The initial operation volumes of the components in F, V_{F1} (Methanol, 2000 ml) and V_{F2} (Ethanol, 2000 ml), were used to calculate the Methanol weight fraction in F. Finally, the volumes of the components in the boiler, $V_{1,12}$ (Methanol, 2000 ml) and $V_{2,12}$ (Ethanol, 2000 ml) were used to calculate the liquid molar mass of Methanol in the boiler using Equation (16).

The sampling period for temperature measurements was 1 min. The samples were taken when all the stages of the distillation column reached the equilibrium. The elapsed time when the process reached this point was discarded, the experiment began ($t = 0$) when the first drop of reflux was obtained.

The comparison between the output of the mathematical model and the experimental measurements are shown in Figure 2. Although there is not a perfect fit of the model, it is useful to test the control strategy. The error between the experimental data and the mathematical model in each tray where between 1.08% (tray 6) and 2.30% (boiler). These errors depend in many factors, mainly in the assumptions **A1** to **A10**. The mathematical model can be improved considering the energy balance, the vertical condenser with subcooling and the specific geometry and configuration of the boiler as in [8].

4. **Controller Design.** The control objective is to regulate the liquid compositions at the condenser (x_1) and the boiler (x_{12}). In this section the steps needed for a successful design are presented. It is noted here that these are particularised for a distillation column but they can be generalised for a range of nonlinear process.

