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ASYMPTOTICAL STABILIZATION FOR TIME-INVARIANT AND TIME-VARYING PORT-HAMILTONIAN SYSTEMS VIA A NEW KINETIC ENERGY-SHAPING

Liangcheng Cai[1,2], Yong He[1,2,*] and Min Wu[1,2]

[1]School of Information Science and Engineering
Central South University
No. 932, Lushan South Road, Changsha 410083, P. R. China
*Corresponding author: heyong08@yahoo.com.cn

[2]Hunan Engineering Laboratory for Advanced Control and Intelligent Automation
Changsha 410083, P. R. China

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Abstract. *This paper investigates the asymptotical stabilization of the time-invariant and the time-varying Port-Hamiltonian (PH) systems via a new kinetic energy-shaping (KES) method. Firstly, a desired kinetic energy function is designed for the non-trivial points. Secondly, applying the KES method to the Hamiltonian function with the desired kinetic energy function builds some matching equations. Thirdly, solving those equations yields some asymptotically stabilized controllers. Finally, numerical examples are given to show the effectiveness of the proposed methods.*
Keywords: Port-Hamiltonian (PH) systems, Kinetic energy-shaping (KES), Desired kinetic energy function, Asymptotical stabilization

1. Introduction.

Port-Hamiltonian (PH) systems can represent many practical systems such as electromechanical systems and mechanical systems [1, 2], which become a hot issue in recent years. Though various methods [3] can control these systems, the interconnection and damping assignment-passivity based control (IDA-PBC) methods [3-6] are the mainstream ways to stabilize the time-invariant and the time-varying PH systems. In a word, the IDA-PBC methods are energy-based and can be classified into three types: parameterized methods, non-parameterized methods, and algebraic methods [4]. Depending on the IDA-PBC methods, many results for the PH systems are introduced [7-14].

However, there are some room to be further investigated for the IDA-PBC methods. Firstly, the original Hamiltonian function is totally energy-shaping by a desired Hamiltonian function in [3-5]. In fact, the original Hamiltonian function consists of a kinetic energy function and a potential energy one, while the kinetic one is positive-definite and the potential one is not. Therefore, the methods in [3-5] may be more complicated and can be developed. On the other hand, a non-increasing condition that the desired Hamiltonian function is non-increasing in the time variable is employed to derive the stabilization controllers for the time-varying PH systems in [8-10]. Unfortunately, this condition is very difficult to be satisfied such that its application is very narrow.

In this paper, a new algebraic method called the kinetic energy-shaping (KES) approach is presented, which designs a new desired kinetic energy function derived from the original kinetic one and matches the new desired kinetic energy function with the original Hamiltonian function. In short, the KES method is different from those in [3-5]. As a result, an improved control law is designed to asymptotically stabilize the time-invariant PH systems. Moreover, the non-increasing condition in [8-10] is avoided by the KES method and an effective control law is presented to asymptotically stabilize the time-varying PH systems, which enlarges its applied ranges over the existing results [8-10]. Finally, two examples and their simulations show the effectiveness of the proposed methods.

2. **Stabilization Control for Time-invariant PH Systems.** Consider the following time-invariant PH system [1]

$$\begin{cases} \dot{x} = [J(x) - R(x)]\dfrac{\partial H(x)}{\partial x} + g(x)u \\ y = g^T(x)\dfrac{\partial H(x)}{\partial x} \end{cases} \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R^m$ $(m \leq n)$ is the control input, $H(x) : R^n \to R$ is the Hamiltonian function, which is not assumed to be positive definite (nor bounded from below), $J(x) : R^n \to R^{n\times n}$, $R(x) : R^n \to R^{n\times n}$, with $J(x) = -J^T(x)$ and $R(x) = R^T(x) \geq 0$, are the natural interconnection and damping matrices, respectively, and $g(x) : R^n \to R^{n\times m}$ is the gain matrix and is assumed full rank. Moreover, the dimensions of the system (1) are finite and the following definition is necessary.

