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A NOVEL SIMULTANEOUS FAULT DETECTION AND CONTROL APPROACH BASED ON DYNAMIC OBSERVER

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Abstract. *The problem of simultaneous fault detection and control (SFDC) for linear continuous-time systems is addressed in this paper. A mixed H_2/H_∞ formulation of the SFDC problem using dynamic observer is presented. In essence, a single unit called detector/controller is designed where the detector is a dynamic observer and the controller is a state feedback controller based on the dynamic observer. Hence, the detector/controller unit produces two signals, i.e., the detection and control signals. It is shown that the dynamic observer can be used effectively to tackle the drawbacks of the existing methods of SFDC design. Indeed, the idea presented in this paper is based on applying the advantages of dynamic observers, which leads to some sufficient conditions for solvability of the SFDC problem in terms of LMI feasibility conditions. Simulation results illustrate the effectiveness of the proposed design technique.*

Keywords: Simultaneous fault detection and control (SFDC), Dynamic observer, Linear matrix inequality (LMI)

1. **Introduction.** Model-based fault detection and isolation (FDI) has attracted considerable interest over the past decades (see, e.g., [1, 2, 3] and the references therein). Among model-based approaches, the most common one is to use state observers or filters to construct residual signal and compare it with a predefined threshold. When the residual evaluation function has a value larger than the threshold, an alarm is generated [4]. However, noises and disturbances may result in significant changes in the residual, leading to false alarms [5]. Hence, fault detection observers have to be sensitive to faults and simultaneously robust to noise and disturbances. Therefore, it is of great significance to design a robust FDI scheme. In [6], different performance indices are given for optimal selection of post-filters as well as optimization of fault detection filters. In [7], the fault detection filter design is formulated as an H_∞-filtering problem, where the errors between residuals and faults are minimized. In [8], the problems of H_- index and multiobjective H_∞/H_- fault detection observer design via LMI conditions are considered.

It should be mentioned that most of the existing fault detection observers have been simply confined in traditional static observers (classic Kalman-Luenberger observer) [9]. In order to distinguish from static observer, the term dynamic observer is used, which is an extension of static observer in its configuration and puts dynamics in the observer gain [10]. In [11], a dynamic observer design method is proposed as a dual of control design for the state estimation. A piece of similar work is the Lipschitz UIO [12], where two dynamic compensators are introduced to tackle Lipschitz nonlinearities. In [10], a zero assignment approach for H_2/H_∞ dynamic filter design with application in fault detection is proposed.

In most of the aforementioned methods, an open-loop model of the process was considered and/or it was assumed that the controller maintains the stability of the closed-loop system upon the failure, the assumption which may not be valid for many practical closed-loop feedback systems [13]. This motivates the problem of simultaneous fault detection and control (SFDC) that has attracted a lot of attention in the last two decades [14, 15]. The simultaneous design unifies the control and the detection units into a single unit which results in less complexity as compared with the case of separate design, and it is a reasonable approach since the design of each unit should take the other into consideration. In [16], the implementation of an integrated control/diagnosis system for an advanced hard disk drive is studied. In [17], the robust integrated control/diagnosis approach using H_∞-optimization techniques is applied to Boeing 747-100/200 aircraft. In [18], the SFDC problem is formulated as a mixed H_2/H_∞ optimization problem and its solution is presented in terms of two coupled Riccati equations. In [19], a brief survey of the integrated design of feedback controllers and fault detectors is presented.

To the best of our knowledge, the problem of SFDC using dynamic filter has not been investigated yet. In this paper, we propose a mixed H_2/H_∞ formulation of the SFDC problem using dynamic observer detector and state feedback controller. In fact a single unit called detector/controller, where detector is a dynamic observer and controller is a state feedback, is designed which produces two signals: detection and control signals, which are used to detect faults and satisfy certain control objectives, respectively. It should be pointed out that the conservatism in the SFDC design problem depends on the type of filter used in the structure of SFDC block. SFDC problem using a dynamic output feedback structure has been studied in [20], where conditions are proposed in terms of Bilinear Matrix Inequality (BMI) which are heavily dependent on initial conditions of the iteration and are not globally convergent. Also, as it is mentioned before, most of the existing fault detection observers have been simply confined in traditional static observers. By using equality constraint [21], LMI conditions can be obtained for solving such observer based SFDC problems. It should be pointed out that applying equality constraint introduces some degrees of conservativeness in the design problem. In this paper, the structure of dynamic observer is employed to eliminate the disadvantages of mentioned observer structures in designing SFDC. Hence, our method has major advantages in contrast with previous results that we obtain strict LMI conditions for designing the dynamic observer parameters and controller gain.

The rest of this paper is organized as follows. In Section 2, the problem statement and definitions are given. The solutions to the simultaneous fault detection and control problem are presented in Section 3. To demonstrate the validity of the proposed approach, a numerical example is given in Section 4 which is followed by a conclusion in Section 5.

Notation: For a matrix A, A^{T} denotes its transpose. I and 0 denote, respectively, the identity and zero matrices with appropriate dimensions. For a symmetric matrix, $A > 0$ and $A < 0$ denote positive-definiteness and negative definiteness. The Hermitioan part of a square matrix M is denoted by $Herm(M) = M + M^{\mathrm{T}}$. The symbol $*$ within a matrix represents the symmetric entries. The symbol $\otimes$ stands for the matrix Kronecker product.

2. **The Problem Statement and Definitions.**

In this section, the system model, problem formulation and some preliminaries are presented.

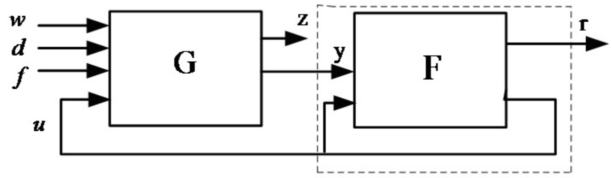

Figure 1. The block diagram of SFDC problem

2.1. **System model.** Consider the following linear time-invariant system:

$$G : \begin{cases} \dot{x}(t) = Ax(t) + B_1u(t) + B_2w(t) + B_3d(t) + B_4f(t) \\ y(t) = Cx(t) + D_1u(t) + D_2w(t) + D_3d(t) + D_4f(t) \\ z(t) = Ex(t) + F_1u(t) + F_2d(t) + F_3f(t) \end{cases}, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is the measured output, and $z(t) \in \mathbb{R}^t$ denotes the regulated output. The unknown input $w(t) \in \mathbb{R}^l$ is assumed to be a fixed spectral density process/measurement noise. $d(t) \in \mathbb{R}^r$ is assumed to be a finite energy disturbance modeling errors due to exogenous signals, linearization or parameter uncertainties. Moreover, the unknown input $f(t) \in \mathbb{R}^q$ is a possible fault. A, B_i's, C, D_i's, E and F_i's are assumed to be known constant matrices of appropriate dimensions.

