International Journal of Innovative Computing, Information and Control
Volume **8**, Number **8**, August **2012**
ICIC International ©2012 ISSN 1349-4198
pp. 5473–5486

METHOD OF STRUCTURAL ANALYSIS FOR STATICALLY INDETERMINATE BEAMS

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Received April 2011; revised August 2011

Abstract. *This paper proposes a method for analysis of statically indeterminate beams, considering the shear deformations, which is an extension to the slope-deflection method, which is used to analyze all kinds of continuous beams. This methodology considers the shear deformation and flexure. The traditional method takes into account only the flexure deformation and without taking into account the shear deformation; this is how it usually develops structural analysis of statically indeterminate beams. Also, it makes a comparison between the proposed method and the traditional method, and the differences between both methods are greater, especially members of short length as can be seen in the results tables of the problems considered, in the traditional method not all values are on the side of safety. Therefore, the usual practice, without considering the shear deformations in short clear between its supports, will not be a recommended solution and it is proposed the use of considering shear deformations and also is more attached to reality.*
Keywords: Shear deformations, Poisson's ratio, Moment of inertia, Elasticity modulus, Shear modulus, Shear area

1. **Introduction.** In the structural systems analysis has been studied by diverse researchers in the past, making a brief historical review of progress in this subject.

In 1857, Benoit Paul Emile Clapeyron presented to the French Academy his "theorem of three moments" for analysis of continuous beams, in the same way Bertot had published two years ago in the Memories of the Society of Civil Engineers of France, but without giving some credit. It can be said that from this moment begins the development of a true "Theory of Structures" [1-3].

In 1854, the French Engineer Jacques Antoine Charles Bresse published his book *"Recherches Analytiques sur la Flexion et la Résistance de Pieces Courbés"* in which he presented practical methods for the analysis of curved beams and arcs [1-3].

In 1867, the "Influence Line" was introduced by the German Emil Winkler (1835-1888). He also made important contributions to the Resistance of materials, especially in the flexure theory of curved beams, flexure of beams, resting on elastic medium [1-3].

James Clerk Maxwell (1830-1879), from the University of Cambridge, published what might be called the first systematic method of analysis for statically indeterminate structures, based on the equality of the internal energy of deformation of a loaded structure and the external work done by applied loads, and equality had been established by Clapeyron. In his analysis presented the Theorem of the Reciprocal Deformations, which, by its brevity and lack of enlightenment, was not appreciated at the time. In another publication later presented his diagram of internal forces to trusses, which combines in one figure all the polygons of forces. The diagram was extended by Cremona, by what is known as the Maxwell-Cremona diagram [1-3].

The Italian Betti in 1872 published a generalized form of Maxwell's theorem, known as the reciprocal theorem of Maxwell-Betti [1-3].

The German Otto Mohr (1835-1918) made great contributions to the Structures Theory. He developed the method for determining the deflections in beam, known as the method of elastic loads or the conjugate beam. He also presented a simple derivation and more extensive, which is the general method of Maxwell for analysis in indeterminate structures, using the principles of virtual work. He made contributions in the graphical analysis of deflections in trusses, complemented by Williot diagram, known as the Mohr-Williot diagram of great practical utility. He also earned his famous Mohr Circle for the graphical representation of the stresses in a biaxial stress state [1-3].

Alberto Castigliano (1847-1884) in 1873 introduced the principle of minimum work, which had been previously suggested by Menabrea, and is known as the First Theorem of Castigliano. Later, it presented the second Theorem of Castigliano to find deflections, as a corollary of the first. Published in Paris in 1879, his famous book "*Thèoreme de l'Equilibre de Systèmes Elastiques et ses Applications*" was remarkable by its originality and very important in the development of analysis of statically indeterminate structures [1-3].

Heinrich Müller-Breslau (1851-1925), published in 1886 a basic method for analysis of indeterminate structures, but was essentially a variation of those presented by Maxwell and Mohr. He gave great importance to Maxwell's Theorem of Reciprocal Deflections in the assessment of displacement. He discovered that the "influence line" for the reaction or an inner strength of a structure was, on some scale, the elastic produced by an action similar to that reaction, or inner strength. Known as the Müller-Breslau theorem, it is the basis for other indirect methods of structural analysis using models [1-3].

Hardy Cross (1885-1959) professor at the University of Illinois, published in 1930 his famous moments distribution method, can be said that revolutionized the analysis of structures of reinforced concrete for continuous frames and can be considered one of the greatest contributions to the analysis from indeterminate structures. This method of successive approximations evades solving systems of equations, as presented in the methods of Mohr and Maxwell. The Method's popularity declined with the availability of computers, with which the resolution of equations systems is no longer a problem. The general concepts of the method were later extended in the study of pipes flow. Later became popular methods of Kani and Takabeya also type iterative and today in disuse [1-4].

In the early 50s, Turner, Clough, Martin and Topp did what may be termed as the beginning of the implementation structures of the stiffness matrix methods, which have gained so much popularity today. Subsequently, it is developed the finite element methods, which have allowed the systematic analysis of large numbers of structures and obtain the forces and deformations in complex systems such as concrete dams used in hydroelectric plants. Among its impellers are: Clough, Wilson, Zienkiewics and Gallagher [1,2,5].

