International Journal of Innovative Computing, Information and Control
Volume **8**, Number **6**, June **2012**

ICIC International ©2012 ISSN 1349-4198
pp. 4015–4024

FURTHER RESULTS ON THE EXPONENTIAL STABILITY CRITERIA FOR TIME DELAY SINGULAR SYSTEMS WITH DELAY-DEPENDENCE

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Received February 2011; revised June 2011

Abstract. *The problem of delay-dependent exponential stability is investigated for singular systems with state delay. In terms of linear matrix inequality (LMI) approach, some improved delay-dependent conditions are presented to ensure the considered system to be regular, impulse free and exponentially stable via an augmented Lyapunov functional and integral inequalities matrix. Numerical examples are given to illustrate the effectiveness and the benefits of the proposed methods. These results are shown to be less conservative than those reported in the literature.*

Keywords: Singular systems, Time-delay systems, Delay-dependent, Exponential stability, Linear matrix inequality (LMI)

1. **Introduction.** Time delay is commonly encountered in various engineering systems, such as manufacturing system, turbojet engine, telecommunication, economic system and chemical engineering system. It is generally regarded as a main source of instability and poor performance. Therefore, the study of stability problem for time-delay systems is of theoretical and practical importance [1-26]. Singular systems, which are also known as descriptor systems, semi-state-space systems and generalized state-space systems are dynamic systems whose behaviors are described by both differential equations (or difference equations) and algebraic equations. Recently, there has been a growing interest in the study of such more general class of delay singular systems [1-3,5,9,10,14,22,25], and singularly perturbed systems [20,21,24] and the references therein. The existing stability criteria for singular time-delay systems can be classified into two types: delay-independent [25] and delay-dependent [2,3,6-9,19,22,23,26]. Generally, delay-dependent conditions are less conservative than the delay-dependent ones, especially when the time delay is small. It should be pointed out that the stability problem for singular systems is much more complicated than that for regular systems because it requires to consider not only stability, but also regularity and absence of impulses (for continuous singular systems) or causality (for discrete singular systems) simultaneously, while the latter two do not arise in the regular ones [5].

All of the above-mentioned stability conditions for time-delay systems are concerned with asymptotic stability instead of exponential ones. But it is very important to estimate the decay rates (i.e., exponential stability degrees) of time-delay systems in many dynamical systems. The issue of exponential stability for delay systems has received considerable attention in recent years. For example, based on the concept of matrix measure, decay rate estimates were investigated in [16], but these conditions are difficult to test. The delay-dependent stability problem was considered in [11-13,18] via linear matrix inequality (LMI) approach and several stability conditions were established. To the best of our knowledge, the problem of delay-dependent exponential stability for singular delay systems has not been fully studied in the literature and still remains open. Motivated by the above-mentioned analysis, in this paper, by using integral inequality matrix, a new delay-dependent criterion for the time-delay singular system to be admissible is established.

In this paper, we revisit the problem of exponential stability analysis for time delay singular system and find the results in [2,3,6-9,19,22,23,26] leave much room for improvement. Based on the fact that such stability conditions are derived via the Lyapunov-Krasovskii functional combining with LMI techniques, simple and delay-dependent exponential stability criteria are derived. The LMI optimization approaches are used to obtain a sufficient condition that is very easy to be checked by using the LMI Toolbox in Matlab. From the illustrated examples, if the delay time lengthens, the decay rate becomes conservative. We claim that the sharpness of the upper bound delay time h varies with the chosen decay rate α. The stability conditions obtained are dependent on the delay values, and are generally less restrictive than those previously presented in the literature.

