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FINITE-TIME CONTROL FOR SWITCHED DELAY SYSTEMS VIA DYNAMIC OUTPUT FEEDBACK

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Abstract. *The problem of finite-time control is addressed in this paper for a class of switched delay systems via dynamic output feedback. First, the concepts of finite-time stability and finite-time boundedness are extended to switched delay systems, respectively. Second, by resorting to the average dwell time approach and Lyapunov-Krasovskii functional technique, some new delay-dependent criteria guaranteeing finite-time boundedness and finite-time stability are developed, respectively. An explicit expression for the desired dynamic output feedback controller is also given. Finally, two numerical examples are provided to demonstrate the effectiveness of the proposed results.*
Keywords: Switched systems, Time delay, Finite-time stability, Finite-time boundedness, Dynamic output feedback controller

1. **Introduction.** Finite-time stability is a definition that, given a bound on the initial condition, the system's state does not exceed a certain threshold during a specified time interval. This concept was first introduced to the control field in 1960s [1]. However, due to the lack of operative test conditions for finite-time stability, the researchers' interest has moved toward the classical Lyapunov stability. Until 1997, with the presentation of the robust finite-time stability problem via linear matrix inequality method, the concept of finite-time stability was revisited [2]. Subsequently, the definition of finite-time stability was generalized to the concept of finite-time boundedness in [3]. Since then, the problems of finite-time stability and finite-time boundedness have been extensively discussed. For instance, finite-time control problem was presented in [4] for linear systems subject to time-varying parametric uncertainties and exogenous constant disturbances. The finite-time stabilization problem was studied for continuous-time linear systems in [5] and discrete-time linear systems in [6], respectively. Now, for various linear systems, the problems of finite-time stability and finite-time boundedness have been further considered. In [7, 8], the finite-time stability analysis was studied for a class of linear singular system and linear time-invariant impulsive systems, respectively. In addition, some results of finite-time stability were presented in [9, 10], which were different from that in this paper and [2, 3, 4, 5, 6, 7, 8].

It should be pointed out that all aforementioned results about finite-time stability and finite-time boundedness focus mainly on non-switched systems. As is well known, switched systems are an important class of hybrid systems, and many real world processes and systems can be modeled as switched systems [11, 12]. Although most switched systems must operate satisfactorily over arbitrarily large intervals of time [13, 14, 15, 16, 17, 18], some systems are required to operate satisfactorily only over fixed time intervals of time.

For example, in order to accomplish a set of experiments, a space vehicle should be guaranteed to remain in a specified orbit for a given length of time. In a chemical process, the temperature, pressure or some other parameters should be kept within a specified bound in a prescribed time interval. For these situations, the only meaningful concept of stability is finite-time stability. And many of these practical problems finally boil down to the finite-time control problem for switched systems, which has inspired some researchers to study the problem of finite-time stability for switched systems. For example, in [19], finite-time stability and stabilization problems were discussed for a class of continuous-time switched linear systems.

It is well known that time delay is the inherent feature of many physical processes, which may degrade the system performance, cause oscillation, and lead to instability. In view of the strong engineering background, switched systems with time delay have attracted special attention during the past decade. Some useful results have been reported in the literature, see, e.g., [20, 21, 22, 23] and the references therein. Up to date, to the best of the authors' knowledge, the problems of finite-time stability and finite-time boundedness for switched delay systems have not been fully investigated, which motivates us to carry out the present study.

In this paper, attention is focused on solving the finite-time control problem for switched delay system via dynamic output feedback. First, the definitions of finite-time stability and finite-time boundedness are extended to switched delay systems, respectively. Second, by resorting to the average dwell time approach and Lyapunov-Krasovskii functional technique, some new delay-dependent criteria guaranteeing finite-time boundedness and finite-time stability for switched delay systems are developed. By virtue of linear matrix inequality approach, the desired dynamic output feedback controller is also given. Finally, two numerical examples are proposed to demonstrate the effectiveness of the obtained results.

2. Problem Formulation and Preliminaries.

Consider the following switched delay system

$$\dot{x}(t) = A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-\tau) + B_{\sigma(t)}u(t) + E_{\sigma(t)}\omega(t), \tag{1a}$$

$$y(t) = C_{\sigma(t)}x(t) + C_{d\sigma(t)}x(t-\tau) + F_{\sigma(t)}\omega(t), \tag{1b}$$

$$\dot{\omega}(t) = G_{\sigma(t)}\omega(t), \tag{1c}$$

$$x(s) = \phi(s), \quad s \in [-\tau, 0],$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the controlled input, $y(t) \in \mathbb{R}^q$ is the output, $\omega(t) \in \mathbb{R}^p$ is the disturbance generated by the exogenous system (1c), τ is the time delay, $\phi(s)$ is the initial value function. $\sigma(t) : [0, \infty) \to \mathcal{N} = \{1, 2, \ldots, N\}$ is the switching signal specifying which subsystem activates at a certain time instant. For each $i \in \mathcal{N}$, $A_i \in \mathbb{R}^{n\times n}$, $A_{di} \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $E_i \in \mathbb{R}^{n\times p}$, $C_i \in \mathbb{R}^{q\times n}$, $C_{di} \in \mathbb{R}^{q\times n}$, $F_i \in \mathbb{R}^{q\times p}$, $G_i \in \mathbb{R}^{p\times p}$ are constant matrices of appropriate dimensions.

In this paper, switching sequence is defined as

$$\zeta = \{x_{t_0}; (i_0, t_0), (i_1, t_1), \cdots, (i_m, t_m), \cdots \mid i_m \in \mathcal{N},\ m = 0, 1, 2, \cdots\},$$

where $t_0 < t_1 < \cdots < t_m < \cdots$. When $t \in [t_m, t_{m+1})$, the i_mth subsystem is activated and the states of system (1a) do not jump when switch occurs. Here we assume that $\sigma(t)$ is not known a priori but its instantaneous value is available in real time.

For system (1a) with $u(t) = 0$, $\omega(t) = 0$, we first present the following definition.

Definition 2.1. *System (1a) with $u(t) = 0$, $\omega(t) = 0$ is said to be finite-time stable (FTS) with respect to $(c_1, c_2, T, R_1, \sigma)$, where $c_2 > c_1 > 0$, $T > 0$ is a given time-constant, $R_1 > 0$*

is a positive definite matrix, $\sigma \in \mathcal{N}$, if

$$\max_{-\tau \le s \le 0} \phi^T(s) R_1 \phi(s) \le c_1 \Rightarrow x^T(t) R_1 x(t) < c_2, \quad \forall\, t \in [0, T].$$

Remark 2.1. *If $\tau = 0$, $A_{di} = 0$, $i \in \mathcal{N}$, the above definition of FTS is reduced to the one in* [19].

