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ROBUST NONLINEAR CONTROL OF ROBOT MANIPULATOR WITH UNCERTAINTIES IN KINEMATICS, DYNAMICS AND ACTUATOR MODELS

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Abstract. *This paper, according to practical method in robot manipulator control, introduces a novel robust control approach for trajectory tracking of electrically-driven robotic manipulators in task space. A new task space control scheme is proposed to overcome uncertainties of actuator dynamics, robot dynamics and kinematics. A robust controller is designed based on Lyapunov method, using dynamic delay, backstepping method and unknown bounds of uncertainties. It is proven that the closed loop system has global uniform ultimate boundedness stability. Although, for overcoming the uncertainties in actuator dynamics, robot dynamics and kinematics is a major advantage, the proposed control cannot pass the singular points in task space and there will be malfunctions when it is applied on the practical implementation. Modifications are used to derive a control law which is free of velocity terms and can pass the singular points into task space. The control approach is applied on a two-link elbow robotic manipulator which is driven by permanent magnet dc motors and can be applied on up to n-links robotic manipulators, too. The performance of proposed control laws is confirmed by simulations.*
Keywords: Robot manipulators, Uncertain kinematics, Uncertain dynamics, Actuator dynamics, Backstepping method, Task space, Practical implementation

1. **Introduction.** It is well known that the kinematics and dynamics of robots are highly nonlinear with coupling existing between joints. There is a problem in the compatibility of the nonlinearity and uncertainty of the robot dynamics, and to cope with this, it has been shown in [1,2] that a simple joint space controller such as the PD or PID feedback is effective for setpoint control.

A great many control schemes for robotic manipulators have been developed in the literature during the past few decades. In most of the control methods [1-5], the controllers are designed at the torque input level and the actuator dynamics has not been considered. As shown by Good et al. [6], the actuator dynamics constitutes an important part of the whole robot system and may cause detrimental effects when neglected in the design procedure, especially, in the cases of highly varying loads. Recently, the actuator dynamics has been explicitly applied in some joint space control schemes, and especially, it has been developed to deal with this problem at rigid-link robots, as can be found in [7,8].

For most robot applications, a desired position for the end-effector is usually specified in task space or Cartesian space. In order to move the robot end-effector to the desired position, the exact knowledge of the kinematics is required. It is needed to solve the inverse kinematics problem to generate the desired position in joint space [3-5]. When the control problem is formulated directly in task space, the necessity to solve the inverse kinematics problem is eliminated [3-5]. However, these kinds of the schemes still require the exact knowledge of a Jacobian matrix from joint space to task space. In a sense, the exact parameters of the kinematics are required to calculate the Jacobian matrix.

Miyazaki and Masutani proposed a feedback control law with imperfect rotation transformation of the mapping from joint space to visual space [9]. Other task space control schemes are proposed later [10,11]. To apply these task space control schemes, it is assumed that the model of manipulator Jacobian matrix from joint space to Cartesian space is exactly known. Therefore, the exact parameters of manipulator kinematics from joint space to Cartesian space such as exact lengths of links and object are still required in the controller. Unfortunately, no physical parameters can be derived exactly. Moreover, when the robot picks up objects or tools in different lengths, unknown orientations and gripping points, the overall kinematics is going to be changed and, therefore, it is going to be difficult to derive exactly. Thus, the robot would not be able to manipulate the tool to a desired path if the length or gripping point of the tool is uncertain.

To overcome the problem of uncertain kinematics, several approximate Jacobian setpoint controllers were proposed recently [12,13]. The proposed controllers do not require the exact knowledge of kinematics and Jacobian matrix that is assumed in the literature of robot control.

However, the results in [12,13] are focusing on setpoint control of robot. Recently, an adaptive Jacobian controller was proposed for trajectory tracking control of robot manipulators [14]. The controller does not require the exact knowledge of kinematics and Jacobian matrix. However, it is assumed that the actuator model is known exactly in [14]. Since the actuator model may be uncertain in practice, calibration is necessary to identify the exact parameters of the actuator in implementing the robot controllers. In addition, the actuator parameters could change as temperature varies due to the overheating of motor or changes in ambient temperature. Hence, in the presence of the modeling uncertainty or calibration error, the convergence of the tracking error may not be guaranteed.

An adaptive Jacobian controller was proposed for trajectory tracking control of robot manipulators recently [15]. The controller does not require the exact knowledge of kinematics, Jacobian matrix, dynamics and actuator models. However, it is assumed in [15] that the dynamic model has parametric uncertainties, whereas, the unstructured uncertainties such as friction and disturbance in model dynamic may make the closed loop system unstable.

The reminder of this paper is organized as follows. In Section 2, the mathematical models of an n-link electrically driven robot manipulator are given and the tracking problem is considered. Section 3 presents control design based on delay technique. Backstepping method and establish of backstepping control are explained in Sections 3.1 and 3.2, respectively. In Section 4, stability proof of closed loop control is illustrated and practical mention of proposed control is considered in Section 5. According to notations of Section 5, proposed control law is modified in Section 6. Section 8 demonstrates the simulation results which are obtained by carrying out a case study of a two-link elbow robot provided in Section 7 and some conclusions are drawn in Section 9, finally.