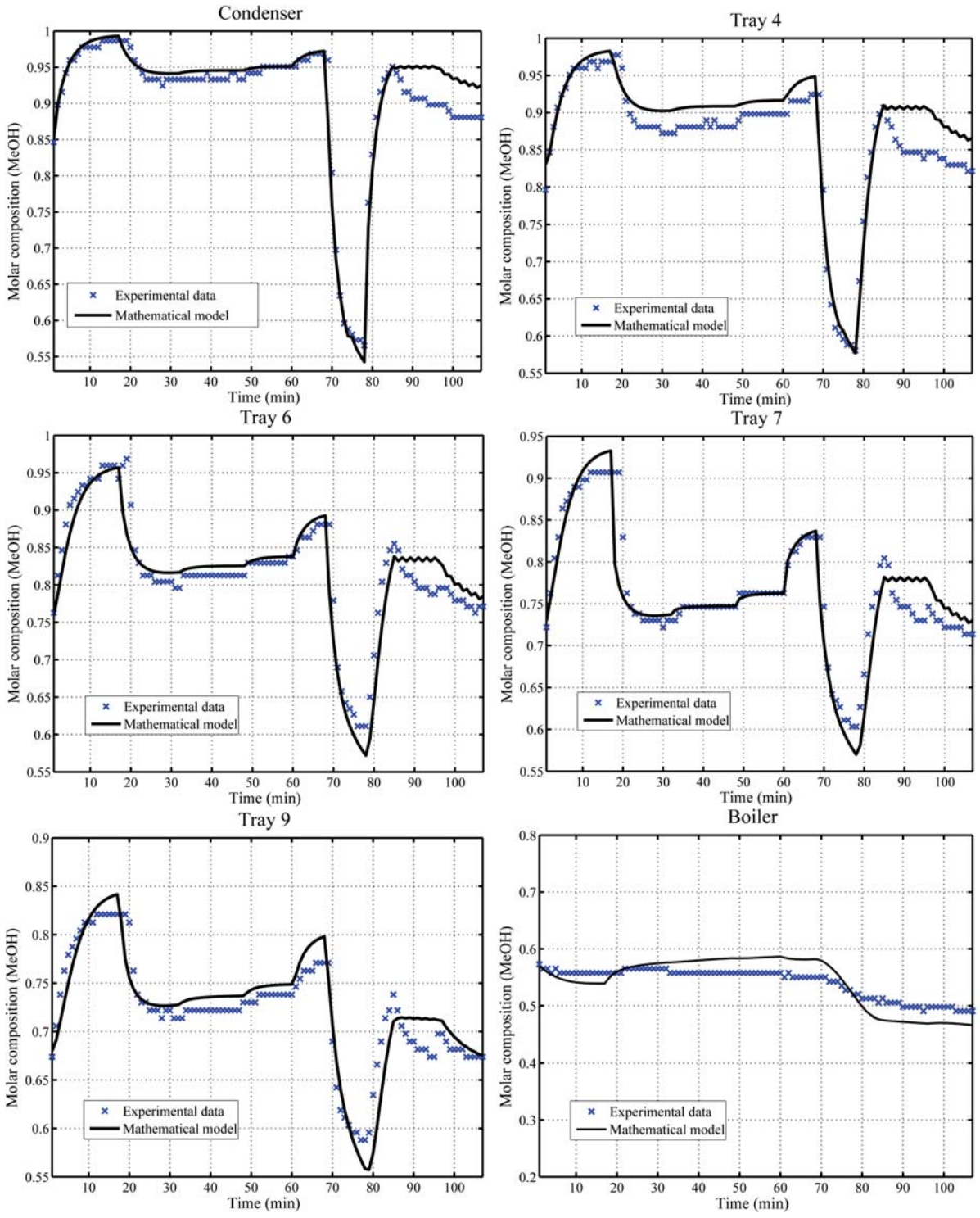

FIGURE 2. Model validation: experimental data *vs.* mathematical model

4.1. **Variable selection.** This step involves the selection of manipulated, controlled and non-manipulated variables (disturbances). Although the design may change over the course of the controller development and implementation, it will be strongly affected by the decisions taken in the initial design.

The standard configuration ($L - V$) of dual control is chosen, so the classification of the process' variables in the distillation column are:

- **Controlled variables**: condenser (x_1) and boiler (x_{12}) compositions.

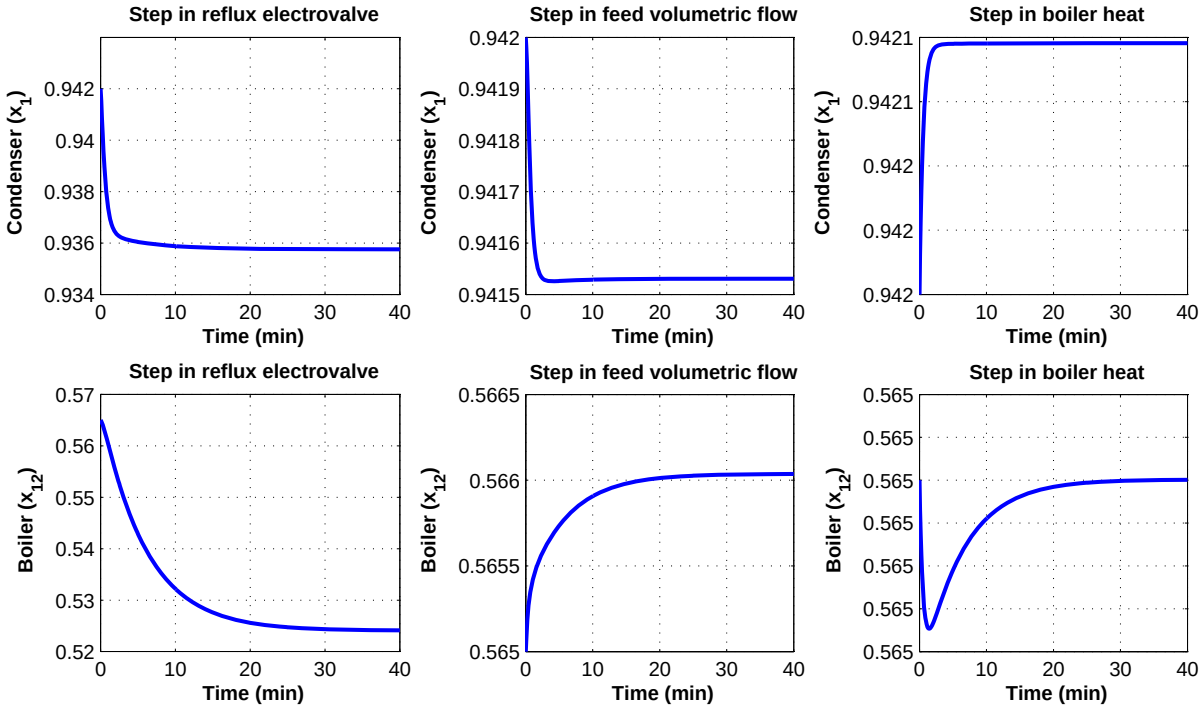

FIGURE 3. Distillation column step

- **Manipulated variables**: the ON/OFF rate of the reflux valve (R) and the heating power applied to the boiler (Q_B).
- **Measurable disturbance**: volumetric feed flow (F_V).