Structural analysis is the study of structures such as discrete systems. The theory of the structures is essentially based on the fundamentals of mechanics with which are formulated the different structural elements. The laws or rules that define the balance and continuity of a structure can be expressed in different ways, including partial differential equations of continuous medium three-dimensional, ordinary differential equations that define a member or the various theories of beams, or simply algebraic equations for a discretized structure. While it is deepened more in the physics of the problem, are developing theories that are most appropriate for solving certain types of structures and that they demonstrate to be more useful for practical calculations. However, in each new theory are doing hypotheses about how the system behaves or element. Therefore, we must always be aware of these hypotheses when evaluating solutions, the result of applying or developing theories [6-8].

Structural analysis can be addressed using three main approaches [9]: a) tensor formulations (Newtonian mechanics and vector), b) formulations based on the principles of virtual work, c) formulations based on classical mechanics [10].

In the design of steel structures, reinforced concrete and prestressed concrete, the study of structural analysis is a crucial stage in its design, since the axial forces, shear forces and moments are those that govern the design of rigid frames and for the case of beams only shear forces and moments, and the damage caused by such effects may become predominant among the various requests to consider for your design.

As regards the conventional techniques of structural analysis of continuous beams, the common practice is to neglect the shear deformations.

This paper proposes to consider the shear deformations and a comparison between the proposed method and the traditional method is realized.

2. Development.

2.1. Theoretical principles.

In the scheme of deformation of a beam that is illustrated in Figure 1, shows the difference between the Timoshenko theory and Euler-Bernoulli theory: the first θ_Z y dy/dx not necessarily coincides, while the second are equal [11].

The fundamental difference between the Euler-Bernoulli theory and Timoshenko's theory is that in the first the relative rotation of the section is approximated by the derivative of vertical displacement, this is an approximation valid only for long parts in relation to the dimensions of cross section, and then it happens that due to shear deformations are negligible compared to the deformations caused by moment. On the Timoshenko theory, which considers the deformation due to the shear and is therefore also valid for short beams, the equation of the elastic curve is given by the complex system of equations:

$$G\left(\frac{dy}{dx} - \theta_Z\right) = \frac{V_y}{A_c} \tag{1}$$

$$E\left(\frac{d\theta_Z}{dx}\right) = \frac{M_z}{I_z} \tag{2}$$

where: G = shear modulus, dy/dx = total rotation around axis "z", θ_Z = rotation around axis "z", due to the flexure, V_y = shear force in direction "y", A_c = shear area, $d\theta_Z/dx = d^2y/dx^2$, E = elasticity modulus, M_z = moment around axis "z", I_z = moment of inertia around axis "z".

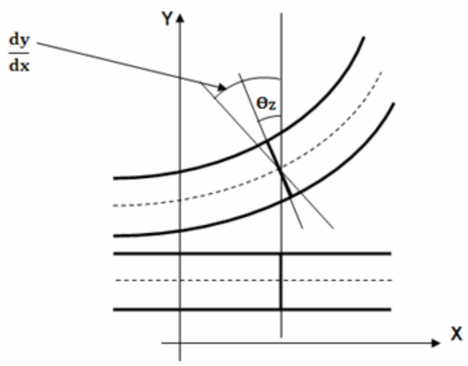

FIGURE 1. Deformation of a beam element

Differentiating Equation (1) and substituting in Equation (2), it is arrived at the equation of the elastic curve including the effect of shear stress:

$$\frac{d^2y}{dx^2} = \frac{1}{GA_c}\frac{dV_y}{dx} + \frac{M_z}{EI_z} \tag{3}$$

From Equation (1), it is obtained dy/dx:

$$\frac{dy}{dx} = \frac{V_y}{GA_c} + \theta_Z \tag{4}$$

And of Equation (2), it is given θ_Z:

$$\theta_Z = \int \frac{M_z}{EI_z}dx \tag{5}$$

Now substituting Equation (5) into Equation (4) is:

$$\frac{dy}{dx} = \frac{V_y}{GA_c} + \int \frac{M_z}{EI_z}dx \tag{6}$$

2.2. **General conditions.** The slope-deflection method can be used to analyze all types of statically indeterminate beams. In this method all joints are considered rigid; i.e., the angles between members at the joints are considered not to change in value, when the loads are applied. Thus the joints at the interior supports of statically indeterminate beams can be considered 180° rigid joints. When beams are deformed, the rigid joints are considered to rotate only as a whole; in other terms, the angles between the tangents to the various branches of the elastic curve meeting at a joint remain the same as those in the original undeformed structure.

In the slope-deflection method the rotations of the joints are treated as unknowns. Then the end moments can be expressed in terms of the rotations. But, to satisfy the condition of equilibrium, the sum of the end moments which any joint on the ends of members exert in meeting must be zero, because the rigid joint in matter is subjected to the sum of these end moments.