2. **Plant Dynamics.** If a direct current (dc) motor driven by an amplifier is used as an actuator at each joint of the robot, the dynamics of the robot with degree of freedom can

be expressed as [16,17]

$$D\left(q\right)\ddot{q}+V_m\left(q,\dot{q}\right)\dot{q}+G\left(q\right)+B\dot{q}+F_d\dot{q}+F_s\left(\dot{q}\right)+T_d=Ku\left(t\right) \tag{1}$$

where $q\left(t\right)\in R^n$ denotes the joint angles of the manipulator, $\dot{q}\left(t\right)$ and $\ddot{q}\left(t\right)$ are the vectors of joint velocity and joint acceleration, respectively. $D\left(q\right)\in R^{n\times n}$ is the inertia matrix which is symmetric and positive definite, $V_m\left(q,\dot{q}\right)\dot{q}\in R^n$ is a vector function containing coriolis and centrifugal forces, $G\left(q\right)\in R^n$ is a vector function consisting of gravitational forces. $B\in R^{n\times n}$ is a diagonal matrix of the effective damping of actuators, $F_d\in R^{n\times n}$ is a diagonal matrix of viscous and dynamic friction coefficients, $F_s\left(\dot{q}\right)\in R^n$ is the vector of unstructured friction effects such as static friction terms. $T_d\in R^n$ is the vector of any generalized input due to disturbances or un-modeled dynamics, $u\left(t\right)\in R^n$ is a voltage or current inputs to the amplifiers and $K\in R^{n\times n}$ is a diagonal transmission matrix that relates the actuator input $u\left(t\right)$ to the control torque τ.

Constraint 2.1. The maximum voltage that joint actuator can supply is $u^{\max}$. So we have:

$$|u_i|\le u_i^{\max},\quad i=1,2,\ldots,n$$

For simplicity Equation (1), $h\left(q,\dot{q}\right)$ can be shown as:

$$h\left(q,\dot{q}\right)=V_m\left(q,\dot{q}\right)\dot{q}+G\left(q\right)+B\dot{q}+F_d\dot{q}+F_s\left(\dot{q}\right)+T_d \tag{2}$$

By substituting Equation (2) into Equation (1), we have:

$$D\left(q\right)\ddot{q}+h\left(q,\dot{q}\right)=Ku\left(t\right) \tag{3}$$

3. **Control Design Based on Delay Technique.** In Equation (3), $\hat{D}(q)\ddot{q}$ is added and subtracted

$$D(q)\ddot{q}+h(q,\dot{q})+\hat{D}(q)\ddot{q}-\hat{D}(q)\ddot{q}=Ku(t) \tag{4}$$

By defining $D(q)-\hat{D}(q)=\Delta D(q)$, Equation (4) is rearranged as:

$$\hat{D}(q)\ddot{q}+h(q,\dot{q})+\Delta D(q)\ddot{q}=Ku(t) \tag{5}$$

By defining $h(q,\dot{q})+\Delta D(q)\ddot{q}=\tilde{h}(t)$, Equation (5) is simplified as:

$$\hat{D}(q)\ddot{q}+\tilde{h}(t)=Ku(t) \tag{6}$$

A control law is presented as:

$$Ku(t)=\hat{D}(q)\hat{J}^{-1}(q)\nu+\hat{h}(t) \tag{7}$$

where $\hat{D}(q)$ can be simplified version of the known parts of $D(q)$, $\hat{h}(t)$ is estimated of $\tilde{h}(t)$, $\hat{J}^{-1}(q)$ is the inverse of Jacobian matrix and ν is a new control vector. We now replace $\tilde{h}(t)$ in (6) with an estimate $\hat{h}(t)$, as $\hat{h}(t)=\tilde{h}(t-\lambda)$, where λ is a small time interval.

Assumption 3.1. The approximation is based on the assumption that $\tilde{h}(t)$ does not change its value much during a small time interval λ, a valid assumption in free space, but possibly a poor assumption for constrained motion, as external torques can be arbitrarily large and fast.

With this assumption, we set $\hat{h}(t)$ from (6) as:

$$\hat{h}(t)=\tilde{h}(t-\lambda)=Ku(t-\lambda)-\hat{D}(q)\ddot{q}(t-\lambda)\cong\tilde{h}(t) \tag{8}$$

According to Equation (7) and Equation (8) we have

$$Ku(t)=\hat{D}(q)\hat{J}^{-1}(q)v+\ Ku(t-\lambda)-\hat{D}(q)\ddot{q}(t-\lambda) \tag{9}$$