Note that F is directly affected by the volumetric flow F_V (see Equation (13)) and is a non-manipulated variable; i.e., it is an uncontrolled input and it can be considered a measurable disturbance to the process.

4.2. **Step response of the process.** In this experiment step changes to the manipulated variables (R and Q_B) and to the measured disturbance (F_V) were applied: a step in the ON/OFF ratio of the reflux valve R (from 30 to 70% using a PWM signal), a step on the heating power Q_B (from 750 to 775 watts) and a step on the volumetric flow F_V (from 20 to 21 ml/min). The steps were applied when the process was in a steady-state operation point. The step responses are shown in Figure 3.

Note that the steps applied on F_V and Q_B have low effects over the outputs x_1 and x_{12} compared with the effect of the step on R. These effects should be compensated with the weights of the control moves (λ) in the cost function Equation (4).

Also, maximum slew rates and constraints for each variable are determined, namely: $0 \leq x_{1,12} \leq 1$, $0 \leq Q_B \leq 2500$, $-10 \leq \Delta Q_B \leq 10$, R is an ON/OFF variable manipulated with a PWM signal.

4.3. **Model for prediction.** The model for prediction is the most important component in this family of controllers. The decision of which model to use and which model to discard depends largely on the complexity of the process dynamics and in the available amount of time for computing the predictions and the optimisation.

In this work an on-line linearisation approach is used. This means that in every sampling period the plant is linearised to calculate an *optimised input* ($\underrightarrow{\Delta \mathbf{u}_k}$). In order have a suitable prediction model for this approach, the mathematical model presented in Section 3 must be linearised.

First, the vapour-liquid equilibrium that describes the relation between the vapour and liquid compositions (y_i and x_i) on each stage i of the column is given by the nonlinear expression:

$$y_i = \frac{\alpha x_i}{1 + (\alpha - 1)\, x_i}, \quad i = 1, \ldots, n, \tag{22}$$

where α is the relative volatility (which depends on the mixture composition). Linearising this equation:

$$\frac{\partial y_i}{\partial x_i} = \frac{\alpha}{\left[1 + (\alpha - 1)\, x_i^*\right]^2}.$$

Then, the system in Equation (21) yields a nonlinear model where $\mathbf{x} \in \mathbb{R}^n$ ($n = 12$) is the state vector composed by the tray compositions x_i ($i = 1 \ldots 12$), the input vector is $\mathbf{u} = (L_R \quad V_S \quad F)^T$ and $\mathbf{y}$ is the process output. The model can be lineralised around the point $(\mathbf{x}, \mathbf{u}) = (\mathbf{x}^*, \mathbf{u}^*)$ performing the following operations: $\mathbf{A} = \frac{\partial f}{\partial \mathbf{x}}(\mathbf{x}^*, \mathbf{u}^*)$, $\mathbf{B} = \frac{\partial f}{\partial \mathbf{u}}(\mathbf{x}^*, \mathbf{u}^*)$, $\mathbf{C} = \frac{\partial g}{\partial \mathbf{x}}(\mathbf{x}^*, \mathbf{u}^*)$ and $\mathbf{D} = \frac{\partial g}{\partial \mathbf{u}}(\mathbf{x}^*, \mathbf{u}^*)$.