Remark 2.2. *Unlike Lyapunov asymptotical stability defined on an infinite time interval, finite-time stability emphasizes the behavior of the system over a fixed finite time interval. In fact, Lyapunov asymptotic stability and finite-time stability are independent concepts: finite-time stability does not mean Lyapunov asymptotical stability; conversely a Lyapunov asymptotical stability system could not be finite-time stability if, during the transients, its state exceeds the prescribed bounds* [4, 5, 8]*. In* [24]*, the authors further demonstrated this point using a numerical example (Example 1).*

The general idea of finite-time boundedness presents the boundedness of the state of systems over a finite time interval given both some initial conditions and an external disturbance working on the systems [3]. For non-switched linear system, many papers have studied the problem of finite-time boundedness, such as [4, 5, 6]. In the sequel, we extend this definition to the case of switched delay systems.

Definition 2.2. *System (1a) ($u(t) = 0$) interconnecting with (1c) is said to be finite-time bounded (FTB) with respect to (c_1, c_0, c_2, T, R_1, R_2, σ), where $c_2 > c_1 > 0$, $T > 0$ is a given time-constant, $R_1 > 0$, $R_2 > 0$ are positive definite matrices, $\sigma \in \mathcal{N}$, if*

$$\left.\begin{array}{r} \max\limits_{-\tau \le s \le 0} \phi^T(s) R_1 \phi(s) \le c_1; \\ \omega^T(0) R_2 \omega(0) \le c_0 \end{array}\right\} \Rightarrow x^T(t) R_1 x(t) < c_2, \quad \forall t \in [0, T].$$

The switched dynamic output feedback controller is designed as:

$$\dot{\hat{x}}(t) = A_{c\sigma(t)} \hat{x}(t) + L_{\sigma(t)} y(t), \tag{2a}$$

$$u(t) = C_{c\sigma(t)} \hat{x}(t) + D_{c\sigma(t)} y(t), \tag{2b}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state of the controller, $A_{ci} \in \mathbb{R}^{n\times n}$, $L_i \in \mathbb{R}^{n\times q}$, $C_{ci} \in \mathbb{R}^{m\times n}$, $D_{ci} \in \mathbb{R}^{m\times q}$, $i \in \mathcal{N}$ are the parameter matrices to be determined. The feedback connection between system (1) and controller (2) leads to the following closed-loop system:

$$\dot{\xi}(t) = \bar{A}_{\sigma(t)} \xi(t) + \bar{A}_{d\sigma(t)} \xi(t-\tau) + \bar{E}_{\sigma(t)} \omega(t), \tag{3a}$$

$$\dot{\omega}(t) = G_{\sigma(t)} \omega(t), \tag{3b}$$

where

$$\bar{A}_{\sigma(t)} = \begin{bmatrix} A_{\sigma(t)} + B_{\sigma(t)} D_{c\sigma(t)} C_{\sigma(t)} & B_{\sigma(t)} C_{c\sigma(t)} \\ L_{\sigma(t)} C_{\sigma(t)} & A_{c\sigma(t)} \end{bmatrix},$$

$$\bar{A}_{d\sigma(t)} = \begin{bmatrix} A_{d\sigma(t)} + B_{\sigma(t)} D_{c\sigma(t)} C_{d\sigma(t)} & 0 \\ L_{\sigma(t)} C_{d\sigma(t)} & 0 \end{bmatrix},$$

$$\bar{E}_{\sigma(t)} = \begin{bmatrix} E_{\sigma(t)} + B_{\sigma(t)} D_{c\sigma(t)} F_{\sigma(t)} \\ L_{\sigma(t)} F_{\sigma(t)} \end{bmatrix},$$

$$\xi(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}.$$

Given four positive scalars c_1, c_0, c_2, T, three positive definite symmetric matrices R_1, R_2, R_3, switching signal $\sigma(t) \in \mathcal{N}$, the purpose of this paper is to find a switched dynamic output feedback controller (2) such that system (3) is FTB with respect to (c_1, c_0, c_2, T, $\mathrm{diag}\{R_1, R_3\}$, R_2, σ), and system (3a) is FTS with respect to (c_1, c_2, T, $\mathrm{diag}\{R_1, R_3\}$, σ) when $\omega(t) = 0$.

3. Main Results.

3.1. Finite-time boundedness and stability. In the sequel, the finite-time boundedness ($u(t) = 0$) and finite-time stability ($u(t) = 0$ and $\omega(t) = 0$) will be presented for system (1), respectively.

Theorem 3.1. *Systems (1a) ($u(t) = 0$) and (1c) are FTB with respect to (c_1, c_0, c_2, T, R_1, R_2, σ) if there exist positive scalars α, τ, and positive definite matrices $P_{1,i} \in \mathbb{R}^{n\times n}$, $P_{2,i} \in \mathbb{R}^{n\times n}$ and $P_{3,i} \in \mathbb{R}^{p\times p}$, $i \in \mathcal{N}$ such that the following inequalities hold*

$$\begin{bmatrix} \Omega_{1,i} & P_{1,i}A_{di} & P_{1,i}E_i \\ * & -e^{\alpha\tau}P_{2,i} & 0 \\ * & * & \Omega_{3,i} \end{bmatrix} < 0, \quad i \in \mathcal{N}, \tag{4a}$$

$$\beta \le \mu < \frac{c_2}{c_1 + c_0 + \frac{c_1}{\alpha}(e^{\alpha\tau} - 1)} e^{-\alpha T}, \tag{4b}$$

$$\tau_a > \tau_a^* = \frac{T \ln \mu}{\ln\left(\frac{c_2}{c_1 + c_0 + \frac{c_1}{\alpha}(e^{\alpha\tau}-1)}\right) - \alpha T - \ln \mu}, \tag{4c}$$

where

$$\begin{aligned} \Omega_{1,i} &= P_{1,i}A_i + A_i^T P_{1,i} + P_{2,i} - \alpha P_{1,i}, \quad \Omega_{3,i} = P_{3,i}G_i + G_i^T P_{3,i} - \alpha P_{3,i}, \\ \beta &= \frac{\lambda_2}{\lambda_1}, \\ \lambda_2 &= \max\left\{\max_{\iota\in\mathcal{N}}(\lambda_{\max}(\widetilde{P}_{1,\iota})), \max_{\iota\in\mathcal{N}}(\lambda_{\max}(\widetilde{P}_{2,\iota})), \max_{\iota\in\mathcal{N}}(\lambda_{\max}(\widetilde{P}_{3,\iota}))\right\}, \quad \iota \in \mathcal{N}, \\ \lambda_1 &= \min\left\{\min_{\kappa\in\mathcal{N}}(\lambda_{\min}(\widetilde{P}_{1,\kappa})), \min_{\kappa\in\mathcal{N}}(\lambda_{\min}(\widetilde{P}_{2,\kappa})), \min_{\kappa\in\mathcal{N}}(\lambda_{\min}(\widetilde{P}_{3,\kappa}))\right\}, \quad \kappa \in \mathcal{N}, \end{aligned} \tag{5}$$

and $\widetilde{P}_{1,i} = R_1^{-1/2}P_{1,i}R_1^{-1/2}$, $\widetilde{P}_{2,i} = R_1^{-1/2}P_{2,i}R_1^{-1/2}$, $\widetilde{P}_{3,i} = R_2^{-1/2}P_{3,i}R_2^{-1/2}$.

