International Journal of Innovative Computing, Information and Control
Volume **8**, Number **8**, August **2012**
ICIC International ©2012 ISSN 1349-4198
pp. 5543–5557

THE CORRELATION AND EIGENANALYSIS MODELS ON AMPLITUDE-LOCKED LOOP EMBEDDED SECURITY SYSTEMS

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Received May 2011; revised September 2011

Abstract. *In this paper, we investigate the utility of correlation and eigenanalysis models for analyzing the performance of frequency modulation (FM) system formed by combining phase-locked loop (PLL) and amplitude-locked loop (ALL) models using the Least-Mean-Square (LMS) algorithm in the presence of co-channel interferences (CCI) and additive white Gaussian noise (AWGN). The ALL system demonstrates various interferences to carrier ratio (ICR) characteristics using the power spectral density (PSD) and can be used as a digital receiver for monitoring systems and signal intercepted applications. These numerically evaluated statistical performances of the cross-correlation algorithm are proposed to characterize the properties and variations of correlation. The results are based on explicit closed-form expressions, which we derive for the eigenvalues distribution of autocorrelated matrices. Finally, we present simulation results to demonstrate separation of the original signals using a good match eigenvalue spread. The system provides a form of secure communication in unfavorable conditions, such as when the transmitted signal is not easily detected or discriminated by unnecessary listeners of the system. The system is also strictly associated with digital communication security technology and improved channel capacity of multiple access (MA) systems.*
Keywords: ALL, PLL, LMS, CCI, PSD, Correlation, Eigenanalysis, MA

1. **Introduction.** In mobile communication systems, CCI is a major interference, because reusing frequencies can increase system capacity. As a method of efficiently suppressing in-band interference, a novel separation algorithm is proposed for canceling CCI in communication systems using an ALL system. Unfortunately, we found that the dominat signal cannot be perfectly separated using the demodulator of a traditional low-pass filter (LPF) when the signals are transmitted with low signal-to-noise ratio (SNR) in the ALL system.

A new co-channel separation algorithm, utilizing digital signal processing (DSP) techniques incorporating PLL and ALL algorithms, has been developed for FM system in order to improve the SNR of the FM demodulator. Here, the ALL algorithm is used to separate the dominant and subdominant signals from each other. This combined algorithm allows us to suppress multi-channel with reduced computational complexity. However, the ALL system was used in environments dominant by additive noise. In this study, we combined the ALL system with an adaptive LMS algorithm to demonstrate successful separation of signals with AWGN. Adaptive filters are widely used in a variety of applications, including noise cancellation, spectral estimation, and linear prediction filtering. It is generally implemented in the time domain in a tapped-delay-line (TDL) form. The optimal solution is to recursively calculate using the Windrow-Hoff adaptive LMS algorithm [1]. However, conventional adaptive control theory can only deal with systems with known dynamic structures but not for unknown (constant or slowly-varying) parameters [2].

One important parameter of a multiple-input–multiple-output (MIMO) channel model is the correlation between the channels, which can result from factors such as spatial or polarization properties [3]. The influence of these parameters on MIMO capacity (or bit error rate) was studied using the correlation characteristics of a channel matrix, position of a mobile terminal, reciprocal coupling between transmitter or receiver antennas, mean effective gains (MEGs) of single antenna elements, and radiation efficiency of an antenna structure [4]. Aside from using well-known cross-correlation-based criteria, such as the multi-channel cross-correlation (MCCC), this synchrony can also be surveyed using the averaged magnitude difference function (AMDF) and averaged magnitude sum function (AMSF), which reflect the lowest calculated cost [5].

A simple and realistic autocorrelation model, based on experimental data, was proposed by Gudmundson, in which the spatial correlation of the shadowing effects is exponentially decayed with an increase in separation distance between any two positions. The variation in shadowing effects is generally modeled as a Gaussian-Markov stochastic process [6]. Using this model, the correlated channel matrix of the point-to-point MIMO channel can be conveyed in terms of the separable variance profile, which relies on the eigenvalues of the correlation matrices [7]. The LMS algorithm used for updating parameters is strongly determined by the eigenvalue spread of the correlation matrix. It has been argued that in order to improve the convergence, a new parameterization has to be used such that the corresponding correlation matrix has better conditioned and well decoupled properties [8]. We also observed that the correlation and eigenanalysis models provided the general solutions for the FM system formed with the combined PLL and ALL models using the LMS algorithm identification application.

In this context, the main contributions of this paper can be summarized as follows: 1) The LMS algorithm, combined with the PLL and ALL systems, can be used to separate two FM signals with the same carrier frequency and suppress the distortion caused by AWGN channel interference; 2) In this paper, we use DSP analysis to simulate and analyze the performance of the separation system. The adaptive finite-impulse response (FIR) filter and LMS algorithm can be used to suppress the environmental interference and obtain better performance than tradition filters, such as the Butterworth filter or the Chebyshev filter; 3) Based on a recent correlation result, the error value calculation problem is transformed into a non-linear programming problem, which can be used to efficiently calculate the minimum mean square error (MMSE) for finite DSP systems; 4) Furthermore, the eigenanalysis of the autocorrelation matrix of this channel model is analyzed based on a direct-detected nonzero mean signal. We discuss how the properties of this signal affect the eigenvalues of the signal when compared with the zero mean additive noise input common in communications applications.

Figure 1 illustrates discrete PLL and ALL separation systems analyzed using correlation results and eigenanalysis of the autocorrelation matrix. Figure 1 shows two signals input into the FM system in CCI with AWGN, which are subsequently separated with the PLL and ALL system using the LMS algorithm. The dominant and subdominant signals from the receiver are recovered by the combined ALL system, and the distortion terms are removed by the LMS filter. We adopt the steepest descent theory to find the optimal weight coefficients of the adaptive FIR filter system for the MMSE value. In the MMSE criterion, the minimum MSE value can be found using an iteration procedure. First, we use eigenanalysis to analyze and classify the cross-correlation techniques. However, the numerical and simulation results show that two signals can adequately characterize the spatial correlation properties of the total CCI. In communication systems, however, two signal transmission formats are seldom used. Therefore, the transmitted signal has non-zero mean, and square-law devices at the receiver introduces non-central and signal-dependent noise into the received signal at the input of the equalizer. In this paper, we provide eigenanalysis of the input autocorrelation matrix when the input is a direct-detected nonzero mean signal. We also consider how the characteristics of the signal would influence the eigenanalysis of the autocorrelation matrix of this channel model. This study is focused on eigenanalysis of the channel matrix, which resolves the MMSE. Finally, we present results which demonstrate that the original signals have been separated using a good match eigenvalue spread. Additionally, the proposed system also has the advantage of being a secure communication system.