Explicitly, the matrix $\mathbf{A}$ has dimension $n \times n$ and is only non-zero for the entries:

$$\mathbf{A}_{i,i-1} = \begin{cases} \dfrac{L_R}{M_i} & i = 2, \ldots, n_f, \\ \dfrac{L_S}{M_i} & i = n_f + 1, \ldots, n - 1, \\ \dfrac{L_S}{M_i} & i = n. \end{cases} \qquad \mathbf{A}_{i,i+1} = \begin{cases} \dfrac{V_R y_{i+1}^*}{M_i} & i = 1, \\ \dfrac{V_R y_{i+1}^*}{M_i} & i = 2, \ldots, n_{f-1}, \\ \dfrac{V_S y_{i+1}^*}{M_i} & i = n_f, \ldots, n - 1. \end{cases}$$

$$\mathbf{A}_{i,i} = \begin{cases} -\dfrac{L_R + D}{M_i} & i = 1, \\ -\dfrac{L_R + V_R y_i^*}{M_i} & i = 2, \ldots, n_f - 1, \\ -\dfrac{L_S + V_R y_i^*}{M_i} & i = n_f, \\ -\dfrac{L_S + V_S y_i^*}{M_i} & i = n_f + 1, \ldots, n - 1, \\ -\dfrac{B + V_S y_i^*}{M_i} & i = n. \end{cases}$$

The matrix $\mathbf{B}$ has dimension $n \times 3$ and its ith row $\mathbf{B}_i$ is:

$$\mathbf{B}_i = \begin{cases} \dfrac{1}{M_i}\left[\; 0 \quad (y_{i+1} - x_i^*) \quad (1 - q)\,(y_{i+1} - x_i^*) \;\right]; & i = 1, \\ \dfrac{1}{M_i}\left[\; (x_{i-1}^* - x_i^*) \quad (y_{i+1} - y_i) \quad (1 - q)\,(y_{i+1} - y_i) \;\right]; & 1 < i < n_f, \\ \dfrac{1}{M_i}\left[\; (x_{i-1}^* - x_i^*) \quad (y_{i+1} - y_i) \quad z_f - q x_i^* - (1 - q)\, y_i \;\right]; & i = n_f, \\ \dfrac{1}{M_i}\left[\; (x_{i-1}^* - x_i^*) \quad (y_{i+1} - y_i) \quad q\,(x_{i-1}^* - x_i^*) \;\right]; & n_f < i < n, \\ \dfrac{1}{M_i}\left[\; (x_{i-1}^* - x_i^*) \quad (x_i^* - y_i) \quad q\,(x_{i-1}^* - x_i^*) \;\right]; & i = n. \end{cases}$$

The matrix $\mathbf{C}$ is an $n \times n$ matrix and $\mathbf{D} = 0$.

4.4. **Tuning parameters of the controller.** The selected prediction and control horizons are $n_p = 10$ and $n_c = 5$. Longer predictions should be avoided because de lineralised

internal model could underestimate the dynamics of the real process. Note that the control horizon should not be greater than the prediction horizon to structure the control signal.

The parameter α is used to build the reference trajectory in Equation (5) and it was selected to be 0.7 since it gives a good ratio *approach-time/smoothness* [18]. Because the process has two inputs, λ in Equation (6), is a 2×2 matrix: diag $([\lambda_R \ \lambda_Q])$, where $\lambda_R = 1$ is the weight used to penalise the control effort of the reflux electro-valve and $\lambda_{Q_B} = 0.001$ is the weight used to penalise the control effort of the power of heating. These weights should compensate the effects noticed in Section 4.2. Finally, the sampling time is 0.2 min to update the linearised model in short periods of time.

The performance of the predictive controller was compared with a standard PI controller. The tuning parameters of the PI controllers are for the reboiler P = 200, I = 1 and for the condenser P = 2000 and I = 5000. The PI design does not take explicit account of constraints, and thus ad hoc mechanisms are required; typically input saturation with some form of anti-wind-up will be used.

Once the control signal for the reflux valve is calculated, it should be converted to a pulse-width modulation signal. This is because the reflux valve is an ON/OFF three-way electro-valve.

Note on closed-loop stability. The methods described in this paper do not guarantee, a priori, closed-loop stability. However, this is normal with GPC/DMC and indeed MPC is normally employed without this guarantee (see Section 7 of [12]). Algorithms which give a priori stability gaurantees, such as those deploying a terminal constraint [9], require far more complex constraint handling and optimisation procedures. The authors believe that the methods presented here are important because of their ability to strategically ensure constraint satisfaction, which is often the main motivation for choosing MPC, and moreover, this is achieved without incurring excessive coding complexity or a high computational burden.