Proof: Choose a Lyapunov-Krasovskii functional candidate as

$$V(t) = V_{\sigma(t)}(t) = V_{1,\sigma(t)}(t) + V_{2,\sigma(t)}(t) + V_{3,\sigma(t)}(t), \tag{6}$$

where

$$\begin{aligned} &V_{1,\sigma(t)}(t) = x^T(t)P_{1,\sigma(t)}x(t), \quad V_{2,\sigma(t)}(t) = \int_{t-\tau}^{t} e^{\alpha(t-s)}x^T(s)P_{2,\sigma(t)}x(s)ds, \\ &V_{3,\sigma(t)}(t) = \omega^T(t)P_{3,\sigma(t)}\omega(t). \end{aligned}$$

When $t \in [t_m, t_{m+1})$, calculating the derivative of $V(t)$ along the trajectory of system (1a) ($u(t) = 0$) and (1c), we have

$$\begin{aligned} \dot{V}_{1,\sigma(t)}(t) &= \dot{x}^T(t)P_{1,\sigma(t)}x(t) + x^T(t)P_{1,\sigma(t)}\dot{x}(t) \\ &= x^T(t)\left(P_{1,\sigma(t)}A_{\sigma(t)} + A_{\sigma(t)}^T P_{1,\sigma(t)} - \alpha P_{1,\sigma(t)}\right)x(t) \\ &\quad + x^T(t)P_{1,\sigma(t)}A_{d\sigma(t)}x(t-\tau) + x^T(t)P_{1,\sigma(t)}E_{\sigma(t)}\omega(t) \\ &\quad + x^T(t-\tau)A_{d\sigma(t)}^T P_{1,\sigma(t)}x(t) + \omega^T(t)E_{\sigma(t)}^T P_{1,\sigma(t)}x(t) + \alpha V_{1,\sigma(t)}(t), \end{aligned} \tag{7a}$$

$$\dot{V}_{2,\sigma(t)}(t) = x^T(t)P_{2,\sigma(t)}x(t) - e^{\alpha\tau}x^T(t-\tau)P_{2,\sigma(t)}x(t-\tau) + \alpha V_{2,\sigma(t)}(t), \tag{7b}$$

$$\begin{aligned} \dot{V}_{3,\sigma(t)}(t) &= \omega^T(t)\left(G_{\sigma(t)}^T P_{3,\sigma(t)} + P_{3,\sigma(t)}G_{\sigma(t)}\right)\omega(t) - \alpha V_{3,\sigma(t)}(t) + \alpha V_{3,\sigma(t)}(t) \\ &= \omega^T(t)\left(G_{\sigma(t)}^T P_{3,\sigma(t)} + P_{3,\sigma(t)}G_{\sigma(t)} - \alpha P_{3,\sigma(t)}\right)\omega(t) + \alpha V_{3,\sigma(t)}(t). \end{aligned} \tag{7c}$$

In view of (4a) and (7), there holds

$$\dot{V}(t) = \dot{V}_{\sigma(t)}(t) < \alpha V_{\sigma(t)}(t). \tag{8}$$

Note that when $t \in [t_m, t_{m+1})$, $\sigma(t) = \sigma(t_m)$. According to (8), we obtain

$$\dot{V}(t) = \dot{V}_{\sigma(t_m)}(t) < \alpha V_{\sigma(t_m)}(t). \tag{9}$$

Integrating (9) from t_m to t reads

$$V(t) = V_{\sigma(t_m)}(t) < e^{\alpha(t-t_m)} V_{\sigma(t_m)}(t_m). \tag{10}$$

In addition, letting $\sigma(t_m) = \iota$, $\sigma(t_m^-) = \kappa$, $\iota, \kappa \in \mathcal{N}$, and $\iota \neq \kappa$, one gets

$$V_\iota(t_m) \le \lambda_2 \left(x^T(t_m) R_1 x(t_m) + \int_{t_m-\tau}^{t_m} e^{\alpha(t_m-s)} x^T(s) R_1 x(s) ds + \omega^T(t_m) R_2 \omega(t_m) \right). \tag{11}$$

In a similar way

$$V_\kappa(t_m^-) \ge \lambda_1 \left(x^T(t_m) R_1 x(t_m) + \int_{t_m-\tau}^{t_m} e^{\alpha(t_m-s)} x^T(s) R_1 x(s) ds + \omega^T(t_m) R_2 \omega(t_m) \right). \tag{12}$$

Taking (11) and (12) into account, we have

$$V_{\sigma(t_m)}(t_m) \le \frac{\lambda_2}{\lambda_1} V_{\sigma(t_m^-)}(t_m^-) = \beta V_{\sigma(t_m^-)}(t_m^-) \le \mu V_{\sigma(t_m^-)}(t_m^-). \tag{13}$$

This together with (10) leads to

$$V(t) < e^{\alpha(t-t_m)} \mu V_{\sigma(t_m^-)}(t_m^-) = e^{\alpha(t-t_m)} \mu V_{\sigma(t_{m-1})}(t_m). \tag{14}$$

For any $t \in [0, T]$, the following inequality holds

$$\begin{aligned} V(t) &< e^{\alpha(t-t_m)} \mu V_{\sigma(t_{m-1})}(t_m) \le e^{\alpha(t-t_m)} \mu e^{\alpha(t_m - t_{m-1})} V_{\sigma(t_{m-1})}(t_{m-1}) \\ &< e^{\alpha(t-t_{m-1})} \mu^2 V_{\sigma(t_{m-2})}(t_{m-1}) < \cdots < e^{\alpha t} \mu^{N_\sigma(0,t)} V_{\sigma(0)}(0) \\ &\le e^{\alpha t} \mu^{N_\sigma(0,T)} V_{\sigma(0)}(0) \le e^{\alpha T} \mu^{\frac{T}{\tau_a}} V_{\sigma(0)}(0). \end{aligned} \tag{15}$$

