International Journal of Innovative
Computing, Information and Control
Volume 8, Number 7(B), July 2012

ICIC International ©2012 ISSN 1349-4198
pp. 5011–5024

ENERGY-SHAPING AND L_2 GAIN DISTURBANCE ATTENUATION CONTROL OF INDUCTION MOTOR

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Received May 2011; revised September 2011

Abstract. *A novel speed tracking control scheme of induction motor (IM) is presented based on the state error Port-Controlled Hamiltonian (PCH) system and L_2 disturbance attenuation theory. Firstly, a PCH system model of IM is established. Secondly, using interconnection and damping assignment, the desired state error PCH structure is assigned to the closed-loop IM control system by the energy-shaping (ES) principle when the load torque is known. Thirdly, following the idea of rotor field orientation, the desired equilibrium of IM system is obtained. The equilibrium stability of the closed-loop system is verified. Moreover, the L_2 gain load torque disturbance attenuation technology is applied to the PCH control of IM system. At last, proportional integral (PI) control is used to eliminate the steady state error brought by the load disturbance. The comparative studies and simulation results show that the proposed scheme has a good performance and application prospects.*
Keywords: Induction motor, Speed control, Hamiltonian systems, L_2 gain, Energy-shaping

1. **Introduction.** Induction motor (IM) drives are widely used in industry for variable-speed applications due to its reliability, ruggedness and low cost. These applications include fans and pumps, textile and paper mills, subway and locomotive propulsions, machine tools and robotics, etc. However, the control problem of IM is rather complicated by the fact that motor model is nonlinear, the motor parameters are time-varying and the load torque disturbances are uncertain during operation. Therefore, the existing scalar control, vector (field-oriented) control and direct torque control methods still have many shortcomings [1].

In order to solve above problems, several globally stable speed controllers for IM have been reported in the control literature. These controllers can be designed using, such as adaptive control [2], sliding model variable structure control [3], intelligent control [4], feedback linearization [5], differential-algebraic nonlinear control [6], backstepping principles [7], passivity-based control (PBC) [8] and L_2 disturbance attenuation control [9]. The above methods have been used in many fields and a series of research results has been acquired. Recently, the energy-shaping (ES) and Port-Controlled Hamiltonian (PCH) systems theory have attracted a lot of attention [10-18]. The PCH system with dissipation has become an important tool in nonlinear control system research. The ES consists in shaping the energy function of the system in order to obtain a new closed-loop energy function that has a minimum point at the desired equilibrium point, preserving the original interconnection and dissipation structure. Thus, the closed-loop system preserves the PCH form with a stable equilibrium point at the state of minimum energy.

A class of electromechanical systems, such as induction motor can be viewed as multiport (electrical port and mechanical port) energy-transformation device. The action of a controller may also be interpreted in energy terms as another dynamical system-interconnected with the electromechanical system to modify its behaviour. The control problem can then be described as finding a dynamical system and an interconnection pattern such that the overall energy function takes the desired form. This energy-shaping approach is used to electromechanical control system. Attention of the new methodology is now focused on the interconnection and damping structures of the system. Energy is injected into the electrical port for determining the full system's behaviour. Many theoretical extensions and practical applications of ES and PCH control theory have been reported in the literature [11-18].

In practical motor control system, disturbance attenuation and parametric uncertainty are also important issues. The PBC design method has been extended by many researchers to achieve γ-dissipativity [8, 9] that not only guarantees asymptotic stability but also renders the L_2-gain from disturbance to a penalty signal less than a given level $\gamma > 0$. As PBC design, a key to solve the disturbance attenuation problem along this line is to construct a proper storage function that ensures the γ-dissipativity. Several effective methods have been reported by [8, 9].

In this paper, a novel speed tracking control scheme of IM is presented based on the ES and L_2 disturbance attenuation principle. The main feature of the method is that the closed-loop system has state error PCH structure. The scheme has the advantage that the closed-loop energy function can be used as Lyapunov (or storage) function rendering the stability analysis more transparent and controller design simpler. Further, the L_2 disturbance attenuation control of the IM system is proposed based on the state error PCH system theory. The L_2-gain from the load torque disturbance to a penalty signal may be reduced to any given level if the penalty signal is defined properly. In order to track the changes of the load torque and eliminate speed steady state error of the IM, the proportional integral (PI) control action of speed error is also added to the system. Applying space vector pulse-width modulation (SVPWM) signal transformation technology, speed regulation of IM system is implemented by controlling the state of each switch in the inverter. Compared with the conventional vector control, the proposed control algorithm has good control performance. The simulation results illustrate the effectiveness of the developed controller.

