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A STATE-SPACE BASED STUDY OF STABILITY, BULLWHIP EFFECT AND TOTAL COSTS IN TWO-STAGE SUPPLY CHAINS

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Abstract. *The main focus of this work is to apply the state-space method used in modern control theory to study the stability/instability of a two-stage supply chain controlled by the order policy called Automatic Pipeline, Variable Inventory and Order Based Production Control System (APVIOBPCS). Because product returns are not permitted, the supply chain may turn out to be an autonomous switched system according to the retailer's order pattern. The stability of each subsystem is examined by analytic method and numerical analysis. The relationship between the supply chain stability/instability and bullwhip effect at different values of input parameters is then examined through simulation. Finally, the impacts of the decision variables on the relationship between the supply chain stability and the chain-wide total cost are analyzed, and the implications for demand forecasting, inventory control, and supply process for improving the supply chain operations are identified.*

Keywords: Supply chain, Lead time, Forecast method, Stability, Bullwhip effect

1. Introduction.

Inventory dynamics exhibit quite complex behaviors in supply chains. Inventory levels result from decision making and product shipment, both of which respond to unpredictable, and sometimes artificial consumer demand. A comprehensive literature review of the economic impacts of bullwhip effect shows that supply chains could decrease the stock expenses by 15 to 30 percent by reducing bullwhip effect [1]. For this reason, different techniques for reducing bullwhip effect have been proposed in the literature. These techniques center on improving demand forecasting, applying a proportional controller to the order policy, filtering the demand to dampen its variability, and others [2-4].

One of the most critical factors affecting effective supply chain management is the delay [5]. The impact of deterministic lead time and information sharing on bullwhip effect and on-hand inventory is studied in [6,7]. Demand forecasting has become a major drive for developing supply chain management strategies and tactics. Under MMSE, MA and EWMA forecasting methods, bullwhip effect is studied in [8,9].

This paper aims to understand how to control the stability of a single-product, one-supplier-one-retailer supply chain in order to reduce the total cost as well as bullwhip effect. In this supply chain, the replenishment lead time is fixed and the review period is determined by the APVIOBPCS policy and the forecasting method called exponential smoothing. We want to examine the relationship between the stability and bullwhip effect in this system. The modern control system theory is used as a basis for our research and allows us to gain important insights about the dynamic behavior of the replenishment rule.

2. A Basic Model.

2.1. Model description. As mentioned previously, the supply chain system under study consists of one retailer and one supplier that carries sufficient inventory for replenishment. If the retailer has enough on-hand inventory, then customer demand is satisfied immediately; if not, the shortage is backlogged. With the assumption that the system is periodically reviewed, the sequence of events during one period is given as follows: receiving shipments and new demand, satisfying the demand, reviewing the inventory and placing an order.

Note that there is a fixed replenishment lead time τ between the placement of an order and the receipt of the ordered goods. The following notations are introduced: L_t is external customer demand; $\hat{L}_t$ is forecasting demand; S_t is on-hand inventory; R_t is the goods received; O_t is order quantity; SL_t is supply line level.

The goods the retailer receives at t is the shipment from the supplier during the previous τ periods, that is $R_t = O_{t-\tau}$. The retailer's inventory is the sum of inventory at the end of the last period and goods received at current period minus the customer demand, that is $S_t = S_{t-1} + R_t - L_t$. The supply line at t is the supply line at $t-1$ plus the order placed at $t-1$ minus the order placed at $t-\tau$, that is $SL_t = SL_{t-1} + O_{t-1} - O_{t-\tau}$. If the simple exponential smoothing method is used to forecast the demand, then the following equation can be obtained:

$$\hat{L}_t = \theta L_t + (1-\theta)\,\hat{L}_{t-1}, \quad 0 < \theta < 1 \tag{1}$$

According to the APVIOBPCS policy, the mathematical representation of this policy is

$$IO_t = \hat{L}_t + \alpha_S\,(S_t^* - S_t) + \alpha_{SL}\,(SL_t^* - SL_t) \tag{2}$$

where IO_t represents the retailer's computational order quantity at t. S_t^* and SL_t^* represent the retailer's desired inventory and the desired supply line, respectively. We set $S_t^* = SL_t^* = \tau\hat{L}_t$. Note that in Equation (2), the parameter, α_S $(0 \le \alpha_S \le 1)$, is the adjustment rate of discrepancy between the retailer's actual and desired inventory levels in each period, and α_{SL} $(0 \le \alpha_{SL} \le 1)$ is the adjustment rate of discrepancy between the retailer's actual and desired supply line.

Finally, according to the APVIOBPCS policy, the order quantity can be negative if returns are permitted. However, in many situations, the return policy is in place, which is assumed in this paper. As a result, the order quantity should satisfy the following condition:

$$O_t = \max\,(0, IO_t) \tag{3}$$

2.2. The state-space model. For the supply chain considered in this paper, L_t is the input and O_t is the output, and both are external variables. We regard $S_t, SL_t, \hat{L}_t$ as the internal variables as well as the system's state variables. Then, we can write the following two functions:

$$S_t = \begin{cases} S_{t-1} - \alpha_S S_{t-\tau} - \alpha_{SL} S_{t-\tau} + (1 + \alpha_S\tau + \alpha_{SL}\tau)\hat{L}_{t-\tau} - L_t, & IO_{t-\tau} \ge 0 \\ S_{t-1} - L_t, & IO_{t-\tau} < 0 \end{cases} \tag{4}$$