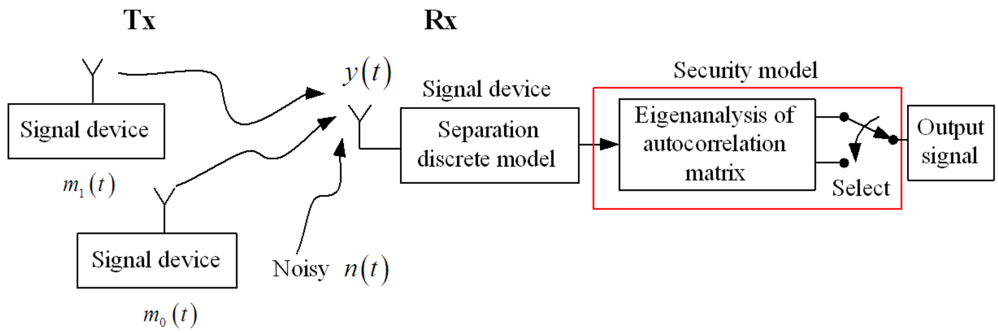

FIGURE 1. The discrete separation system analyzed using correlation results and eigenanalysis of the autocorrelation matrix

The remainder of this paper is structured as follows. Section 2 describes the mathematical theory of the adaptive filter model, correlation techniques obtained using time delay estimation, and the autocorrelation coefficient. Section 3 reviews the novel separation system for simulating FM signal separation in an AWGN channel. Section 4 presents an eigenanalysis for the autocorrelation matrix of an input with signal-dependent noise as a function of the eigenvalues of a noiseless input autocorrelation matrix. Section 5 evaluates the performance of the proposed methods with numerical examples and simulations. Section 6 concludes the paper.

2. **Related Work.** The correlation figures for the transmitter and receiver took into account the combined effects of spatial correlation and low-SNR. The empirical eigenvalue distribution for the channel covariance matrix is derived and used to calculate the MMSE filtering capacity [9]. In this paper, it is shown that the performance of environmental interference suppression is greatly compromised by correlation.

2.1. **LMS algorithm.** In this section, we develop the theory of a widely used algorithm which is called the LMS algorithm. The LMS algorithm is an important member of the family of stochastic gradient algorithms. The LMS filter block is estimated using filter weights, or coefficients, needed to convert an input signal into a desired signal. Moreover, the algorithm does not require measurements of the pertinent correlation functions, nor does it require matrix inversion. Indeed, it is the simplicity of the LMS algorithm that has made it the standard against which other adaptive filter algorithms are benchmarked. The LMS algorithm is a stochastic adaptive filter algorithm, and it seeks to find the minimum point of the error surface by following a zig-zag path to find the MMSE and improve the system performance [10]. FIR digital filter synthesis can be established by employing adaptive modeling techniques [11]. This algorithm is a linear adaptive filtering algorithm that consists of filtering, and adaptive two basic processes. The two combined processes work together to create a feedback loop around the LMS algorithm. If a suitable step-size parameter μ is chosen, then the tap-weight vector computed using the steepest-descent algorithm would converge to the optimum Wiener solution [12]:

$$y(n) = \hat{W}^H(n)x(n) \tag{1}$$

$$e(n) = d(n) - y(n) \tag{2}$$

$$\hat{W}(n+1) = \hat{W}(n) + \mu x(n)e^*(n) \tag{3}$$

where $y(n)$ is the output of the ALL system, and $\hat{W}(n)$ is the current estimate of the tap-weight vector. The desired response $d(n)$ is supplied for processing, alongside the tap-input vector $x(n)$. Equation (1) and Equation (2) are defined as the estimation error $e(n)$. μ is the step size, which governs the rate of convergence and ensures stability of the adaptive process. The value of μ can be constrained to satisfy the condition

$$0 < \mu < \frac{1}{\lambda_{\max}} \tag{4}$$

where $\lambda_{\max}$ is the maximum eigenvalue of $R = E\left[x(n)x^T(n)\right]$. Nest, the estimate of the gradient vector $\nabla J(n)$ can be computed as:

$$\nabla J(n) = -2p(n) + 2R\hat{W}(n) \tag{5}$$

where $p(n)$ is the cross-correlation vector.

2.2. **Cross-correlation functions.** Let us consider two stationary, zero mean signals, $d_0(n)$ and $d_1(n)$, with a time duration T. The cross-correlation functions of these signals are

$$R_{01}(\tau) = \frac{1}{T}\int_0^T d_0(n)d_1(n+\tau)dn \tag{6}$$

$$R_{10}(\tau) = \frac{1}{T}\int_0^T d_1(n)d_0(n+\tau)dn \tag{7}$$

The normalized version is defined as

$$\rho_{01} = \frac{R_{01}(\tau)}{\sqrt{R_{00}(0)R_{11}(0)}} \tag{8}$$

where $R_{00}(\tau)$ and $R_{11}(\tau)$ are autocorrelation functions of $d_0(n)$ and $d_1(n)$, respectively. For any τ, the relationship $-1 \leq \rho_{01}(\tau) \leq +1$ is satisfied. For this study, we use the discrete definition of this function:

$$\rho_{01}(\tau) = \frac{\sum\limits_{i=1}^{N-1} d_0(r+i)d_1(r+i+\tau)}{\sqrt{\sum\limits_{j=0}^{N-1}[d_0(r+j)]^2 \sum\limits_{k=0}^{N-1}[d_1(r+k+\tau)]^2}} \tag{9}$$

where N is the window length over which the correlation is calculated. If random processes $d_0(n)$ and $d_1(n)$ are each stationary and jointly stationary, then the correlation matrix can be written as

$$R(\tau) = \begin{bmatrix} R_{00}(\tau) & R_{01}(\tau) \\ R_{10}(\tau) & R_{11}(\tau) \end{bmatrix} \tag{10}$$

In this system, correlations obey the following symmetry relationship:

$$R_{01}(\tau) = R_{10}(-\tau) \tag{11}$$

2.3. **Autocorrelation matrix.** The main objective is to calculate the error between the desired responses, $d_0(n)$ and $d_1(n)$, and the filter estimation outputs, $y_0(n)$ and $y_1(n)$, respectively. From Equation (24), we can assume that $S_0(n) = \cos\omega_0 n$, $S_1(n) = \cos\omega_1 n$, $\cos\omega_d n$, and $\cos\omega_n n$ are uncorrelated with each other. The total expectations values of $E[\cos 2\omega_0 n]$ and $E[\cos 2\omega_1 n]$ are assumed to be zero. The corresponding parameters for the dominant signal can be written as follows:

$$\mathrm{E}[d_0^2(n)] = \mathrm{E}[\cos^2(\omega_0 n)] = \frac{1}{2} \tag{12}$$

The autocorrelation matrix can be written as

$$\mathbf{R}_0 = \begin{bmatrix} d_0(n) & d_0^H(n) \end{bmatrix} = \begin{bmatrix} \dfrac{1+m^4}{2} & 0 \\ 0 & \dfrac{1+m^4}{2} \end{bmatrix} \tag{13}$$