5. Numerical Simulations.

The MPC controller designed in Section 4 will be used to control the distillation process given by the mathematical model in Section 3. In order to test the regulation and tracking capabilities of the proposed controller, two numerical simulations were carried out. These are described below.

5.1. Regulation capabilities.

The aim of this test is to show the controller performance in presence of disturbances (changes in the feed flow F_V) without changes in the set-point.

The simulation began when the column have reached the steady-state operation point. The initial feed flow was 20 ml/min and the initial conditions of the states (compositions) were:

$$\mathbf{x}_0 = [0.942, 0.915, 0.9, 0.865, 0.82, 0.81, 0.804, 0.73, 0.722, 0.65, 0.624, 0.565]^T$$

At $t = 50$ min, the system was disturbed with a volumetric feed step from $F_V = 20$ ml/min to $F_V = 30$ ml/min. At $t = 150$ min the volumetric feed flow was changed to $F_V = 50$ ml/min (a double increase than the previous one). Finally, at $t = 250$ min the feed flow was changed to $F_V = 30$ ml/min.

The simulation results are shown in Figure 4. Different changes in the volumetric feed flow F_V can be compensated by the predictive controller. This can be done because the volumetric feed flow is used as a feedforward variable to compute the output prediction and the control signal. The compositions are practically not affected by the step disturbances in F_V using the predictive control.

On the other hand, compensation of the volumetric feed flow can not be done with the PI controller because, for the the PI controller design, F_V is assumed as an operation

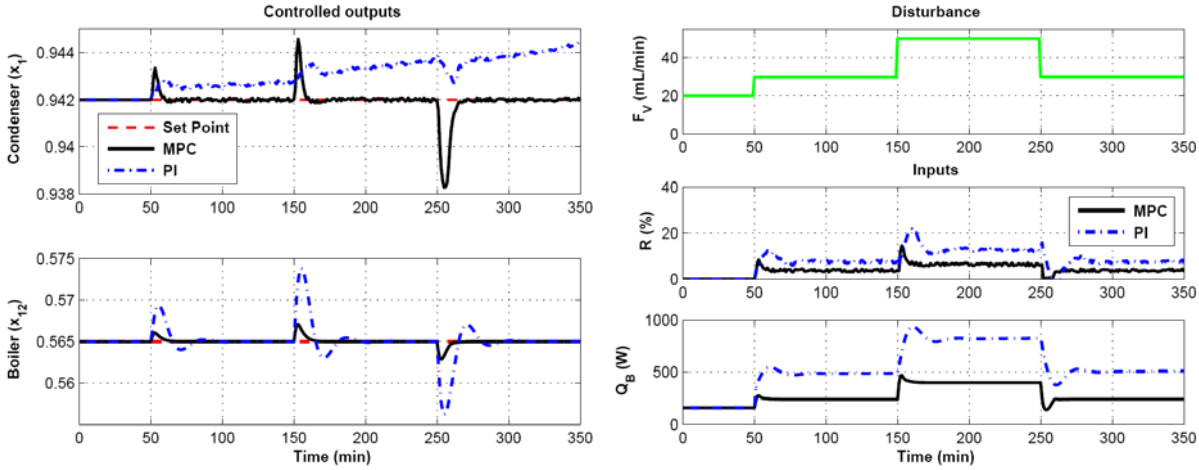

FIGURE 4. Regulation case

condition. Large disturbances (changes of the volumetric feed flow), as presented in Figure 4, destabilises the PI controller.

5.2. **Tracking capabilities.** The aim of this test is to show the controller performance in presence of set-point changes; i.e., how MPC allows low sensitivity to set-point changes for a nonlinear process.

As in the previous case, the simulation began when all the stages of the column have reached the steady stage operation. The initial feed flow F_V was 20 ml/min and did not change during the simulation. The initial conditions of the states (compositions) were the same as in the previous case.