$$SL_t = \begin{cases} -\alpha_S S_{t-1} + (1-\alpha_{SL})\,SL_{t-1} + (1 + \alpha_S\tau + \alpha_{SL}\tau)\,\hat{L}_{t-1} & \\ \qquad + \alpha_S S_{t-\tau} + \alpha_{SL} SL_{t-\tau} - (1 + \alpha_S\tau + \alpha_{SL}\tau)\,\hat{L}_{t-\tau}, & IO_{t-1} \ge 0,\ IO_{t-\tau} \ge 0 \\ -\alpha_S S_{t-1} + (1-\alpha_{SL})\,SL_{t-1} + (1 + \alpha_S\tau + \alpha_{SL}\tau)\,\hat{L}_{t-1}, & IO_{t-1} \ge 0,\ IO_{t-\tau} < 0 \\ SL_{t-1} + (1-\alpha_{SL})\,SL_{t-1} + (1 + \alpha_S\tau + \alpha_{SL}\tau)\,\hat{L}_{t-1}, & IO_{t-1} < 0,\ IO_{t-\tau} \ge 0 \\ SL_{t-1}, & IO_{t-1} < 0,\ IO_{t-\tau} < 0 \end{cases} \tag{5}$$

If we define the system's switching rules as: $\sigma_t^{(1)} = \text{sign}\left(IO_{t-1}\right)$ and $\sigma_t^{(2)} = \text{sign}\left(IO_{t-\tau}\right)$, then we obtain four subsystems. Combining the two possibilities of $(IO_{t-1}, IO_{t-\tau})$, we see that the supply chain system under study will switch among the following four subsystems.

$$\begin{cases} \text{When } \sigma_t^{(1)} \geq 0,\ \sigma_t^{(2)} \geq 0, & \text{switch to subsystem 1} \\ \text{When } \sigma_t^{(1)} \geq 0,\ \sigma_t^{(2)} < 0, & \text{switch to subsystem 2} \\ \text{When } \sigma_t^{(1)} < 0,\ \sigma_t^{(2)} \geq 0, & \text{switch to subsystem 3} \\ \text{When } \sigma_t^{(1)} < 0,\ \sigma_t^{(2)} < 0, & \text{switch to subsystem 4} \end{cases} \tag{6}$$

Furthermore, setting $x_t = \left[S_t, SL_t, \hat{L}_t\right]^T$ and combining equations yield the following state-space model

$$x_t = A_k x_{t-1} + B_k x_{t-\tau} + C_k L_t, \quad k = 1, 2, 3, 4 \tag{7}$$

$$A_1 = A_2 = \begin{bmatrix} 1 & 0 & 0 \\ -\alpha_S & 1-\alpha_{SL} & 1+\alpha_S\tau+\alpha_{SL}\tau \\ 0 & 0 & 1-\theta \end{bmatrix}, \quad A_3 = A_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\theta \end{bmatrix} \tag{8}$$

$$B_1 = B_3 = \begin{bmatrix} -\alpha_S & -\alpha_{SL} & 1+\alpha_S\tau+\alpha_{SL}\tau \\ \alpha_S & \alpha_{SL} & -\left(1+\alpha_S\tau+\alpha_{SL}\tau\right) \\ 0 & 0 & 0 \end{bmatrix}, \quad B_2 = B_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{9}$$

$$C_1 = C_2 = C_3 = C_4 = \begin{bmatrix} -1 & 0 & \theta \end{bmatrix}^T \tag{10}$$

3. **Analysis of the System Stability.** Stability is the fundamental issue of a dynamic system. An instable policy will react to any demand pattern with a response that oscillates with ever-increasing amplitude and is thus inherently undesirable. A stable supply chain ordering policy on the other hand will react to any finite demand signal and after a finite period of time return to steady-state conditions. Obviously, this is a necessary property of a practical supply chain replenishment decision [10-12]. We can examine the system stability based on whether the roots of the characteristic equation lie within the unit circle or not. Equation (6) is identified for such a system with time-delay for the supply chain under study, and the characteristic equation of each subsystem is

$$\left|I - A_k\lambda^{-1} - B_k\lambda^{-\tau}\right| = 0, \quad \forall k = 1, 2, 3, 4. \tag{11}$$

If all the roots of Equation (11) lie within the unit circle and there is only one single root located on the unit circle, then the discrete dynamic system is stable.

3.1. **Stability of subsystem 1.** Based on Equation (11), the characteristic equation of subsystem 1 can be written as

$$\left|I - A_1\lambda^{-1} - B_1\lambda^{-\tau}\right| = \lambda^{-\tau-2}\left(\lambda-1\right)\left[\lambda-(1-\theta)\right]\left[\lambda^\tau - \left(1-\alpha_{SL}\right)\lambda^{\tau-1} + \alpha_S - \alpha_{SL}\right] = 0 \tag{12}$$

It is easy to see that $\lambda_1 = 1$, $\lambda_2 = 1-\theta$ are the two Eigen-values. Therefore, for the subsystem 1 to be stable, the following condition must be met

$$\lambda^\tau - \left(1-\alpha_{SL}\right)\lambda^{\tau-1} + \alpha_S - \alpha_{SL} = 0 \tag{13}$$

and all the roots of Equation (13) must stay inside the unit circle.

When $\tau = 1$, the root of Equation (13) is $\lambda_3 = 1-\alpha_S$, so subsystem 1 is stable. When $\tau = 2$, Equation (13) can be re-written as $\lambda^2 - \left(1-\alpha_{SL}\right)\lambda + \alpha_S - \alpha_{SL} = 0$. There are two cases:

Case 1: If $\left(1-\alpha_{SL}\right)^2 - 4\left(\alpha_S - \alpha_{SL}\right) = \left(1+\alpha_{SL}\right)^2 - 4\alpha_S \geq 0$. The roots are $\lambda'_{3,4} = \left(1-\alpha_{SL} \pm \sqrt{\left(1+\alpha_{SL}\right)^2 - 4\alpha_S}\right)\Big/2 \leq \left(1-\alpha_{SL}+1+\alpha_{SL}\right)/2 = 1$.