These procedures solve the equation system of rotations for statically indeterminate beams or continuous beams. Therefore, it is important to remember the hypotheses under which the equations are deduced: a) the material is homogeneous, isotropic and behaves as linear elastic, i.e., the material is of the same nature, have identical physical properties in all directions and efforts, which resists, are directly proportional to the deformations that suffering, and the proportionality factor is called the elasticity modulus, E, i.e., $\sigma = E\varepsilon$ (Hooke's Law), b) the principle of the small deformations, which once loaded structure, the deformations or linear displacements and angular of the joints and of each of the points of its members are rather small in such a way that form do not change, nor are altered appreciably, c) the principle of effects superposition, that supposes the totals displacements and internals forces totals of the structure under a system of loads, can be found separately by the sum of the effects of each one of the considered loads, d) you can only take into account the first order effects such as: internal deformations by flexure always, while the shear deformations can be taken into account or not.

2.3. **Slope-deflection equations.** The slope-deflection equations, the moments acting at the ends of the members are expressed in terms of rotations and the loads on members. Then, the member AB is shown in Figure 2(a) can be expressed in terms of θ_A and θ_B and the applied loads P_1, P_2 and P_3. Counterclockwise the end moments that acting on the members are considered to be positive, and clockwise end moments that acting on the members are considered to be negative. Now, with the applied loading on the member, the fixed-end moments, M_{FAB} and M_{FBA} are required to hold the tangents at the ends fixed in Figure 2(b). The additional end moments, M'_{AB} and M'_{BA}, should be such as to cause rotations of θ_A and θ_B, respectively. If θ_{A1} and θ_{B1} are the end rotations caused by M'_{AB}, according to Figure 2(c), and θ_{A2} and θ_{B2} by M'_{BA}, they are observed in Figure 2(d).

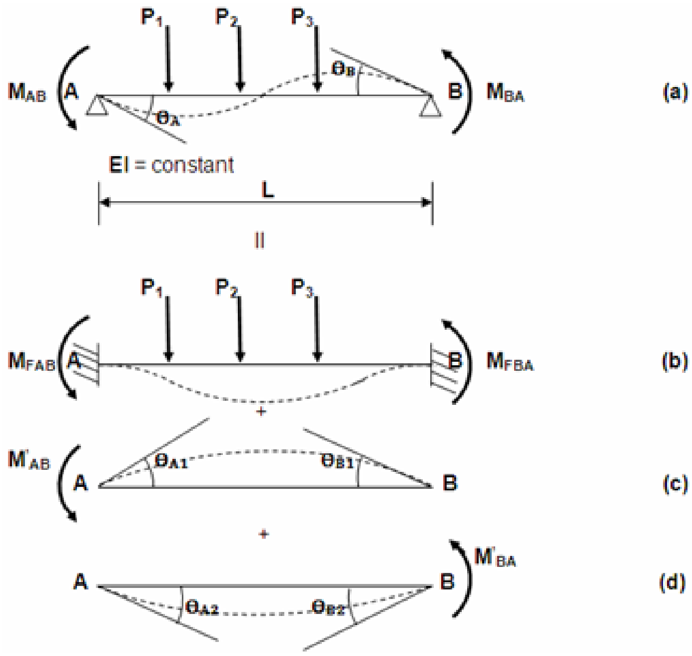

Figure 2. Derivation of slope-deflection equations

The conditions required of geometry are [12-16]:

$$\begin{aligned} \theta_A &= -\theta_{A1} + \theta_{A2} \\ \theta_B &= \theta_{B1} - \theta_{B2} \end{aligned} \tag{7}$$

By superposition:

$$\begin{aligned} M_{AB} &= M_{FAB} + M'_{AB} \\ M_{BA} &= M_{FBA} + M'_{BA} \end{aligned} \tag{8}$$

The beam of Figure 2(c) is analyzed to find θ_{A1} and θ_{B1} in function of M'_{AB}:

It is considered that $V_A = V_B$, doing the sum of moments in B and obtaining M'_{AB}, in function of V_A, it obtains:

$$M'_{AB} = V_A L \tag{9}$$

Therefore, the shear forces and moments at a distance "x" are:

$$V_x = \frac{M'_{AB}}{L} \tag{10}$$

$$M_x = \frac{M'_{AB}}{L}(L - x) \tag{11}$$

Substituting M_x and V_x in function of M'_{AB} into Equation (6), and are separated shear deformation and flexure to obtain the stiffness and is presented as follows:

Shear deformation:

$$\frac{dy}{dx} = \frac{M'_{AB}}{GA_c L} \tag{12}$$

Integrating Equation (12) is presented as follows:

$$y = \frac{M'_{AB}}{GA_cL}x + C_1 \tag{13}$$

Considering the conditions of border, when $x = 0$, $y = 0$, it is of the following way $C_1 = 0$.

$$y = \frac{M'_{AB}}{GA_cL}x \tag{14}$$

Flexure deformation:

$$\frac{dy}{dx} = \frac{M'_{AB}}{EI_zL}\int (L - x)dx \tag{15}$$

Developing the integral, it is obtained:

$$\frac{dy}{dx} = \frac{M'_{AB}}{EI_zL}(Lx - \frac{x^2}{2} + C_2) \tag{16}$$

Integrating Equation (16), it is obtained:

$$y = \frac{M'_{AB}}{EI_zL}\left(\frac{L}{2}x^2 - \frac{x^3}{6} + C_2x + C_3\right) \tag{17}$$

Considering the conditions of border, when $x = 0$, $y = 0$, it is of the following way $C_3 = 0$.