By (6), we derive

$$\begin{aligned} V(t) &\ge x^T(t) P_{1,\sigma(t)} x(t) = x^T(t) R_1^{\frac{1}{2}} R_1^{-\frac{1}{2}} P_{1,\sigma(t)} R_1^{-\frac{1}{2}} R_1^{\frac{1}{2}} x(t) \\ &\ge \lambda_{\min} \left(R_1^{-\frac{1}{2}} P_{1,\sigma(t)} R_1^{-\frac{1}{2}} \right) x^T(t) R_1 x(t) \\ &= \lambda_{\min}(\widetilde{P}_{1,\sigma(t)}) x^T(t) R_1 x(t), \end{aligned} \tag{16}$$

and

$$\begin{aligned} V_{\sigma(0)}(0) &= x^T(0) P_{1,\sigma(0)} x(0) + \int_{-\tau}^{0} e^{-\alpha s} x^T(s) P_{2,\sigma(0)} x(s) ds + \omega^T(0) P_{3,\sigma(0)} \omega(0) \\ &\le \max \left\{ \lambda_{\max}(\widetilde{P}_{1,\sigma(0)}), \lambda_{\max}(\widetilde{P}_{3,\sigma(0)}), \lambda_{\max}(\widetilde{P}_{2,\sigma(0)}) \right\} \left(c_1 + c_0 + \frac{c_1}{\alpha}(e^{\alpha\tau} - 1) \right). \end{aligned} \tag{17}$$

Then, from (15), (16) and (17), we obtain

$$x^T(t) R_1 x(t) \le e^{\alpha T} \mu^{\frac{T}{\tau_a}+1} \left(c_1 + c_0 + \frac{c_1}{\alpha}(e^{\alpha\tau} - 1) \right), \tag{18}$$

which, combining (4c), further implies that $x^T(t) R_1 x(t) < c_2$. This completes the proof of Theorem 3.1.

Remark 3.1. *Based on $\beta \leq \mu$, we can know that at the switching point t_m, the Lyapunov-Krasovskii functional satisfies*

$$\frac{V_{\sigma(t_m)}(t_m)}{V_{\sigma(t_m^-)}(t_m)} = \frac{V_{\sigma(t_m)}(t_m)}{V_{\sigma(t_{m-1})}(t_m)} \leq \mu \Leftrightarrow V_{\sigma(t_m)}(t_m) \leq \mu V_{\sigma(t_{m-1})}(t_m),$$

which is just the condition needed using average dwell time approach [16].

When $\omega(t) = 0$, choose a Lyapunov-Krasovskii functional candidate as

$$V(t) = V_{\sigma(t)}(t) = V_{1,\sigma(t)}(t) + V_{2,\sigma(t)}(t). \tag{19}$$

For system (1a) with $u(t) = 0$, $\omega(t) = 0$, we derive the following result.

Theorem 3.2. *System (1a) ($u(t) = 0$, $\omega(t) = 0$) is FTS with respect to (c_1, c_2, T, R_1, σ) if there exist positive scalars α, τ, positive definite matrices $P_{1,i} \in \mathbb{R}^{n\times n}$ and $P_{2,i} \in \mathbb{R}^{n\times n}$, $i \in \mathcal{N}$ such that the following inequalities hold*

$$\begin{bmatrix} \Omega_{1,i} & P_{1,i}A_{di} \\ * & -e^{\alpha\tau}P_{2,i} \end{bmatrix} < 0, \quad i \in \mathcal{N}, \tag{20a}$$

$$\beta \leq \mu < \frac{c_2}{c_1 + \frac{c_1}{\alpha}(e^{\alpha\tau} - 1)} e^{-\alpha T}, \tag{20b}$$

$$\tau_a > \tau_a^* = \frac{T \ln \mu}{\ln\left(\frac{c_2}{c_1 + \frac{c_1}{\alpha}(e^{\alpha\tau}-1)}\right) - \alpha T - \ln \mu}, \tag{20c}$$

where

$$\beta = \frac{\lambda_2}{\lambda_1}, \quad \lambda_2 = \max\left\{\max_{\iota\in\mathcal{N}}(\lambda_{\max}(\widetilde{P}_{1,\iota})), \max_{\iota\in\mathcal{N}}(\lambda_{\max}(\widetilde{P}_{2,\iota}))\right\}, \quad \iota \in \mathcal{N},$$

$$\lambda_1 = \min\left\{\min_{\kappa\in\mathcal{N}}(\lambda_{\min}(\widetilde{P}_{1,\kappa})), \min_{\kappa\in\mathcal{N}}(\lambda_{\min}(\widetilde{P}_{2,\kappa}))\right\}, \quad \kappa \in \mathcal{N}.$$

When $\tau = 0$, $u(t) = 0$, $\omega(t) = 0$, system (1a) becomes

$$\dot{x}(t) = A_{\sigma(t)}x(t), \tag{21}$$

then we can obtain the following result from Theorem 3.2.

Corollary 3.1. *System (21) is FTS with respect to $(c_1, c_2, T, R_1, \sigma)$ if there exist a positive scalar α, and positive definite matrices $P_{1,i} \in \mathbb{R}^{n\times n}$, $i \in \mathcal{N}$ such that*

$$P_{1,i}A_i + A_i^T P_{1,i} - \alpha P_{1,i} < 0, \quad i \in \mathcal{N}, \tag{22a}$$

$$\frac{\max_{\iota\in\mathcal{N}}(\lambda_{\max}(\widetilde{P}_{1,\iota}))}{\min_{\kappa\in\mathcal{N}}(\lambda_{\min}(\widetilde{P}_{1,\kappa}))} \leq \mu < \frac{c_2}{c_1}e^{-\alpha T}, \quad \iota \in \mathcal{N}, \quad \kappa \in \mathcal{N}, \tag{22b}$$

$$\tau_a > \tau_a^* = \frac{T \ln \mu}{\ln \frac{c_2}{c_1} - \alpha T - \ln \mu}. \tag{22c}$$

3.2. **Dynamic output feedback controller design.** Now, we are in a position to present a solution to the problem of finite-time dynamic output feedback control for system (3). By Theorem 3.1, we can derive the following result.