Case 2: If $(1-\alpha_{SL})^2 - 4(\alpha_S - \alpha_{SL}) = (1+\alpha_{SL})^2 - 4\alpha_S < 0$. The roots are $\lambda'_{3,4} = \left(1 - \alpha_{SL} \pm j\sqrt{4\alpha_S - (1+\alpha_{SL})^2}\right)\Big/2$, and $\left|\lambda'_{3,4}\right| = \left((1-\alpha_{SL})^2 + 4\alpha_S - (1-\alpha_{SL})^2\right)/4 = \alpha_S - \alpha_{SL} < 1$. Hence, subsystem 1 is stable when $\tau = 2$.

When $\tau > 2$, finding the exact roots of Equation (13) becomes difficult. Hence, we choose to use a numerical analysis method to obtain the range of the parameters (α_S, α_{SL}) that produces the roots of Equation (13) within the unit circle. Specifically, we set each of (α_S, α_{SL}) in the interval of $[0, 1]$ and divide the interval into 20 equal segments, or 0.05 each, which consequently results in 400 points for each given lead time. Note that the values of (α_S, α_{SL}) can be essentially any positive numbers, but most practically are selected from the interval of $[0, 1]$. A set of numerical results with $\tau = 3$ through 8 are shown in Figure 1, where the black dots represent all the roots of Equation (13) lying within a unit circle, suggesting the situations under which the system is stable.

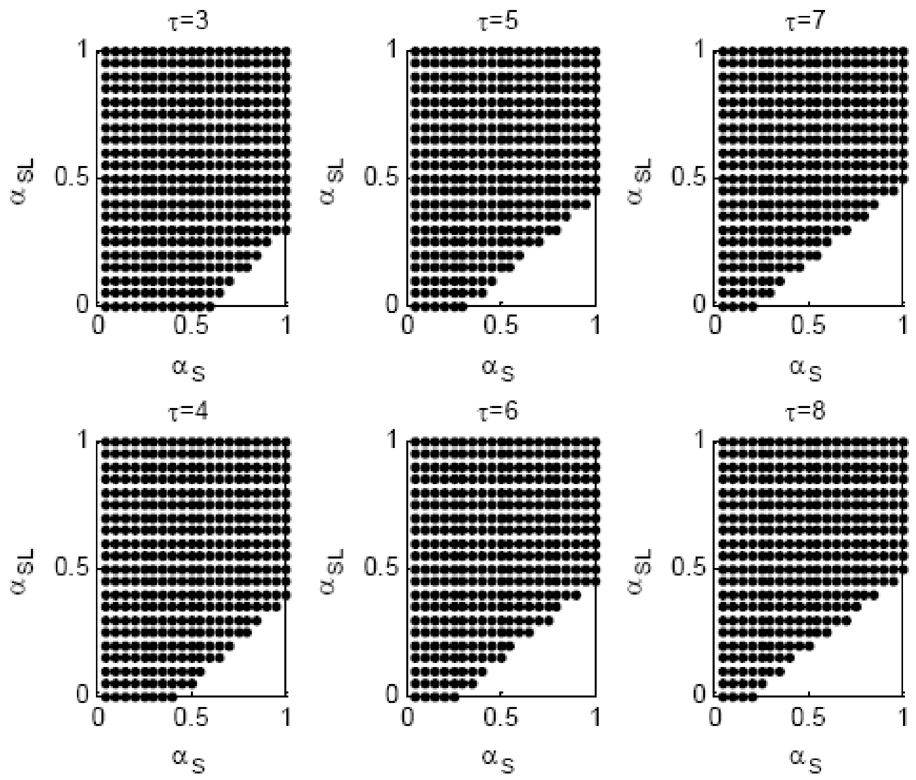

Figure 1. Range of (α_S, α_{SL}) suggesting a stable subsystem 1

As Figure 1 indicates, the stability of the subsystem 1 is more sensitive to the factor, α_S; in particular, with high α_{SL} values, the subsystem is usually stable, and as the lead time increases, the blank area (lower right corner where the value of α_S is high and the value of α_{SL} is low) enlarges and the number of stable points decreases. According to the numerical results, we can get practical implications: being very reactive to the supply line and less reactive to inventory would yield more stability.

Using the numerical analysis results for various lead times, we are also able to identify an approximate relationship between the number of stable points (NS) and the lead time by Excel, which is shown in Figure 2. Specifically, the relationship is given by a polynomial function with the following coefficients with excellent goodness of fit ($R^2 = 0.989$):

$$NS_1(t) = -0.042\tau^3 + 1.853\tau^2 - 27.370\tau + 451.987, \quad \forall t > 2 \tag{14}$$