$$y = \frac{M'_{AB}}{EI_zL}\left(\frac{L}{2}x^2 - \frac{x^3}{6} + C_2x\right) \tag{18}$$

Now considering the conditions of border, when $x = L$, $y = 0$, it is of the following way:

$$C_2 = -\frac{L^2}{3} \tag{19}$$

Then, substituting Equation (19) in Equations (16) and (18) is shown as follows:

$$\frac{dy}{dx} = \frac{M'_{AB}}{EI_zL}\left(Lx - \frac{x^2}{2} - \frac{L^2}{3}\right) \tag{20}$$

$$y = \frac{M'_{AB}}{EI_zL}\left(\frac{L}{2}x^2 - \frac{x^3}{6} - \frac{L^2}{3}x\right) \tag{21}$$

Substituting $x = 0$, into Equation (20) to find the rotation in support A due to the flexure deformation θ_{A1F}, it is as follows:

$$\theta_{A1F} = -\frac{M'_{AB}L}{3EI_z} \tag{22}$$

Substituting $x = L$, into Equation (20) to find the rotation in support B due to the flexure deformation θ_{B1F}, it is obtained as follows:

$$\theta_{B1F} = \frac{M'_{AB}L}{6EI_z} \tag{23}$$

If it is considered that they have his curvature radius in the inferior part. Then, the rotations are positive:

$$\begin{aligned} \theta_{A1F} &= +\frac{M'_{AB}L}{3EI_z} \\ \theta_{B1F} &= +\frac{M'_{AB}L}{6EI_z} \end{aligned} \tag{24}$$

The rotation due to shear deformation θ_{A1C} and θ_{B1C}, taking into account the curvature radius is:

$$\begin{aligned}\theta_{A1C} &= \frac{dy}{dx} = \frac{M'_{AB}}{GA_cL} \\ \theta_{B1C} &= \frac{dy}{dx} = -\frac{M'_{AB}}{GA_cL}\end{aligned} \tag{25}$$

Adding the shear rotation and flexure in the joint A, it is obtained:

$$\theta_{A1} = \theta_{A1F} + \theta_{A1C} \tag{26}$$

Substituting Equations (24) and (25) into Equation (26), it is as follows:

$$\theta_{A1} = +\frac{M'_{AB}L}{3EI_z} + \frac{M'_{AB}}{GA_cL} \tag{27}$$

The common factor is obtained in Equation (27) for M'_{AB}, is as follows:

$$\theta_{A1} = \frac{M'_{AB}L}{12EI_z}\left(4 + \frac{12EI_z}{GA_cL^2}\right) \tag{28}$$

By replacing [17,18]:

$$\emptyset = \frac{12EI_z}{GA_cL^2} \tag{29}$$

It is obtained G as follows:

$$G = \frac{E}{2(1+\nu)} \tag{30}$$

where: $\emptyset$ = form factor, and ν = Poisson's ratio.

Then, substituting Equation (29) into Equation (28), it is obtained:

$$\theta_{A1} = \frac{M'_{AB}L}{12EI_z}(4+\emptyset) \tag{31}$$

Adding the shear rotation and flexure in the joint B, and make the simplifications corresponding, it is presents:

$$\theta_{B1} = \frac{M'_{AB}L}{12EI_z}(2-\emptyset) \tag{32}$$

Analyzing the beam in Figure 2(d) to find θ_{A2} and θ_{B2} in function of M'_{BA} of the same way as was done in Figure 2(c), it is obtain the following:

$$\theta_{A2} = \frac{M'_{BA}L}{12EI_z}(2-\emptyset) \tag{33}$$

$$\theta_{B2} = \frac{M'_{BA}L}{12EI_z}(4+\emptyset) \tag{34}$$

Now, substituting Equations (33) and (34) into Equation (7), it is as follows:

$$\theta_A = -\frac{M'_{AB}L}{12EI_z}(4+\emptyset) + \frac{M'_{BA}L}{12EI_z}(2-\emptyset) \tag{35}$$

$$\theta_B = \frac{M'_{AB}L}{12EI_z}(2-\emptyset) - \frac{M'_{BA}L}{12EI_z}(4+\emptyset) \tag{36}$$

Developing Equations (35) and (36), to find M'_{AB} and M'_{BA} in function of θ_A and θ_B, it is as it follows:

$$M'_{AB} = \frac{EI_z}{L}\left[-\left(\frac{4+\emptyset}{1+\emptyset}\right)\theta_A - \left(\frac{2-\emptyset}{1+\emptyset}\right)\theta_B\right] \tag{37}$$

$$M'_{BA} = \frac{EI_z}{L}\left[-\left(\frac{4+\emptyset}{1+\emptyset}\right)\theta_B - \left(\frac{2-\emptyset}{1+\emptyset}\right)\theta_A\right] \tag{38}$$

Finally the substituting Equations (37) and (38) in Equation (8), respectively, it is obtained the slope-deflection equations for statically indeterminate beams:

$$M_{AB} = M_{FAB} + \frac{EI_z}{L}\left[-\left(\frac{4+\emptyset}{1+\emptyset}\right)\theta_A - \left(\frac{2-\emptyset}{1+\emptyset}\right)\theta_B\right] \tag{39}$$

$$M_{BA} = M_{FBA} + \frac{EI_z}{L}\left[-\left(\frac{4+\emptyset}{1+\emptyset}\right)\theta_B - \left(\frac{2-\emptyset}{1+\emptyset}\right)\theta_A\right] \tag{40}$$