Assuming that the directional components and ambient noise are uncorrelated, R_0 can be written as

$$\mathbf{R}_0 = A_0 P_0 A_0^H + K_0 \tag{14}$$

In this equation, P_0 is the cross-spectrum matrix and A_0 denotes the gain. The matrix K_0 is the spatial correlation matrix of the ambient noise, which is defined as $K_0 = E\left[n_0(n) n_0^H(n)\right]$. The cross-correlation matrix can be shown as

$$P_0 = \begin{bmatrix} -\dfrac{1}{2} \\ -\mathrm{E}\left[\cos\left(\omega_0 n\right)\cos\left(\omega_0(n-1)\right)\right] \end{bmatrix} \tag{15}$$

The derivation of the optimal tap-weight vector can be obtained by solving the Wiener-Hopf equation. The optimal weight is given by

$$\mathbf{W}_{o0} = \mathbf{R}_0^{-1} P_0 = \begin{bmatrix} \dfrac{1}{1+m^4} \\ \dfrac{2\mathrm{E}\left[\cos\left(\omega_0 n\right)\cos\left(\omega_0(n-1)\right)\right]}{1+m^4} \end{bmatrix} \tag{16}$$

Using Equation (12), Equation (13), Equation (15), and Equation (16), we can derive the minimum mean square error value is calculated as follows

$$\xi_{\min 0} = \mathrm{E}\left[d_0^2(n)\right] - P_0^H \mathbf{R}_0^{-1} P_0 = \frac{1}{2} - \frac{1 + 4\mathrm{E}\left\{\left[\cos\left(\omega_0 n\right)\cos\left(\omega_0(n-1)\right)\right]\right\}^2}{2(1+m^4)} \tag{17}$$

Secondly, subdominant signal separation in an AWGN channel can also be derived, and has a minimum mean square error value of

$$\xi_{\min 1} = \mathrm{E}[d_1^2(n)] - P_1^H \mathbf{R}_1^{-1} P_1 = -2\left\{\mathrm{E}\left[\cos\left(\omega_1 n\right)\cos\left(\omega_1(n-1)\right)\right]\right\}^2 \tag{18}$$

3. **Main Results.** A new separation algorithm utilizing a DSP technique using PLL and ALL algorithms in FM system has been developed in order to improve the SNR of FM demodulators. The ALL algorithm can be used to separate dominant and subdominant signals from each other. Furthermore, we can reduce the computational complexity and suppress AWGN interference using this algorithm. In order to decrease the effects of AWGN channel interference, we use an adaptive filter to track time-varying range signals.

3.1. **The model of signal/noise.** In radio communication systems, CCI occurs when two or more FM signals strike a radio antenna with nearly the same carrier amplitude and mutually overlapping frequencies [13]. Briefly, CCI occurs when there are two or more simultaneous transmissions on a single channel. If the transmitted FM signals operate at the same carrier frequency, CCI occurs in the transmission channel. For convenience, we adopt to analyze the FM separation system using the following FM scheme:

$$\begin{aligned} y(t) &= y_0(t) + my_1(t) \\ &= A_c \cos\left[2\pi f_c t + \theta_0(t)\right] + mA_c \cos\left[2\pi f_c t + \theta_1(t)\right] \\ &= l(t)\cos\phi(t) \end{aligned} \tag{19}$$

where

$$\theta_i(t) = 2\pi k_f \int_0^t m_i(\tau)d\tau \quad i = 0, 1 \tag{20}$$

Parameters A_c, f_c, m, k_f, and $m(\tau)$ are the amplitude of the carrier signal, carrier frequency, interference to carrier ratio, frequency deviation constant, and modulation signal, respectively. This composite signal passes through an ideal band-pass filter (BPF) and wideband noise naturally becomes the narrow band noise. Band-limited noise can be calculated in terms of the in-phase and quadrature components and represented as

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \tag{21}$$

where $n_I(t)$ and $n_Q(t)$ are the in-phase and quadrature noise components of $n(t)$, respectively. Equivalently, we can express $n(t)$ in terms of its envelope and phase as

$$n(t) = n_n(t)\cos\left[2\pi f_c t + \varphi(t)\right] \tag{22}$$

where $n_n(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$ and $\phi_n(t) = \tan^{-1}\frac{n_Q(t)}{n_I(t)}$.

The mathematical representation of two modulated co-channel FM signals under an AWGN channel can be described as

$$\begin{aligned} y_n(t) &= l(t)\cos\phi(t) + n_n(t)\cos\left[2\pi f_c t + \varphi_n(t)\right] \\ &= Y(t)\cos\phi_2(t) \end{aligned} \tag{23}$$

In this paper, the signals are mixed with the co-channel transmitter, and we considered the co-channel as the one with AWGN interference. To recover the original signals, the PLL system can be used to demodulate the transmission signals. Figure 2 shows the modulator and demodulator for FM signals formed with the combined PLL and ALL models using the LMS algorithm.

3.2. **PLL model.** The PLL is a negative feedback system, the operation of which is closely linked to FM system. The method can track the differential phase between the transmission signals, and the value of the phase error is notably small. Therefore, we use the PLL model to demodulate mixed signals, where the output of the PLL can be written as:

$$f_{PLL}(t) = \frac{1 + m\cos\omega_d t}{1 + 2m\cos\omega_d t + m^2}S_0(t) + \frac{m^2 + m\cos\omega_d t}{1 + 2m\cos\omega_d t + m^2}S_1(t) \tag{24}$$

$$\omega_d t = \Theta_1(t) - \Theta_0(t) \tag{25}$$

$$S_i(t) = k_f \cdot s_i(t), \quad i = 0, 1 \tag{26}$$

$S_0(t)$ is the dominant signal, $S_1(t)$ is the subdominant signal, k_f is the frequency deviation constant, $S_i(t)$ is the ith modulating signal, $\Theta_i(t) = 2\pi k_f \int_0^t m_i(\tau)d\tau$ $i = 0, 1$ and m is the interference to carrier ratio.

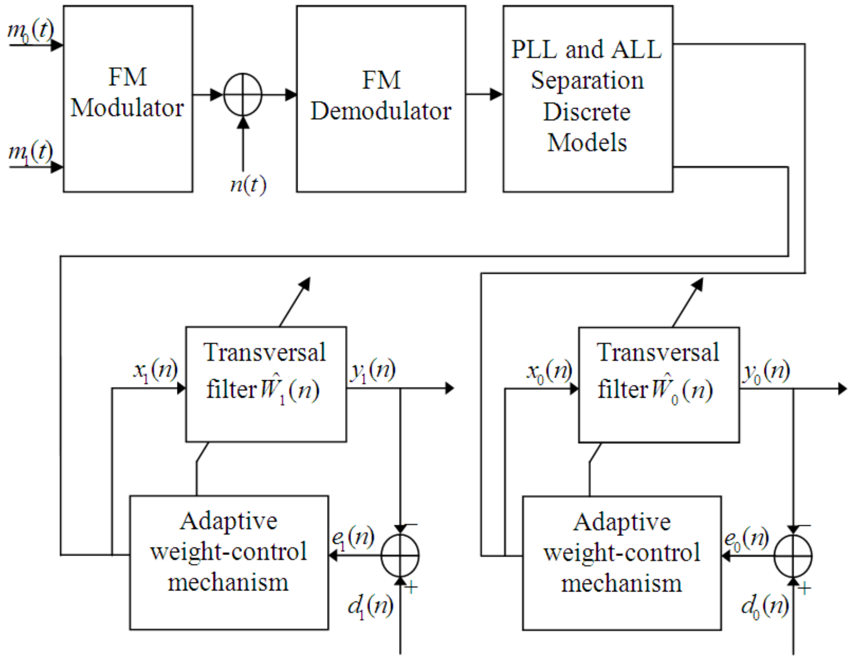

FIGURE 2. The discrete time separation model with AWGN

We suppose that the transmission signals are transmitted with AWGN interference. Such phenomena as the Rician spikes, cross modulation distortion, and high noise interference can be found in the receiver. An ALL system can be used to eliminate channel interference in order to separate the CCI signals from the PLL system.