Definition 2.1. *$g^+(x)$ is the Moore-Penrose pseudo-inverse of matrix $g(x)$, that is, $g^+(x) = [g^T(x)g(x)]^{-1}g^T(x)$.*

For the property of the $H(x)$, it cannot be a Lyapunov function candidate. Thus, in the traditional IDA-PBC methods [3, 4], a desired Hamiltonian function $H_d(x)$ is designed to be a Lyapunov function candidate as follows:

$$H_d(x) = H(x) + H_a(x) \tag{2}$$

where $H_a(x)$ is an assigning Hamiltonian function. In order to obtain the function (2), it is necessary to solve the inequality

$$\det \frac{\partial^2 H_d(x)}{\partial x^2} > 0 \tag{3}$$

Depending on the relationship between the positive definite function and its extreme value, the desired Hamiltonian function $H_d(x)$ is a Lyapunov function candidate and naturally satisfies the following condition

$$x_\star = \arg\min H_d(x) \tag{4}$$

where the equilibrium $x_\star \in R^n$ is to be stabilized.

However, it is tedious to solve the inequality (3). Additionally, the Hamiltonian function $H(x)$ contains a kinetic energy function $K(x)$ and a potential energy function $P(x)$, while the $K(x)$ is positive definite and can be a Lyapunov function candidate. Inspired by those, a kinetic energy-shaping (KES) method is presented. It firstly designs a novel desired kinetic energy function $K_d(x)$ derived from the $K(x)$. Secondly, the original Hamiltonian function $H(x)$ is energy-shaping with the $K_d(x)$ via some new matching equations. Let us illustrate the KES method in details.

Firstly, the KES method designs $K_d(x)$ satisfying the following properties.

Property 2.1. *The number of state variables in $K_d(x)$ is the same as that in system (1).*

Property 2.2. *$K_d(x)$ is a positive definite quadratic form function, that is*

$$x^\star = \arg\min K_d(x) \tag{5}$$

where the non-trivial point $x^\star \in R^n$ is to be stabilized.

Then, $K_d(x)$ is constructed in three steps as follows.

Step 1: Take out the kinetic energy function $K(x)$ from the Hamiltonian function $H(x)$.

Step 2: It is necessary to test whether the Property 2.1 holds or not. If it is true, the kinetic energy function $K(x)$ can be chosen as a transitive kinetic energy function $K_t(x)$. On the contrary, the absent state variables with the forms of the positive definite should be added into the $K(x)$ to yield $K_t(x)$.

Step 3: Assigning any non-trivial point to the transitive kinetic energy $K_t(x)$ generates a desired kinetic energy function $K_d(x)$, that is $K_d(x) = K_t(x - x^\star)$.

Remark 2.1. *It is clear that $K_d(x)$ can be chosen as a Lyapunov function candidate, which is not the same as that in (2) and avoids computing the inequality (3). Meantime, an integrability condition $\frac{\partial^2 H_a(x)}{\partial x^2} = \left(\frac{\partial^2 H_a(x)}{\partial x^2}\right)^T$ in* [3] *is presented to design a $H_d(x)$. However, it is unnecessary for $K_d(x)$ to satisfy this condition as it is a quadratic form function, which assures its integrability.*

Moreover, the Step 2 is important to construct $K_d(x)$, which assures that the state variables of system (1) are fully remained. Taking a PH system with three state variables x_1, x_2 and x_3 as an example, if only x_1 and x_2 are contained in the $K(x)$, it is necessary to construct a transitive kinetic energy function $K_t(x) = K(x) + \frac{x_3^2}{2}$, which contains all the state variables.

Secondly, the KES method matches the desired kinetic energy function $K_d(x)$ with the Hamiltonian function $H(x)$ of the system (1) as follows:

$$[J(x) - R(x)]\frac{\partial H(x)}{\partial x} + g(x)u = [J_d(x) - R_d(x)]\frac{\partial K_d(x)}{\partial x} \tag{6}$$

which yields

$$u = g^+(x)\left\{[J_d(x) - R_d(x)]\frac{\partial K_d(x)}{\partial x} - [J(x) - R(x)]\frac{\partial H(x)}{\partial x}\right\} \tag{7}$$

where

$$J_d(x) = J_a(x) + J(x) = -J_d^T(x) \tag{8}$$

$$R_d(x) = R_a(x) + R(x) = R_d^T(x) \geq 0 \tag{9}$$

and $J_a(x)$ and $R_a(x)$ are the assigning matrices for the original ones, respectively.