3. **Application.** It developed the following structural analysis of continuous steel beam, in four part with equal length, and three different problems, as shown in Figure 3, by the traditional method and the proposed method, i.e., without taking into account and considering the shear deformation, based on the following data, that are presented below:
$w = 3500\text{kg/m}$
$L = 10.00\text{m}; 5.00\text{m}; 3.00\text{m}$
$E = 2040734\text{kg/cm}^2$
Properties of the beamW24X94
$A = 178.71\text{cm}^2$
$A_C = 80.83\text{cm}^2$
$I = 111966\text{cm}^4$
$\nu = 0.32$
Unknowns
θ_A, θ_B, θ_C, θ_D y θ_E

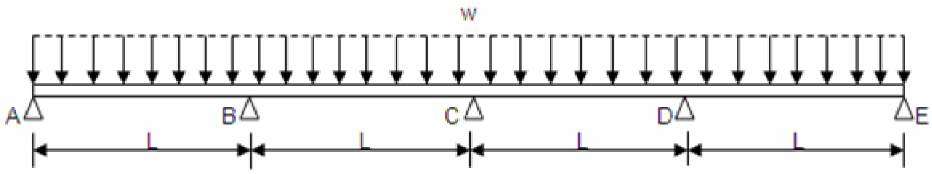

FIGURE 3. Continuous beam in four parts in equal length with uniformly distributed load

Using Equation (30), it is obtained the shear modulus, as follows:

$$G = \frac{2040734}{2(1+0.32)} = 773005.303\text{kg/cm}^2$$

Once that is obtained the shear modulus is found the form factor through Equation (29) as follows:

To 10.00m is:

$$\emptyset_{AB} = \emptyset_{BC} = \emptyset_{CD} = \emptyset_{DE} = \frac{12(2040734)(111966)}{(773005.303)(80.83)(1000)^2} = 0.04388324731$$

To 5.00m is:

$$\emptyset_{AB} = \emptyset_{BC} = \emptyset_{CD} = \emptyset_{DE} = \frac{12(2040734)(111966)}{(773005.303)(80.83)(500)^2} = 0.1755329892$$

To 3.00m is:

$$\text{Ø}_{AB} = \text{Ø}_{BC} = \text{Ø}_{CD} = \text{Ø}_{DE} = \frac{12(2040734)(111966)}{(773005.303)(80.83)(300)^2} = 0.4875916368$$

The fixed-end moments for beams with uniformly distributed load are:
To 10.00m is:

$$M_{FAB} = M_{FBC} = M_{FCD} = M_{FDE} = \frac{wL^2}{12} = +\frac{(3500)(10.00)^2}{12} = +29166.67\text{kg-m}$$

$$M_{FBA} = M_{FCB} = M_{FDC} = M_{FED} = -\frac{wL^2}{12} = -\frac{(3500)(10.00)^2}{12} = -29166.67\text{kg-m}$$

To 5.00m is:

$$M_{FAB} = M_{FBC} = M_{FCD} = M_{FDE} = \frac{wL^2}{12} = +\frac{(3500)(5.00)^2}{12} = +7291.67\text{kg-m}$$

$$M_{FBA} = M_{FCB} = M_{FDC} = M_{FED} = -\frac{wL^2}{12} = -\frac{(3500)(5.00)^2}{12} = -7291.67\text{kg-m}$$

To 3.00m is:

$$M_{FAB} = M_{FBC} = M_{FCD} = M_{FDE} = \frac{wL^2}{12} = +\frac{(3500)(3.00)^2}{12} = +2625\text{kg-m}$$

$$M_{FBA} = M_{FCB} = M_{FDC} = M_{FED} = -\frac{wL^2}{12} = -\frac{(3500)(3.00)^2}{12} = -2625\text{kg-m}$$

Calculation of "EI", for all beams is:

$$EI = (2,040,734)(111,966) = 228492823000\text{kg-cm}^2 = 22849282.3\text{kg-cm}^2$$

Then, substituting, all these values into the corresponding equations for each beam in the traditional method and the proposed method.

The slope-deflection equations, neglecting shear deformations (traditional method) are:

$$M_{AB} = M_{FAB} + \frac{EI}{L}[-4\theta_A - 2\theta_B]$$
$$M_{BA} = M_{FBA} + \frac{EI}{L}[-4\theta_B - 2\theta_A]$$

The slope-deflection equations, considering shear deformations (proposed method) are:

$$M_{AB} = M_{FAB} + \frac{EI}{L}\left[-\left(\frac{4+\text{Ø}}{1+\text{Ø}}\right)\theta_A - \left(\frac{2-\text{Ø}}{1+\text{Ø}}\right)\theta_B\right]$$
$$M_{BA} = M_{FBA} + \frac{EI}{L}\left[-\left(\frac{4+\text{Ø}}{1+\text{Ø}}\right)\theta_B - \left(\frac{2-\text{Ø}}{1+\text{Ø}}\right)\theta_A\right]$$