Then, the following model is proposed for the detector (dynamic observer)/controller (state feedback) throughout the paper:

$$F : \begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + B_1u(t) + n(t) \\ \hat{y}(t) = C\hat{x}(t) + D_1u(t) \\ u(t) = -K\hat{x}(t) \end{cases}, \tag{2}$$

where $n(t) \in \mathbb{R}^n$ is the correction signal, the dynamics of which is given by:

$$\begin{cases} \dot{x}_d(t) = A_dx_d(t) + B_dr(t) \\ n(t) = C_dx_d(t) + D_dr(t) \\ r(t) = y(t) - \hat{y}(t) \end{cases}, \tag{3}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimation of $x(t)$, $\hat{y}(t) \in \mathbb{R}^p$ is the observer output, $x_d(t) \in \mathbb{R}^n$ is an auxiliary vector, $K \in \mathbb{R}^{m\times n}$ is the controller gain, $r(t) \in \mathbb{R}^p$ is the residual signal and the constant matrices A_d, B_d, C_d, D_d are the observer parameters to be designed later.

Now, substituting the detector/controller (2) into the system Equations (1), results in the following closed-loop system equations:

$$G : \begin{cases} \dot{\xi}(t) = \overline{A}\xi(t) + \overline{B}_ww(t) + \overline{B}_dd(t) + \overline{B}_ff(t) \\ r(t) = \overline{C}_1\xi(t) + D_2w(t) + D_3d(t) + D_4f(t) \\ z(t) = \overline{C}_2\xi(t) + F_2d(t) + F_3f(t) \end{cases}, \tag{4}$$

where

$$\begin{gathered} \overline{A} = \begin{bmatrix} A - B_1K & D_dC & C_d \\ 0 & A - D_dC & -C_d \\ 0 & B_dC & A_d \end{bmatrix}, \ \xi = \begin{bmatrix} \hat{x}^{\mathrm{T}}(t) & e^{\mathrm{T}}(t) & x_d^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}, \\ \overline{B}_d = \begin{bmatrix} D_dD_3 \\ B_3 - D_dD_3 \\ B_dD_3 \end{bmatrix}, \ \overline{B}_f = \begin{bmatrix} D_dD_4 \\ B_4 - D_dD_4 \\ B_dD_4 \end{bmatrix}, \ \overline{B}_w = \begin{bmatrix} D_dD_2 \\ B_2 - D_dD_2 \\ B_dD_2 \end{bmatrix}, \\ \overline{C}_1 = \begin{bmatrix} 0 & C & 0 \end{bmatrix}, \ \overline{C}_2 = \begin{bmatrix} E - F_1K & E & 0 \end{bmatrix}, \ e(t) = x(t) - \hat{x}(t). \end{gathered} \tag{5}$$

The block diagram of the SFDC problem is depicted in Figure 1.

In the next subsection the SFDC design problem to be addressed in this paper will be transformed into a mixed H_2/H_∞ optimization problem.

2.2. **Problem formulation and preliminaries.** The simultaneous fault detection and control problem to be addressed in this paper can be stated as follows.

SFDC Problem:

Given system (1), design a detector/controller (2) such that the closed-loop system (4) is stable, the effects of disturbance and noise on regulated output $z(t)$ and residual output $r(t)$ are minimized, and the effects of fault on $z(t)$ are minimized, while the effects of fault on residual output $r(t)$ are maximized. More specifically, we are to find a filter such that the closed-loop system is stable and the following conditions are satisfied:

$$\begin{array}{llll} \text{(i)} & \|G_{zd}(s)\|_\infty < \gamma_1, & \text{(iv)} & \|G_{rd}(s)\|_\infty < \gamma_4, \\ \text{(ii)} & \|G_{zw}(s)\|_2 < \gamma_2, & \text{(v)} & \|G_{rw}(s)\|_2 < \gamma_5, \\ \text{(iii)} & \|G_{zf}(s)\|_\infty < \gamma_3, & \text{(vi)} & \|G_{rf}(s)\|_- < \gamma_6, \end{array} \tag{6}$$

where

$$\begin{aligned} G_{zd}(s) &= \overline{C}_2(sI - \overline{A})^{-1}\overline{B}_d + F_2 \\ G_{zf}(s) &= \overline{C}_2(sI - \overline{A})^{-1}\overline{B}_f + F_3 \\ G_{zw}(s) &= \overline{C}_2(sI - \overline{A})^{-1}\overline{B}_w \\ G_{rd}(s) &= \overline{C}_1(sI - \overline{A})^{-1}\overline{B}_d + D_3 \\ G_{rw}(s) &= \overline{C}_1(sI - \overline{A})^{-1}\overline{B}_w \\ G_{rf}(s) &= \overline{C}_1(sI - \overline{A})^{-1}\overline{B}_f + D_4. \end{aligned} \tag{7}$$

Remark 2.1. *Note that, the matrix D_2 is excluded from the transfer matrix G_{rw} for the same reasons mentioned in* [22].

For simplification, the condition (vi) is replaced by a standard H_∞ model matching problem as follows [23]:

$$\|W_f - G_{rf}\|_\infty < \gamma_6. \tag{8}$$

This condition means that the residual signal $r(t)$ robustly tracks a filtered version of the fault signals, $W_f f$, with W_f appropriately chosen. Assume we select W_f in the following form:

$$W_f = \left[\begin{array}{c|c} A_F & B_F \\ \hline C_F & D_F \end{array}\right], \tag{9}$$

where A_F is a Hurwitz matrix. Then,

$$W_f - G_{rf}(s) = \tilde{C}(sI - \tilde{A})^{-1}\tilde{B} + \tilde{D}, \tag{10}$$

where

$$\begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix} = \left[\begin{array}{c:ccc|c} A_F & & 0 & & B_F \\ \hdashline & A - B_1K & D_dC & C_d & D_dD_4 \\ 0 & 0 & A - D_dC & -C_d & B_4 - D_dD_4 \\ & 0 & B_dC & A_d & B_dD_4 \\ \hline C_F & 0 & -C & 0 & D_F - D4 \end{array}\right]. \tag{11}$$

The following lemmas are used in the next section.

Lemma 2.1. *Suppose the system* (12) *is asymptotically stable:*

$$\begin{cases} \dot{x}(t) = Ax(t) + Bw(t) \\ z(t) = Cx(t) + Dw(t) \end{cases}, \tag{12}$$

and let $T(s) = C(sI - A)^{-1}B + D$ denote its transfer function, if $D = 0$ then the following statements are equivalent:

1. *There exits a prescribed positive constant γ such that:*

$$||T(s)||_2 < \gamma, \tag{13}$$

2. *There exists $P = P^{\mathrm{T}}$ and Z such that:*

$$\begin{bmatrix} A^{\mathrm{T}}P + PA & PB \\ * & -\gamma I \end{bmatrix} < 0, \ \begin{bmatrix} P & C^{\mathrm{T}} \\ * & Z \end{bmatrix} > 0, \ \mathrm{trace}(Z) < \gamma. \tag{14}$$

Lemma 2.2. *(Projection lemma)* [24]. *Given a symmetric matrix $Z \in S_m$ and two matrices U and V of column dimension m; there exists an unstructured matrix X that satisfies:*

$$U^{\mathrm{T}}XV + V^{\mathrm{T}}X^{\mathrm{T}}U + Z < 0, \tag{15}$$

if and only if the following projection inequalities with respect to X are satisfied:

$$N_U^{\mathrm{T}} Z N_U < 0, \tag{16a}$$

$$N_V^{\mathrm{T}} Z N_V < 0, \tag{16b}$$

where N_U and N_V are arbitrary matrices whose columns form a basis of the null spaces of U and V, respectively.