Depending on the matching Equation (6), an equivalent system of time-invariant PH system (1) is

$$\dot{x} = [J_d(x) - R_d(x)]\frac{\partial K_d(x)}{\partial x} \tag{10}$$

where $K_d(x)$ is a Lyapunov function candidate and its differential is

$$\begin{aligned}\dot{K}_d(x) &= -\frac{\partial K_d^T(x)}{\partial x}R_d(x)\frac{\partial K_d(x)}{\partial x} \\ &\leq 0\end{aligned} \tag{11}$$

which implies that the time-invariant PH system (1) is stable. Furthermore, if the largest invariant set is contained in

$$\left\{x \in R^n \left| \frac{\partial K_d^T(x)}{\partial x}R_d(x)\frac{\partial K_d(x)}{\partial x} = 0\right.\right\} \tag{12}$$

then, the time-invariant PH system (1) is asymptotical stabilization via the La Salle's invariance principle. In other words, the following theorem is true.

Theorem 2.1. *Consider the time-invariant PH system (1). If there exists a desired kinetic energy function $K_d(x)$ satisfying the Properties 2.1 and 2.2, and the matrices $J_a(x)$ and $R_a(x)$, such that the matching Equation (6) holds, the control law (7) stabilizes the time-invariant PH system (1). Additionally, if the largest invariant set is contained in the condition (12), the control law (7) asymptotically stabilizes the time-invariant PH system (1).*

In fact, if a matrix $R(x)$ is already included in the system (1), it is unnecessary to add a matrix $R_a(x)$ to the matrix $R(x)$ because the latter is non-negative definite; thus, another matching equation is established

$$[J(x) - R(x)]\frac{\partial H(x)}{\partial x} + g(x)u = [J_d(x) - R(x)]\frac{\partial K_d(x)}{\partial x} \tag{13}$$

which draws forth the following corollary as a by-product.

Corollary 2.1. *Consider the time-invariant PH system (1). If there exists a desired kinetic energy function $K_d(x)$ satisfying the Properties 2.1 and 2.2, and the matrix $J_a(x)$, such that the matching Equation (13) holds, then the control law*

$$u = g^{+}(x)\left\{[J_d(x) - R(x)]\frac{\partial K_d(x)}{\partial x} - [J(x) - R(x)]\frac{\partial H(x)}{\partial x}\right\} \tag{14}$$

stabilizes the time-invariant PH system (1). Additionally, if the largest invariant set is contained in

$$\left\{x \in R^n \,\middle|\, \frac{\partial K_d^T(x)}{\partial x}R(x)\frac{\partial K_d(x)}{\partial x} = 0\right\} \tag{15}$$

the control law (14) asymptotically stabilizes the time-invariant PH system (1).

Proof: The proof is similar to the proof of Theorem 2.1, which is omitted here.

Remark 2.2. *According to the matching Equations (6) and (13), the Hamiltonian function $H(x)$ of the system (1) is energy-shaping by the desired kinetic energy function $K_d(x)$, which is addressed as the KES method and is much simpler than the traditional energy-shaping* [3-5].

Furthermore, some improved conclusions will be drawn to the time-varying PH systems following the similar line in the next section.

3. **Stabilization Control for Time-varying PH Systems.**

Consider the following time-varying PH system [8]

$$\begin{cases} \dot{x} = [J(x,t) - R(x,t)]\dfrac{\partial H(x,t)}{\partial x} + g(x,t)u \\ y = g^T(x,t)\dfrac{\partial H(x,t)}{\partial x} \end{cases} \tag{16}$$

where $x \in R^n$ is the state vector, $u \in R^m$ $(m \le n)$ is the control input. $H(x,t) : R^n \times [0,\ +\infty) \to R$ is the Hamiltonian function, which is not assumed to be positive definite (nor bounded from below), $J(x,t) : R^n \times [0,\ +\infty) \to R^{n\times n}$, $R(x,t) : R^n \times [0,\ +\infty) \to R^{n\times n}$, with $J(x,t) = -J^T(x,t)$ and $R(x,t) = R^T(x,t) \ge 0$, are the natural interconnection and damping matrices, respectively, and $g(x,t) : R^n \times [0,\ +\infty) \to R^{n\times m}$ is the gain matrix and is assumed full rank. Moreover, the dimensions of the system (16) are finite and the following definition is necessary.