Once that is obtained the moments in each beam as a function of rotations, it is applied the condition equilibrium of moments at the joints, which are:

Joint A:

$$M_{AB} = 0 \tag{I}$$

Joint B:

$$M_{BA} + M_{BC} = 0 \tag{II}$$

Joint C:

$$M_{CB} + M_{CD} = 0 \tag{III}$$

Joint D:

$$M_{DC} + M_{DE} = 0 \tag{IV}$$

Joint E:

$$M_{ED} = 0 \tag{V}$$

These equations are presented in terms of the rotations and in this case, there are 5 equations and 5 rotations (unknowns), these are developed to find their values. Once, that are found rotations, were subsequently substituted into the slope-deflection equations to localize the final moments at the ends of the beams. Now by static equilibrium, shear forces are obtained for each beam. Then, it is obtained the diagrams of shear forces and moments.

Below are the tables and figures with the results.

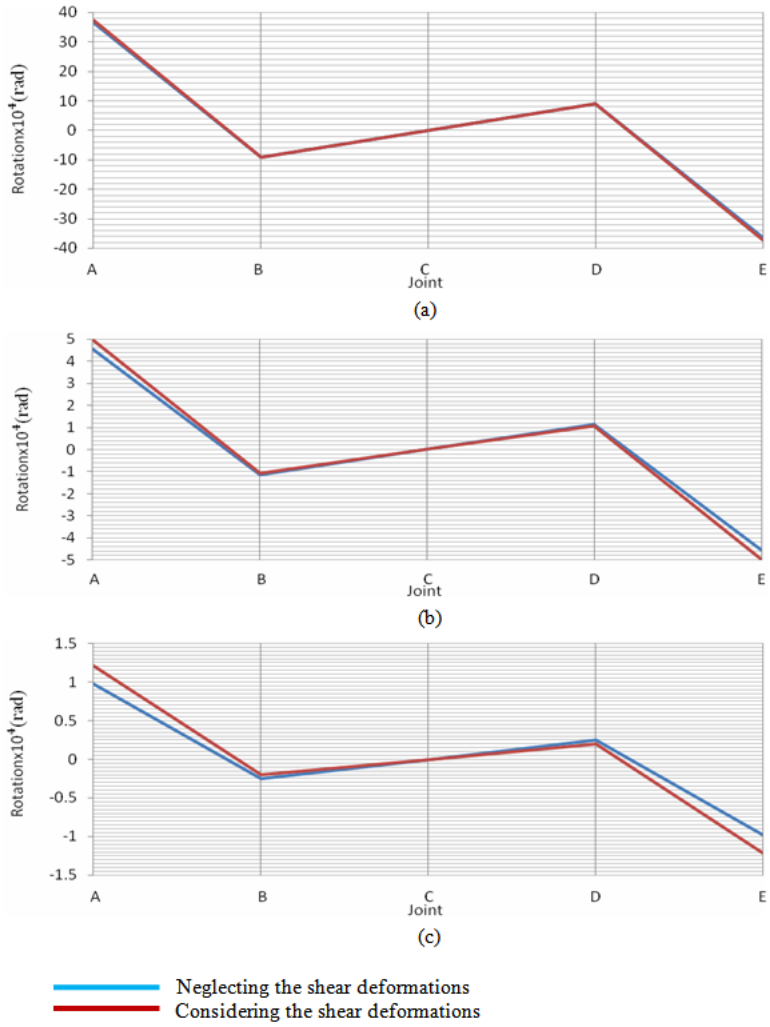

Figure 4. Deformations of the beam: (a) for $L = 10.00$m, (b) for $L = 5.00$m, (c) for $L = 3.00$m

4. **Conclusions.** According to Table 1 and Figure 4, which presents the rotations in each of the supports, it is observed that the difference in the slope-deflection method, neglecting and considering the shear deformations, is quite considerable when the light is reduced between supports of beams and all are not within the safety in the traditional method. This implies that should be taken into account the deformations permitted by the rules of construction, because in some situations could be the case does not comply.

In Table 2 and Figure 5 show the shear forces at the ends of the beams between the two methods, the differences being larger, when the length between supports is reduced.