When the simulation began (at $t = 0$ min), step changes in the condenser-composition ($x_1^{set} = 0.98$) and in the boiler-composition ($x_{12}^{set} = 0.55$) were demanded. Once a new steady-state operation point has been reached (at $t = 50$ min), a gradual change in the boiler-composition was demanded from $x_{12}^{set} = 0.55$ until $x_{12}^{set} = 0.5$. A new gradual change was demanded in the condenser-composition (at $t = 140$ min) from $x_1^{set} = 0.98$ to $x_1^{set} = 0.96$. Two more step changes were demanded in the composition: the first at $t = 200$ min, from $x_1^{set} = 0.96$ to $x_1^{set} = 0.92$ and the second at $t = 300$ min, from $x_{12}^{set} = 0.5$ to $x_{12}^{set} = 0.4$. Finally, a smooth condenser's composition change was demanded at $t = 330$ min from $x_1^{set} = 0.92$ to $x_1^{set} = 0.94$.

Figure 4 shows that, with the predictive control, the process takes a shorter time (compared with the PI controller) to become stabilised. The regulation of the condenser composition is a harder task for the controller due the ON/OFF effect of the reflux electrovalve, nevertheless, the process outputs reach the set-points during all the simulation time.

The PI control reach the set-points in the first part of the simulation ($t < 200$ min). When changes in the set-points are smooth the PI controller does not have tracking problems, leading the process outputs to reach them. But it is not performant when big changes in the set-points are made ($t = 200$ min, $t = 300$ min). That is because in the attempt to take the outputs to the new set-points, the PI controller calculates control signals that exceed the allowed limits of these variables. Then, PI control signals saturate at their limits, whereas the MPC "anticipates" the violation of these constraints. Also, the PI controller was designed for a specific operating point, while the MPC controller is redesigned in each sampling time because of the on-line linearisation of the prediction model.

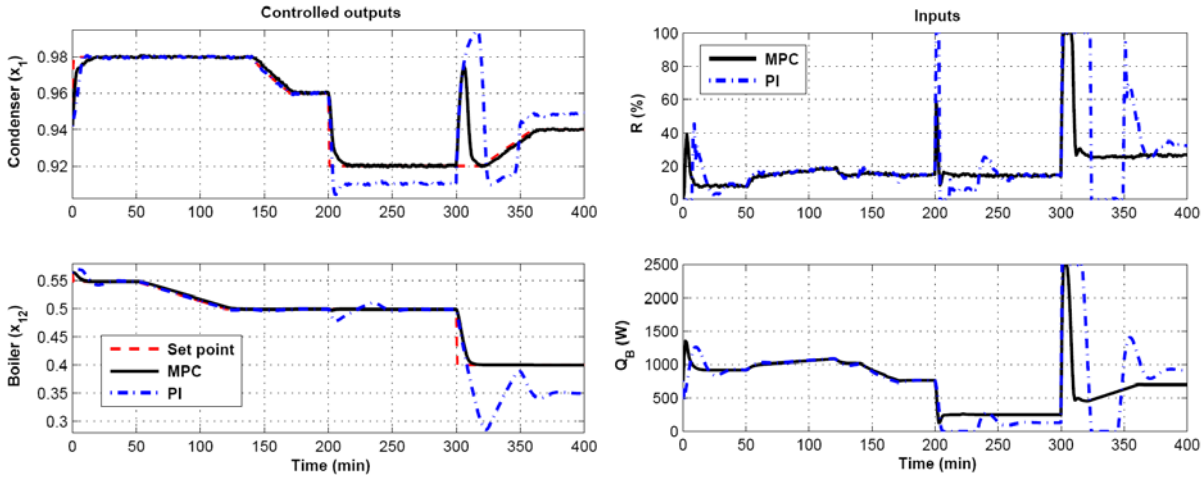

FIGURE 5. Tracking case

TABLE 1. Mean values of the cost J for the control simulations

	MPC Controller		**PI Controller**	
	x_1	x_{12}	x_1	x_{12}
Regulation	0.22	0.12	1.23	0.57
Tracking	0.85	0.53	6.45	2.5

In Figure 5, it can be seen that slow gradual changes are an easier task for both controllers. This is because the changes in the set-points are smoother. For that reason, the PI controller calculates smoother control signals, with no need to violate the constraints. In spite of it, the PI control has a steady-state error because it is working in a different operating point for witch it was designed. Whereas, for the MPC control, the greater problem is the inherent noise caused by the loss of information when the signal control is converted to pulses.