Definition 3.1. *$g^{+}(x,t)$ is the Moore-Penrose pseudo-inverse of matrix $g(x,t)$, that is, $g^{+}(x,t) = [g^T(x,t)g(x,t)]^{-1}g^T(x,t)$.*

Since $H(x,t)$ is not positive definite, energy-shaping should be carried out by $K_d(x,t)$, which is assumed to have the following properties.

Property 3.1. *The number of state variables in $K_d(x,t)$ is the same as that in system (16).*

Property 3.2. *$K_d(x,t)$ is a positive definite quadratic form function, that is*

$$x^\star = \arg\min K_d(x,t) \tag{17}$$

where the non-trivial point $x^\star \in R^n$ is to be stabilized.

Once a $K_d(x,t)$ is obtained satisfying the Properties 3.1 and 3.2, the KES method can give out the following matching equation

$$[J(x,t) - R(x,t)]\frac{\partial H(x,t)}{\partial x} + g(x,t)u = [J_d(x,t) - R_d(x,t)]\frac{\partial K_d(x,t)}{\partial x} \tag{18}$$

which yields an effective control law

$$u = g^+(x,t)\left\{[J_d(x,t) - R_d(x,t)]\frac{\partial K_d(x,t)}{\partial x} - [J(x,t) - R(x,t)]\frac{\partial H(x,t)}{\partial x}\right\} \tag{19}$$

where

$$J_d(x,t) = J_a(x,t) + J(x,t) = -J_d^T(x,t) \tag{20}$$

$$R_d(x,t) = R_a(x,t) + R(x,t) = R_d^T(x,t) \geq 0 \tag{21}$$

and the assigning matrices $J_a(x,t)$ and $R_a(x,t)$ for the original ones, respectively. Due to the above contents, the following theorem is established.

Theorem 3.1. *Consider the time-varying PH system (16). If there exists a desired kinetic energy function $K_d(x,t)$ satisfying the Properties 3.1 and 3.2 and the non-increasing condition $\frac{\partial K_d(x,t)}{\partial t} \leq 0$, and the matrices $J_a(x,t)$ and $R_a(x,t)$, such that the matching Equation (18) holds, the control law (19) stabilizes the time-varying PH system (16). Additionally, if the largest invariant set is contained in*

$$\left\{x \in R^n \left| \frac{\partial K_d^T(x,t)}{\partial x} R_d(x,t) \frac{\partial K_d(x,t)}{\partial x} = 0 \right.\right\} \tag{22}$$

the control law (19) asymptotically stabilizes the time-varying PH system (16).

Proof: Depending on the matching Equation (18), there is

$$\dot{x} = [J_d(x,t) - R_d(x,t)]\frac{\partial K_d(x,t)}{\partial x} \tag{23}$$

where $K_d(x,t)$ is a Lyapunov function and its differential is

$$\begin{aligned}\dot{K}_d(x,t) &= \frac{\partial K_d(x,t)}{\partial t} - \frac{\partial K_d^T(x,t)}{\partial x} R_d(x,t) \frac{\partial K_d(x,t)}{\partial x} \\ &\leq 0\end{aligned} \tag{24}$$

which means that the time-varying PH system (16) is stable. And the La Salle's invariance principle and the condition (22) guarantee that the time-varying PH system (16) is asymptotically stabilized by the control law (19).

Remark 3.1. *For Theorem 3.1, there exists a non-increasing condition* $\frac{\partial K_d(x,t)}{\partial t} \le 0$. *Similarly, some other non-increasing conditions were employed such as the non-increasing conditions* $\frac{\partial H(x,t)}{\partial t} \le 0$ *[8, 9] and* $\frac{\partial H_a(x,t)}{\partial t} \le 0$ *and* $H_a(x,t) = H(x,t) + H_c(c_1(x) + d_1, \cdots, c_{n_c}(x) + d_n, t)$ *[10]. Unfortunately, some time-varying Hamiltonian functions naturally cannot satisfy the non-increasing condition* $\frac{\partial H(x,t)}{\partial t} \le 0$, *for example* $H(x,t) = -cx_3 \cos x_1 - ax_1 + \frac{x_2^2}{2} + \frac{cd}{2e}x_3^2 - x_3 \sin t$. *Hence, these non-increasing conditions will be difficult to be satisfied.*