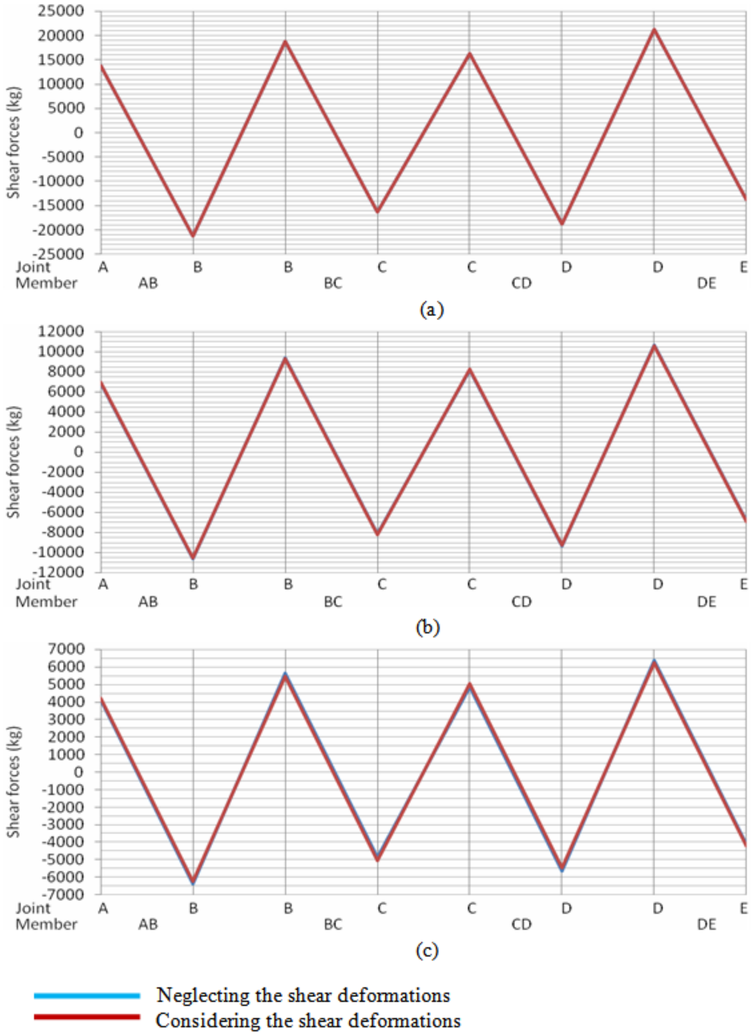

FIGURE 5. Shear forces of the beam: (a) for $L = 10.00$m, (b) for $L = 5.00$m, (c) for $L = 3.00$m

TABLE 1. The rotations in each one of the joints in radians

Rotations	Case 1 $L = 10.00$m			Case 2 $L = 5.00$m			Case 3 $L = 3.00$m		
	N S D	C S D	$\frac{\text{NSD}}{\text{CSD}}$	N S D	C S D	$\frac{\text{NSD}}{\text{CSD}}$	N S D	C S D	$\frac{\text{NSD}}{\text{CSD}}$
$\theta_A \times 10^4$	+36.47	+37.32	0.9772	+4.56	+4.97	0.9175	+0.98	+1.21	0.8099
$\theta_B \times 10^4$	−9.12	−9.03	1.0100	−1.14	−1.08	1.0556	−0.25	−0.20	1.2500
$\theta_C \times 10^4$	0	0	0	0	0	0	0	0	0
$\theta_D \times 10^4$	+9.12	9.03	1.0100	+1.14	+1.08	1.0556	+0.25	+0.20	1.2500
$\theta_E \times 10^4$	−36.47	−37.32	0.9772	−4.56	−4.97	0.9175	−0.98	−1.21	0.8099

θ_i = the angle that forms the tangent due to the deformation in the joint i.

N S D = neglecting the shear deformations

C S D = considering the shear deformations

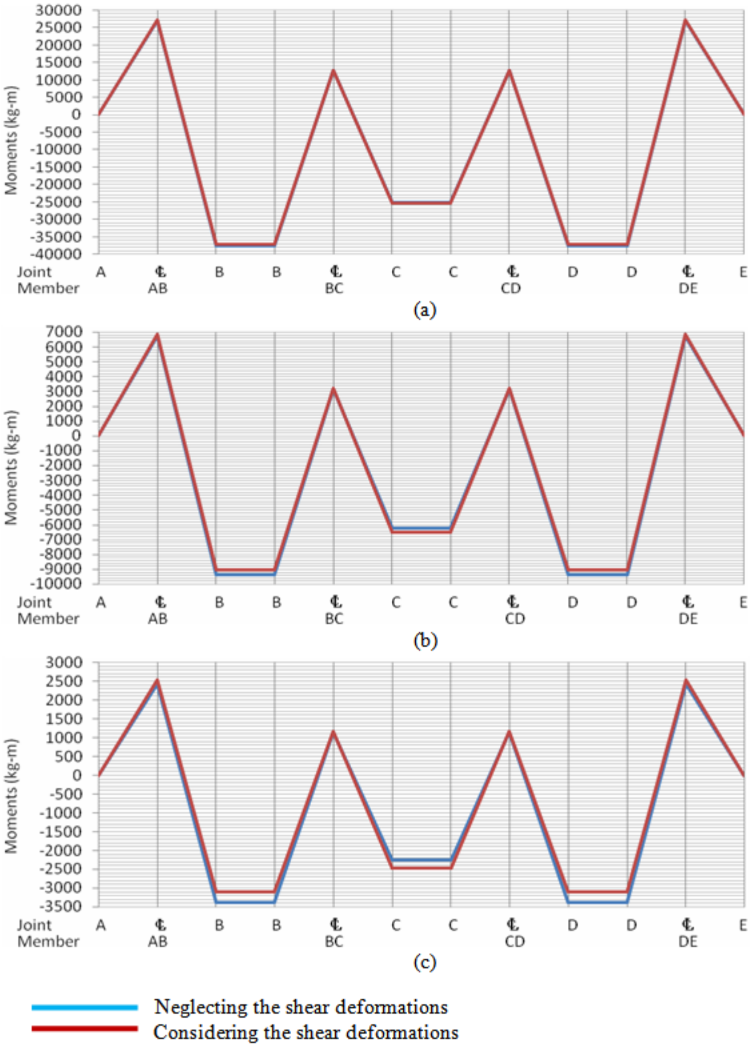

FIGURE 6. Final moments of the beam: (a) for L = 10.00m, (b) for L = 5.00m, (c) for L = 3.00m