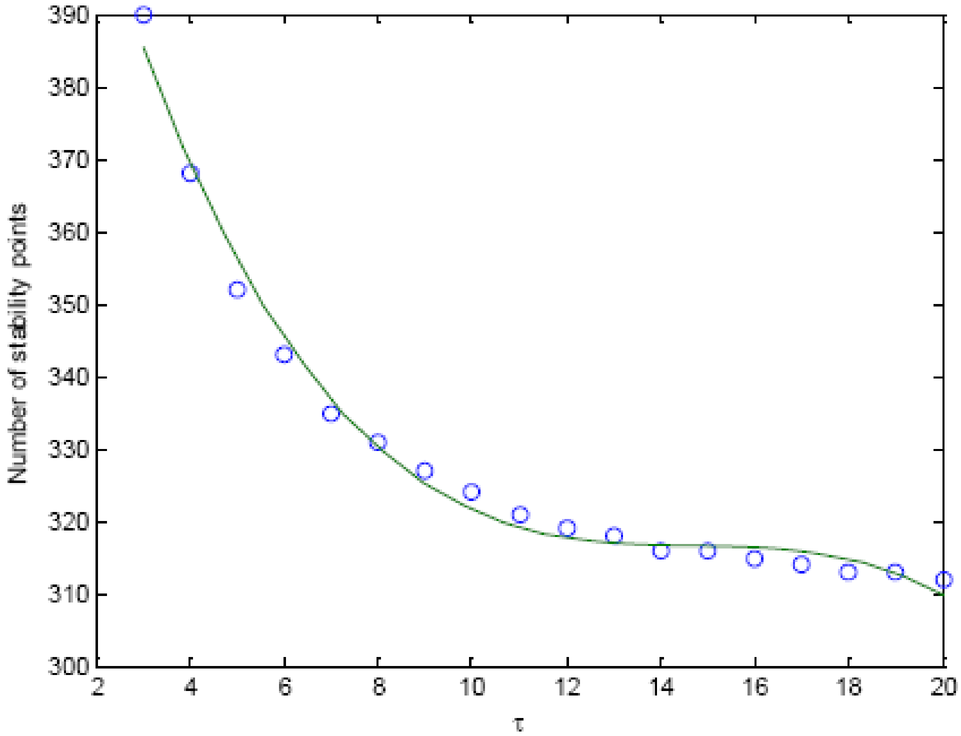

FIGURE 2. Approximated relationship between the stable points and τ ($\tau > 2$)

From a practical point of view, Equation (14) provides a quick tool for checking the stability/instability of subsystem 1 when lead time is greater than 2 periods.

3.2. **Stability of subsystem 2.** Since matrix B_2 in (9) contains only zeroes, state-space model of subsystem 2 does not have time-delay, and its stability is hence not related to the replenishment lead time. As a result, the characteristic equation of subsystem 2 based on Equation (6) can be written as

$$|\lambda I - A_2| = (\lambda - 1)[\lambda - (1 - \theta)][\lambda - (1 - \alpha_{SL})] = 0 \tag{15}$$

Three roots are found easily as $\lambda_1 = 1$, $\lambda_2 = 1 - \theta$, $\lambda_3 = 1 - \alpha_{SL}$. It is seen clearly the subsystem is stable. However, because of $\sigma_t^{(2)} < 0$, it is impossible for the supply chain to stay at this subsystem for long. That means the retailer does not make order, therefore the retailer have stock-out soon, and it is unrealistic. Then, the subsystem will likely switch to other states during most of τ periods. The likelihood of being in this situation is dependent on the lead time and decision parameters.

3.3. **Stability of subsystem 3.** Based on Equation (7), the characteristic equation of subsystem 3 has the following format:

$$\left|I - A_3\lambda^{-1} - B_3\lambda^{-\tau}\right| = \lambda^{-\tau-2}(\lambda - 1)[\lambda - (1 - \theta)]\left[\lambda^{\tau} - \lambda^{\tau-1} + \alpha_S - \alpha_{SL}\right] = 0 \tag{16}$$

It is straightforward to see that $\lambda_1 = 1$, $\lambda_2 = 1 - \theta$ are the two Eigen-values. Therefore, for subsystem 3 to be stable, the following relationship must be true:

$$\lambda^{\tau} - \lambda^{\tau-1} + \alpha_S - \alpha_{SL} = 0 \tag{17}$$

When $\tau = 1$, the roots is $\lambda_3 = 1 - \alpha_S + \alpha_{SL}$. Thus, the subsystem 3 is stable. When $\tau = 2$, Equation (17) becomes $\lambda^2 - \lambda + \alpha_S - \alpha_{SL} = 0$. If $1 - 4(\alpha_S - \alpha_{SL}) \geq 0$, that is $\alpha_S - \alpha_{SL} \leq 1/4$, the Eigen-values are $\lambda'_{3,4} = \left(1 \pm \sqrt{1 - 4(\alpha_S - \alpha_{SL})}\right)\Big/2$, and

$\left|\lambda'_{3,4}\right| \leq 1$. If $4(\alpha_S - \alpha_{SL}) < 0$, the Eigen-values are $\lambda'_{3,4} = \left(1 \pm j\sqrt{4(\alpha_S - \alpha_{SL}) - 1}\right)\Big/2$, and $\left|\lambda'_{1,2}\right|^2 = (1/2)^2 + (4(\alpha_S - \alpha_{SL}) - 1)/4 = 4(\alpha_S - \alpha_{SL})/4 < 1$. Thus, when lead time is 2 time periods, the subsystem 3 is stable.