2. **Stability Analysis.** Consider the following continuous-time singular system with time delay in the state

$$E\dot{x}(t) = Ax(t) + Bx(t-h) \quad t > 0 \tag{1a}$$

with the initial condition

$$x(t_0 + \theta) = \Phi(\theta), \quad -h \le \theta \le 0, \quad h > 0, \tag{1b}$$

where $x(t) \in R^n$ is the state vector of the system; A, $B \in R^{n\times n}$ are constant matrices. The matrix $E \in R^{n\times n}$ maybe singular, without loss generality, we suppose $rankE = r \le n$; $h > 0$ denotes time delay. $\Phi(\theta)$ is a continuous vector-valued initial function. Many papers provide delay-dependent criteria to evaluate the allowable delay magnitude for the asymptotic stability of time delay singular systems (1). When the time delay is unknown, how long time delay can be tolerated to keep the system stable. To do this, one fundamental lemma is reviewed.

The main objective is to find the range of h and guarantee stability for the time delay singular systems (1). Here, definitions and fundamental lemmas are reviewed.

Definition 2.1. *[5] The pair (E, A) is said to regular if $\det(sE - I)$ is not identically zero.*

Definition 2.2. *[5] The pair (E, A) is said to be impulse free if $\deg(\det(sE - I)) =$ rank E.*

Definition 2.3. *For a given scalar $\overline{h} > 0$, the singular delay system (1) is said to be regular and impulse free for any constant time delay h satisfying $0 \le h \le \overline{h}$, it the pairs (E, A) and $(E, A + B)$ are regular and impulse free.*

Remark 2.1. *The regularity and the absence of impulses of the pair (E, A) ensures the system (1) with time delay $h \ne 0$ to be regular and impulse free, while the fact that the pair $(E, A + B)$ is regular and impulse free ensures the system (1) with time delay $h = 0$ to be regular and impulse free.*

Lemma 2.1. *[10] The singular system $E\dot{x}(t) = Ax(t)$ is regular, impulse free, and stable, if and only if there exists a matrix P such that*

$$P^T E = E^T P \ge 0 \tag{2a}$$

$$A^T P + PA < 0 \tag{2b}$$

Lemma 2.2. *[13] For any positive semi-definite matrices*

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \geq 0 \tag{3a}$$

the following integral inequality holds

$$-\int_{t-h}^{t} \dot{x}^T(s)\, X_{33}\, \dot{x}(s) ds \leq$$
$$\int_{t-h}^{t} \begin{bmatrix} x^T(t) & x^T(t-h) & \dot{x}^T(s) \end{bmatrix} \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \\ \dot{x}(s) \end{bmatrix} ds \tag{3b}$$

We now present a delay-dependent criterion for asymptotic stability of the systems (1).

Lemma 2.3. *[4] The following matrix inequality*

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} < 0 \tag{4a}$$

where $Q(x) = Q^T(x)$, $R(x) = R^T(x)$ and $S(x)$ depend affinely on x, is equivalent to

$$R(x) < 0 \tag{4b}$$

$$Q(x) < 0 \tag{4c}$$

and

$$Q(x) - S(x)\, R^{-1}(x)\, S^T(x) < 0 \tag{4d}$$

Theorem 2.1. *For a give scalar $h > 0$, the time-delay singular system (1) is regular, impulse free and asymptotically stable if there exist positive-definite symmetric matrices $P = P^T > 0$, $Q = Q^T > 0$, $R = R^T > 0$, and matrix S of appropriate dimensions and a positive semi-definite matrix $X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \geq 0$ such that the following LMIs hold:*

$$P^T E = E^T P \geq 0 \tag{5a}$$

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ \Omega_{12}^T & \Omega_{22} & \Omega_{23} \\ \Omega_{13}^T & \Omega_{23}^T & \Omega_{33} \end{bmatrix} < 0 \tag{5b}$$

and

$$E^T(R - X_{33})E > 0 \tag{5c}$$

where $Z \in R^{n\times(n-r)}$ is any matrix satisfying $E^T Z = 0$ and

$$\begin{aligned}
&\Omega_{11} = A^T P + PA + A^T Z S^T + S Z^T A + Q + E^T(hX_{11} + X_{13} + X_{13}^T)E, \\
&\Omega_{12} = PA_1 + SZ^T B + E^T(hX_{12} - X_{13} + X_{23}^T)E, \\
&\Omega_{13} = hA^T R, \quad \Omega_{22} = -Q + E^T(hX_{22} - X_{23} - X_{23}^T)E, \\
&\Omega_{23} = hB^T R, \quad \Omega_{33} = -hR.
\end{aligned}$$

Based on that, a convex optimization problem is formulated to find the bound on the allowable delay time h which maintains the delay-dependent stability of the time delay singular system (1).