By defining $Ku(t-\lambda) - \hat{D}(q)\ddot{q}(t-\lambda) - \tilde{h}(t) = \Delta h$, (9) is substituted into (6) and is simplified as:

$$\ddot{q} = \hat{J}^{-1}(q)\nu + \hat{D}^{-1}(q)\Delta h \tag{10}$$

$\hat{J}(q)$ is multiplied in both side of (10)

$$\hat{J}(q)\ddot{q} = \nu + \hat{J}(q)\hat{D}^{-1}(q)\Delta h \tag{11}$$

The velocity vector in task space $\dot{X}$ is therefore related to the velocity vector $\dot{q}$ as [17]:

$$\dot{X} = J(q)\dot{q} \tag{12}$$

where $J(q) \in R^{n\times n}$ is the jacobian matrix of mapping from joint space to task space. Notice that if the robots kinematics is uncertain, the jacobian matrix becomes uncertain, too. Thus $J(q)$ is estimated by $\hat{J}(q)$. The derivative of Equation (12) respect to time can be written as:

$$\ddot{\hat{X}} = \hat{J}(q)\ddot{q} + \dot{\hat{J}}(q)\dot{q} \tag{13}$$

where $\ddot{\hat{X}}$ is acceleration in the task space. By defining $\nu = w - \dot{\hat{J}}(q)\dot{q}$ and according to (13), Equation (11) is transferred to Task space as:

$$\ddot{\hat{X}} = w + \hat{J}(q)\hat{D}^{-1}(q)\Delta h \tag{14}$$

Control law w is selected to the following form

$$w = \Delta w + \ddot{X}_d \tag{15}$$

where $\ddot{X}_d$ is desired acceleration in the task space and Δw is a new control law. By defining position error $\hat{e} = \hat{X} - X_d$, acceleration error $\ddot{\hat{e}} = \ddot{\hat{X}} - \ddot{X}_d$, structured and unstructured uncertainty $\eta = \hat{J}(q)\hat{D}^{-1}(q)\Delta h$, (14) is simplified as:

$$\ddot{\hat{e}}(t) = \Delta w + \eta \tag{16}$$

By defining $\hat{X}_1 = \hat{e}(t)$ and $\hat{X}_2 = \dot{\hat{e}}(t)$, (16) is expressed as:

$$\begin{cases} \dot{\hat{X}}_1(t) = \hat{X}_2(t) \\ \dot{\hat{X}}_2(t) = \Delta w + \eta \end{cases} \tag{17}$$

3.1. **Backstepping method.** The form of Equation (17) is shown that the backstepping method can be used for control of closed loop system (17) [18]. Therefore, $\hat{X}_2(t)$ is designed as the input which causes $\hat{X}_1(t)$ converges to zero. Hence, $\hat{X}_2(t)$ is expressed as:

$$\hat{X}_2(t) = -\mu \hat{X}_1(t), \quad \forall \mu > 0 \tag{18}$$

To prove the stability of system presented by Equation (17), the Lyapunov function candidate is presented as:

$$V\left(\hat{X}_1\right) = \frac{1}{2}\hat{X}_1^T(t)\hat{X}_1(t) \tag{19}$$

The derivative of Equation (19) respect to time can be written as:

$$\dot{V}\left(\hat{X}_1\right) = \dot{\hat{X}}_1^T(t)\hat{X}_1(t) \tag{20}$$

According to (17), (18) and (20), we have

$$\dot{V}\left(\hat{X}_1\right) = -\mu \hat{X}_1^T(t)\hat{X}_1(t) \le 0 \tag{21}$$

(21) is shown $\hat{X}_1(t)$ that is error in task space convergence to zero.

3.2. **Establish of backstepping control.** To establish (18), we select sliding surfaces Z to the following form

$$Z = X_2\left(t\right) + \mu X_1(t) \tag{22}$$

where $X_1(t)$ and $X_2(t)$ are $X - X_d$ and $\dot{X} - \dot{X}_d$, respectively. X, $\dot{X}$, X_d and $\dot{X}_d$ are end-effector position, end-effector velocity, desired position in task space and desired velocity in task space, respectively. The derivative of Equation (22) respect to time can be written as:

$$\dot{Z} = \dot{X}_2\left(t\right) + \mu\,\dot{X}_1\left(t\right) \tag{23}$$

According to (17), it is clear that jacobian matrix has uncertainties therefore $\dot{X}_2(t) \cong \Delta w + \eta$ because $X_1\left(t\right)$ and $X_2\left(t\right)$ can be measured by sensors or vision technique instead of we compute them. $\dot{X}_2(t) \cong \Delta w + \eta$ is substituted into (23)

$$\dot{Z} = \Delta w + \eta + \mu\,\dot{X}_1\left(t\right) \tag{24}$$

Control law is expressed as:

$$\Delta w = \gamma - \mu\,\dot{X}_1\left(t\right) \tag{25}$$

where γ is new robust control that is proposed for compensation of structured and unstructured uncertainties. (25) is substituted into (24)

$$\dot{Z} = \gamma + \eta \tag{26}$$

According to $\eta = \hat{J}(q)\hat{D}^{-1}(q)\Delta h$, that is contained of the structured and unstructured uncertainties therefore we can expressed the second assumption of proposed control.