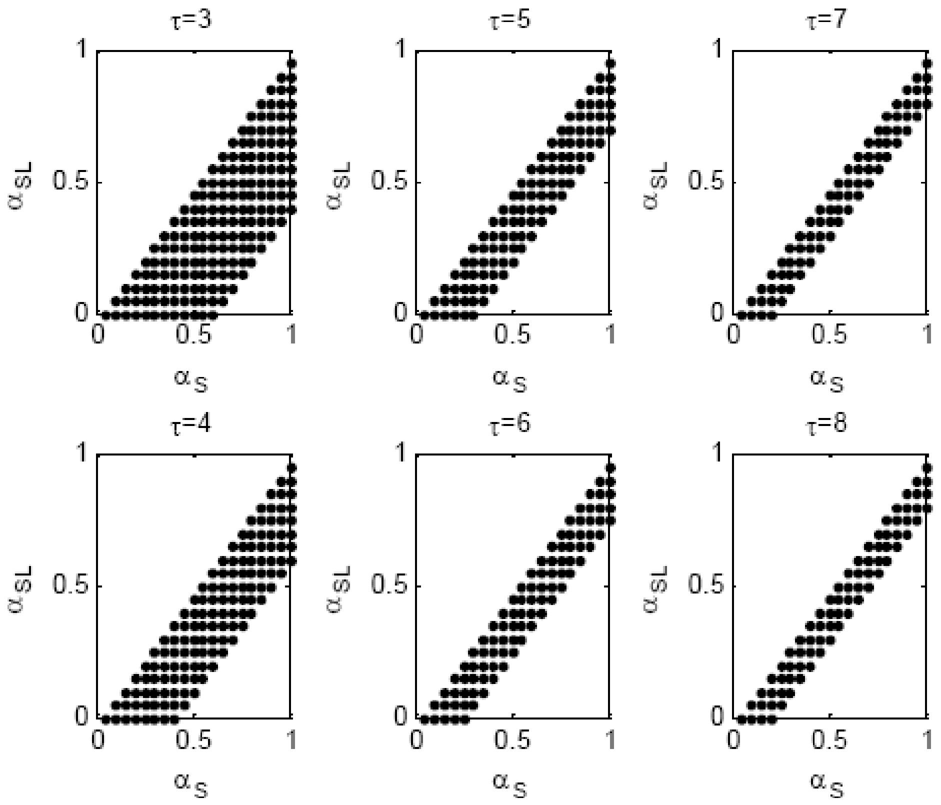

FIGURE 3. Range of (α_S, α_{SL}) at which subsystem 3 is stable

As is analyzed in subsystem 1, we use numerical analysis to show the results when $\tau > 2$ in Figure 3. The stable points have always stayed in the lower right corner. For practical, the decision makers would give much more weights to the inventory than to supply line.

Meanwhile, we can also identify the approximate relationship between the number of stable points and the lead time:

$$NS_3(t) = -0.023\tau^3 + 1.487\tau^2 - 31.779\tau + 240.384, \quad \forall t > 2 \tag{18}$$

According to Figure 4, it is seen that the number of stable points is zero when lead time is greater than 31 periods. This implies that the subsystem is instable when $\tau > 31$.

3.4. **Stability of subsystem 4.** We give the characteristic equation of subsystem 4 in the following format:

$$|\lambda I - A_4| = (\lambda - 1)(\lambda - 1)[\lambda - (1 - \theta)] = 0 \tag{19}$$

Three roots of (19) are found: $\lambda_1 = \lambda_2 = 1$, $\lambda_3 = 1 - \theta$. Since there exist multiple roots located on the unit circle, subsystem 4 is instable. Furthermore, the fact that $O_{t-1} = O_{t-\tau} = 0$ in this subsystem implies that the retailer does not place any order and relies on the on-hand inventory to satisfy the external customer demand. Since the inventory level decreases during this period, the subsystem cannot be stable.

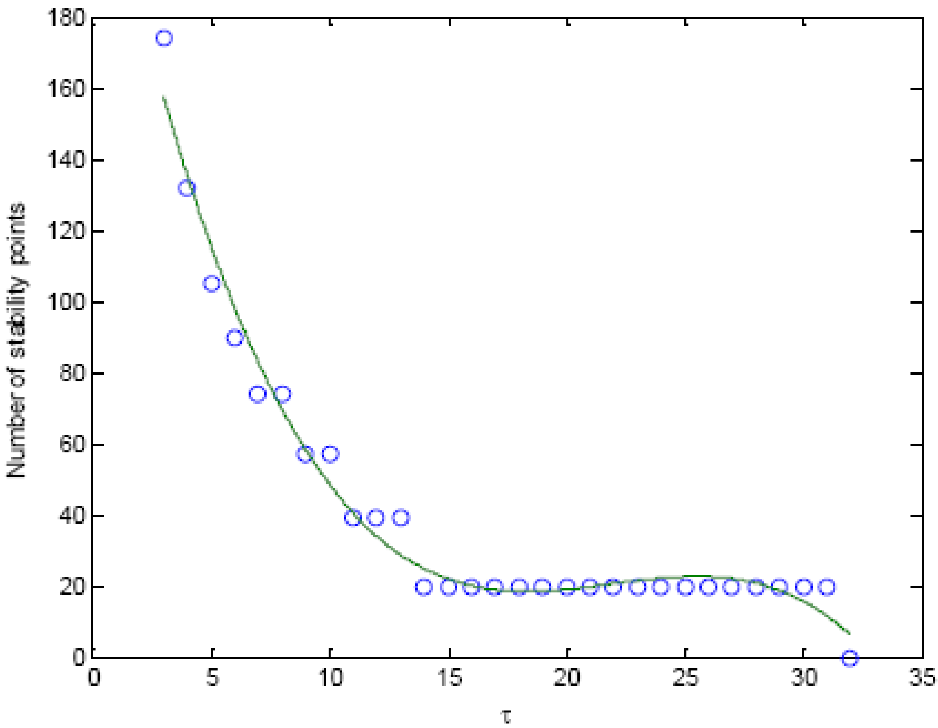

FIGURE 4. Number of stability points versus lead time

4. **Bullwhip Effect and Supply Chain Stability.** To see if mode instability or stability affects bullwhip metric, we next go to calculate bullwhip effect. In this section, we ignore the assumption, $O_t = \max(0, IO_t)$, do not consider the sign of IO_t, $IO_{t-\tau}$, and keep all others unchanged in the supply chain system, and then analyze the relationship between bullwhip effect and stability. According to Equation (7), the state-space of this supply chain system is the same as that of subsystem 1. Thus, the characteristic function in Equation (13) applies here as well.