3.3. **ALL models.** The ALL is a system of high gain, high bandwidth servo-loops. When this output is multiplied by the PLL output, the ALL can be achieve a perfect dominant signal. The resulting system can be used to demodulate the dominant signal and suppress the subdominant signal. The ALL functions are defined as

$$f_{ALL}(t) = \frac{1 - m'^2}{1 + m' \cos \omega_d t} \tag{27}$$

$$f_{ALL-1}(t) = \frac{-m'^2 - m' \cos \omega_d t}{1 + m' \cos \omega_d t} \tag{28}$$

$$f_{ALL-2}(t) = \frac{-1 - 2m' \cos \omega_d t - m'^2}{1 + m' \cos \omega_d t} \tag{29}$$

The relationship between nonlinear parameter m' and linear parameter m in practical ALL circuit is defined as:

$$m' = \frac{2m}{1 + m^2} \tag{30}$$

The simulated PLL output was found to contain Rician spikes. Although the proposed ALL algorithm can be used to cancel these spikes in the receiver, in an AWGN channel, the signal separation performance will be affected. An adaptive filter is provided to track the time-varying interference signal and compare the estimated signal with the desired signal. The PLL output is multiplied by a DC shift of the ALL function, which can be written as:

$$
\begin{aligned}
f_{PLL}(t) \times f_{ALL-2}(t) = & - \frac{1 + \alpha(t)\cos\omega_n t}{1 + 2\alpha(t)\cos\omega_n t + \alpha^2(t)} S_0(t) \\
& + \frac{1 + \alpha(t)\cos\omega_n t}{1 + 2\alpha(t)\cos\omega_n t + \alpha^2(t)} f_{ALL-1}(t) S_1(t)
\end{aligned} \tag{31}
$$

The subdominant signal is derived as follows:

$$
\begin{aligned}
f_{PLL}(t) \times f_{-(ALL-1)}(t) = & - \frac{1 + \alpha(t)\cos\omega_n t}{1 + 2\alpha(t)\cos\omega_n t + \alpha^2(t)} \frac{f_{ALL-1}(t)}{f_{ALL-2}(t)} S_0(t) \\
& + \frac{1 + \alpha(t)\cos\omega_n t}{1 + 2\alpha(t)\cos\omega_n t + \alpha^2(t)} \frac{f_{-(ALL-1)}(t) f_{ALL-1}(t)}{f_{ALL-2}(t)} S_1(t)
\end{aligned} \tag{32}
$$

To simply the analysis and simulation, we use a DSP model to transform the continuous time signal into a discrete signal. The mathematical theory of the DSP separation algorithm is described in this section. A FIR adaptive filter is proposed for detecting variations in dominant and subdominant signals. Numerical analysis can be calculated either in the continuous or discrete time domain. It is assumed that the dominant and subdominant signals pass through an analogue/digital (A/D) converter [14,15]. The discrete time mathematical representation of the PLL output is defined as follows:

$$f_{PLL}(n) = [1 - \alpha(n)\cos\omega_n n]\, f_{PLL1}(n) + [1 - \alpha(n)\cos\omega_n n]\, f_{PLL2}(n) \tag{33}$$

$$f_{PLL1}(n) = [1 - m\cos(\omega_n n)]\, S_0(n) \tag{34}$$

$$f_{PLL2}(n) = [m\cos(\omega_n n)]\, S_1(n) \tag{35}$$

ω_n: The instantaneous frequency of the noise phase and transmission phase. An algorithm is proposed that can be used to separate the mixed signals and eliminated the channel interference effects. Using DSP analysis, we can apply a z-transform to analyze Equation (33), which can be rewritten as

$$
\begin{aligned}
f_{PLL}(z) = \sum_{n=0}^{\infty} \Bigg[& \frac{1 + m\cos(\varpi_d n)}{1 + 2m\cos(\varpi_d n) + m^2} \beta\varpi_0 \cos(\varpi_0 n) \\
& + \frac{m^2 + m\cos(\varpi_d n)}{1 + 2m\cos(\varpi_d n) + m^2} \beta\varpi_1 \cos(\varpi_1 n) \Bigg] \cdot z^{-n}
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
f_{PLL}(z) = & \sum_{k=0}^{\infty} \left\{ \frac{\beta\varpi_0 (-m)^k}{2} \left[\frac{1 - \cos(\varpi_0 - k\varpi_d) z^{-1}}{1 - 2\cos(\varpi_0 - k\varpi_d) z^{-1} + z^{-2}} \right.\right. \\
& \left.\left. + \frac{1 - \cos(\varpi_0 + k\varpi_d) z^{-1}}{1 - 2\cos(\varpi_0 + k\varpi_d) z^{-1} + z^{-2}} \right] \right\} \\
& + \sum_{i=1}^{\infty} \left\{ (-1)^{i+1} \frac{\beta\varpi_1 m^i}{2} \left[\frac{1 - \cos(\varpi_1 - i\varpi_d) z^{-1}}{1 - 2\cos(\varpi_1 - i\varpi_d) z^{-1} + z^{-2}} \right.\right. \\
& \left.\left. + \frac{1 - \cos(\varpi_1 + i\varpi_d) z^{-1}}{1 - 2\cos(\varpi_1 + i\varpi_d) z^{-1} + z^{-2}} \right] \right\}
\end{aligned} \tag{37}
$$

In the ALL algorithm, these aforementioned equations can be rewritten as

$$f_{ALL-1}(m') = -\left[m'^2 + (1-m'^2)\sum_{n=1}^{\infty}(-1)^{n+1}m'^n\cos^n\theta\right] \tag{38}$$

$$f_{-(ALL-1)}(m') = \left[m'^2 + (1-m'^2)\sum_{n=1}^{\infty}(-1)^{n+1}m'^n\cos^n\theta\right] \tag{39}$$

$$f_{ALL-2}(m') = -1 - m\cos\theta - \sum_{n=2}^{\infty} m'^n\cos^{n-2}\theta(1-\cos^2\theta) \tag{40}$$