For both cases (regulation and tracking), the running costs which quantifies the performance of the controllers, are measured using (4) and are presented in Table 1.

It is clear that this strategy can cope with measurable disturbances and different operating points of the process with no need of readjusting the controller. The proposed algorithm constitutes an alternative to nonlinear MPC with the advantage that the online linearisation keeps the computational burden low solving a linear MPC problem (instead a nonlinear one) each sampling time.

6. **Conclusions.** An improved discrete-time model-based predictive controller with constraints for a distillation process was developed. The control objective was to regulate the liquid compositions at the condenser and the boiler. The MPC controller was compared with a conventional PI controller. The implementation of the algorithms were presented in a didactic way which allows meaningful and versatile learning of this technique. The methodology presented for the incorporation of constrained predictive algorithms to the distillation process can be easily extended to other systems.

The model used for the simulation of the distillation column is a simplified model that reproduces the nonlinearities and the directionality of the process. The model was successfully validated with experimental data.

The main feature of the proposed MPC control lies in the fact that it uses an on-line linearisation of the nonlinear model of the column. This ensures a solution to the minimisation problem (QP), saving computing time and avoiding the solution of a nonlinear optimisation problem. In contrast, when nonlinear programming is used, the number of numerical operations increases, leading to computational delays (with no guarantee to find an optimal solution) that can easily arise from accumulated errors and, with that, a degradation of the process performance. On the other hand, when linear MPC is used to control a nonlinear process the tuning normally is done for an operating point; as the set-points are modified, the poor tuning originates a loss of yield in the controller, causing a high variability in the controlled outputs that can even arise in instability.

The proposed controller was simulated for continuous distillation and it performed well despite the nonlinearities in the model and step perturbations (as already noted in Figures 4 and 5). A further point to be discussed as a potential advantage is the easy of tuning, using mainly the step response of the process. On the other hand, the main disadvantage of this method is the selection of the process model. In fact, this is the most difficult part as it is in any algorithm of model-based control.

These results demonstrate the potential for implementing MPC based on online linearisation for a nonlinear process. Nevertheless, there are several important issues the authors intend to pursue next of which some significant ones are: (i) implement this controller in standard industrial hardware; (ii) consider cases with more demanding constraint handling requirements; (iii) consider the impact of uncertainty such as unmeasurable disturbances and (iv) implement more advanced dual-mode type MPC algorithms that provide a priori stability guarantees.

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Appendix A. Nomenclature.

Parameters

B	Molar flow rate of the bottom product (mol/min).
Cp_c	Specific heat (kJ/mol°C).
D	Molar flow rate of the distillate product (mol/min).
E_i, e_i	Murphree stage efficiency
F	Molar flow rate at the feed tray (mol/min).
F_V	Volumetric flow rate at the feed tray (ml/min).
K_{ci}	Equilibrium coefficient.
L	Liquid molar flow rate (mol/min).
M_i	Molar holdup at stage i (mol).
M_W	Molecular weight (g).
P_T	Total pressure (kPa).
P_i	Pressure at stage i (kPa).
Q_B	Heat input in the boiler (W).
R	Reflux, binary value {0-1}.
T_i	Temperature at stage i (°C).
T_b	Boiling temperature (°C).
V	Vapour molar flow rate (mol/min).
Wt	Weight fraction.
n_c	Control horizon.
n_p	Prediction horizon.
q	Feed quality.
x_i	Liquid molar composition at stage i.
y_i	Vapour molar composition at stage i.
ΔH_c^{vap}	Vaporisation enthalpy (kJ/mol).
ρ_c	Density (g/cm^3).

Subscripts and superscripts

$\{\cdot\}_c$	Component.
$\{\cdot\}_f$	Feed tray.
$\{\cdot\}_i$	ith stage of the column.
$\{\cdot\}_R$	Rectifying section.
$\{\cdot\}_S$	Stripping section.
$\{\cdot\}^{equ}$	In equilibrium.
$\{\cdot\}^{set}$	Set-point value.