3. **Simultaneous Fault Detection and Control Problem.** There are six performance indices (i)-(vi) that must be satisfied simultaneously for solving the SFDC problem. At first, each performance index will be transformed into the LMI feasibility conditions in Theorems 3.1-3.6. Then, in Corollary 3.1 a feasible solution to the SFDC problem is obtained by considering all of Theorems 3.1-3.6 simultaneously. Since the LMI conditions in each theorem involve the product of Lyapunov matrices and system state space matrices, we have to take equal Lyapunov matrices in Corollary 3.1 which lead to a conservatism problem. Hence, in Theorem 3.7, we will employ projection lemma to reduce conservativeness in the SFDC problem by the introduction of additional matrix variables, so as to avoid the coupling of Lyapunov matrices with the system matrices.

First, design objective (i) is transformed to LMI feasibility constraints in the following theorem.

Theorem 3.1. *The closed-loop system* (4) *is stable and the condition:*

$$||G_{zd}(s)||_\infty < \gamma_1 \tag{17}$$

holds, if there exist symmetric positive-definite matrices Q_{11}, P_{11}, X, matrices A_k, B_k, C_k, D_k, M and a prescribed positive constant γ_1, such that the following inequalities are satisfied:

$$\begin{bmatrix} Q_{11} & I \\ I & P_{11} \end{bmatrix} > 0, \tag{18}$$

$$\begin{bmatrix} E_{11} & E_{12} & E_{13} & D_kD_3 & XE^{\mathrm{T}} - M^{\mathrm{T}}F_1^{\mathrm{T}} \\ * & E_{22} & E_{23} & B_3 - D_kD_3 & Q_{11}E^{\mathrm{T}} \\ * & * & E_{33} & P_{11}B_3 + B_kD_3 & E^{\mathrm{T}} \\ * & * & * & -\gamma_1^2 I & F_2^{\mathrm{T}} \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{19}$$

where

$$\begin{aligned} E_{11} &= AX - B_1M + XA^{\mathrm{T}} - M^{\mathrm{T}}B_1^{\mathrm{T}}, \\ E_{12} &= -C_k, \quad E_{13} = D_kC, \\ E_{22} &= AQ_{11} + C_k + C_k^{\mathrm{T}} + Q_{11}A^{\mathrm{T}}, \\ E_{23} &= A + A_k^{\mathrm{T}} - D_kC, \\ E_{33} &= P_{11}A + B_kC + C^{\mathrm{T}}B_k^{\mathrm{T}} + A^{\mathrm{T}}P_{11}. \end{aligned} \tag{20}$$

The control gain K and the dynamic observer parameters A_d, B_d, C_d, D_d are given by:

$$\begin{aligned} A_d &= P_{12}^{-1}\left(A_k - P_{11}(A - D_dC)Q_{11} - P_{12}B_dCQ_{11} + P_{11}C_dQ_{12}^{\mathrm{T}}\right)(Q_{12}^{\mathrm{T}})^{-1}, \\ B_d &= P_{12}^{-1}(B_k + P_{11}D_d), \quad K = MX^{-1}, \\ C_d &= -(C_k + D_dCQ_{11})(Q_{12}^{\mathrm{T}})^{-1}, \quad D_d = D_k, \end{aligned} \tag{21}$$

where P_{12} and Q_{12} are invertible matrices satisfying the following condition:

$$P_{12}Q_{12}^{\mathrm{T}} = I - P_{11}Q_{11}. \tag{22}$$

Proof: First, note that by applying bounded-real lemma (BRL), condition (17) is satisfied if and only if the following inequality holds:

$$\begin{bmatrix} \overline{A}^{\mathrm{T}}P + P\overline{A} & P\overline{B}_d & \overline{C}_2^{\mathrm{T}} \\ * & -\gamma_1^2 I & F_2^{\mathrm{T}} \\ * & * & -I \end{bmatrix} < 0, \tag{23}$$

assume that P has the following structure:

$$P = \begin{bmatrix} P_{1(n\times n)} & 0 \\ 0 & P_{2(2n\times 2n)} \end{bmatrix}, \quad P_2 = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^{\mathrm{T}} & P_{22} \end{bmatrix}. \tag{24}$$

Using the structure defined for P in (24), condition (23) can be rewritten as:

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} \\ * & * & \Omega_{33} & \Omega_{34} & \Omega_{35} \\ * & * & * & \Omega_{44} & \Omega_{45} \\ * & * & * & * & \Omega_{55} \end{bmatrix} < 0, \tag{25}$$

where

$$\begin{aligned} &\Omega_{11} = P_1(A - B_1K) + (A - B_1K)^{\mathrm{T}}P_1, \;\; \Omega_{12} = P_1D_dC, \;\; \Omega_{13} = P_1C_d \\ &\Omega_{14} = P_1D_dD_3, \;\; \Omega_{15} = E - F_1K, \;\; \Omega_{25} = E, \;\; \Omega_{35} = 0, \;\; \Omega_{55} = -I \\ &\begin{bmatrix} \Omega_{22} & \Omega_{23} \\ * & \Omega_{33} \end{bmatrix} = P_2\begin{bmatrix} A - D_dC & -C_d \\ B_dC & A_d \end{bmatrix} + \begin{bmatrix} A - D_dC & -C_d \\ B_dC & A_d \end{bmatrix}^{\mathrm{T}} P_2 \\ &\begin{bmatrix} \Omega_{24} \\ \Omega_{34} \end{bmatrix} = P_2\begin{bmatrix} B_3 - D_dD_3 \\ B_dD_3 \end{bmatrix}, \;\; \Omega_{44} = -\gamma_1^2 I, \;\; \Omega_{45} = F_2^{\mathrm{T}}. \end{aligned} \tag{26}$$

Suppose that $X = P_1^{-1}$ and $Q = P_2^{-1}$, and Q is partitioned as:

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^{\mathrm{T}} & Q_{22} \end{bmatrix}. \tag{27}$$

Define the matrices Π_1 and Π_2 as follows:

$$\Pi_1 = \begin{bmatrix} Q_{11} & I \\ Q_{12}^{\mathrm{T}} & 0 \end{bmatrix}, \quad \Pi_2 = P_2\Pi_1 = \begin{bmatrix} I & P_{11} \\ 0 & P_{12}^{\mathrm{T}} \end{bmatrix}. \tag{28}$$