Without loss of generality, we set that the lead time is equal to 3 time periods. Let $\lambda = (S+1)/(S-1)$, where the factor, S, is selected to give such λ. Substituting it to Equation (18) yields the following equation

$$\alpha_S S^3 + (2 + 4\alpha_{SL} - 3\alpha_S)\, S^2 + (4 - 4\alpha_{SL} + 3\alpha_S)\, S + 2 - \alpha_{SL} = 0 \tag{20}$$

Notice that the parameter θ associated with demand forecasting is not present in Equation (20). This means that demand forecasting has no effect on the stability of the supply chain system. The necessary and sufficient condition that the roots of Equation (18) are within unit circle is equivalent to the condition that all the roots of Equation (20) are located on left half of the complex plane. Two parametes must meet the following Equation (21).

$$\begin{cases} 2 + 4\alpha_{SL} - 3\alpha_S > 0 \\ 2 - \alpha_{SL} > 0 \\ (2 + 4\alpha_{SL} - 3\alpha_S) \times (4 - 4\alpha_{SL} + 3\alpha_S) > \alpha_S \times (2 - \alpha_{SL}) \end{cases} \tag{21}$$

Since $0 \le \alpha_S \le 1$, $0 \le \alpha_{SL} \le 1$, we can obtain the following range,

$$\left\{ (\alpha_S, \alpha_{SL}) \middle| \alpha_{SL} > \left(3\alpha_S + 1 - \sqrt{\alpha_S^2 - 2\alpha_S + 9} \right) \Big/ 4,\ 0 \le \alpha_S \le 1, 0 \le \alpha_{SL} \le 1 \right\} \tag{22}$$

Figures 5 illustrates the stable (grey area) region and instable region on the parameter plane when α_S, α_{SL} changes from 0 to 1. The system guarantees to be stable when the values of α_S, α_{SL} are restricted to the stable region. It is seen from Figure 5 that the system stays stable much more often than instable, and generally speaking, the system is instable when α_S is greater than 0.6 and α_{SL} is between 0 and 0.3.

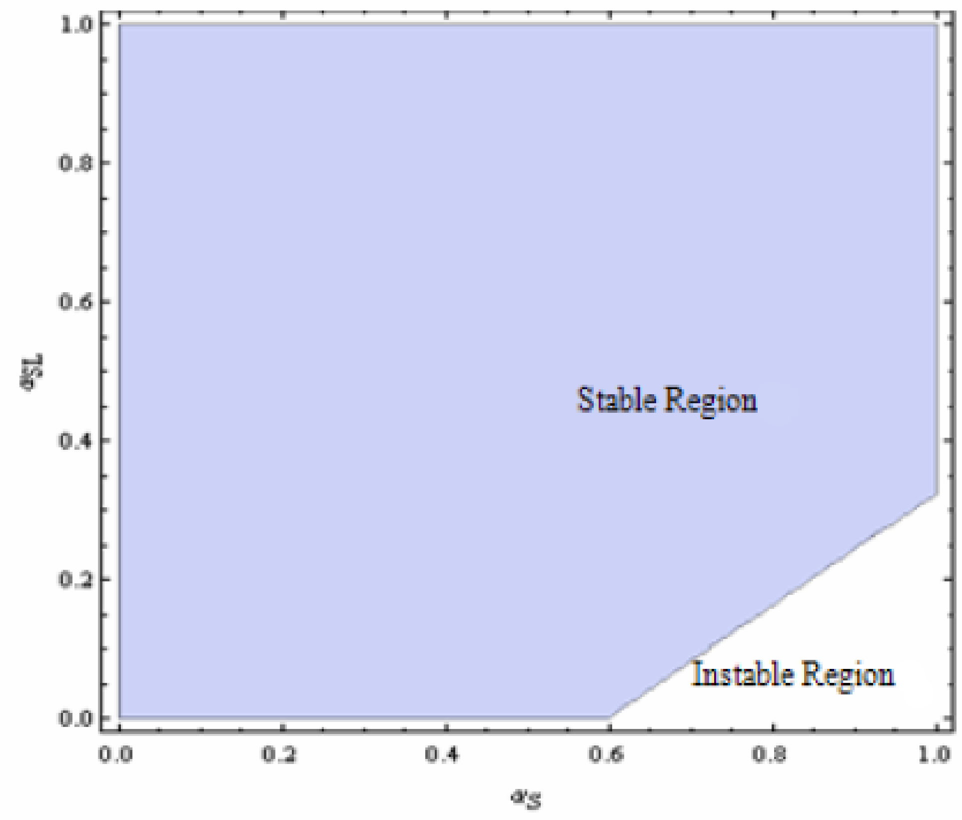

Figure 5. Stability and instability regions for (α_S, α_{SL})

We choose to examine bullwhip effect in the stable and instable region, respectively. To find the numerical results of bullwhip effect calculated in (20), we rely on a simulation approach. In particular, we assume that the customer demand follows a normal distribution with an average of 10 units and standard deviation of 3 units; we simulate each scenario 100 times and calculate the average values of the simulation results; and the initial inventory is set to 8 units and each simulation run is 1000-period long.

We first list the values of bullwhip effect calculated by Equation (20) in the instable region in Table 1. It can be observed from Table 1 that when α_{SL} is fixed, bullwhip effect increases with the other parameter, α_S; in contrast, the effect decreases with the increase of α_{SL}. In particular, when $(\alpha_S, \alpha_{SL}) = (0.65, 0.21)$, bullwhip effect in the instable region reduces to minimum. Hence, based on this simulation study, we conclude that adjusting the parameter α_{SL} can more effectively decrease bullwhip effect.