TABLE 2. The shear forces in kg

Shear forces	Case 1 L = 10.00m			Case 2 L = 5.00m			Case 3 L = 3.00m		
	N S D	C S D	$\frac{NSD}{CSD}$	N S D	C S D	$\frac{NSD}{CSD}$	N S D	C S D	$\frac{NSD}{CSD}$
V_{AB}	+13750.0	+13784.4	0.9975	+6875.0	+6939.4	0.9907	+4125.0	+4218.7	0.9778
V_{BA}	−21250.0	−21215.6	1.0016	−10625.0	−10560.6	1.0061	−6375.0	−6218.3	1.0252
V_{BC}	+18750.0	+18685.3	1.0035	+9375.0	+9256.1	1.0128	+5625.0	+5459.0	1.0304
V_{CB}	−16250.0	−16314.7	0.9960	−8125.0	−8243.9	0.9856	−4875.0	−5041.0	0.9671
V_{CD}	+16250.0	+16314.7	0.9960	+8125.0	+8243.9	0.9856	+4875.0	+5041.0	0.9671
V_{DC}	−18750.0	−18685.3	1.0035	−9375.0	−9256.1	1.0128	−5625.0	−5459.0	1.0304
V_{DE}	+21250.0	+21215.6	1.0016	+10625.0	+10560.6	1.0061	+6375.0	+6218.3	1.0252
V_{ED}	−13750.0	−13784.4	0.9975	−6875.0	−6939.4	0.9907	−4125.0	−4218.7	0.9778

V_{ij} = Shear forces of the beam ij in end i

V_{ji} = Shear forces of the beam ji in end j

Table 3. The final moments in kg-m

Moments	Case 1 L = 10.00m			Case 2 L = 5.00m			Case 3 L = 3.00m		
	N S D	C S D	N S D / C S D	N S D	C S D	N S D / C S D	N S D	C S D	N S D / C S D
M_{AB}	0	0	0	0	0	0	0	0	0
$M_{℄AB}$	+27008.9	+27144.4	0.9950	+6752.2	+6879.4	0.9854	+2430.8	+2542.5	0.9561
M_{BA}	−37500.0	−37155.6	1.0093	−9375.0	−9052.8	1.0356	−3375.0	−3094.0	1.0908
M_{BC}	−37500.0	−37155.6	1.0093	−9375.0	−9052.8	1.0356	−3375.0	−3094.0	1.0908
$M_{℄BC}$	+12722.8	+12721.8	1.0001	+3180.8	+3186.6	0.9982	+1145.1	+1163.3	0.9844
M_{CB}	−25000.0	−25302.2	0.9881	−6250.0	−6522.2	0.9583	−2250.0	−2466.9	0.9121
M_{CD}	−25000.0	−25302.2	0.9881	−6250.0	−6522.2	0.9583	−2250.0	−2466.9	0.9121
$M_{℄CD}$	+12722.8	+12721.8	1.0001	+3180.8	+3186.6	0.9982	+1145.1	+1163.3	0.9844
M_{DC}	−37500.0	−37155.6	1.0093	−9375.0	−9052.8	1.0356	−3375.0	−3094.0	1.0908
M_{DE}	−37500.0	−37155.6	1.0093	−9375.0	−9052.8	1.0356	−3375.0	−3094.0	1.0908
$M_{℄DE}$	+27008.9	+27144.4	0.9950	+6752.2	+6879.4	0.9854	+2430.8	+2542.5	0.9561
M_{ED}	0	0	0	0	0	0	0	0	0

M_{ij} = Negative moment of the beam ij in end i
$M_{℄AB}$ = Positive moment of the beam ij
M_{BA} = Negative moment of the beam ji in end j

With regard to Table 3 and Figure 6, it is illustrating the final moments, both negative and positive, there are big differences when you reduce the member length between the two methods and not all are on the side of safety.

As for Tables 2 and 3, and in Figures 5 and 6 where they are presented, shear forces and final moments, acting on the beams, these elements are those governing the design of a structure, were studied by traditional method and the proposed method. The results showed that differences between the two methods, when members tend to be shorter, the differences are increased, as in the conservative side as the unsafe side.

This means that this is poorly designed; on the one hand some members are bigger in their transverse dimension, according to what are needed and in another situation does not meet the minimal conditions for a satisfactory beam. Since there are two fundamentals principles of civil engineering, for structural conditions, that have to be safe and economical.

Therefore, the usual practice of using the slope-deflection method (Neglecting shear deformations) is not a recommended solution when having short length between supports.

So taking into account the numerical approximation, the slope-deflection method (considering shear deformations), happens to be the more appropriate method for structural analysis of continuous beams and also more attached to the real conditions.

Acknowledgment. This work is totally supported by the Facultad de Ingenieria, Ciencias y Arquitectura de la Universidad Juárez del Estado de Durango, Gómez Palacio, Durango and México. The author also gratefully acknowledges the helpful comments and suggestions of the reviewers, which have improved the presentation.

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