Proof: Consider the time-delay singular system (1), using the Lyapunov-Krasovskii functional candidate in the following form, we can write

$$V(x_t) = x^T(t)PEx(t) + \int_{t-h}^{t} x^T(s)Qx(s)ds + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)E^TRE\dot{x}(s)dsd\theta \tag{6}$$

The time derivative of (6) along the trajectory of (1) is given by

$$\begin{aligned}\dot{V}(x_t) =& x^T(t)(PA + A^TP)x(t) + x^T(t)PBx(t-h) + x^T(t-h)B^TPx(t) + x^T(t)Qx(t) \\ & - x^T(t-h)Qx(t-h) + \dot{x}^T(t)hE^TRE\dot{x}(t) - \int_{t-h}^{t} \dot{x}^T(s)E^TRE\dot{x}(s)ds \\ =& x^T(t)(PA + A^TP + Q)x(t) + x^T(t)PBx(t-h) + x^T(t-h)B^TPx(t) \\ & - x^T(t-h)Qx(t-h) + \dot{x}^T(t)hR\dot{x}(t) - \int_{t-h}^{t} \dot{x}^T(s)E^T(R - X_{33})E\dot{x}(s)ds \\ & - \int_{t-h}^{t} \dot{x}^T(s)E^TX_{33}E\dot{x}(s)ds\end{aligned} \tag{7}$$

Using the Leibniz-Newton formula $x(t) - x(t-h) = \int_{t-h}^{t} \dot{x}(s)ds$, and Lemma 2.2, we obtain

$$\begin{aligned}& - \int_{t-h}^{t} \dot{x}^T(s)X_{33}\dot{x}(s)ds \\ \leq & \int_{t-h}^{t} \begin{bmatrix} x^T(t) & x^T(t-h) & \dot{x}^T(s) \end{bmatrix} \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \\ \dot{x}(s) \end{bmatrix} ds \\ \leq & x^T(t)hX_{11}x(t) + x^T(t)hX_{12}x(t-h) + x^T(t)X_{13}\int_{t-h}^{t} \dot{x}(s)ds + x^T(t \\ & - h)hX_{12}^Tx(t) + x^T(t-h)hX_{22}x(t-h) + x^T(t-h)X_{23}\int_{t-h}^{t} \dot{x}(s)ds \\ & + \int_{t-h}^{t} \dot{x}^T(s)dsX_{13}^Tx(t) + \int_{t-h}^{t} \dot{x}^T(s)dsX_{23}^Tx(t-h) \\ = & x^T(t)[hX_{11} + X_{13}^T + X_{13}]x(t) + x^T(t)[hX_{12} - X_{13} + X_{23}^T]x(t-h) \\ & + x^T(t-h)[hX_{12}^T - X_{13}^T + X_{23}]x(t) + x^T(t-h)[hX_{22} - X_{23} - X_{23}^T]x(t-h)\end{aligned} \tag{8}$$

Furthermore, noting $E^TZ = 0$, we can deduce

$$0 = 2\dot{x}^T(t)E^TZS^Tx(t) \tag{9}$$

Substituting the above Equations (8) and (9) into (7), we obtain

$$\dot{V}(x_t) < \xi^T(t)\Xi\xi(t) - \int_{t-h}^{t} \dot{x}^T(s)E^T(R - X_{33})E\dot{x}(s)ds \tag{10}$$

where $\xi^T(t) = \begin{bmatrix} x^T(t) & x^T(t-h) \end{bmatrix}$ and $\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{12}^T & \Xi_{22} \end{bmatrix}$, with

$$\begin{aligned}\Xi_{11} &= A^TP + PA + A^TZS^T + SZ^TA + Q + E^T(hX_{11} + X_{13} + X_{13}^T)E + hA^TRA, \\ \Xi_{12} &= PB + SZ^TB + E^T(hX_{12} - X_{13} + X_{23}^T)E + hA^TRB, \\ \Xi_{22} &= -Q + E^T(hX_{22} - X_{23} - X_{23}^T)E + hB^TRB.\end{aligned}$$