Theorem 3.3. *System (3) is FTB with respect to $(c_1, c_0, c_2, T, diag\{R_1, R_3\}, R_2, \sigma)$ if there exist positive scalars α, τ, positive definite matrices $X_{1,i} \in \mathbb{R}^{n\times n}$, $Y_{1,i} \in \mathbb{R}^{n\times n}$,*

$Q_{2,i} \in \mathbb{R}^{2n\times 2n}$, $P_{3,i} \in \mathbb{R}^{p\times p}$, *and matrices* $N_{1,i} \in \mathbb{R}^{n\times n}$, $\widetilde{A}_i \in \mathbb{R}^{n\times n}$, $\widetilde{B}_i \in \mathbb{R}^{n\times q}$, $\widetilde{C}_i \in \mathbb{R}^{m\times q}$, $\widetilde{D}_i \in \mathbb{R}^{m\times q}$, $i \in \mathcal{N}$ *such that (4c) and the following inequalities hold*

$$\begin{bmatrix} \Psi_{1,i} & \Psi_{2,i} & \Psi_{3,i} & \Pi_{1,i}^T \\ * & -e^{\alpha\tau}Q_{2,i} & 0 & 0 \\ * & * & \Omega_{3,i} & 0 \\ * & * & * & -Q_{2,i}^{-1} \end{bmatrix} < 0, \quad i \in \mathcal{N}, \tag{23a}$$

$$\beta \le \mu < \frac{c_2}{c_1 + c_0 + \frac{c_1}{\alpha}(e^{\alpha\tau} - 1)} e^{-\alpha T}, \tag{23b}$$

where

$$\Psi_{1,i} = \begin{bmatrix} \Psi_{1,i}^1 & \Psi_{1,i}^2 \\ * & \Psi_{1,i}^3 \end{bmatrix}, \quad \Psi_{2,i} = \begin{bmatrix} \Psi_{2,i}^1 & 0 \\ \Psi_{2,i}^2 & 0 \end{bmatrix},$$

$$\Psi_{3,i} = \begin{bmatrix} \Psi_{3,i}^1 \\ \Psi_{3,i}^2 \end{bmatrix}, \quad \Pi_{1,i} = \begin{bmatrix} Y_{1,i} & I \\ N_{1,i}^T & 0 \end{bmatrix}, \quad \beta = \frac{\lambda_2}{\lambda_1},$$

and

$$\begin{aligned}
\Psi_{1,i}^1 &= A_iY_{1,i} + Y_{1,i}A_i^T + B_i\widetilde{C}_i + \widetilde{C}_i^TB_i^T - \alpha Y_{1,i}, \quad \Psi_{1,i}^2 = A_i + B_i\widetilde{D}_iC_i + \widetilde{A}_i^T - \alpha I, \\
\Psi_{1,i}^3 &= X_{1,i}A_i + \widetilde{B}_iC_i + A_i^TX_{1,i} + C_i^T\widetilde{B}_i^T - \alpha X_{1,i}, \quad \Psi_{2,i}^1 = A_{di} + B_i\widetilde{D}_iC_{di}, \\
\Psi_{2,i}^2 &= X_{1,i}A_{di} + \widetilde{B}_iC_{di}, \quad \Psi_{3,i}^1 = E_i + B_i\widetilde{D}_iF_i, \quad \Psi_{3,i}^2 = X_{1,i}E_i + \widetilde{B}_iF_i, \\
\widetilde{A}_i &= X_{1,i}A_iY_{1,i} + X_{1,i}B_iD_{ci}C_iY_{1,i} + M_{1,i}L_iC_iY_{1,i} + X_{1,i}B_iC_{ci}N_{1,i}^T + M_{1,i}A_{ci}N_{1,i}^T, \\
\widetilde{B}_i &= X_{1,i}B_iD_{ci} + M_{1,i}L_i, \quad \widetilde{C}_i = D_{ci}C_iY_{1,i} + C_{ci}N_{1,i}^T, \quad \widetilde{D}_i = D_{ci}, \\
\lambda_2 &= \max\left\{ \max_{\iota\in\mathcal{N}}(\lambda_{\max}(\widetilde{Q}_{1,\iota})), \max_{\iota\in\mathcal{N}}(\lambda_{\max}(\widetilde{Q}_{2,\iota})), \max_{\iota\in\mathcal{N}}(\lambda_{\max}(\widetilde{P}_{3,\iota})) \right\}, \quad \iota \in \mathcal{N}, \\
\lambda_1 &= \min\left\{ \min_{\kappa\in\mathcal{N}}(\lambda_{\min}(\widetilde{Q}_{1,\kappa})), \min_{\kappa\in\mathcal{N}}(\lambda_{\min}(\widetilde{Q}_{2,\kappa})), \min_{\kappa\in\mathcal{N}}(\lambda_{\min}(\widetilde{P}_{3,\kappa})) \right\}, \quad \kappa \in \mathcal{N}, \\
\widetilde{Q}_{1,i} &= diag\left\{R_1^{-1/2}, R_3^{-1/2}\right\} Q_{1,i}\, diag\left\{R_1^{-1/2}, R_3^{-1/2}\right\}, \\
\widetilde{Q}_{2,i} &= diag\left\{R_1^{-1/2}, R_3^{-1/2}\right\} Q_{2,i}\, diag\left\{R_1^{-1/2}, R_3^{-1/2}\right\}.
\end{aligned}$$

Proof: Choose a Lyapunov-Krasovskii functional candidate as

$$\overline{V}(t) = \overline{V}_{\sigma(t)}(t) = \overline{V}_{1,\sigma(t)}(t) + \overline{V}_{2,\sigma(t)}(t) + V_{3,\sigma(t)}(t), \tag{24}$$

where

$$\overline{V}_{1,\sigma(t)}(t) = \xi^T(t)Q_{1,\sigma(t)}\xi(t), \quad \overline{V}_{2,\sigma(t)}(t) = \int_{t-\tau}^{t} e^{\alpha(t-s)}\xi^T(s)Q_{2,\sigma(t)}\xi(s)ds.$$

Replacing A_i, A_{di}, E_i, $P_{1,i}$, $P_{2,i}$ with $\bar{A}_i$, $\bar{A}_{di}$, $\bar{E}_i$, $Q_{1,i}$, $Q_{2,i}$ in (4a), respectively, yields

$$\begin{bmatrix} \overline{\Omega}_{1,i} & Q_{1,i}\bar{A}_{di} & Q_{1,i}\bar{E}_i \\ * & -e^{\alpha\tau}Q_{2,i} & 0 \\ * & * & \Omega_{3,i} \end{bmatrix} < 0, \tag{25}$$

where

$$\overline{\Omega}_{1,i} = Q_{1,i}\bar{A}_i + \bar{A}_i^TQ_{1,i} + Q_{2,i} - \alpha Q_{1,i}.$$