Motivated by those facts, another desired kinetic energy function $\bar{K}_d(x)$ satisfying the Properties 2.1 and 2.2 is presented, which avoids applying the non-increasing condition and enlarges its applied fields. Depending on $\bar{K}_d(x)$, the KES method presents another matching equation as follows:

$$[J(x,t) - R(x,t)]\frac{\partial H(x,t)}{\partial x} + g(x,t)u = [J_d(x,t) - R_d(x,t)]\frac{\partial \bar{K}_d(x)}{\partial x} \tag{25}$$

which yields an effective control law

$$u = g^+(x,t)\left\{[J_d(x,t) - R_d(x,t)]\frac{\partial \bar{K}_d(x)}{\partial x} - [J(x,t) - R(x,t)]\frac{\partial H(x,t)}{\partial x}\right\} \tag{26}$$

As a result, the following theorem is responded to it.

Theorem 3.2. *Consider the time-varying PH system (16). If there exists a desired kinetic energy function* $\bar{K}_d(x)$ *satisfying the Properties 2.1 and 2.2, and the matrices* $J_a(x,t)$ *and* $R_a(x,t)$, *such that the matching Equation (25) holds, the control law (26) stabilizes the time-varying PH system (16). Additionally, if the largest invariant set is contained in*

$$\left\{x \in R^n \left| \frac{\partial \bar{K}_d^T(x)}{\partial x} R_d(x,t) \frac{\partial \bar{K}_d(x)}{\partial x} = 0 \right.\right\} \tag{27}$$

the control law (26) asymptotically stabilizes the time-varying PH system (16).

Proof: Since $\bar{K}_d(x)$ can be a Lyapunov function candidate, calculating its differential yields

$$\begin{aligned}\dot{\bar{K}}_d(x) &= -\frac{\partial \bar{K}_d^T(x)}{\partial x} R_d(x,t) \frac{\partial \bar{K}_d(x)}{\partial x} \\ &\le 0\end{aligned} \tag{28}$$

which means that the time-varying PH system (16) is stable. And the La Salle's invariance principle and the condition (27) guarantee that the time-varying PH system (16) is asymptotically stabilized by the control law (26).

Remark 3.2. *For Theorem 3.2, the proposed method avoids applying the non-increasing condition* $\frac{\partial K_d(x,t)}{\partial t} \le 0$, *whose applied ranges are larger than those in the existing results [8-10].*

4. **Numerical Example.** Consider the three-phase synchronous generator model [7]

$$\begin{cases} \dot{\delta} = \omega - \omega_0 \\ \dot{\omega} = \dfrac{\omega_0}{M}P_m - \dfrac{D}{M}(\omega - \omega_0) - \dfrac{\omega_0 E_q' V_s}{M x_{d\Sigma}'}\sin\delta \\ \dot{E}_q' = -\dfrac{E_q'}{T_d'} + \dfrac{x_d - x_d'}{T_{do} x_{d\Sigma}'} V_s \cos\delta + \dfrac{V_f}{T_{do}} \end{cases} \tag{29}$$

where

δ: rotor angle [rad]
ω: rotor speed [rad/s]
ω_0: $= 2\pi f_0$
E'_q: q axis internal transient voltage [p.u.]
x_d: d-axis synchronous reactance of the generator [p.u.]
x'_d: d-axis transitive reactance [p.u.]
V_f: voltage of the field circuit of the generator model [p.u.]
M: inertia coefficient of generator
D: damping constant [p.u.]
T_{do}: excitation circuit time constant [s]
T'_d: stator closed-loop time constant [s]
P_m: mechanical power [p.u.]
V_s: voltage of infinite-bus [V]
$x'_{d\Sigma}$: $= x'_d + \frac{1}{2}x_L + x_T$
x_L: reactance of transmission line [p.u.]
x_T: transformer reactance [p.u.].