Now, pre- and post-multiplication of (25) by $diag(X, \Pi_1^{\mathrm{T}}, I, I)$ and $diag(X, \Pi_1, I, I)$ respectively, yields in:

$$\begin{bmatrix} Herm(AX-B_1M) & -C_k & D_kC & D_kD_3 & XE^{\mathrm{T}}-M^{\mathrm{T}}F_1^{\mathrm{T}} \\ * & Herm(AQ_{11}+C_k) & A-D_kC+A_k^{\mathrm{T}} & B_3-D_kD_3 & Q_{11}E^{\mathrm{T}} \\ * & * & Herm(P_{11}A+B_kC) & P_{11}B_3+B_kD_3 & E^{\mathrm{T}} \\ * & * & * & -\gamma_1^2 I & F_2^{\mathrm{T}} \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{29}$$

where

$$\begin{aligned} A_k &= P_{11}(A - D_dC)Q_{11} + P_{12}B_dCQ_{11} - P_{11}C_dQ_{12}^{\mathrm{T}} + P_{12}A_dQ_{12}^{\mathrm{T}}, \\ B_k &= -P_{11}D_d + P_{12}B_d, \quad C_k = -D_dCQ_{11} - C_dQ_{12}^{\mathrm{T}}, \\ D_k &= D_d, \quad M = KX. \end{aligned} \tag{30}$$

Note that $X > 0$ and $P_2 > 0$. Using the definition of Π_1 and Π_2 in (28), $P_2 > 0$ is equivalent to:

$$\Pi_1^{\mathrm{T}} P_2 \Pi_1 = \Pi_2^{\mathrm{T}} \Pi_1 = \begin{bmatrix} Q_{11} & I \\ I & P_{11} \end{bmatrix} > 0. \tag{31}$$

This completes the proof.

The LMI constraints for condition (iv) are given in the following theorem.

Theorem 3.2. *The closed-loop system* (4) *is stable and guarantees the performance index (iv) if there exist symmetric positive-definite matrices Q_{11}, P_{11}, X, matrices A_k, B_k, C_k, D_k, M and a prescribed positive constant γ_4, such that the following inequalities are satisfied:*

$$\begin{bmatrix} Q_{11} & I \\ I & P_{11} \end{bmatrix} > 0, \tag{32}$$

$$\begin{bmatrix} E_{11} & E_{12} & E_{13} & D_kD_3 & 0 \\ * & E_{22} & E_{23} & B_3 - D_kD_3 & Q_{11}C^{\mathrm{T}} \\ * & * & E_{33} & P_{11}B_3 + B_kD_3 & C^{\mathrm{T}} \\ * & * & * & -\gamma_4^2 I & D_3^{\mathrm{T}} \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{33}$$

where E_{11}, E_{12}, E_{13}, E_{22}, E_{23}, E_{33} are defined in (20).

The control gain K and the dynamic observer parameters A_d, B_d, C_d, D_d are obtained from (21).

Proof: The proof of this theorem is similar to that of Theorem 3.1, so it is omitted for the sake of brevity.

The following theorem gives the LMI constraints for condition (iii).

Theorem 3.3. *The closed-loop system* (4) *is stable and guarantees the performance index (iii) if there exist symmetric positive-definite matrices Q_{11}, P_{11}, X, matrices A_k, B_k, C_k, D_k, M and a prescribed positive constant γ_3, such that the following inequalities are satisfied:*

$$\begin{bmatrix} Q_{11} & I \\ I & P_{11} \end{bmatrix} > 0, \tag{34}$$

$$\begin{bmatrix} E_{11} & E_{12} & E_{13} & D_kD_4 & XE^{\mathrm{T}} - M^{\mathrm{T}}F_1^{\mathrm{T}} \\ * & E_{22} & E_{23} & B_4 - D_kD_4 & Q_{11}E^{\mathrm{T}} \\ * & * & E_{33} & P_{11}B_4 + B_kD_4 & E^{\mathrm{T}} \\ * & * & * & -\gamma_3^2 I & F_3^{\mathrm{T}} \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{35}$$

where E_{11}*,* E_{12}*,* E_{13}*,* E_{22}*,* E_{23}*,* E_{33} *are defined in* (20)*.*

The control gain K *and the dynamic observer parameters* A_d*,* B_d*,* C_d*,* D_d *are obtained from* (21)*.*

Proof: The proof of this theorem is similar to that of Theorem 3.1, so it is omitted for the sake of brevity.

The LMI constraints for condition (vi) are given in the following theorem.

Theorem 3.4. *The closed-loop system* (4) *is stable and guarantees the performance index (vi) if there exist symmetric positive-definite matrices* Q_{11}*,* P_{11}*,* X*,* P_F*, matrices* A_k*,* B_k*,* C_k*,* D_k*,* M *and a prescribed positive constant* γ_6*, such that the following inequalities are satisfied:*

$$\begin{bmatrix} Q_{11} & I \\ I & P_{11} \end{bmatrix} > 0, \tag{36}$$

$$\begin{bmatrix} A_F^{\mathrm{T}} P_F + P_F A_F & 0 & 0 & 0 & P_F B_F & C_F^{\mathrm{T}} \\ * & E_{11} & E_{12} & E_{13} & D_k D_4 & 0 \\ * & * & E_{22} & E_{23} & B_4 - D_k D_4 & -Q_{11} C^{\mathrm{T}} \\ * & * & * & E_{33} & P_{11} B_4 + B_k D_4 & -C^{\mathrm{T}} \\ * & * & * & * & -\gamma_6^2 I & D_F^{\mathrm{T}} - D_4^{\mathrm{T}} \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{37}$$

where E_{11}*,* E_{12}*,* E_{13}*,* E_{22}*,* E_{23}*,* E_{33} *are defined in* (20)*.*

The filter gains A_d*,* B_d*,* C_d*,* D_d*, and the controller gain* K *are obtained from* (21)*.*

Proof: This theorem can be proved by employing the same techniques as in the proof of Theorem 3.1; hence, the detailed procedure is omitted here.

The LMI constraints for condition (ii) are presented in the following theorem.

Theorem 3.5. *The closed-loop system* (4) *is stable and the condition:*

$$||G_{zw}(s)||_2 < \gamma_2 \tag{38}$$

holds, if there exist symmetric positive-definite matrices Q_{11}*,* P_{11}*,* X*, matrices* A_k*,* B_k*,* C_k*,* D_k*,* M*,* Z *and a prescribed positive constant* γ_2*, such that the following inequalities are satisfied:*

$$\begin{bmatrix} E_{11} & E_{12} & E_{13} & D_k D_2 \\ * & E_{22} & E_{23} & B_2 - D_k D_2 \\ * & * & E_{33} & P_{11} B_2 + B_k D_2 \\ * & * & * & -\gamma_2 I \end{bmatrix} < 0, \tag{39}$$

$$\begin{aligned} &\begin{bmatrix} X & 0 & 0 & X E^{\mathrm{T}} - M^{\mathrm{T}} F_1^{\mathrm{T}} \\ * & Q_{11} & I & Q_{11} E^{\mathrm{T}} \\ * & * & P_{11} & E^{\mathrm{T}} \\ * & * & * & Z \end{bmatrix} < 0, \\ &trace(Z) < \gamma_2, \end{aligned} \tag{40}$$

where E_{11}*,* E_{12}*,* E_{13}*,* E_{22}*,* E_{23}*,* E_{33} *are defined in* (20)*.*