Using DSP analysis, we can apply a z-transform to analyze Equations (39) and (40), which can be rewritten as

$$\begin{aligned} f_{-(ALL-1)}(z) = \Bigg\{ & \frac{m'^2}{1-z^{-1}} + (1-m'^2)\left[\frac{m'(1-\cos\omega_d z^{-1})}{1-2\cos\varpi_d z^{-1}+z^{-2}}\right. \\ & - \frac{m'^2}{2}\left(\frac{1}{1-z^{-1}} + \frac{1-\cos\varpi_d z^{-1}}{1-2\cos\varpi_d z^{-1}+z^{-2}}\right) \\ & \left. + \frac{m'^3}{4}\left[\frac{3(1-\cos\omega_d z^{-1})}{1-2\cos\omega_d z^{-1}+z^{-2}} + \frac{1-\cos 3\omega_d z^{-1}}{1-2\cos 3\omega_d z^{-1}+z^{-2}}\right]\right]\cdots\Bigg\} \end{aligned} \tag{41}$$

and

$$\begin{aligned} f_{ALL-2}(z) = & \frac{-1}{1-z^{-1}} - m'\frac{1-\cos\omega_d z^{-1}}{1-2\cos\omega_d z^{-1}+z^{-2}} \\ & - \frac{1}{2}\cdot\left[\frac{1}{(1-z^{-1})} - \frac{1-\cos 2\omega_d z^{-1}}{1-2\cos\omega_d z^{-1}+z^{-2}}\right] \\ & \cdot\left[\frac{m'^2}{1-z^{-1}} + \frac{m'^3(1-\cos\omega_d z^{-1})}{1-2\cos\omega_d z^{-1}+z^{-2}}\right. \\ & \left. + \frac{m'^4}{2}\left(\frac{1}{1+z^{-1}} + \frac{1-\cos\omega_d z^{-1}}{1-2\cos\omega_d z^{-1}+z^{-2}}\right)\cdots\right] \end{aligned} \tag{42}$$

A DSP infinite-impulse response (IIR) filter can be designed using the coefficients of this equation. However, the separated signals at the ALL output exhibit noise distortion. In order to remove this noise, we adopt the LMS filter algorithm to search for the optimal and most stable value of the system [16].

4. **Eigenanalysis of Autocorrelation Matrix.** The autocorrelation matrix of the input characterizes the behavior of the MMSE in terms of their misadjustment and convergence characteristics when gradient optimization is used. In this analysis, we are concerned with the performance deviation of the adaptive algorithm in the presence of noise, as well as signal. The notation used in the analysis is discrete because, despite using a digital filter, both its tapped delay line implementation structure and the sampling rate determine the behavior of the adaptive algorithm used.

4.1. **Autocorrelation matrix in the presence of signal dependence noise.** It has been shown that signal-dependent noise interference cancellation corresponds to autocorrelation matrix methods for solving a set of nonlinear equations [17]. Based on aforementioned system model, the set of equations to be solved can be written as

$$u(n) = d(n) + n(n) + v(n) \tag{43}$$

where $d(n)$ is the noiseless input, $n(n)$ is the additive noise component, and $v(n)$ is the signal dependent noise component. $v(n)$ is defined as $g(d(n))h(n(n))$, where $g(\cdot)$ and $h(\cdot)$ indicate two functions.

The two autocorrelation matrices $\tilde{R} = E\left\{u(n)u(n)^T\right\}$ and $\tilde{R}_0 = E\left\{D(n)D(n)^T\right\}$ are noisy and noiseless input vectors. The sample vectors are defined such that they include the last M samples, e.g., $D(n) = [d(n), d(n-1), \cdots, d(n-M+1)]^T$

$$\tilde{R} = \gamma\tilde{R}_0 + \alpha I + \beta ZZ^T \tag{44}$$

where Z is an $M \times 1$ vector of all Zs, and γ, α, and β are coefficients dependent on the signal and noise components. We assume that $n(n)$ is independent and identically distributed (i.i.d.). Three cases that can be delineated using Equation (44) are defined as follows:

The noiseless signal $d(n)$ is i.i.d. In this case, $\tilde{R}_0$ is a diagonal matrix, and $g(d(n))$ and $h(n(n))$ can be of any form. The coefficients in Equation (44) for this case are

$$\begin{aligned}&\gamma = 1\\&\alpha = \sigma_n^2 + 2\mu_h(\gamma_{dg} - \mu_d\mu_g) + 2\mu_g(\gamma_{nh} - \mu_n\mu_h) + \mu_h^2\sigma_g^2 + \mu_g^2\sigma_h^2 + \sigma_g^2\sigma_h^2\\&\beta = \mu_n^2 + 2\mu_d\mu_n + 2\mu_h\mu_d\mu_g + 2\mu_g\mu_n\mu_h + \mu_h^2\mu_g^2\end{aligned} \tag{45}$$

where μ_d, μ_n, μ_g, μ_h, γ_{dg}, and γ_{nh} are the expectation values of $d(n)$, $n(n)$, $g(d(n))$, $h(n(n))$, $d(n)g(d(n))$, and $n(n)h(n(n))$, respectively, and σ_d^2, σ_n^2, σ_g^2, and σ_h^2 are variances of $d(n)$, $n(n)$, $g(d(n))$, and $h(n(n))$, respectively.

4.2. **Approximation of eigenvalues.** The generalized eigenvalue decomposition [17] can be applied to R_k as

$$R_kE = K_kE\Lambda_0 \tag{46}$$

In this equation, eigenvector matrix E consists of eigenvectors $\{e_m\}$ as $E = [e_1, \cdots, e_M]$, and K_k denotes the spatial correlation matrix of n_k (*noise vector*). The eigenvalue matrix has diagonal eigenvalues, $\Lambda_0 = diag(\lambda_0^1, \cdots, \lambda_M^0)$, for real eigenvalues $\lambda_0^1 < \lambda_2^0 < \cdots < \lambda_M^0$, and X is an orthogonal matrix with columns that correspond to eigenvectors. The matrix A is defined as

$$A \equiv X^T\tilde{R}X = \gamma\Lambda_0 + \alpha I + \beta X^TZZ^TX \equiv \Lambda + \beta xx^T \tag{47}$$

where the diagonal entries of Λ are given by $c_i = \gamma\lambda_i^0 + \alpha$. $x \equiv X^TZ = [x_1, x_2, \cdots, x_M]^T$, where x_i is the sum of the entries of the ith eigenvector. Note that A and $\tilde{R}$ are similar matrices and must have the same eigenvalues. The proximity of zero crossings to the poles in $f(\lambda)$ will provide a useful approximation. To obtain an approximation for the maximum eigenvalue, $f(\lambda)$ can be written as

$$f(\lambda) = 1 + \beta\sum_{i=2}^{M}\frac{x_i^2}{c_i - \lambda} + \frac{\beta x_1^2}{c_1 - \lambda} \tag{48}$$