Assumption 3.2. According to physical properties of robot dynamics, we can express as [16,17]:

$$\mu_1 I \le D\left(q\right) \le \mu_2 I \quad \text{or} \quad \mu_1 \le \left\|D\left(q\right)\right\| \le \mu_2 \tag{27}$$

where $\|\circ\|$ stand for the Euclidean norm, μ_1 and μ_2 are positive constant. It is clear that $\hat{J}(q)$ is bound therefore we can select a properly delay λ until the following assumption is established

$$\|\eta\| \le \rho \tag{28}$$

where ρ is a positive constant.

4. **Stability Proof.** To prove the stability of closed loop system, the Lyapunov function candidate is presented as:

$$V(Z) = \frac{1}{2}Z^T Z \tag{29}$$

The derivative of Equation (29) respect to time can be written as:

$$\dot{V}(Z) = Z^T\dot{Z} \tag{30}$$

(26) is substituted into (30)

$$\dot{V}(Z) = Z^T\gamma + Z^T\eta \tag{31}$$

In order to satisfy $\dot{V}(Z) \le 0$, it is sufficient to propose a following control law:

$$\gamma = \frac{-Z\,\rho}{\|Z\|} \tag{32}$$

Under the above control law and with attention of Section 3.1, $V(Z)$ is then a Lyapunov function and thus the equilibrium point $e = 0$ is asymptotically stable except for $Z = 0$ since the control law is not defined at $Z = 0$ as stated by Equation (32). Thus, $e \to 0$ as

$t \to \infty$ but never reaches $e = 0$. Chattering is then appeared as a side effect of switching rule (32). In order to attenuate chattering problem, the control law (32) is modified as

$$\gamma = \begin{cases} \frac{-\rho Z}{\|Z\|} & \|Z\| > \sigma \\ \frac{-\rho Z}{\sigma} & \|Z\| < \sigma \end{cases} \tag{33}$$

where σ is a small positive constant. We can express totally control law as

$$\begin{cases} Ku(t) = \hat{D}(q)\hat{J}^{-1}(q)\nu + \left\{Ku(t-\lambda) - \hat{D}(q)\ddot{q}(t-\lambda)\right\} \\ \nu = (\gamma - \mu\,\dot{e}(t) + \ddot{X}_d) - \dot{\hat{J}}(q)\dot{q} \\ Z = \dot{e}(t) + \mu\,e(t) \\ \gamma = \begin{cases} \frac{-\rho Z}{\|Z\|} & \|Z\| > \sigma \\ \frac{-\rho Z}{\sigma} & \|Z\| < \sigma \end{cases} \end{cases} \tag{34}$$

5. **Practical Mention of the Proposed Control Law.** Sensing requirements is another important problem which has to be considered. The control law (34) is formed by measuring joint positions q, the joint velocities $\dot{q}$ and the end-effector positions X and the end-effector velocities $\dot{X}$. A joint position is commonly measured by an optical encoder and a joint velocity may be measured directly or by soft derivative of joint position. Meanwhile, many commercial sensors are available for measurement of X, such as vision systems, electromagnetic measurement systems, position sensitive detectors or laser tracking systems. However, $\dot{X}$ is rarely measured in robotic applications while vision technique can be used for this purpose. Alternatively, vision technique was used to measure the end-effector position X, precisely, and then $\dot{X}$ can be computed.

In addition of above sentences, there is an inverse jacobian matrix in the control law (34), so, it is assumed that there are no singular points in the desired path in task space like the one which the jacobian matrix becomes a full rank matrix. In the next section, above constraints are modified in the proposed control law.

6. **Modifying the Control Law.** In practical purposes aspect, in (34), sliding surface can be modified as

$$Z = \hat{X}_2(t) + \mu X_1(t) \tag{35}$$

where $X_1(t)$ and $\hat{X}_2(t)$ are $X - X_d$ and $\dot{\hat{X}} - \dot{X}_d$, respectively. In (35), end-effector positions X can be measured by sensors or vision system and end-effector velocities $\dot{\hat{X}}$ can be computed by Equation (12). In the case of passing the singular points, to avoid singularity problem, transpose jacobian can be used in substitution of inverse jacobian in the control law (34) [19]. Thus, we can express modified control law to the following form

$$\begin{cases} Ku(t) = \hat{D}(q)\hat{J}^{T}(q)\nu + \left\{Ku(t-\lambda) - \hat{D}(q)\ddot{q}(t-\lambda)\right\} \\ \nu = \left(\gamma - \mu\left(\hat{J}\,\dot{q} - \dot{X}_d\right) + \ddot{X}_d\right) - \dot{\hat{J}}(q)\dot{q} \\ Z = \left(\hat{J}\,\dot{q} - \dot{X}_d\right) + \mu e(t) \\ \gamma = \begin{cases} \frac{-\rho Z}{\|Z\|} & \|Z\| > \sigma \\ \frac{-\rho Z}{\sigma} & \|Z\| < \sigma \end{cases} \end{cases} \tag{36}$$

Remark 6.1. *The control law (36) does not have the practical constraint problems of the control law (34) and robot manipulator also can pass the singular points in task space by this control law.*