The filter gains A_d*,* B_d*,* C_d*,* D_d*, and the controller gain* K *are obtained from* (21)*.*

Proof: First, condition (38), is transformed to the following inequalities using (14):

$$\begin{bmatrix} \overline{A}^{\mathrm{T}}P + P\overline{A} & P\overline{B}_w \\ * & -\gamma_2 I \end{bmatrix} < 0, \tag{41}$$

$$\begin{gathered} \begin{bmatrix} P & \overline{C}_2^{\mathrm{T}} \\ * & Z \end{bmatrix} > 0, \\ trace(Z) < \gamma_2. \end{gathered} \tag{42}$$

Inequality (41) is non-convex because of the nonlinear terms $P\overline{A}$ and $P\overline{B}_w$. Therefore, the matrix P is partitioned as in (24), and P_2^{-1}, Π_1 and Π_2 are defined the same as in (27) and (28), respectively. With pre-and post-multiplying inequality (42) by $diag(X, \Pi_1^{\mathrm{T}}, I)$ and $diag(X, \Pi_1, I)$, respectively, inequality (40) is obtained. Similarly, LMI condition (39) is derived from (41) by pre- and post-multiplication by $diag(X, \Pi_1^{\mathrm{T}}, I)$ and $diag(X, \Pi_1, I)$, respectively.

The LMI constraints for condition (v) are presented in the following theorem.

Theorem 3.6. *The closed-loop system* (4) *is stable and guarantees the performance index (v) if there exist symmetric positive-definite matrices Q_{11}, P_{11}, X, matrices A_k, B_k, C_k, D_k, M, Z and a prescribed positive constant γ_5, such that the following inequalities are satisfied:*

$$\begin{bmatrix} E_{11} & E_{12} & E_{13} & D_k D_2 \\ * & E_{22} & E_{23} & B_2 - D_k D_2 \\ * & * & E_{33} & P_{11}B_2 + B_k D_2 \\ * & * & * & -\gamma_5 I \end{bmatrix} < 0, \tag{43}$$

$$\begin{gathered} \begin{bmatrix} X & 0 & 0 & 0 \\ * & Q_{11} & I & Q_{11}C^{\mathrm{T}} \\ * & * & P_{11} & C^{\mathrm{T}} \\ * & * & * & Z \end{bmatrix} < 0, \\ trace(Z) < \gamma_5, \end{gathered} \tag{44}$$

where E_{11}, E_{12}, E_{13}, E_{22}, E_{23}, E_{33} are defined in (20).

The filter gains A_d, B_d, C_d, D_d, and the controller gain K are obtained from (21).

Proof: This theorem can be proved by employing the same techniques as in the proof of Theorem 3.5; hence, the detailed procedure is omitted here.

At this point, all control and detection objectives given in (6) have been transformed to LMI feasibility constraints. The next corollary unifies the above theorems and provides a procedure for solving SFDC problem.

Corollary 3.1. *Given γ_1, γ_2, γ_3, γ_4, γ_5, a feasible solution to the SFDC problem is obtained by solving a sequence of convex optimization problems:*

$$\begin{aligned} &\min_{X, P_{11}, Q_{11}, P_F, A_k, B_k, C_k, D_k, M, Z} \gamma_6 \\ &\text{s.t. } (18), (19), (33), (35), (37), (39), (40), (43), (44). \end{aligned} \tag{45}$$

Proof: This corollary can be easily proved by collecting all the previous theorems (Theorems 3.1-3.6).

Remark 3.1. *For more perception of applying dynamic observer in SFDC design, we compare its structure with the static observer-based SFDC design. Consider, the static* *observer detector/state feedback controller for system* (1) *as follows:*

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + B_1u(t) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t) + D_1u(t) \\ r(t) = y(t) - \hat{y}(t) \\ u(t) = -K\hat{x}(t) \end{cases} . \tag{46}$$

By (1) *and* (46) *the closed-loop system dynamics can be derived as follows:*

$$\begin{cases} \dot{\zeta}(t) = \overline{A}\zeta(t) + \overline{B}_w w(t) + \overline{B}_d d(t) + \overline{B}_f f(t) \\ r(t) = \overline{C}_1\zeta(t) + D_2w(t) + D_3d(t) + D_4f(t) \\ z(t) = \overline{C}_2\zeta(t) + F_2d(t) + F_3f(t) \end{cases} , \tag{47}$$

where

$$\overline{A} = \begin{bmatrix} A - B_1K & B_1K \\ 0 & A - LC \end{bmatrix}, \quad \zeta = \begin{bmatrix} x^{\mathrm{T}}(t) & e^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}},$$
$$\overline{B}_d = \begin{bmatrix} B_3 \\ B_3 - LD_3 \end{bmatrix}, \quad \overline{B}_f = \begin{bmatrix} B_4 \\ B_4 - LD_4 \end{bmatrix}, \quad \overline{B}_w = \begin{bmatrix} B_2 \\ B_2 - LD_2 \end{bmatrix},$$
$$\overline{C}_1 = \begin{bmatrix} 0 & C \end{bmatrix}, \quad \overline{C}_2 = \begin{bmatrix} E - F_1K & F_1K \end{bmatrix}, \quad e(t) = x(t) - \hat{x}(t).$$

For comparison, consider a static observer-based SFDC for performance index (i).

The closed loop system (47) *is asymptotically stable and satisfy performance index (i), if there exist positive definite symmetric matrices P_1, P_2 and matrices K and N such that the following inequality is satisfied:*

$$\begin{bmatrix} Herm\Big(P_1(A - B_1K)\Big) & P_1B_1K & P_1B_3 & E^{\mathrm{T}} - K^{\mathrm{T}}F_1^{\mathrm{T}} \\ * & Herm(P_2A - NC) & P_2B_3 - ND_3 & K^{\mathrm{T}}F_1^{\mathrm{T}} \\ * & * & -\gamma_1^2 I & F_2^{\mathrm{T}} \\ * & * & * & -I \end{bmatrix} < 0, \tag{48}$$
$$N = P_2L,$$

where (48) *is derived from a procedure similar to the procedure given in the proof of Theorem 3.1. Note that* (48) *is not a strict LMI because of the nonlinear term P_1B_1K. In this situation, the SFDC design problem can be solved in one step using equality constraint* [21]*. It should be pointed out that the equality constraint is applicable only if $F_1 = 0$ in* (48)*. However, if $F_1 \neq 0$ then the generically two step procedure* [20] *can be considered, whereas by using dynamic observer, one does not face such problems by introducing an extra auxiliary dynamics with the new state variable x_d. In fact, the advantage of using this new auxiliary state variable is that we can apply more degrees of freedom by employing A_d, B_d, C_d and D_d in the closed-loop system dynamics and therefore the conditions for designing the controller and observer parameters can be presented in term of strict LMIs conditions.*