Define

$$Q_{1,i} = \begin{bmatrix} X_{1,i} & M_{1,i} \\ M_{1,i}^T & W_{1,i} \end{bmatrix}, \quad Q_{1,i}^{-1} = \begin{bmatrix} Y_{1,i} & N_{1,i} \\ N_{1,i}^T & V_{1,i} \end{bmatrix}, \quad \Pi_{2,i} = \begin{bmatrix} I & X_{1,i} \\ 0 & M_{1,i}^T \end{bmatrix},$$

from which we can obtain

$$X_{1,i}Y_{1,i} + M_{1,i}N_{1,i}^T = I, \quad X_{1,i}N_{1,i} + M_{1,i}V_{1,i} = 0, \quad Q_{1,i}\Pi_{1,i} = \Pi_{2,i}.$$

Let $T_i = \mathrm{diag}\{\Pi_{1,i}, I, I\}$. Multiplying (25) by T_i^T and T_i on the left and on the right, respectively, leads to

$$\begin{bmatrix} \Psi_{1,i} + \Pi_{1,i}^T Q_{2,i}\Pi_{1,i} & \Psi_{2,i} & \Psi_{3,i} \\ * & -e^{\alpha\tau}Q_{2,i} & 0 \\ * & * & \Omega_{3,i} \end{bmatrix} < 0. \tag{26}$$

Using Schur complement formula, (23a) can be obtained. Replacing $\widetilde{P}_{1,\iota}$, $\widetilde{P}_{2,\iota}$, $\widetilde{P}_{1,\kappa}$, $\widetilde{P}_{2,\kappa}$ with $\widetilde{Q}_{1,\iota}$, $\widetilde{Q}_{2,\iota}$, $\widetilde{Q}_{1,\kappa}$, $\widetilde{Q}_{2,\kappa}$ in (4b), respectively, we have (23b). This completes the proof of Theorem 3.3.

It is noted that (23a) is not a linear matrix inequality due to the existence of the terms $Q_{2,i}$ and $Q_{2,i}^{-1}$. In order to obtain the desired dynamic output feedback controller (2), we propose the following method.

Performing a congruence transformation to the matrix in (23a) via $\mathrm{diag}\{I, I, I, Q_{1,i}\}$ leads to

$$\begin{bmatrix} \Psi_{1,i} & \Psi_{2,i} & \Psi_{3,i} & \Pi_{1,i}^T Q_{1,i} \\ * & -e^{\alpha\tau}Q_{2,i} & 0 & 0 \\ * & * & \Omega_{3,i} & 0 \\ * & * & * & -Q_{1,i}Q_{2,i}^{-1}Q_{1,i} \end{bmatrix} < 0, \quad i \in \mathcal{N}. \tag{27}$$

Note that

$$(Q_{1,i} - Q_{2,i})Q_{2,i}^{-1}(Q_{1,i} - Q_{2,i}) \geq 0, \tag{28}$$

which means that

$$-Q_{1,i}Q_{2,i}^{-1}Q_{1,i} \leq Q_{2,i} - 2Q_{1,i}. \tag{29}$$

Denote $Q_{2,i} = \begin{bmatrix} X_{2,i} & M_{2,i} \\ M_{2,i}^T & W_{2,i} \end{bmatrix}$. Based on (27) and (29), we can derive the following theorem immediately.

Theorem 3.4. *System (3) is FTB with respect to $(c_1, c_0, c_2, T, diag\{R_1, R_3\}, R_2, \sigma)$ if there exist positive scalars α, τ, positive definite matrices $X_{1,i} \in \mathbb{R}^{n\times n}$, $Y_{1,i} \in \mathbb{R}^{n\times n}$, $W_{1,i} \in \mathbb{R}^{n\times n}$, $X_{2,i} \in \mathbb{R}^{n\times n}$, $W_{2,i} \in \mathbb{R}^{n\times n}$, $P_{3,i} \in \mathbb{R}^{p\times p}$, and matrices $M_{1,i} \in \mathbb{R}^{n\times n}$, $M_{2,i} \in \mathbb{R}^{n\times n}$, $\widetilde{A}_i \in \mathbb{R}^{n\times n}$, $\widetilde{B}_i \in \mathbb{R}^{n\times q}$, $\widetilde{C}_i \in \mathbb{R}^{m\times q}$ and $\widetilde{D}_i \in \mathbb{R}^{m\times q}$, $i \in \mathcal{N}$ such that (4c), (23b) and the following inequalities hold*

$$\begin{bmatrix} \Psi_{1,i} & \Psi_{2,i} & \Psi_{3,i} & \Pi_{2,i}^T \\ * & -\Psi_{4,i} & 0 & 0 \\ * & * & \Omega_{3,i} & 0 \\ * & * & * & \Psi_{5,i} \end{bmatrix} < 0, \quad i \in \mathcal{N}, \tag{30}$$

where

$$\Psi_{4,i} = \begin{bmatrix} X_{2,i} & M_{2,i} \\ M_{2,i}^T & W_{2,i} \end{bmatrix}, \quad \Psi_{5,i} = \begin{bmatrix} X_{2,i} - 2X_{1,i} & M_{2,i} - 2M_{1,i} \\ M_{2,i}^T - 2M_{1,i}^T & W_{2,i} - 2W_{1,i} \end{bmatrix}.$$

Moreover, dynamic output feedback controller gains are given by (2) with

$$D_{ci} = \widetilde{D}_i, \quad C_{ci} = \left(\widetilde{C}_i - D_{ci}C_iY_{1,i}\right)N_{1,i}^{-T}, \quad L_i = M_{1,i}^{-1}\left(\widetilde{B}_i - X_{1,i}B_iD_{ci}\right), \tag{31}$$

$$A_{ci} = M_{1,i}^{-1}\left(\widetilde{A}_i - X_{1,i}A_iY_{1,i} - X_{1,i}B_iD_{ci}C_iY_{1,i} - M_{1,i}L_iC_iY_{1,i} - X_{1,i}B_iC_{ci}N_{1,i}^T\right)N_{1,i}^{-T}.$$

Based on Theorem 3.2 and Theorem 3.4, we have the following result.