Finally, using the Schur complement of Lemma 2.3, with some effort we can show that (10) guarantees of $\dot{V}(x_t)$ <0. In condition (5) of the present Theorem 2.1 is satisfied, if $\dot{V}(x_t)$ <0, then $\Xi < 0$ and $E^T(R - X_{33})E \geq 0$, if and only if (5) holds. Therefore, the time delay singular system (1) is asymptotically stable.

3. **Extension to Exponential Stability for Time Delay Systems.**

We now present a delay-dependent criterion for exponential asymptotic stability of the time delay singular systems (1).

Theorem 3.1. *For any given positive scalars $h > 0$ and $\alpha > 0$, the time delay singular system (1) is regular, impulse free and exponential asymptotically stable with decay rate α if there exist symmetry positive-definite matrices $P = P^T > 0$, $Q = Q^T > 0$, $R = R^T > 0$, and matrix S of appropriate dimensions and positive semi-definite matrix $X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \geq 0$ which satisfy the following inequalities:*

$$P^T E = E^T P \geq 0 \tag{11a}$$

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} \\ \Psi_{12}^T & \Psi_{22} & \Psi_{23} \\ \Psi_{13}^T & \Psi_{23}^T & \Psi_{33} \end{bmatrix} < 0 \tag{11b}$$

and

$$E^T(R - X_{33})E \geq 0 \tag{11c}$$

where Z follows the same definition as that in Theorem 2.1, and

$$\begin{aligned}
\Psi_{11} = &(A + 0.5\alpha E)^T P + P(A + 0.5\alpha E) + A^T Z S^T + S Z^T A \\
&+ Q + e^{-\alpha h} E^T (hX_{11} + X_{13} + X_{13}^T)E, \\
\Psi_{12} = &PB + SZ^T A + e^{-\alpha h} E^T (hX_{12} - X_{13} + X_{23}^T)E, \quad \Psi_{13} = hA^T R, \\
\Psi_{22} = &e^{-\alpha h}[E^T (hX_{22} - X_{23} - X_{23}^T)E - Q], \quad \Psi_{23} = hB^T R, \quad \Omega_{33} = -hR.
\end{aligned}$$

Based on that, a convex optimization problem is formulated to find the bound on the allowable delay time h and delay decay rate α which maintains the delay- dependent stability of the time delay singular system (1).

Proof: Consider the time-delay singular system (1), using the Lyapunov-Krasovskii functional candidate in the following form, we can write

$$V(x_t) = e^{\alpha t} x^T(t) P E x(t) + \int_{t-h}^{t} e^{\alpha s} x^T(s) Q x(s) ds + \int_{-h}^{0} \int_{t+\theta}^{t} e^{\alpha s} \dot{x}^T(s) E^T R E \dot{x}(s) ds d\theta \tag{12}$$

The time derivative of (12) along the trajectory of (1) is given by

$$\begin{aligned}
\dot{V}(x_t) = &e^{\alpha t}\{x^T(t)\alpha E P x(t) + \dot{x}^T(t) P x(t) + x^T(t) P \dot{x}(t) + x^T(t) Q x(t) - x^T(t \\
&- h) e^{-\alpha h} Q x(t-h) + \dot{x}^T(t) h E^T R E \dot{x}(t) - \int_{t-h}^{t} e^{\alpha(s-t)} \dot{x}^T(s) E^T R E \dot{x}(s) ds\} \\
= &e^{\alpha t}\{x^T(t)[P(A + 0.5\alpha E) + (A + 0.5\alpha E)^T P] x(t) + x^T(t) P B x(t-h) \\
&+ x^T(t-h) B^T P x(t) + x^T(t) Q x(t) - x^T(t-h) e^{-\alpha h} Q x(t-h) \\
&+ \dot{x}^T(t) h E^T R E \dot{x}(t) - \int_{t-h}^{t} e^{\alpha(s-t)} \dot{x}^T(s) E^T R E \dot{x}(s) ds\}
\end{aligned} \tag{13}$$