Define the following transformations

$$x_1 = \delta, \quad x_2 = \omega - \omega_0 \tag{30}$$

$$x_3 = E'_q, \quad a = \frac{\omega_0}{M}P_m \tag{31}$$

$$b = \frac{D}{M}, \quad c = \frac{\omega_0 V_s}{M x'_{d\Sigma}} \tag{32}$$

$$d = \frac{1}{T'_d}, \quad e = \frac{x_d - x'_d}{T_{do} x'_{d\Sigma}} V_s \tag{33}$$

$$h = \frac{1}{T_{do}}, \quad y = -c\cos x_1 + \frac{cd}{e}x_3, \tag{34}$$

which represent the system (29) as a time-invariant PH system (1)

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -b & 0 \\ 0 & 0 & -\frac{e}{c} \end{bmatrix} \frac{\partial H(x)}{\partial x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \frac{\partial H(x)}{\partial x} \end{cases} \tag{35}$$

where the Hamiltonian function is

$$H(x) = -cx_3\cos x_1 - ax_1 + \frac{cd}{2e}x_3^2 + \frac{x_2^2}{2} \tag{36}$$

It is clear that $\det\left(\frac{\partial^2 H(x)}{\partial x^2}\right) = \frac{c^2 d}{e} x_3 \cos x_1 - c^2 \sin^2 x_1$ is sign indefinite, which means that the Hamiltonian function (36) cannot be a Lyapunove function candidate. So, energy-shaping is necessary. If the existing methods in [3, 4] are applied, it is necessary to solve the inequality (3). However, the KES method is applied to the time-invariant PH system (35), which avoids solving the inequality (3). Firstly, reviewing $H(x)$ generates a kinetic energy function $K(x)$ as follows:

$$K(x) = \frac{cd}{2e}x_3^2 + \frac{x_2^2}{2} \tag{37}$$

which is without the state variable x_1. So, a transitive kinetic energy function $K_t(x)$ is derived as follows:

$$K_t(x) = \frac{cd}{2e}x_3^2 + \frac{x_2^2}{2} + \frac{x_1^2}{2} \tag{38}$$

which remedies this problem. Assigning the equilibrium $x^\star = (x_{1e}, x_{2e}, x_{3e})$ of the system (35) into $K_t(x)$ yields

$$K_d(x) = \frac{cd}{2e}(x_3 - x_{3e})^2 + \frac{(x_2 - x_{2e})^2}{2} + \frac{(x_1 - x_{1e})^2}{2} \tag{39}$$

which can be chosen as a Lyapunov function candidate. Employing Theorem 2.1 establishes the following matching equation

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & -b & 0 \\ 0 & 0 & -\frac{e}{c} \end{bmatrix} \frac{\partial H(x)}{\partial x} + \begin{bmatrix} 0 \\ 0 \\ u \end{bmatrix} = \begin{bmatrix} 0 & 1+j_1 & 0 \\ -1-j_1 & -b & j_2 \\ 0 & -j_2 & -\frac{e}{c} - r_1 \end{bmatrix} \begin{bmatrix} x_1 - x_{1e} \\ x_2 - x_{2e} \\ \frac{cd}{e}(x_3 - x_{3e}) \end{bmatrix} \tag{40}$$

solving Equation (40) yields the desired matrices and control law as follows:

$$J_d(x) = J_a(x) + J(x) = \begin{bmatrix} 0 & j_1 & 0 \\ -j_1 & 0 & j_2 \\ 0 & -j_2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1+j_1 & 0 \\ -1-j_1 & 0 & j_2 \\ 0 & -j_2 & 0 \end{bmatrix}$$

$$R_d(x) = R_a(x) + R(x) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & r_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & \frac{e}{c} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & \frac{2e}{c} \end{bmatrix}$$

$$u = -dx_3 + 2dx_{3e} - e\cos x_1 - j_2(x_2 - x_{2e}) \tag{41}$$

where $r_1 = \frac{e}{c}$, $j_1 = \frac{x_2}{x_2 - x_{2e}} - 1$ and $j_2 = \frac{(1+j_1)(x_1 - x_{1e}) + a - cx_3 \sin x_1 - bx_{2e}}{\frac{cd}{e}(x_3 - x_{3e})}$.