Theorem 3.5. *System (3) with $\omega(t) = 0$ is FTS with respect to $(c_1, c_2, T, diag\{R_1, R_3\}, \sigma)$ if there exist positive scalars α, τ, positive definite matrices $X_{1,i} \in \mathbb{R}^{n\times n}$, $Y_{1,i} \in \mathbb{R}^{n\times n}$, $W_{1,i} \in \mathbb{R}^{n\times n}$, $X_{2,i} \in \mathbb{R}^{n\times n}$, $W_{2,i} \in \mathbb{R}^{n\times n}$, and matrices $M_{1,i} \in \mathbb{R}^{n\times n}$, $M_{2,i} \in \mathbb{R}^{n\times n}$, $\widetilde{A}_i \in \mathbb{R}^{n\times n}$, $\widetilde{B}_i \in \mathbb{R}^{n\times q}$, $\widetilde{C}_i \in \mathbb{R}^{m\times q}$, $\widetilde{D}_i \in \mathbb{R}^{m\times q}$, $i \in \mathcal{N}$ such that (20c) and the following inequalities hold*

$$\begin{bmatrix} \Psi_{1,i} & \Psi_{2,i} & \Pi_{2,i}^T \\ * & -\Psi_{4,i} & 0 \\ * & * & \Psi_{5,i} \end{bmatrix} < 0, \quad i \in \mathcal{N}, \tag{32a}$$

$$\beta \le \mu < \frac{c_2}{c_1 + \frac{c_1}{\alpha}(e^{\alpha\tau} - 1)} e^{-\alpha T}, \tag{32b}$$

where

$$\beta = \frac{\lambda_2}{\lambda_1}, \quad \lambda_2 = \max\left\{\max_{\iota\in\mathcal{N}}(\lambda_{\max}(\widetilde{Q}_{1,\iota})), \max_{\iota\in\mathcal{N}}(\lambda_{\max}(\widetilde{Q}_{2,\iota}))\right\}, \quad \iota \in \mathcal{N},$$

$$\lambda_1 = \min\left\{\min_{\kappa\in\mathcal{N}}(\lambda_{\min}(\widetilde{Q}_{1,\kappa})), \min_{\kappa\in\mathcal{N}}(\lambda_{\min}(\widetilde{Q}_{2,\kappa}))\right\}, \quad \kappa \in \mathcal{N}.$$

Moreover, dynamic output feedback controller gains are given by (2) with (31).

4. Numerical Examples.

In this section, two examples are presented to show the effectiveness of the main results in this paper.

Example 4.1. *Consider system (1a) and (1c) with the following parameters*

$$A_1 = \begin{bmatrix} 0 & -0.5 \\ 0.5 & 0 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.05 & -0.05 \\ 0.15 & 0.05 \end{bmatrix}, \quad E_1 = \begin{bmatrix} -0.1 \\ -0.1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0 & -1.2 \\ 1.2 & 0 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.04 & -0.2 \\ 0.08 & 0.04 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix},$$
$$G_1 = G_2 = -1, \quad B_1 = B_2 = 0.$$

Let $c_1 = 0.1$, $c_0 = 0.1$, $c_2 = 20$, $R_1 = R_2 = I$. Firstly, we give the following two tables to demonstrate the relations between the parameters $\tau_{\max}$ and T for the fixed $\alpha = 0.4$, and the relations between $\tau_{\max}$ and α under $T = 3$ by solving Theorem 3.1, respectively.

Table 1. The $\tau_{\max}$ and time interval T when $\alpha = 0.4$

T	*1*	*1.5*	*2*	*2.5*	*3*
$\tau_{\max}$	*4.5*	*3.4*	*2.4*	*1.4*	*0.4*

From Table 1, we can know that $\tau_{\max}$ is related to the time interval T, and a larger $\tau_{\max}$ allows a smaller T for the given α, c_1, c_0 and c_2.

Table 2. The $\tau_{\max}$ and α when $T = 3$

α	*0.4*	*0.42*	*0.45*	*0.452*	*0.455*	*0.465*	*0.47*	*0.48*
$\tau_{\max}$	*0.43*	*0.3*	*0.12*	*0.11*	*0.09*	*0.03*	*0.004*	−

Table 2 demonstrates that a larger α will lead to a smaller $\tau_{\max}$, and the feasible maximum value of α is 0.48 *for the given T, c_1, c_0 and c_2.*

When $c_1 = 0.1$, $c_0 = 0.1$, $c_2 = 20$, $R_1 = R_2 = I$, $\alpha = 0.4$, $\tau = 0.1$, the time interval T satisfies $T \le 3.23$. In this case, choosing $T = 3$ and solving Theorem 3.1, we can obtain

$$\beta = 1.8735, \quad \mu = 2.2272, \quad \frac{c_2}{c_1 + \frac{c_1}{\alpha}(e^{\alpha\tau} - 1)} e^{-\alpha T} = 28.6575, \quad \tau_a^* = 0.9572,$$

which shows that condition (4b) is satisfied. According to (4c), for any switching signal $\sigma(t) \in \mathcal{N}$ with average dwell time $\tau_a > \tau_a^* = 0.9572$, system (1) is FTB with respect to $(0.1, 0.1, 20, 3, I, I, \sigma)$.

Choose $\tau_a = 0.958$, and the initial value function $\phi(s) = \begin{bmatrix} 0.2 & -0.2 \end{bmatrix}^T$, $\omega(0) = 0.1$. Figure 1 and Figure 2 present the phase plot of state and the switching signal, respectively.