Obviously, for any a scalar $s \in [t-h, t]$, we have $e^{-\alpha h} \le e^{\alpha(s-t)} \le 1$, and

$$-\int_{t-h}^{t} e^{\alpha(s-t)}\dot{x}^T(s)R\dot{x}(s)ds \le -e^{-\alpha h}\int_{t-h}^{t}\dot{x}^T(s)R\dot{x}(s)ds \tag{14}$$

Applying the proof of Theorem 2.1, we obtain

$$\dot{V}(x_t) < e^{\alpha t}\{\xi^T(t)\Upsilon\xi(t) - \int_{t-h}^{t}\dot{x}^T(s)e^{-\alpha h}E^T(R-X_{33})E\dot{x}(s)ds\} \tag{15}$$

where $\xi^T(t) = \begin{bmatrix} x^T(t) & x^T(t-h) \end{bmatrix}$ and $\Upsilon = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} \\ \Upsilon_{12}^T & \Upsilon_{22} \end{bmatrix}$ with

$$\begin{aligned}
\Upsilon_{11} &= (A+0.5\alpha E)^T P + P(A+0.5\alpha E) + A^T Z S^T + S Z^T A + Q \\
&\quad + e^{-\alpha h}E^T(hX_{11} + X_{13} + X_{13}^T)E + hA^T R A, \\
\Upsilon_{12} &= PB + SZ^T B + e^{-\alpha h}E^T(hX_{12} - X_{13} + X_{23}^T)E + hA^T R B, \\
\Upsilon_{22} &= e^{-\alpha h}[E^T(hX_{22} - X_{23} - X_{23}^T)E - Q] + hB^T R B.
\end{aligned}$$

Finally, using the Schur complement of Lemma 2.3, with some effort we can show that (15) guarantees of $\dot{V}(x_t)$ <0. In condition (11) of the present Theorem 3.1 is satisfied, if $\dot{V}(x_t) < 0$, then $\Upsilon < 0$ and $E^T(R - X_{33})E \ge 0$, if and only if (11) holds. Therefore, the time delay singular system (1) is exponential stable.

Remark 3.1. *Theorem 3.1 provides delay-dependent asymptotic stability criteria for the time delay singular systems (1) in terms of solvability of LMIs [4]. Based on them, we can obtain the maximum allowable delay bound (MADB) $\overline{h}$ such that (1) is stable by solving the following convex optimization problem:*

$$\begin{cases} \text{Maximize} & \overline{h} \\ \text{Subject to} & \text{(11) and P} > 0,\ \text{Q} > 0,\ \text{R} > 0,\ \alpha > 0. \end{cases} \tag{16}$$

Inequality (16) is a convex optimization problem and can be obtained efficiently using the MATLAB LMI Toolbox.

4. **Illustrative Examples.**

To show usefulness of our result, let us consider the following numerical examples.

Example 4.1. *Consider the following time delay singular systems*

$$E\dot{x}(t) = Ax(t) + Bx(t-h) \tag{17}$$

where

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\quad A = \begin{bmatrix} 0.5 & 0 \\ 0 & -1 \end{bmatrix},\quad B = \begin{bmatrix} -1.1 & 1 \\ 0 & 0.5 \end{bmatrix}.$$

Now, our problem is to estimate the bound of delay time h to keep the stability of system (17).