Remark 3.2. *The SFDC design is a multiobjective problem since there exist some control and detection objectives that must be satisfied simultaneously. The multiobjective problem is a conservatism problem, due to the fact that the involved Lyapunov matrices are constrained to be equal and are coupled with system matrices. Over the past ten years, extensive research has been devoted to reduce conservatism in hard problems like multiobjective problems, robust stability and performance analysis. Oliveira et al.* [25, 26] *showed how extended LMI characterization can dramatically reduce the conservatism through the introduction of additional matrix variables, so as to eliminate the coupling of Lyapunov* *variables with the system matrices. In* [27]*, extended LMI characterization for stability and performance of linear systems are considered.*

In the following theorem, motivated by the work of Pipeleers et al. [27], the LMI feasibility conditions for solving performance index (i) are presented such that they do not involve products of the Lyapunov matrix and the systems state space matrices. Indeed, we will use the extended LMIs to reduce conservativeness in the SFDC problem.

Theorem 3.7. *Consider the closed-loop system* (4) *and let* $\alpha > 0$*,* $\lambda > 0$ *and* $\gamma_1 > 0$ *be given constants. The corresponding system is stable and guarantees the performance index (i) if there exist symmetric positive-definite matrix* T_1 *and matrices* X_{11}*,* Y_{11}*,* Q*,* S*,* A_k*,* B_k*,* C_k*,* D_k*,* M *such that the following inequality is satisfied:*

$$\begin{bmatrix} \alpha T_1 + E_{11} & T_1 + E_{12} & E_{13} & E_{14} \\ * & E_{22} & E_{23} & 0 \\ * & * & E_{33} & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{49}$$

where

$$\begin{aligned}
E_{11} &= \begin{bmatrix} Herm(Q^{\mathrm{T}}A^{\mathrm{T}} - M^{\mathrm{T}}B_1^{\mathrm{T}}) & * & * \\ C_k^{\mathrm{T}} & Herm(Y_{11}^{\mathrm{T}}A^{\mathrm{T}} - C_k^{\mathrm{T}}) & * \\ C^{\mathrm{T}}D_k^{\mathrm{T}} & A^{\mathrm{T}} - C^{\mathrm{T}}D_k^{\mathrm{T}} + A_k & Herm(A^{\mathrm{T}}X_{11} - C^{\mathrm{T}}B_k^{\mathrm{T}}) \end{bmatrix}, \\
E_{12} &= \lambda \begin{bmatrix} Q^{\mathrm{T}}A^{\mathrm{T}} - M^{\mathrm{T}}B_1^{\mathrm{T}} & 0 & 0 \\ C_k^{\mathrm{T}} & Y_{11}^{\mathrm{T}}A^{\mathrm{T}} - C_k^{\mathrm{T}} & A_k^{\mathrm{T}} \\ C^{\mathrm{T}}D_k^{\mathrm{T}} & A^{\mathrm{T}} - C^{\mathrm{T}}D_k^{\mathrm{T}} & A^{\mathrm{T}}X_{11} - C^{\mathrm{T}}B_k^{\mathrm{T}} \end{bmatrix} - \begin{bmatrix} Q & 0 & 0 \\ 0 & Y_{11} & I \\ 0 & S^{\mathrm{T}} & X_{11}^{\mathrm{T}} \end{bmatrix}, \\
E_{22} &= -\lambda \begin{bmatrix} Q + Q^{\mathrm{T}} & 0 & 0 \\ * & Y_{11} + Y_{11}^{\mathrm{T}} & S + I \\ * & * & X_{11} + X_{11}^{\mathrm{T}} \end{bmatrix}, \quad E_{23} = \lambda \begin{bmatrix} D_k D_3 \\ B_3 - D_k D_3 \\ X_{11}^{\mathrm{T}} B_3 - B_k D_3 \end{bmatrix}, \\
E_{13} &= \begin{bmatrix} D_k D_3 + (QE^{\mathrm{T}} - M^{\mathrm{T}}F_1^{\mathrm{T}})F_2 \\ B_3 - D_k D_3 + Y_{11}^{\mathrm{T}}E^{\mathrm{T}}F_2 \\ X_{11}^{\mathrm{T}}B_3 - B_k D_3 + E^{\mathrm{T}}F_2 \end{bmatrix}, \quad E_{14} = \begin{bmatrix} QE^{\mathrm{T}} - M^{\mathrm{T}}F_1^{\mathrm{T}} \\ Y_{11}^{\mathrm{T}}E^{\mathrm{T}} \\ E^{\mathrm{T}} \end{bmatrix}, \\
E_{33} &= F_2^{\mathrm{T}}F_2 - \gamma_1^2 I.
\end{aligned} \tag{50}$$

The control gain K *and the dynamic observer parameters* A_d*,* B_d*,* C_d*,* D_d *are given by:*

$$\begin{aligned}
A_d &= (X_{21}^{\mathrm{T}})^{-1}\left(A_k - X_{11}^{\mathrm{T}}(A - D_dC)Y_{11} - X_{21}^{\mathrm{T}}B_dCY_{11} + X_{11}^{\mathrm{T}}C_dY_{21}\right)(Y_{21})^{-1}, \\
B_d &= (X_{21}^{\mathrm{T}})^{-1}(X_{11}^{\mathrm{T}}D_d - B_k), \quad K = MQ^{-1}, \\
C_d &= (C_k - D_dCY_{11})(Y_{21})^{-1}, \quad D_d = D_k,
\end{aligned} \tag{51}$$

where X_{21} *and* Y_{21} *are invertible matrices satisfying the following condition:*

$$Y_{21}^{\mathrm{T}}X_{21} = S - Y_{11}X_{11}. \tag{52}$$

Proof: System (4) is asymptotically stable and satisfies performance index (i), if there exists positive definite Lyapunov function $V(t) = \xi^{\mathrm{T}}(t)P_1\xi^{\mathrm{T}}$, where P_1 is the positive definite symmetric matrix and $\xi(t)$ is defined in (5), such that:

$$\dot{V}(t) \leq -\alpha V(t) - z^{\mathrm{T}}(t)z(t) + \gamma_1^2 d^{\mathrm{T}}(t)d(t). \tag{53}$$

From (53) following inequality is obtained:

$$\begin{bmatrix} I & 0 \\ \overline{A} & \overline{B}_d \end{bmatrix}^{\mathrm{T}} \left(\begin{bmatrix} \alpha & 1 \\ 1 & 0 \end{bmatrix} \otimes P_1 \right) \begin{bmatrix} I & 0 \\ \overline{A} & \overline{B}_d \end{bmatrix} + \begin{bmatrix} 0 & I \\ \overline{C}_2 & F_2 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\gamma_1^2 I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} 0 & I \\ \overline{C}_2 & F_2 \end{bmatrix} < 0, \tag{54}$$

where $\overline{A}$, $\overline{B}_d$, $\overline{C}_2$ and F_2 are defined in (5).