On the other hand, consider a time-varying PH system

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -b - b\sin t & 0 \\ 0 & 0 & -\frac{e}{c} \end{bmatrix} \frac{\partial H(x,t)}{\partial x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \frac{\partial H(x,t)}{\partial x} \end{cases} \tag{42}$$

where the Hamiltonian function is

$$H(x,t) = -cx_3 \cos x_1 - ax_1 + \frac{x_2^2}{2} + \frac{cd}{2e}x_3^2 - x_3 \sin t \tag{43}$$

It is clear that $\frac{\partial H(x,t)}{\partial t} = -x_3 \cos t$ is sign indefinite, which cannot satisfy the non-increasing condition $\frac{\partial H(x,t)}{\partial t} < 0$. Thus, the method in [8] is invalid for the system (42). Similarly, it is hard to design an $H_d(x,t)$ satisfying the non-increasing condition $\frac{\partial H_d(x,t)}{\partial t} < 0$, which is also not easy to apply the method in [10] to stabilize the system (42). Fortunately, Theorem 3.2 conquers these obstacles in the following procedures. Firstly, a desired kinetic energy function is designed as follows:

$$\bar{K}_d(x) = \frac{(x_1 - \bar{x}_1)^2}{2} + \frac{(x_2 - \bar{x}_2)^2}{2} + \frac{cd}{2e}(x_3 - \bar{x}_3)^2 \tag{44}$$

where $\bar{x} = (\bar{x}_1,\ \bar{x}_2,\ \bar{x}_3)$ is a non-trivial point for the system (42). Secondly, another matching equation is established

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & -b - b\sin t & 0 \\ 0 & 0 & -\frac{e}{c} \end{bmatrix} \frac{\partial H(x,t)}{\partial x} + \begin{bmatrix} 0 \\ 0 \\ u \end{bmatrix} = [J_d(x,t) - R_d(x,t)] \begin{bmatrix} x_1 - \bar{x}_1 \\ x_2 - \bar{x}_2 \\ \frac{cd}{e}(x_3 - \bar{x}_3) \end{bmatrix} \tag{45}$$

where $J_d(x,t) = \begin{bmatrix} 0 & 1+j_3 & 0 \\ -1-j_3 & 0 & j_4 \\ 0 & -j_4 & 0 \end{bmatrix}$ and $R_d(x,t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & b+b\sin t & 0 \\ 0 & 0 & \frac{e}{c}+r_2 \end{bmatrix}$. At last, solving Equation (45) yields a control law as follows:

$$u = -dx_3 + 2d\bar{x}_3 - e\cos x_1 - j_4(x_2 - \bar{x}_2) - \frac{e}{c}\sin t \tag{46}$$

where $r_2 = \frac{e}{c}$, $j_3 = \frac{x_2}{x_2 - \bar{x}_2} - 1$ and $j_4 = \frac{(1+j_3)(x_1-\bar{x}_1)+a-cx_3\sin x_1 - b\bar{x}_2 - b\bar{x}_2\sin t}{\frac{cd}{e}(x_3-\bar{x}_3)}$.

In order to demonstrate the effectiveness of the control laws (41) and (46), the systemic parameters are given in Table 1 [7] and both of the systems (35) and (42) choose point (0.1075, 0, 2.2370) as their desired points $x^\star$ and $\bar{x}$. As a result, Figures 1.1 and 1.2 show the asymptotical stabilization of the state curves x, which belong to the systems (35) and (42), respectively. Furthermore, Figure 2 shows the comparison between the proposed method and the existing method in [7], where the curve x is obtained by the proposed method and the curve x' is obtained by the latter. It is clear that the convergence rate of x is faster than that of x'. Taking the curve x_2 as an example, its physical meaning is the difference between ω and ω_0. Thus, the proposed method makes ω track the desired ω_0 more quickly, which means that the proposed method is better than the existing method in [7].

Table 1. Parameters of the three-phase synchronous generator model

ω_0	M	P_m	D	V_s	x_d	x_d'	$x_{d\Sigma}'$	T_{do}	T_d'
1	7.6	1	3	1.5	0.9	0.36	0.36	5	5

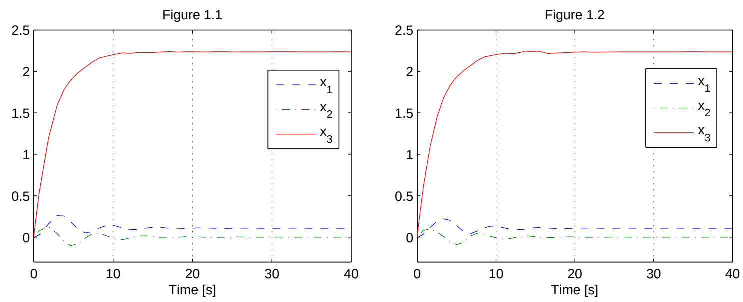

Figure 1. The response state curves x of closed-loop systems (35) and (42)