Solution: Choosing $Z = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$ and applying the LMI Toolbox in MATLAB (with accuracy 0.01), this above time delay singular system (17) is asymptotically stable for delay time satisfying $h \le 1.0660$. The upper bounds on the time delay form Theorem 2.1 is shown in Table 1, in which "–" means that the results are not applicable to the corresponding cases. Note that the results of [2,3,22] fail to deal with this system since the matrix describing the relationship between the slow and fast variables can not be determined beforehand. For comparison, the table also lists the upper bounds obtained from the criteria in [2,3,6-9,19,22,23,26]. It can be seen that our methods are less conservative. The simulation of the system (17) for $h = 1.06$ is depicted in Figure 1.

TABLE 1. Comparison of delay-dependent stability condition of Example 4.1

Methods	[2,3,22]	[23,26]	[9]	[7]	[6,8]	[19]	Example 4.1
h	–	0.5567	0.8708	0.9091	0.9680	1.0423	1.0660

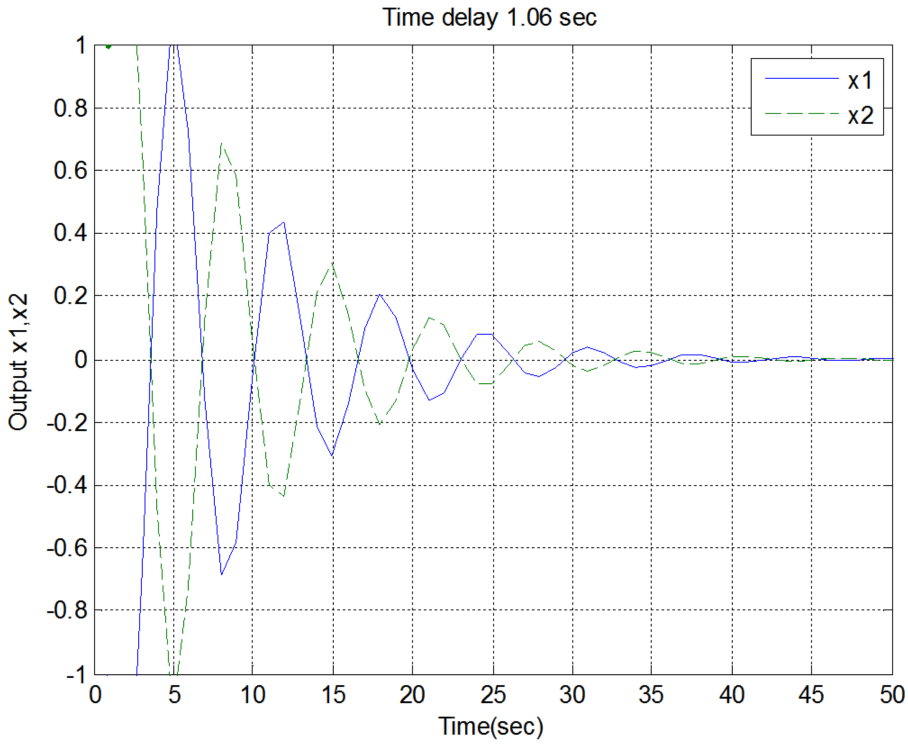

FIGURE 1. The simulation of Example 4.1 for $h = 1.06$ sec

Example 4.2. *Consider the following time delay singular systems*

$$E\dot{x}(t) = Ax(t) + Bx(t-h) \tag{18}$$

where

$$E = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} -2.4 & 2 \\ 0 & 1 \end{bmatrix}.$$

Now, our problem is to estimate the bound of delay time h to keep the stability of system (18).

Solution: Choosing $\alpha = 0$, $Z = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$ and applying the LMI Toolbox in MATLAB (with accuracy 0.01), this above time delay singular system (17) is asymptotically stable for delay time satisfying $h \leq \overline{h} = 0.9897$. Furthermore, by taking the decay rate α, and from Theorem 3.1, we obtain the upper bound of delay time h as shown in Table 2. From the above results of Table 2, if the decay rate α increases the delay time length decreases. As shown in Figure 2, the simulation of system (18) for $h = 0.98$. As the diagram indicates, system (18) would be asymptotically stable if the delay time h is less than 0.98.