The matrix inequality (54) is reformulated as:

$$N_U^{\mathrm{T}} Z N_U < 0, \tag{55}$$

where N_U and Z are defined below:

$$Z = \begin{bmatrix} \alpha P_1 + \overline{C}_2^{\mathrm{T}} \overline{C}_2 & P_1 & \overline{C}_2^{\mathrm{T}} F_2 \\ * & 0 & 0 \\ * & * & F_2^{\mathrm{T}} F_2 - \gamma_1^2 I \end{bmatrix}, \quad N_U = \begin{bmatrix} I & 0 \\ \overline{A} & \overline{B}_d \\ 0 & I \end{bmatrix}. \tag{56}$$

If we choose N_V in (16b) as follows:

$$N_V = \begin{bmatrix} \lambda I & 0 \\ -I & 0 \\ 0 & I \end{bmatrix} \to V = \begin{bmatrix} I & \lambda I & 0 \end{bmatrix}, \tag{57}$$

and use Lemma 2.2 then it can be concluded that inequality (55) is equivalent to:

$$Z + \begin{bmatrix} \overline{A}^{\mathrm{T}} \\ -I \\ \overline{B}_d^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} X & \lambda X & 0 \end{bmatrix} + \begin{bmatrix} X^{\mathrm{T}} \\ \lambda X^{\mathrm{T}} \\ 0 \end{bmatrix} \begin{bmatrix} \overline{A} & -I & \overline{B}_d \end{bmatrix} < 0. \tag{58}$$

By partitioning X as follows:

$$X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix}, \quad X_2 = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}, \tag{59}$$

and using Schur complement, the following inequality is obtained:

$$\left[\begin{array}{c|c|c|c} \alpha P_1 + Herm\left(\overline{A}^{\mathrm{T}} \left[\begin{smallmatrix} X_1 & 0 \\ 0 & X_2 \end{smallmatrix}\right]\right) & P_1 + \lambda \overline{A}^{\mathrm{T}} \left[\begin{smallmatrix} X_1 & 0 \\ 0 & X_2 \end{smallmatrix}\right] - \left[\begin{smallmatrix} X_1^{\mathrm{T}} & 0 \\ 0 & X_2^{\mathrm{T}} \end{smallmatrix}\right] & \overline{C}_2^{\mathrm{T}} F_2 + \left[\begin{smallmatrix} X_1^{\mathrm{T}} & 0 \\ 0 & X_2^{\mathrm{T}} \end{smallmatrix}\right] \overline{B}_d & \overline{C}_2^{\mathrm{T}} \\ \hline * & \lambda \left[\begin{smallmatrix} -X_1 - X_1^{\mathrm{T}} & 0 \\ 0 & -X_2 - X_2^{\mathrm{T}} \end{smallmatrix}\right] & \lambda \left[\begin{smallmatrix} X_1^{\mathrm{T}} & 0 \\ 0 & X_2^{\mathrm{T}} \end{smallmatrix}\right] \overline{B}_d & 0 \\ \hline * & * & F_2^{\mathrm{T}} F_2 - \gamma_1^2 I & 0 \\ \hline * & * & * & -I \end{array}\right] < 0. \tag{60}$$

Now define new matrices Y, Q, Π_1, Π_2 and $\tilde{\Pi}_1$ as follows:

$$\begin{aligned} Y &= X_2^{-1} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}, \quad \Pi_1 = \begin{bmatrix} Y_{11} & I \\ Y_{21} & 0 \end{bmatrix}, \quad Q = X_1^{-1}, \\ \Pi_2 &= X_2 \Pi_1 = \begin{bmatrix} I & X_{11} \\ 0 & X_{21} \end{bmatrix}, \quad \tilde{\Pi}_1 = \begin{bmatrix} Q & 0 \\ 0 & \Pi_1 \end{bmatrix}. \end{aligned} \tag{61}$$

Note that from (60), we find $X_1 + X_1^{\mathrm{T}} > 0$, $X_2 + X_2^{\mathrm{T}} > 0$ and therefore, $Y + Y^{\mathrm{T}} > 0$, $X_{11} + X_{11}^{\mathrm{T}} > 0$, $X_{22} + X_{22}^{\mathrm{T}} > 0$ and $Y_{11} + Y_{11}^{\mathrm{T}} > 0$, which imply non-singularity of X_1, X_2, X_{11}, X_{22}, Y_{11}. Also without loss of generality, we assume X_{21} and Y_{21} are nonsingular. Therefore, $\tilde{\Pi}_1$ is nonsingular. It should be mentioned that non-singularity of X_{21} is guaranteed by assuming $X_{21} = I$ easily. Then matrix inversion formula [28] yields $Y_{21} = -X_{22}^{-1} X_{21} Y_{11}$, which is nonsingular because the matrices X_{21}, X_{22} and Y_{11} are invertible.

Now, if we perform congruence transformation with $diag(\tilde{\Pi}_1^{\mathrm{T}}, \tilde{\Pi}_1^{\mathrm{T}}, I, I)$ on inequality (60) inequality (49) is obtained. Note that from $P_1 > 0$, we find that T_1 must be positive definite. This completes the proof.

It should be noted that similar results can be obtained for performance indices (ii)-(vi) like Theorem 3.7, but they are not presented here for the sake of brevity.

Remark 3.3. *In some previous works (e.g.,* [8, 29]*), the iterative LMI algorithms which need more computational time than LMI methods are applied for conservatism reduction in FD problems. Furthermore, these algorithms heavily depend on the initial conditions of*

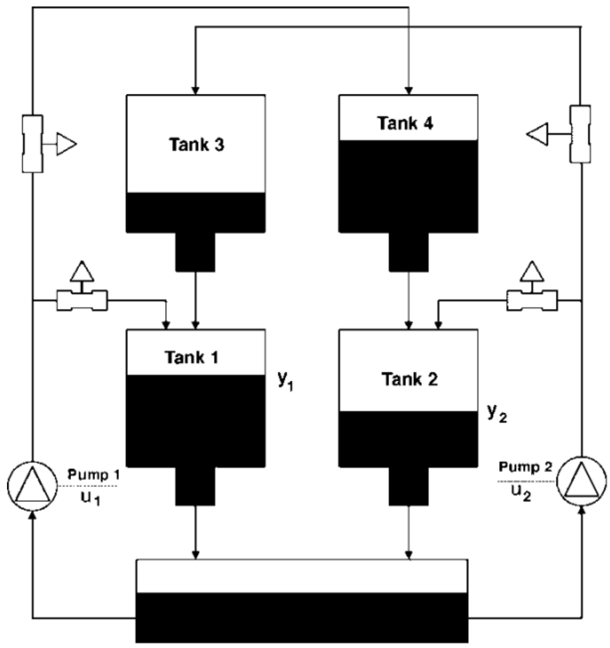

FIGURE 2. Schematic of the four-tank process

the iterations and are not globally convergent. To avoid these problems, the extended LMIs are applied to present strict LMI conditions for solving our problem. Indeed, as can be seen from Theorem 3.7, we reduce the conservatism in our SFDC problem by introducing additional matrix variables (X_1, X_2) *and eliminating the coupling of Lyapunov matrices with the system state space matrices.*

4. **Simulation Results.** To illustrate the effectiveness of the proposed method, a numerical example is given in this section.