Remark 6.2. *In comparison with the control laws (34) and (36), it is shown that in the select of sliding surface Z and due to the mentioned reasons in the Section 5, "$\hat{J}\dot{q}$" is used instead of $\dot{X}$. So the practical implementation problems of control law (34) are solved by this modification.*

Remark 6.3. *According to practical considerations in robot manipulator control design, another significant advantage of modification control law is the simplicity of its implementation. In addition, its design does not rely on the boundaries of dynamic and kinematic uncertainties and disturbances in the robot manipulator model. While, in many proposed tracking controller for robot manipulator, the boundaries of uncertainties must be known in joint space and also in task space.*

7. **Case Study of Two-Link Elbow Robot Manipulator.** In order to verify the performance of proposed control scheme, as an illustration, we will apply the above presented controller to a two-link elbow robot manipulator driven by permanent dc motors as shown in Figure 1. The dynamic of the two-link elbow robot manipulator can be described in the following differential equations [16,17]:

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + B\dot{q} + F_d\dot{q} + F_s + T_d = Ku(t) \tag{37}$$

$$D_{11} = m_1 l_{c_1}^2 + m_2(l_1^2 + l_{c_2}^2 + 2l_1 l_{c_2}\cos(q_2)) + I_1 + I_2 + \frac{J_{m_1}}{r_1^2} \tag{38}$$

$$D_{12} = D_{21} = m_2\left(l_{c_2}^2 + l_1 l_{c_2}\cos(q_2)\right) + I_2 \tag{39}$$

$$D_{22} = m_2 l_{c_2}^2 + I_2 + \frac{J_{m_2}}{r_2^2} \tag{40}$$

$$V(q,\dot{q}) = \begin{bmatrix} -m_2 l_1 l_{c_2}\dot{q}_2\sin(q_2) & -m_2 l_1 l_{c_2}(\dot{q}_1+\dot{q}_2)\sin(q_2) \\ m_2 l_1 l_{c_2}\dot{q}_1\sin(q_2) & 0 \end{bmatrix} \tag{41}$$

$$G(q) = \begin{bmatrix} (m_1 l_{c_1} + m_2 l_1) g\cos(q_1) + m_2 l_{c_2} g\cos(q_1+q_2) \\ m_2 l_{c_2} g\cos(q_1+q_2) \end{bmatrix} \tag{42}$$

$$F_d = \begin{bmatrix} F_{d_1} & 0 \\ 0 & F_{d_2} \end{bmatrix} \tag{43}$$

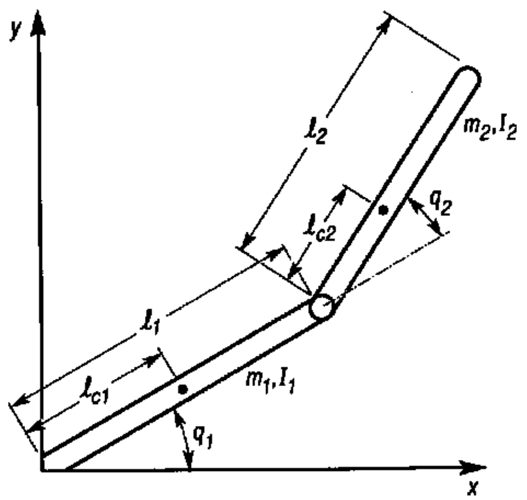

FIGURE 1. Two-link elbow robot manipulator

$$F_s = \begin{bmatrix} F_{s_1} \\ F_{s_2} \end{bmatrix} \tag{44}$$

$$T_d = \begin{bmatrix} T_{d_1}\sin(t) \\ T_{d_2} \end{bmatrix} \tag{45}$$

$$B = \begin{bmatrix} \frac{\left(B_{m_1}+\frac{K_{b_1}K_{m_1}}{R_1}\right)}{r_1^2} & 0 \\ 0 & \frac{\left(B_{m_2}+\frac{K_{b_2}K_{m_2}}{R_2}\right)}{r_2^2} \end{bmatrix} \tag{46}$$

$$K = \begin{bmatrix} \frac{K_{m_1}}{r_1R_1} & 0 \\ 0 & \frac{K_{m_2}}{r_2R_2} \end{bmatrix} \tag{47}$$

where q_i for $i = 1, 2$ denotes the joint angle, l_i is the link length, m_i is the link mass, I_i is the link's moment of inertia given in center of mass, l_{c_i} is the distance between the center of mass of link and the "i"th joint, J_{m_i} is the sum of the actuator and gear inertias, r_i is gear ratio, F_{d_i} is dynamic friction, F_{s_i} is static friction, T_{d_i} is disturbance and un-model dynamic, B_{m_i} is the coefficient of motor friction and includes friction in the brushes and gears, K_{m_i} is the torque constant, K_{b_i} is the back emf constant, R_i is armature resistance and $u(t)$ is armature voltage. The Jacobian matrix is in the form of

$$J(q) = \begin{bmatrix} -l_1\sin(q_1) - l_2\sin(q_1+q_2) & -l_2\sin(q_1+q_2) \\ l_1\cos(q_1) + l_2\cos(q_1+q_2) & l_2\cos(q_1+q_2) \end{bmatrix} \tag{48}$$