Next, the values of bullwhip effect in the stable region are reported in Table 2. Notice that when $0 \le \alpha_S \le 0.1$, $0 \le \alpha_{SL} \le 0.3$, the value of the order variance is less than the demand variance, indicating that bullwhip effect is not present. Moreover, three other observations can be made: 1) bullwhip effect increases with one parameter when the other is fixed; 2) when α_S is between 0.3 and 0.6, bullwhip effect starts to increase; and 3) when α_S is larger than α_{SL}, bullwhip effect is relatively small. From a managerial perspective, this simulation study implies that if the retailer pays more attention to the inventory level

TABLE 1. Bullwhip effect in the instable range

α_S \ α_{SL}	0.00	0.03	0.06	0.09	0.12	0.15	0.18	0.21
0.65	5.87×10^2	1.23×10^2	49.55	31.55	24.80	20.30	18.05	16.10
0.70	1.37×10^4	1.21×10^3	187.18	66.01	41.20	29.92	24.93	21.26
0.75	5.30×10^6	3.50×10^4	2.40×10^3	3.20×10^2	120.85	61.40	36.50	28.85
0.80	1.51×10^7	1.18×10^6	6.64×10^4	4.75×10^3	755.98	177.68	66.87	44.75
0.85	4.65×10^8	3.30×10^7	2.38×10^6	1.70×10^5	1.29×10^4	1.26×10^3	219×10^2	89.70
0.90	1.11×10^{10}	7.39×10^8	6.65×10^7	5.31×10^6	3.44×10^5	2.47×10^4	2.04×10^3	3.32×10^2
0.95	1.91×10^{11}	1.70×10^{10}	1.57×10^9	1.14×10^8	7.68×10^7	6.00×10^6	4.32×10^4	3.60×10^3
1.00	3.48×10^{12}	4.10×10^{11}	3.25×10^{10}	2.26×10^9	2.02×10^8	1.69×10^7	1.13×10^6	8.08×10^4

TABLE 2. Bullwhip effect in the stable range

α_S \ α_{SL}	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.34	0.50	0.70	0.96	1.31	1.79	2.53	3.77	6.27	13.77	771.94
0.1	0.87	1.05	1.28	1.57	1.96	2.49	3.26	4.46	6.59	11.32	29.94
0.2	1.77	1.87	2.06	2.35	2.74	3.29	4.07	5.24	7.16	10.75	19.72
0.3	3.34	3.13	3.17	3.37	3.73	4.26	5.04	6.17	7.94	10.93	16.95
0.4	6.47	5.18	4.78	4.76	4.99	5.46	6.19	7.28	8.92	11.54	16.18
0.5	15.09	8.96	7.26	6.70	6.65	6.96	7.59	8.59	10.11	12.46	16.32
0.6	125.44	17.95	11.56	9.58	8.91	8.88	9.31	10.18	11.56	13.69	17.04

than to the lead-time demand, bullwhip effect can be reduced. However, in an extreme case where the inventory level is perfect ($\alpha_{SL} = 1$) and the lead-time demand is zero ($\alpha_S = 0$), bullwhip effect ($= 771.94$) can be very large. This implies that the stability of system cannot represent low bullwhip effect.

Comparing Table 1 with Table 2, we can observe that bullwhip effect is still present when the system is stable, but the effect is much smaller than that when the system is instable. To avoid system instability and large bullwhip effect, decision makers should carefully select the adjustment parameters for inventory discrepancy and supply line discrepancy, such that they are more comparable.

5. **Analysis of the Total Costs.** In this section, we analyze and discuss the impacts of the decision parameters in the forecasting method and the retailer's order policy on supply chain's total costs. The following numerical example is used. Assumptive customer demand, the simulation run and the number of simulation is the same as in Section 4. The retailer's unit ordering cost is $C_0 = \$100$; unit holding cost per period is $C_h = \$10$, and unit backlog cost is $C_b = \$50$. The sum of these costs yields the total cost per period as follows:

$$TC_t = \begin{cases} C_0 * O_t + C_h * S_t, & S_t \geq 0 \\ C_0 * O_t + C_b * (-S_t), & S_t < 0 \end{cases} \tag{23}$$

Hence, the supply chain's total costs in a 1000-period simulation run are calculated as $TC = \sum_{t=1}^{1000} TC_t$.

5.1. **Sensitive analysis of individual decision parameter.** The total costs are plotted in Figure 6, which shows how each parameter affects the total costs. For all three cases, the minimum cost is almost the same, but the impact of the inventory adjustment parameter on the maximum cost is most significant, followed by the forecasting parameter, and finally the supply line adjustment parameter. Specifically, the following three facts can be revealed: (1) when $\alpha_S = 0.75$, total supply chain cost reaches the minimum ($TC = \$5.26 \times 10^5$); (2) if the parameter α_S is fixed, then the larger the parameter α_{SL}, the higher the total costs; and (3) when $\theta = 0.25$, the total cost is optimized at ($TC = \$1.40 \times 10^6$).

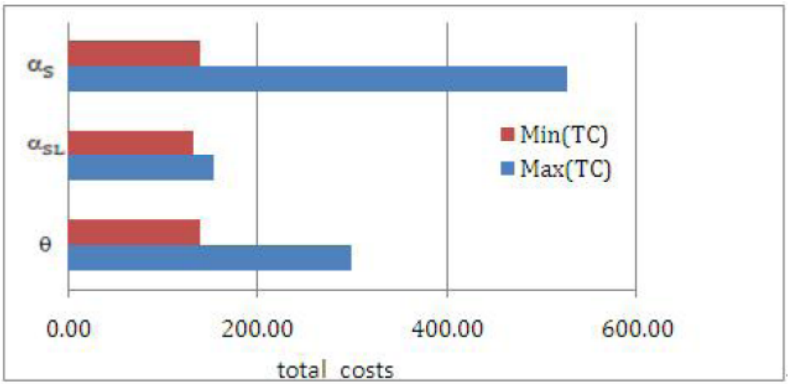

FIGURE 6. Impact of individual parameter on the total costs (in \$10,000)

5.2. **Sensitive analysis of the joint impacts of the decision parameters.** The total costs with various (θ, α_S) are summarized in Table 3, which shows that the total costs tend to be high when the values of α_S and θ go opposite directions. Practically, this means that if the retailer is fully concerned about the demand forecast of the last period but ignores the actual customer demand, then the resulted total cost can be very large. This confirms the importance of information sharing and the value of obtaining real-time data for actual customer demand. It can be also observed that under small α_S, the total costs decrease first and then increase with θ; but under large α_S, the total costs always increase as θ goes up. Putting this into perspective, we see that if the retailer places extra emphasis on forecast demand, he must choose larger values for the adjustment parameters for both inventory and supply line to reduce the total costs; on the other hand, if the retailer assumes that the next period's demand is the same as before, then he must choose small values of the two adjustment parameters in order to reduce the total costs.