Because $f(\lambda)$ has a pole at x_1, the last term dominates the behavior of $f(\lambda)$ for $\lambda \cong c_1$. The second term, which is the sum of terms associated with the other eigenvalues, is almost constant in the region around $\lambda_{\max}$. Thus, we can write

$$f(\lambda_{\max}) \approx 1 + \beta\sum_{i=2}^{M}\frac{x_i^2}{c_i - c_1} + \frac{\beta c_1^2}{c_1 - \lambda_{\max}} \tag{49}$$

The solution of $f(\lambda_{\max} = 0)$ yields

$$\overline{\lambda}_{\max} = \gamma\lambda^0_{\max} + \alpha + \frac{\beta x_1^2}{1 + \beta \sum_{i=2}^{M} \frac{x_i^2}{(c_i - c_1)}} \tag{50}$$

We have replaced c_1 in the last term in Equation (47) with its definition, which is given in Equation (43). Note that $\bar{\lambda}_{\max} < \lambda_{\max}$, where $\lambda_{\max}$ is the true maximum eigenvalue of $\tilde{R}$.

Thus, the maximum eigenvalue can be further approximated as

$$\overline{\lambda}_{\max} \approx \gamma\lambda^0_{\max} + \alpha + \beta x_1^2 \tag{51}$$

This finding suggests the use of similar arguments to approximate the other eigenvalues of the correlation matrix $\tilde{R}$. For example, the approximate minimum eigenvalue of $\tilde{R}$ can be written as

$$\overline{\lambda}_{\min} = \gamma\lambda^0_{\min} + \alpha + \frac{\beta x_M^2}{1 + \beta \sum_{i=1}^{M-1} \frac{x_i^2}{(c_i - \lambda_{\min})}} \approx \gamma\lambda^0_{\min} + \alpha \tag{52}$$

The eigenvalue spread $\chi(R)$ of the equalizer input signal covariance matrix is given by

$$\chi(R) = \frac{\lambda_{\max}}{\lambda_{\min}} \tag{53}$$

where $\lambda_{\max}$ and $\lambda_{\min}$ denote the maximum and minimum eigenvalues of the input autocorrelation matrix $R = E\left[r_{fullband}[n] \quad r^H_{fullband}[n]\right]$. Thus, $\chi(R)$ can be approximated by the ratio of the maximum and minimum of the equalizer input PSD, $\gamma_{\max}$ and $\gamma_{\min}$, as [18]

$$\chi(R) \cong \frac{\gamma_{\max}}{\gamma_{\min}} \tag{54}$$

5. **Numerical Example and Simulation.** We use the concept of majorization as a mathematical tool to characterize different spatial correlation environments. Using majorization theory, an analytical framework was established to assess the performance of separation systems with different profiles. In particular, suppression theorems were derived for various performances parameters, such as output SNR.

The advantage of the ALL is that it is a system that does not require coding, has high gain and high bandwidth servo-loop operation, and is in the amplitude, rather than frequency, domain. The ALL system can prove to be an effective improvement in communication systems for filtering out signal distortion and AWGN interference.

The following simulation demonstrates the performance of the proposed signal separation model for AWGN channels based on the LMS algorithm. The MMSE criteria of the step size and optimal weight coefficient values can be defined using the Wiener-Hopf Equation (16) with a steepest descent algorithm. The filter output response can be used to approximate the desired response. Impairment of the channel noise will affect the MMSE value. The PSD performances of separated signals with SNR of -5 dB are illustrated in Figure 3 for m located at 0.9999 and 1.0001. Figure 4 shows the 100 Hz component of the dominant signal and 500 Hz component of the subdominant signal for an m value of "10" and -5 dB SNR [22].

The simulation results show successful separation of FM signals by the PLL and ALL algorithms. In an AWGN channel, we propose the LMS algorithm for accelerating the convergence rate and obtaining the MMSE value [19]. Figures 5 and 6 show the simulation results of the dominant and subdominant signals, respectively. Figure 5 shows the dominant separated signal with minimal SNR values between -20 dB and 0 dB, which improved by 2.79 dB and 2.8 dB for m values of 0.9 and 0.1, respectively. By the same