The kinematic equation is given by

$$X = \begin{bmatrix} l_1\cos(q_1) + l_2\cos(q_1+q_2) \\ l_1\sin(q_1) + l_2\sin(q_1+q_2) \end{bmatrix} \tag{49}$$

The link's parameters are estimated by a gain of 0.9 from real values given in Table 1. We set the controller with $\mu = 10$, $\sigma = 0.1$, $\lambda = 0.001$ and $\rho = 5$. Desired path in task space and initial condition are expressed in Table 2.

Table 1. Parameters of two link elbow robot

$l_1 = 1$	$l_2 = 1$	$l_{c_1} = 0.5$
$l_{c_2} = 0.5$	$m_1 = 15$	$m_2 = 6$
$I_1 = 5$	$I_2 = 2$	$F_{d_1} = F_{d_2} = 1$
$F_{s_1} = F_{s_2} = 1$	$T_{d_1} = T_{d_2} = 10$	$g = 9.8$
$J_{m_1} = J_{m_2} = 0.0001$	$r_1 = r_2 = 0.01$	$R_1 = R_2 = 1$
$B_{m_1} = B_{m_2} = 0.01$	$K_{m_1} = K_{m_2} = 0.01$	$K_{b_1} = K_{b_2} = 0.01$

Table 2. Desired path and initial condition

$X_d = 0.95 + 0.05\sin(3t)$	$Y_d = 0.95 + 0.05\cos(3t)$
$X_d(0) = 0.95$	$Y_d(0) = 1$

8. **Simulation Results.**

Simulation 1. The task space control given by (34) is simulated with $\rho = 5$. We cannot figure out any differences between the desired and actual trajectories as shown in Figure 2. The control inputs are under the permitted values of 150 V as shown in Figure 3. The norm of tracking error in the task space has been reduced efficiently as it has a maximum value of 1.6 mm as shown in Figure 4. The tracking performance is enhanced by increasing the bounding function ρ as illustrated in Figure 5. The simulation results show norm of errors for given values of 5, 10 and 20 to ρ, respectively.

According to above results, it is concluded that the proposed control without chattering as well as overcome to dynamic, kinematic and actuator model uncertainties. In Figure 3, we conclude that the input controls do not saturate and remain at an acceptable level. Despite all these advantages, the proposed control cannot pass the singular points, consequently, it is difficult for practical implementation of this controller due to the reasons stated in Section 5. In the next section, the simulation results of modified control (36) are going to show that the above defects have been resolved.

Simulation 2. The modified control law (36) is compared with (34) on norm of tracking error while parameters are the same as sim1 and $\rho = 5$. However, its norm of tracking error is larger than control law (34) with a maximum value of 2 mm as shown in Figure 6. We can reduce the maximum norm of tracking error by increasing ρ in the modified control law as is shown in Figure 7. The simulation results show norm of errors for given values of 5 and 25 to ρ, respectively.

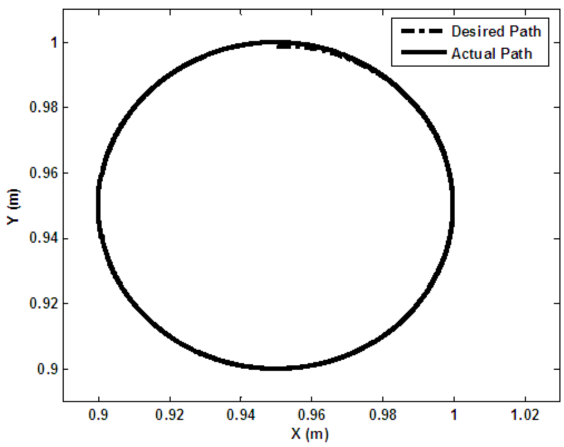

FIGURE 2. Trajectory tracking by control law (34)