Table 4 presents the impacts of (α_S, α_{SL}) on the total costs when $\theta = 0.25$. The total costs in the column with $\alpha_{SL} = 0$ are much lower than those in the row with $\alpha_S = 0$, which implies that it is more important for the retailer to adjust the inventory level than the supply line for the purpose of cost reduction. It is also seen that as α_S increases from 0.1 to 0.7, the total costs are relatively small if α_{SL} is fairly small (except that $\alpha_{SL} = 0$), but as α_S continues to increase till 1.0, the total costs are no longer small even if α_{SL} is small. For the retailer, the best decision for (α_S, α_{SL}) is to choose small values such that $(\alpha_S > \alpha_{SL})$ to keep the total costs low.

The above studies and results help identify the impacts of the parameters on the total costs and suggest the following three important decision guidelines for: 1) the retailer should pay more attention to the inventory adjustment parameter and keep its value large; 2) since either a too small or a too large supply line adjustment, α_{SL}, will induce high costs, it is better to ensure α_S to be slightly larger than α_{SL}; and 3) although the forecasting parameter has no impact on the supply chain's stability, it plays a great role in the total costs. In sum, a reasonable choice of decision parameters not only makes the system stable, but also reduces bullwhip effect and lowers the total costs in the supply chain.

6. **Conclusions.** This paper focuses on analyzing the stability of a two-stage supply chain controlled by the APVIOBPCS order policy and investigates the effects of decision parameters on bullwhip effect and total costs. Three interesting results are obtained that have important implications for practices. First, an approximate relationship between the number of stable points of the system and lead time under a given range of (α_S, α_{SL}) is found. Second, it is obtained a stable system does not necessarily have low bullwhip effect. Finally, the key parameters affecting the total costs of the supply chain under study are found.

There are a few limitations in this study. First, we considered the impact of only the order decision parameters and the lead time on the stability of the subsystems. The future research step is to study the relationship between the overall supply chain system and the subsystems. Second, we examined only deterministic replenishment lead time and ignored the delays and uncertainties in other areas such as transportation and decision making. Third, other decision variables such as the forecasting method and the replenishment

TABLE 3. Total costs (in \$100.000) versus (θ, α_S)

α_S \ θ	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	2500.30	70.50	45.90	37.70	33.50	31.10	29.40	28.20	27.30	26.60	25.90
0.1	69.27	14.28	14.15	14.11	14.09	14.08	14.07	14.07	14.07	14.08	14.10
0.2	39.28	14.12	14.09	14.08	14.08	14.08	14.08	14.09	14.12	14.16	14.22
0.3	27.22	14.06	14.07	14.07	14.08	14.09	14.11	14.15	14.20	14.27	14.35
0.4	19.69	14.04	14.06	14.07	14.08	14.11	14.16	14.22	14.29	14.38	14.49
0.5	14.72	14.03	14.05	14.07	14.10	14.15	14.21	14.29	14.38	14.49	14.63
0.6	12.48	14.03	14.05	14.08	14.12	14.18	14.26	14.36	14.48	14.61	14.77
0.7	12.01	14.03	14.06	14.09	14.15	14.23	14.32	14.44	14.57	14.73	14.90
0.8	12.29	14.04	14.07	14.11	14.18	14.27	14.38	14.52	14.67	14.84	15.04
0.9	12.84	14.05	14.08	14.13	14.21	14.32	14.45	14.59	14.76	14.96	15.17
1.0	13.48	14.06	14.09	14.16	14.25	14.37	14.51	14.68	14.86	15.07	15.30

TABLE 4. Total costs (in \$100,000) at different values of (α_S, α_{SL})

α_S \ α_{SL}	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	25.95	49.62	79.82	102.56	120.27	134.46	146.16	156.24	165.70	176.12	191.23
0.1	13.07	14.05	15.03	16.00	16.98	17.96	18.96	19.99	21.09	22.30	23.69
0.2	13.08	13.57	14.06	14.56	15.06	15.56	16.08	16.62	17.18	17.79	18.48
0.3	13.09	13.42	13.75	14.08	14.42	14.76	15.11	15.48	15.86	16.27	16.72
0.4	13.15	13.37	13.61	13.86	14.11	14.37	14.64	14.91	15.20	15.51	15.85
0.5	13.27	13.39	13.56	13.75	13.94	14.15	14.36	14.58	14.82	15.06	15.33
0.6	13.43	13.46	13.56	13.69	13.85	14.01	14.19	14.37	14.56	14.77	14.99
0.7	13.63	13.58	13.61	13.69	13.80	13.94	14.08	14.23	14.39	14.57	14.75
0.8	13.87	13.73	13.69	13.73	13.79	13.89	14.01	14.14	14.28	14.42	14.58
0.9	14.13	13.91	13.81	13.79	13.82	13.89	13.97	14.08	14.19	14.32	14.46
1.0	14.39	14.10	13.95	13.89	13.88	13.90	13.96	14.04	14.14	14.25	14.37

policies and their impacts on the stability, bullwhip effect and supply chain costs can be also of interest for future research efforts.

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