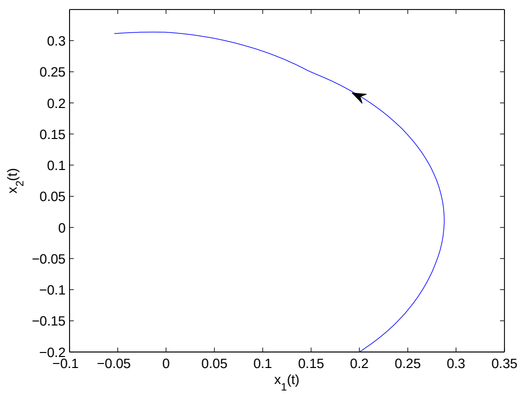

Figure 1. Phase plot of state $x(t)$

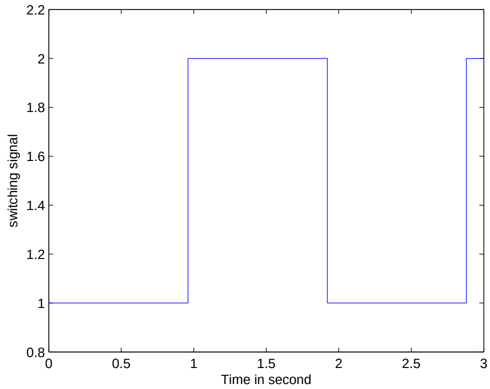

Figure 2. Switching signal $\sigma(t)$ with $\tau_a = 0.958$

Example 4.2. *Consider systems (1) with the following parameters*

$$\begin{aligned}
&A_1 = \begin{bmatrix} -0.9 & 0.2 & -0.2 \\ 0.2 & -0.6 & 0.3 \\ -0.3 & 0.1 & -0.1 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.2 & 0 & 0.1 \\ 0.1 & 0.3 & 0.1 \\ 0.3 & 0.1 & 0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.3 \\ 0.5 \\ 0.2 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.02 \\ 0.01 \\ 0.04 \end{bmatrix}, \\
&C_1 = \begin{bmatrix} -1.2 & 0.5 & 0.9 \end{bmatrix}, \quad C_{d1} = \begin{bmatrix} 0.3 & 0.1 & 0.2 \end{bmatrix}, \quad F_1 = -0.1, \quad G_1 = -2, \qquad (34) \\
&A_2 = \begin{bmatrix} -0.8 & -0.1 & -0.2 \\ 0.2 & -0.7 & 0.3 \\ 0.2 & -0.1 & 0.1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.4 \\ -0.2 \\ 0.3 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.1 \\ 0.07 \\ 0.015 \end{bmatrix}, \\
&C_2 = \begin{bmatrix} -0.1 & 0.12 & 0.5 \end{bmatrix}, \quad C_{d2} = \begin{bmatrix} 0.1 & 0.3 & 0.4 \end{bmatrix}, \quad F_2 = -0.1, \quad G_2 = -1.
\end{aligned}$$

Letting

$$c_1 = 0.1, \quad c_0 = 0.1, \quad c_2 = 20, \quad diag\{R_1, R_3\} = I, \quad R_2 = I, \quad \alpha = 0.05, \quad \tau = 0.1, \qquad (35)$$

Theorem 3.4 has a feasible solution when $T \le 4.12$. Choosing $T = 4$ and solving Theorem 3.4, we can obtain

$$\beta = 2.4136, \quad \mu = 2.6337, \quad \frac{c_2}{c_1 + \frac{c_1}{\alpha}(e^{\alpha\tau} - 1)} e^{-\alpha T} = 27.9651, \quad \tau_a^* = 1.2297,$$

which shows that condition (23b) is satisfied. Moreover, the parameters of dynamic output feedback controller (2) given by (31) as follows:

$$A_{c1} = \begin{bmatrix} -0.3680 & -0.0407 & -0.7364 \\ 0.6777 & -0.8536 & -0.2681 \\ 0.6780 & -0.4050 & -0.9813 \end{bmatrix}, \quad L_1 = \begin{bmatrix} -0.9260 \\ -0.7092 \\ -2.3379 \end{bmatrix},$$

$$C_{c1} = \begin{bmatrix} -0.2364 & 0.1319 & 0.2859 \end{bmatrix}, \quad D_{c1} = 0.2275,$$

$$A_{c2} = \begin{bmatrix} -0.5501 & -0.2425 & -0.4145 \\ 0.1445 & -0.6903 & 0.2161 \\ 0.4970 & -0.3714 & -0.8783 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 2.3065 \\ -2.4720 \\ -2.6711 \end{bmatrix},$$

$$C_{c2} = \begin{bmatrix} -0.1628 & 0.0997 & 0.164 \end{bmatrix}, \quad D_{c2} = -2.1263.$$

According to (4c), for any switching signal $\sigma(t) \in \mathcal{N}$ *with average dwell time* $\tau_a > \tau_a^* = 1.2297$*, system (1) is FTB with respect to* $(0.1, 0.1, 20, 4, diag\{I, I\}, I, \sigma)$*.*

Choosing $\tau_a = 1.23$*, the initial value function* $\phi(s) = \begin{bmatrix} 0.2 & -0.2 & 0.1 \end{bmatrix}^T$*, and* $\omega(0) = 0.1$*, Figure 3 and Figure 4 present the phase plot of state and the switching signal, respectively.*

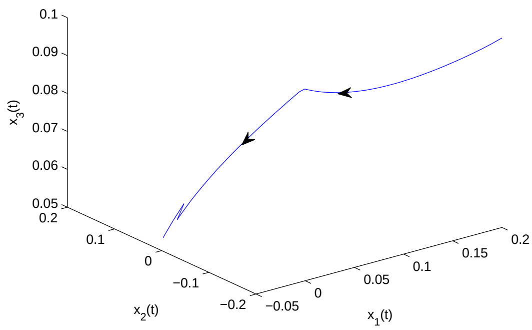

FIGURE 3. Phase plot of state $x(t)$

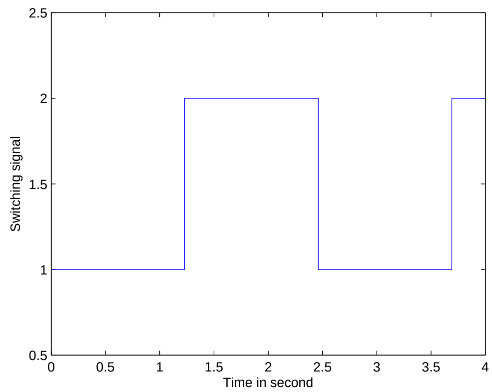

FIGURE 4. Switching signal $\sigma(t)$ with $\tau_a = 0.958$

Moreover, with the parameters (34), (35) and $T = 4$*, solving Theorem 3.5, we can obtain the parameters of dynamic output feedback controller (2) given by (31) as follows:*

$$A_{c1} = \begin{bmatrix} -1.8330 & 0.7322 & 0.4849 \\ -1.2529 & 0.1612 & 1.3220 \\ 0.0364 & -0.0515 & -0.4668 \end{bmatrix}, \quad L_1 = \begin{bmatrix} 2.7709 \\ 4.1594 \\ -0.6778 \end{bmatrix},$$

$$C_{c1} = \begin{bmatrix} 1.0164 & -0.5774 & -0.8323 \end{bmatrix}, \quad D_{c1} = -3.0668,$$

$$A_{c2} = \begin{bmatrix} -0.3929 & -0.4049 & -1.1240 \\ 0.0542 & -0.6010 & 0.5647 \\ 0.5952 & -0.4726 & -1.3561 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -1.5202 \\ -0.5413 \\ -5.2242 \end{bmatrix},$$

$$C_{c2} = \begin{bmatrix} -0.2865 & 0.2158 & 0.7241 \end{bmatrix}, \quad D_{c2} = 0.8810.$$

5. **Conclusions.** In this paper, the problem of finite-time control via dynamic output feedback has been studied for a class of switched delay system. The concepts of finite-time stability and finite-time boundedness have been generalized to switched delay system, respectively. Some sufficient criteria have been developed to solve the problem of finite-time boundedness, finite-time stability and finite-time dynamic output feedback control.

A feasible dynamic output feedback controller has also been given. Finally, two numerical examples have been provided to show the effectiveness of the results.

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