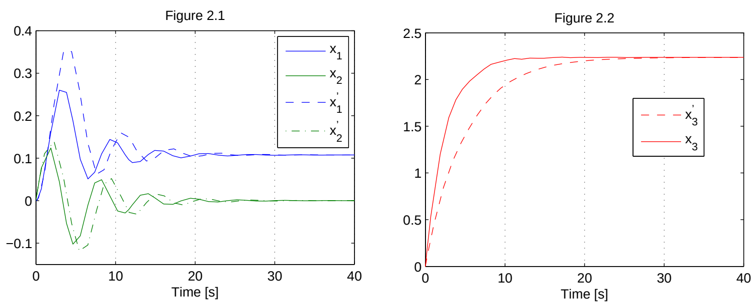

Figure 2. The comparative response state curves x of closed-loop systems (35)

5. **Conclusions.** This paper has presented a new KES method to asymptotically stabilize the time-invariant and time-varying PH systems. A desired kinetic energy function has been constructed without the computing inequality (3). Meantime, for the time-varying PH systems, the non-increasing conditions have been avoided by the KES method whose applied ranges are enlarged compared with the existing literature [8-10].

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REFERENCES

[1] A. van der Schaft, *L_2-Gain and Passivity Techniques in the Nonlinear Control*, Springer, London, 2000.

[2] M. Deng, A. Inoue, M. Kosugi and T. Henmi, Swing-up control of a cart-type single inverted pendulum with parasitic dynamics, *International Journal of Innovative Computing, Information and Control*, vol.3, no.6(B), pp.1501-1510, 2007.

[3] R. Ortega, A. van der Schaft, B. Maschke and G. Escobar, Interconnection and damping assignment passivity-based control of Port-controlled Hamiltonian systems, *Automatica*, vol.38, no.4, pp.585-596, 2002.

[4] R. Ortege and E. Garcia-Canseco, Interconnection and damping assignment passivity-based control: A survey, *European Journal of Control*, vol.10, no.2, pp.432-450, 2004.

[5] R. Ortega, A. van der Schaft, F. Castanos and A. Astolfi, Control by interconnection and standard passivity-based control of Port-Hamiltonian systems, *IEEE Transactions on Automatic Control*, vol.53, no.11, pp.2527-2542, 2008.

[6] F. Castanos and R. Ortega, Energy-balancing passivity-based control is equivalent to dissipation and output invariance, *Systems and Control Letters*, vol.58, no.8, pp.533-560, 2009.

[7] Z. Xi and D. Cheng, Passivity-based stabilization and H_∞ control of the Hamiltonian control systems with dissipation and its applications to power systems, *International Journal of Control*, vol.73, no.18, pp.1686-1691, 2000.

[8] K. Fujimoto and T. Sugie, Canonical transformation and stabilization of generalized Hamiltonian systems, *System and Control Letters*, vol.42, no.3, pp.217-227, 2001.

[9] K. Fujimoto and T. Sugie, Stabilization of Hamiltonian systems with nonholonomic constraints based on time-varying generalized canonical transformations, *System and Control Letters*, vol.44, no.4, pp.309-319, 2001.

[10] Y. Guo and D. Cheng, Stabilization of time-varying Hamiltonian systems, *IEEE Transactions Control Systems Technology*, vol.14, no.5, pp.871-880, 2006.

[11] R. Ortega, M. W. Spong, F. Gomez-Estern and G. Blankenstein, Stabilization of a class of underactuated mechanical systems via interconnection and damping assignment, *IEEE Transactions on Automatic Control*, vol.47, no.8, pp.1218-1233, 2002.

[12] H. Ramire, D. Sbarbaro and R. Ortega, On the control of non-linear processes: An IDA-PBC approach, *Journal of Process Control*, vol.19, no.3, pp.405-414, 2009.

[13] A. Astolfi, R. Ortega and A. Venkatraman, A globally exponentially convergent immersion and invariance speed observer for machanical systems with non-holomic constraints, *Automatica*, vol.46, no.1, pp.182-189, 2010.

[14] A. Venkatraman, R. Ortega, I. Sarras and A. van der Schaft, Speed observation and position feedback stabilization of partially linearizable mechanical systems, *IEEE Transactions on Automatic Control*, vol.55, no.5, pp.1059-1074, 2010.