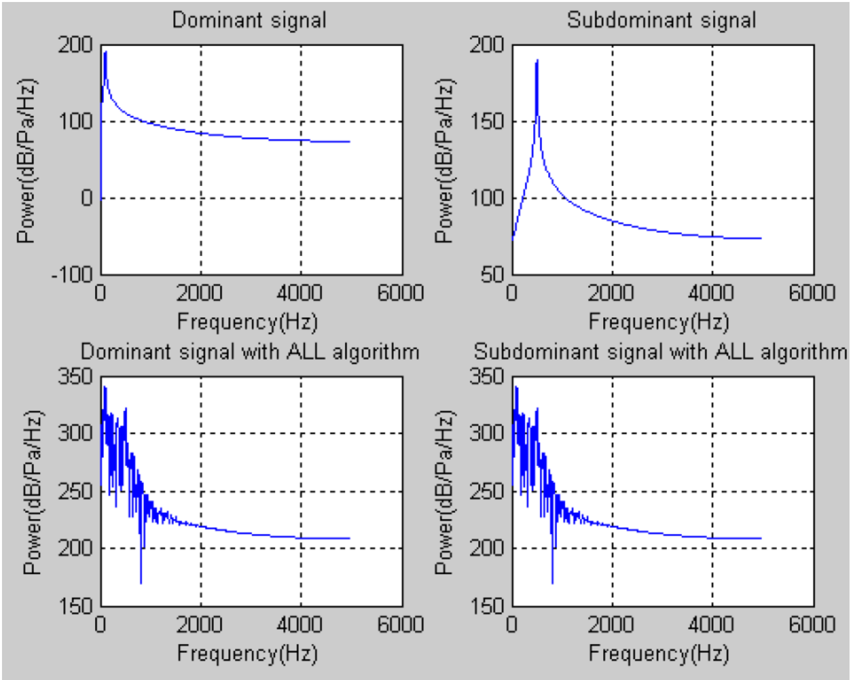

FIGURE 3. The separated signals for an m of 0.9999 and 1.0001

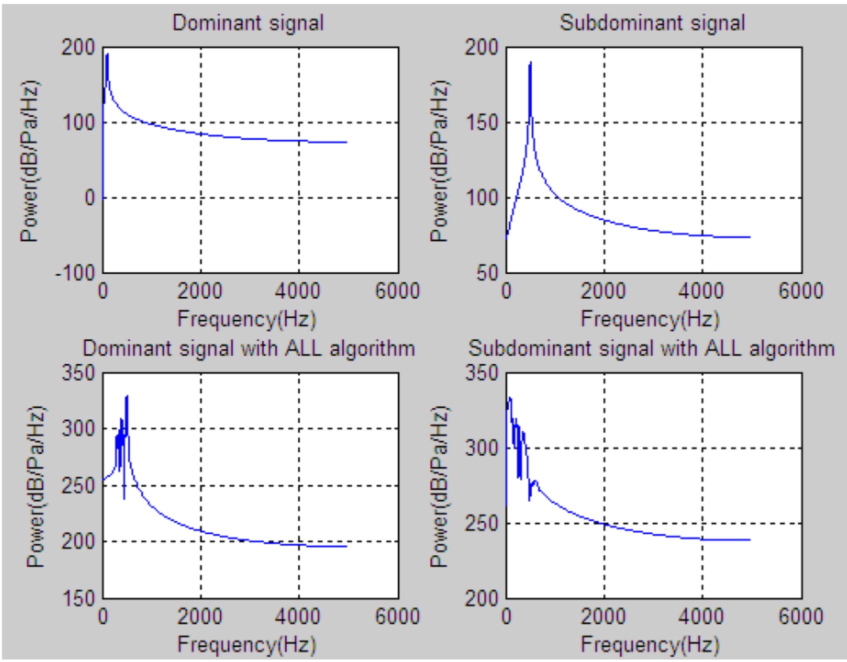

FIGURE 4. The PSD output signals with $m = 10$ and -5 dB SNR

method, the subdominant separation signal SNR values varies between -20 dB and 0 dB, and improved by 1.8 dB and 3.3 dB for m values of 0.9 and 0.1, respectively. The MSE values can be found using these figures, and the adaptive filter algorithm is introduced to solve this problem.

The correlation function versus total channel SNR for the dominant signal, S_0, and subdominant signal, S_1, is shown in Figure 7. The correlation functions for SNR between 0 dB and 20 dB shows a dominant signal S_0 correlation coefficient of close to 1. The blue curve depicts the ideal channel simulator with numerical variations of the CCI. Figure 8 shows the eigenvalues of the autocorrelation matrix versus received SNR for the dominant signal, S_0, and subdominant signal, S_1, under two transmit correlation scenarios, as given by

$$H_0 = 1.203U_{\min}U_{\min}^H + 0.7187U_{\max}U_{\max}^H$$
$$H_1 = \begin{bmatrix} 1 & 1-\lambda_{\max} \\ 1-\lambda_{\min} & 1 \end{bmatrix} \text{ or } H_1 = \begin{bmatrix} 1 & 1-\lambda_{\min} \\ 1-\lambda_{\max} & 1 \end{bmatrix} \quad (55)$$

where $U_{\max}$ and $U_{\min}$ are the eigenvectors corresponding to the minimum and maximum eigenvalue of the matrix A, which is given in Equation (47).

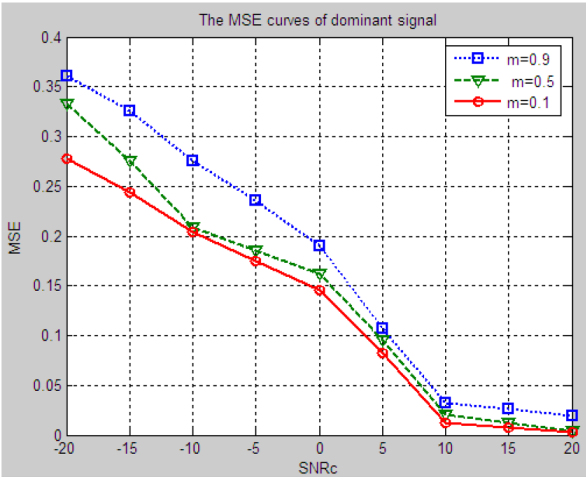

FIGURE 5. The MSE and SNR_c for the dominant signal S_0

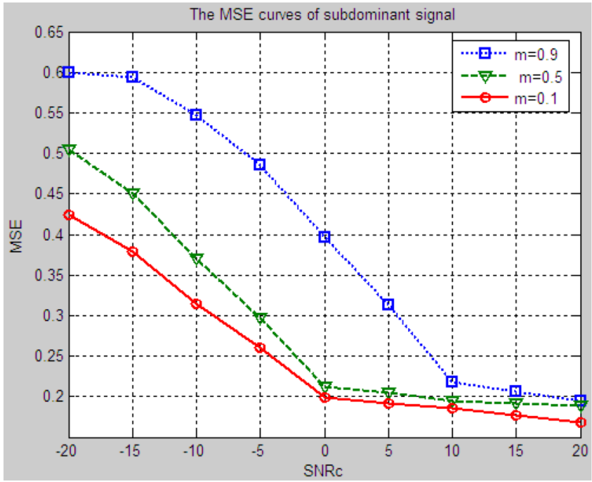

FIGURE 6. The MSE and SNR_c for the subdominant signal S_1

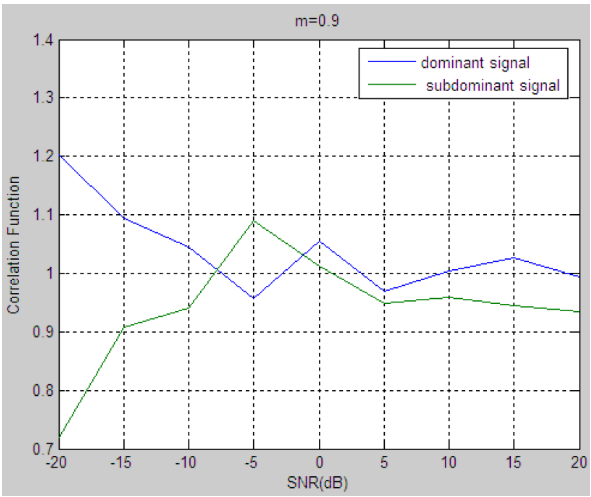

FIGURE 7. The correlation function versus total received SNR for the dominant signal S_0, and the subdominant S_1

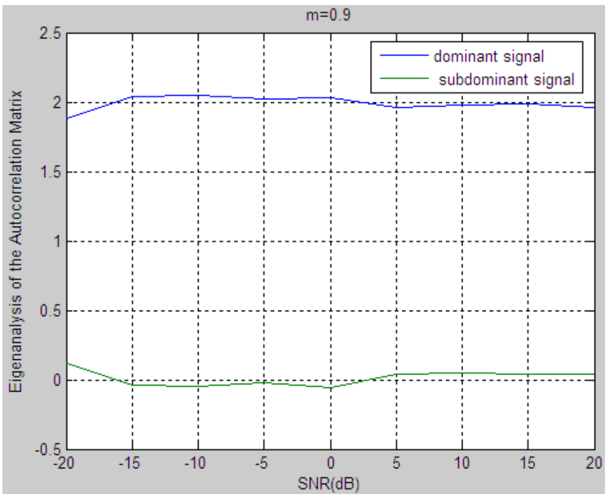

FIGURE 8. The eigenvalues of the autocorrelation matrix versus received SNR for the dominant signal S_0, and the subdominant S_1

H_0 and H_1 have the same set of eigenvalues $\{1.9298, 0.0702\}$ when the ICR m is equaled to 0.9 and the SNR is -20 dB. Using [20], when μ is equal to 0.002, the eigenvalue spread for our system (m is equaled to 0.1 and SNR is 0 dB) and the Volterra model are found to be 116.6 and 81.343 (compared with the experimental value), respectively. Comparing our system to the Volterra model, the eigenvalue spread of our system is reduced approximately 1.43-fold. There are many algorithms that successfully compensate for the eigenvalue spread and have low computational complexities that enhance the performance of adaptive filters. For example, our proposed algorithm is more efficient than the normalized LMS (NLMS) algorithm in cases in which the eigenvalue spread is large [21].