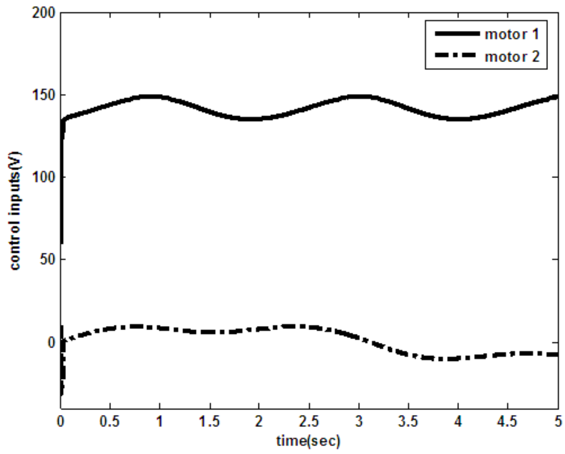

FIGURE 3. Control inputs

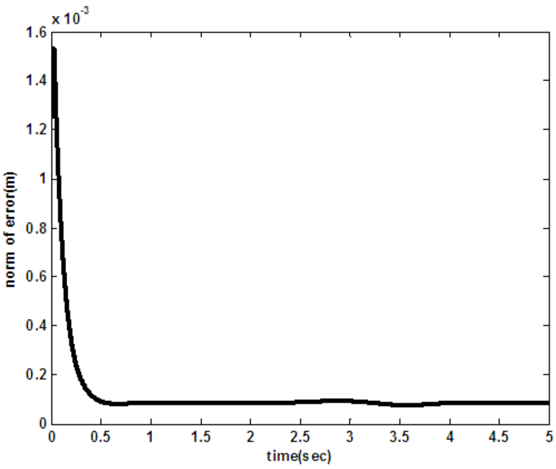

FIGURE 4. Norm of tracking error in task

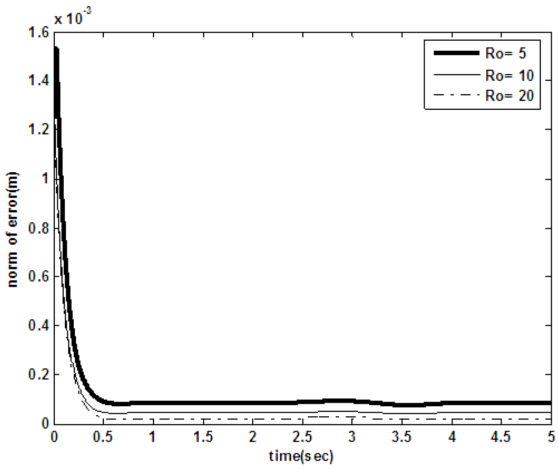

FIGURE 5. Comparing tracking error according to increasing ρ

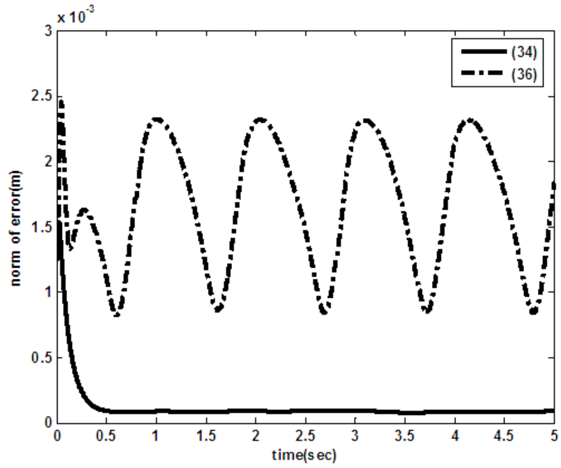

FIGURE 6. Comparing control approach

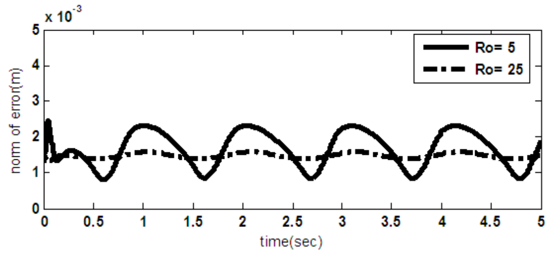

FIGURE 7. Comparing tracking error according to increasing ρ

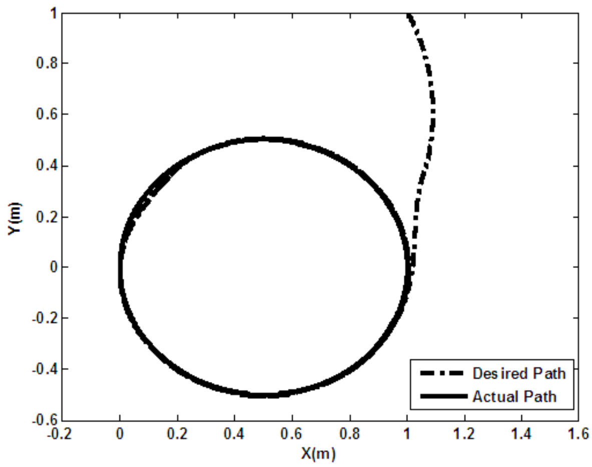

FIGURE 8. Trajectory tracking by control law (36)

The specific usage of control law (36) is when we cannot use Jacobian inverse for example for redundant robots or passing the singular points. For this purpose, a circle with radius of 0.5 m centered at $(0.5, 0)$ is given to control system as a desired trajectory. The desired trajectory passes a singular point $(0, 0)$ where the determinant of Jacobian matrix is zero. The control approach (36) works well and we can see a little difference between the desired and actual trajectories as shown in Figure 8 while the control approach based on inverse Jacobian cannot be applied.

According to above simulation results, it is concluded that the modified control (36) compared with the control input (34) have a higher tracking error, but it still have acceptable performance. Tracking error can be reduced by selecting the appropriate coefficients of input control without the saturated actuators. Unlike the control law (34), modified control (36) can pass the singular points in the task space and implementation problems have been solved in its design.