Example 4.1. *Consider the four-tank process which is shown in Figure 2. A linearized model of the four-tank process is given by* [22]*:*

$$\dot{x} = \begin{bmatrix} \frac{-1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0 \\ 0 & \frac{-1}{T_2} & 0 & \frac{A_4}{A_2T_4} \\ 0 & 0 & \frac{1}{T_3} & 0 \\ 0 & 0 & 0 & \frac{-1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\alpha_1 k_1}{A_1} & 0 \\ 0 & \frac{\alpha_2 k_2}{A_3} \\ 0 & \frac{(1-\alpha_2)k_2}{A_3} \\ \frac{(1-\alpha_1)k_1}{A_4} & 0 \end{bmatrix} (u + f) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{-k_{d1}}{A_3} & 0 \\ 0 & \frac{-k_{d2}}{A_4} \end{bmatrix} d,$$

$$y = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x + w, \quad z = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x,$$

where x is the level of water in the tanks, $u = (u_1, u_2)^{\mathrm{T}}$ is the voltage applied to Pumps 1 and 2, $f = (f_1, f_2)^{\mathrm{T}}$ is the actuator fault associated with Pumps 1 and 2 and $d = (d_1, d_2)^{\mathrm{T}}$ is the disturbance representing flow out of Tanks 3 and 4. $v = (v_1, v_2)^{\mathrm{T}}$ is the measurement noise which is assumed to be a zero-mean white noise process with covariance $0.5I_2$ and $T_i = (A_i/a_i)\sqrt{2h_{0i}/g}$. The following nominal parameter values are used: $A_1 = A_3 = 28$ $[cm^2]$, $A_2 = A_4 = 32$ $[cm^2]$, $a_1 = a_3 = 0.071$ $[cm^2]$, $a_2 = a_4 = 0.057$ $[cm^2]$, $k_c = 0.5$ $[V/cm]$, $g = 981$ $[cm/s^2]$, $k_1 = 3.33$ $[cm^3/Vs]$, $k_2 = 3.35$ $[cm^3/Vs]$, $k_{d1} = k_{d2} = 1$ $[cm^3/Vs]$, $\alpha_1 = 0.7$, $\alpha_2 = 0.6$, $h_{01} = 12.4$, $h_{02} = 12.7$, $h_{03} = 1.8$ and $h_{04} = 1.4$. It is desired to detect the actuator fault f in presence of the disturbance d and the measurement noise w.

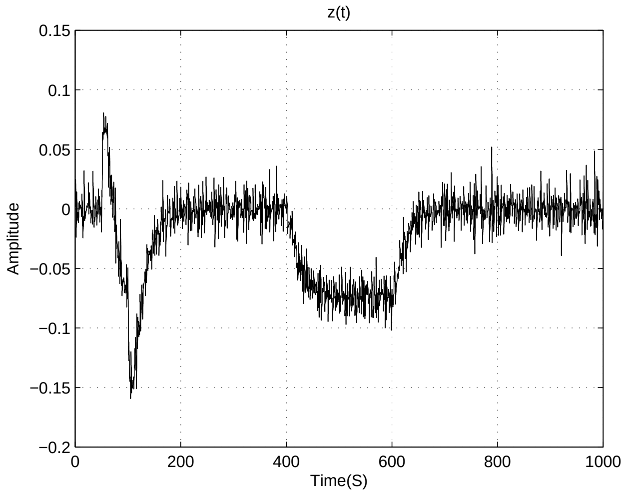

FIGURE 3. The performance output $z(t)$

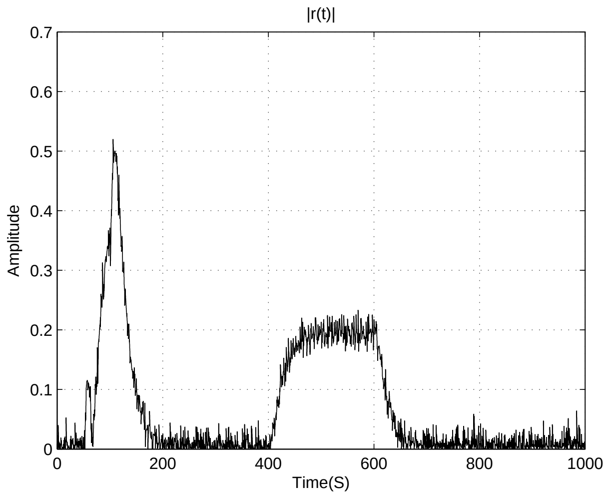

FIGURE 4. The residual signal $|r(t)|$

Reference model parameters for residual are selected as:

$$A_F = \begin{bmatrix} -3 & 0 \\ 0 & -5 \end{bmatrix}, \quad B_F = \begin{bmatrix} 1 & 1 \\ 0.5 & 0.5 \end{bmatrix},$$
$$C_F = \begin{bmatrix} 0.1 & 1 \\ 0.1 & 1 \end{bmatrix}, \quad D_F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

For a given $\gamma_1 = 0.7$, $\gamma_2 = 1.5$, $\gamma_3 = 0.7$, $\gamma_4 = 0.7$, $\gamma_5 = 1.5$, optimization problem (45), was solved and γ_6 is obtained as 0.87. The regulated output $z(t)$ of the closed-loop system is shown in Figure 3. From Figure 3, it can be concluded that the effects of disturbance $d(t)$, noise $w(t)$ and fault $f(t)$ on the regulated output $z(t)$ have been attenuated. The residual signal is shown in Figure 4, where $|r(t)|$ is adopted instead of $r(t)$. From Figure 4, it can be seen that the robustness against disturbance and noise, and the fault sensitivity are both enhanced, and the faults are well discriminated from disturbance and noise. Hence, by using a threshold test, the fault $f(t)$ can be effectively detected.

These results demonstrate the better performance of the proposed technique in comparison with the study performed by [22]. This is because sensitivity of residual signal to fault signal that was ignored in [22], has been considered in our method.

On the other hand, if we employ observer based controller structure [21] in our SFDC design, the problem becomes infeasible. As it is mentioned in Remark 3.1, the amount of conservativeness that is imposed to problem by equality constraint is the reason of this infeasibility.

5. **Conclusion.** A mixed H_2/H_∞ formulation of the SFDC problem using dynamic observer detector and state feedback controller has been considered. Dynamic observer is recommended to overcome some disadvantages of other filters for designing SFDC. In SFDC problem, the proposed method has major advantages in contrast with previous results that the presented conditions have been obtained in terms of LMIs. A numerical example has been given to demonstrate the effectiveness of proposed approach. Finally, let us remark that the presented method will have other advantages in designing SFDC for uncertain LTI systems that have not been considered in this paper and we will study them in our future work.

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