6. **Conclusions.** We propose a signal separation system with an adaptive FIR filter based on LMS theory for AWGN channels. The proposed technique can effectively separate the desired signals in the presence of interference noise. The adaptive filter was found to have better tracking ability, and all of the weights are updated using an iterative procedure. The algorithm proposed in this paper is suited for high performance separation in both low-SNR and high-ICR conditions.

The ALL system was used in combination with a PLL and additional circuitry to achieve a major improvement in frequency modulation technology. The ALL system is used to filter out signal distortion, CCI, and AWGN interference, and should be a significant improvement in communication systems. According to Figure 4, this paper adjust the ICR (i.e., m value), which can exchanges the dominant and subdominant signals. In the future, the ALL system can be applied to a monitored system and signal intercepted applications. Moreover, the all-digital phase-locked loop (ADPLL) and all-digital amplitude-locked loop (ADALL) system structures [23] can be applied for binary frequency-shift keying (BFSK) modulation of signals for a co-channel transmission separation system.

In this paper, we proposed two signals for characterizes the correlation properties and variations of the SNR for communication systems. The numerical and simulation results show that these two signals can characterize the correlation properties of SNR notably well. We utilize the eigenvalues distribution of autocorrelated matrix in the form of determinants with special functions. The polynomial expression was used to simplify mathematical operations. Before transmitting the original signal, we used a good match eigenvalue spread because our system provides a form of secure communication in unfriendly surroundings such that the transmitted signal is not easily detected or discriminated by unwanted listeners and visibly separated the original signal. The advantages of our system are that it doubles the useful range of the FM spectrum and can easily increase the channel capacity of an MA system or any other existing schemes. Additionally, the suggested system also creates a secure communication system, which can potentially supersede or displace encryption (or decryption) systems.

REFERENCES

[1] E. Walach and B. Widrow, Appendix A: Stability and misadjustment of the LMS adaptive filter, in *Adaptive Inverse Control, Reissue Edition: A Signal Processing Approach*, IEEE Book Chapter, 2008.

[2] B. Widrow, Thinking about thinking: The discovery of the LMS algorithm, *IEEE Signal Process Magazine*, vol.22, no.1, pp.100-106, 2005.

[3] T. Jamsa, P. Kyosti and J. Iinatti, Correlation error metrics of simulated MIMO channels, *IEEE the 65th Vehicular Technology Conference*, pp.407-412, 2007.

[4] J. Villanen, P. Suvikunnas, C. Icheln, J. Ollikainen and P. ainikainen, Performance analysis and design aspects of mobile-terminal multiantenna configurations, *IEEE Trans. on Vehicular Technology*, vol.57, no.3, pp.1664-1674, 2008.

[5] M. Souden, J. Benesty and S. Affes, Broadband source localization from an eigenanalysis perspective, *IEEE Trans. on Audio, Speech, and Language Processing*, vol.18, no.6, pp.1575-1587, 2010.

[6] Y.-R. Tsai, Spatial correlation models for total co-channel interference and carrier-to-interference ratio in mobile cellular systems, *IEEE Trans. on Wireless Comm.*, vol.5, no.10, pp.2752-2759, 2006.

[7] S. Chatzinotas, M. A. Imrana and R. Hoshyar, On the multicell processing capacity of the cellular MIMO uplink channel in correlated rayleigh fading environment, *IEEE Trans. on Wireless Comm.*, vol.8, no.7, pp.3704-3715, 2009.

[8] G. Li and L. H. Xie, Comments on performance analysis of signed self-orthogonalizing adaptive lattice filter, *IEEE Trans. on Circuits and Systems II: Analog and Digital Signal Processing*, vol.48, no.10, pp.994-996, 2001.

[9] H. Shin and M. Z. Win, MIMO diversity in the presence of double scattering, *IEEE Trans. on Information Theory*, vol.54, no.7, pp.2976-2996, 2008.

[10] G. Miramontes, E. Garcia and A. Richardson, A comparison between the Wiener and LMS methods in FIR digital filter synthesis using adaptive modeling, *IEEE Pacific Rim Conference*, pp.375-378, 1995.

[11] S. Haykin, *Adaptive Filter Theory*, 4th Edition, 2002.

[12] J. G. Proakis, *Digital Signal Processing*, Wiely & Sons, 2000.

[13] T. S. Rappaport, *Wireless Communications: Principles and Practice (Prentice Hall Communications Engineering and Emerging Technologies Series)*, Prentice Hall, USA, 2001.

[14] P. A. Lyn and W. Fuerst, *Introductory Digital Signal Processing With Computer Applications*, 2nd Edition, Wiely, 2000.

[15] S. Haykin, *Communication System*, 4th Edition, Wiley, 2001.

[16] G.-J. Jong, P.-J. Liao, C.-Y. Jung and T.-J. Su, Multi-channel interference separation for the AWGN channel, *Int. Symposium on Intelligent Signal Processing and Comm. Systems*, pp.581-584, 2005.

[17] W. Wang and T. Adali, Eigenanalysis of autocorrelation matrices in the presence of noncentral and signal-dependent noise, *IEEE Signal Processing Letter*, vol.12, no.2, pp.85-88, 2005.

[18] Y. H. Ng, H. Mohamad and T. C. Chuah, On sub-band adaptive chip equalisation for direct sequence code division multiple access downlink over dispersive channels, *IET Comm.*, vol.1, no.4, pp.802-809, 2007.

[19] T. E. Hunter and D. A. Linebarger, An alternative formulation for low rank transform domain adaptive filtering, *ICASSP*, vol.1, pp.29-32, 2000.

[20] T. Ogunfunmi and S. L. Chang, Second-order adaptive Volterra system identification based on discrete nonlinear Wiener model, *IEE Vision, Image and Signal Processing*, vol.148, no.1, pp.21-29, 2001.

[21] S. Gazor and T. Liu, Adaptive filtering with decorrelation for coloured AR environments, *IEE Proceedings – Vision, Image and Signal Processing*, vol.152, no.6, pp.806-818, 2005.

[22] G.-J. Jong and J.-C. Huang, *Combined Phase-Locked Loop and Amplitude-Locked Loop Module for Switching FM Signals*, USA Patent, No.US7598803B2.

[23] G.-J. Jong and G.-J. Horng, The improvement of all-digital amplitude-locked loop separation analysis combined MIMO system, *International Journal of Innovative Computing, Information and Control*, vol.7, no.3, pp.1001-1015, 2011.