Example 4.3. *Consider the following time delay singular systems*

$$E\dot{x}(t) = Ax(t) + Bx(t-h) \tag{19}$$

where

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 & 1 \\ -1 & -1 & -0.1 \\ -1 & 1 & -0.1 \end{bmatrix}.$$

Now, our problem is to estimate the bound of delay time h to keep the stability of system (19).

TABLE 2. Bound of delay time h for various decay rates α

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Theorem 3.1	0.9292	0.8761	0.8278	0.7830	0.7407	0.6999	0.6565	0.5056	0.3847

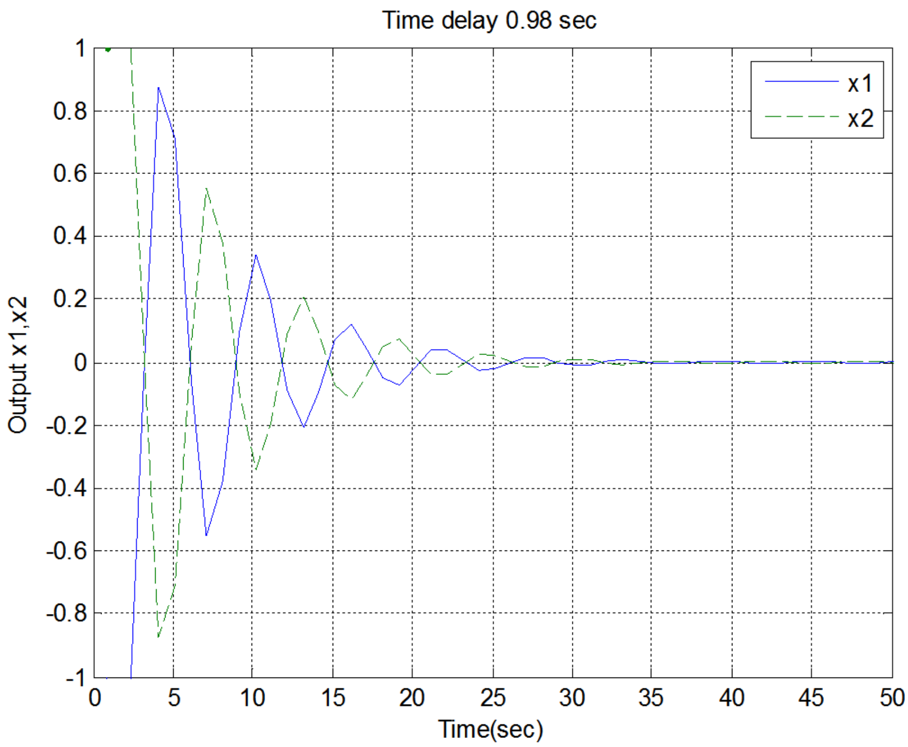

FIGURE 2. The simulation of Example 4.2 for $h = 0.98$ sec

Solution: Choosing $Z = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ and applying the LMI Toolbox in MATLAB (with accuracy 0.01), this above time delay singular system (19) is asymptotically stable for delay time satisfying $h \le 2.3619$. The upper bounds on the time delay form Theorem 2.1 is shown in Table 3. The result in this paper is less conservative.

TABLE 3. Comparison of delay-dependent stability conditions of Example 4.3

Method	Maximum $\bar{h}$ allowed
Gao et al. [9]	1.06
Zhu et al. [26]	1.274
Theorem 2.1	2.3619

5. **Conclusion.** The objective of this paper is to obtain exponential stability condition that provide better insights into the effects of delay terms on the singular system behavior, and to use these conditions and insights in control problems. The proof is based on the Lyapunov-Krasovskii functional techniques, and the conditions are expressed in terms of linear matrix inequality. From the obtained results, if the delay time lengthens, the decay rate becomes conservative. We claim that the sharpness of the upper bound delay time h varies with the chosen the decay rate α. The stability conditions obtained are dependent of the delay values, and are generally less restrictive than those previously presented in the literature. Many complex systems with uncertainties and neutral types, as well as time-varying or state-dependent delays are still inviting further investigation.

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