9. **Conclusion.** A novel approach was developed for trajectory tracking control of electrically-driven robotic manipulators in task space. In this approach, we do not be informed

about uncertainty boundaries, certainly. The simulation results confirmed that the proposed control law can provide a desired tracking performance for a robotic manipulator with uncertain dynamics, uncertain kinematics and uncertainties in actuator models. It is concluded that the proposed approach can be used for task-space tracking control of a normal-cost due to overcoming uncertainties. In contrast, a perfect joint space control approach will never provide a desired tracking performance in task space for such a robot. Moreover, applying feedback linearization technique without realizing and canceling the uncertainties cannot operate well.

The modifications were presented in order to simplify sensing problem such the one on the control law to be independent of velocities. In addition, we can use transpose Jacobian in replace of inverse Jacobian whenever the inverse Jacobian cannot be applied for example in the cases of singularity and redundancy. The use of switching rule was efficient to cancel the effects of estimations and obtaining uniform ultimate boundedness tracking error.

REFERENCES

[1] M. Takogaki and S. Arimoto, A new feedback method for dynamic control of manipulators, *Journal of Dynamic Systems, Measurement and Control*, vol.1, pp.119-125, 1981.

[2] A. Z. Rio and V. Santibanez, Simple extensions of the PD-with-gravity-compensation control law for robot manipulators with bounded inputs, *IEEE Transactions on Control Systems Technology*, vol.14, no.5, pp.958-965, 2006.

[3] M. M. Fateh and M. R. Soltanpour, Robust task-space control of robot manipulators under imperfect transformation of control space, *International Journal of Innovative Computing, Information and Control*, vol.5, no.11(A), pp.3949-3960, 2009.

[4] M. R. Soltanpour and S. E. Shafiei, Robust adaptive control of manipulators in the task space by dynamical partitioning approach, *Journal of Electronics and Electrical Engineering*, vol.101, no.5, pp.73-78, 2010.

[5] M. R. Soltanpour and M. M. Fateh, Adaptive robust tracking control of robot manipulators in the task-space under uncertainties, *Australian Journal of Basic and Applied Sciences*, vol.3, no.1, pp.308-322, 2009.

[6] M. C. Good, L. M. Sweet and K. L. Strobel, Dynamic models for control system design of integrated robot and drive systems, *Transactions of ASME, Journal of Dynamics Systems, Measurement and Control*, vol.107, pp.53-59, 1985.

[7] S. E. Shafiei and M. R. Soltanpour, Robust neural network control of electrically driven robot manipulator using backstepping approach, *International Journal of Advanced Robotic Systems*, vol.6, no.4, pp.285-292, 2009.

[8] M. M. Fateh, On the voltage based control of robot manipulators, *International Journal of Control, Automation, and Systems*, vol.6, no.5, pp.702-712, 2008.

[9] F. Miyasaki and Y. Masutani, Robustness of sensory feedback control based on imperfect jacobian, *Proc. of the 5th International Symposium on Robotic Research*, pp.201-208, 1990.

[10] F. L. Lewis, C. T. Abdallah and D. M. Dawson, *Control of Robot Manipulators*, Macmillan Publishing Inc., New York, 1993.

[11] R. Kelly, Regulation of manipulators in generic task space: An energy shaping plus damping injection approach, *IEEE Transactions on Robotics and Automation*, vol.15, no.2, pp.381-386, 1999.

[12] C. C. Cheah, S. Kawamura and S. Arimoto, Feedback control for robotic manipulator with an uncertain Jacobian matrix, *Journal of Robotic Systems*, vol.16, no.2, pp.119-134, 1999.

[13] C. C. Cheah, M. Hirano, S. Kawamura and S. Arimoto, Approximate jacobian robot control with uncertain kinematics and dynamics, *IEEE Transactions on Robotics and Automation*, vol.19, no.4, pp.692-702, 2003.

[14] C. C. Cheah, C. Liu and J. J. E. Slotine, Approximate jacobian adaptive control for robot manipulators, *Proc. of IEEE International Conference Robotics Automation*, pp.3075-3080, 2004.

[15] C. C. Cheah, C. Liu and J. J. E. Slotine, Adaptive jacobian tracking control of robots with uncertainties in kinematic, dynamic and actuator models, *IEEE Transactions on Automatic Control*, vol.51, no.6, pp.1024-1029, 2006.

[16] Z. Qu and D. M. Dawson, *Robust Tracking Control of Robot Manipulators*, IEEE Press Inc., New York, 1996.
[17] M. Spong, S. Hutchinson and M. Vidyasagar, *Robot Modeling and Control*, Wiley, New York, 2006.
[18] K. H. Khalil, *Nonlinear Systems*, Prentice Hall Inc., New York, 2001.
[19] M. M. Fateh and H. Farhangfard, On the transforming of control space by manipulator jacobian, *International Journal of Control, Automation, and Systems*, vol.6, no.1, pp.101-108, 2008.