The remainder of the paper is organized as follows. In Section 2, the state error PCH control principle of IM is presented. Based on the state error PCH control of IM, load disturbance attenuation control is developed according to L_2 gain and PI control principle in Section 3. In Section 4, the theoretical analysis is verified to be right by the comparative studies and simulation results. Finally, some conclusions are presented in Section 5.

2. The State Error PCH Control of IM.

2.1. Control principle of the state error PCH system.

Consider an affine nonlinear system

$$\begin{cases} \dot{x} = f(x) + g(x)u, \\ y = h(x). \end{cases} \tag{1}$$

The system (1) is passive if it is possible to find a non-negative function $V(x)$ such that $V(0) = 0$ and

$$V(x(t)) - V(x(0)) \leq \int_0^t y^T(\tau)u(\tau)d\tau. \tag{2}$$

The PCH systems can be described as follows [8, 10]:

$$\dot{x} = [J(x) - R(x)]\frac{\partial H(x)}{\partial x} + g(x)u$$
$$y = g^T(x)\frac{\partial H(x)}{\partial x} \tag{3}$$

where $x \in \Re^n$ is the state vector, u, $y \in \Re^m$ representing the input and output vector respectively, are conjugated variables whose product has units of power, $R(x) = R^T(x) \geq 0$ represents the dissipation, the interconnection structure is captured in matrix $g(x)$ and the skew-symmetric matrix $J(x) = -J^T(x)$, $H(x)$ is the Hamiltonian function of the system.

The most important feature of PCH systems is its input-output passivity and stability properties. Given a dynamical system (3), the variation of internal energy equals the dissipated power plus the power provided to the system by the environment. The energy balance equation for system (3) is given by

$$\frac{dH(x)}{dt} = \left[\frac{\partial H(x)}{\partial x}\right]^T \dot{x} = y^T u - \left[\frac{\partial H(x)}{\partial x}\right]^T R(x)\frac{\partial H(x)}{\partial x} \leq y^T u \tag{4}$$

then, integrating (4) over an time-interval $[0, t]$ results in the following well-known dissipative inequality below,which ascertains the passivity properties of PCH systems

$$H(x(t)) - H(x(0)) \leq \int_0^t y^T(\tau)u(\tau)d\tau$$

which is the same as (2).

If disturbance input is added to system (3), then PCH systems with disturbance input are given by

$$\dot{x} = [J(x) - R(x)]\frac{\partial H(x)}{\partial x} + g(x)u + g_w(x)w \tag{5}$$
$$y = g^T(x)\frac{\partial H(x)}{\partial x}$$

where w is the disturbance input, $g_w(x)$ is the disturbance input matrix.

The feedback control principle of state error PCH system is given as follows.

Proposition 2.1. *Consider the PCH system (3), let x_0 be a desired equilibrium and $\tilde{x} = x - x_0$ be the state error. Given $H(x)$, $J(x)$, $R(x)$, $g(x)$ and*

$$H(x) = \frac{1}{2}x^T D^{-1} x \tag{6}$$

$$J(\tilde{x} + x_0) = J(\tilde{x}) + J(x_0) \tag{7}$$

If we can find $\alpha(x)$, $H_d(\tilde{x})$, J_a and R_a, satisfying

$$u = \alpha(x) \tag{8}$$

$$H_d(\tilde{x}) = \frac{1}{2}\tilde{x}^T D^{-1}\tilde{x} \tag{9}$$

$$J_d(\tilde{x}) = J(\tilde{x}) + J_a = -J_d^T(\tilde{x}), \quad R_d(\tilde{x}) = R(\tilde{x}) + R_a = R_d^T(\tilde{x}) > 0 \tag{10}$$

$$g(x)\alpha(x) = [J_a - R_a - J(x_0)]D^{-1}\tilde{x} - J(\tilde{x})D^{-1}x_0 + g(x_0)u_0 \tag{11}$$

Then, the closed-loop PCH system (3) with $u = \alpha(x)$ takes a the state error PCH form

$$\dot{\tilde{x}} = [J_d(\tilde{x}) - R_d(\tilde{x})]\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}} \tag{12}$$

Furthermore, $\tilde{x} = 0$ is globally asymptotically stable equilibrium point of the closed-loop system (12), and the state x does converge to the desired equilibrium x_0.

Proof: Define $\tilde{x} = x - x_0$. Then $x = \tilde{x} + x_0$, substituting into (3), we get

$$\dot{\tilde{x}} = [J(\tilde{x} + x_0) - R]D^{-1}(\tilde{x} + x_0) + g(\tilde{x} + x_0)\alpha(x) - \dot{x}_0 \tag{13}$$

where $\frac{\partial H(x)}{\partial x} = D^{-1}x$ and $\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}} = D^{-1}\tilde{x}$.

From (3), we have

$$\dot{x}_0 = [J(x_0) - R]D^{-1}x_0 + g(x_0)u_0 \tag{14}$$

Substitution of above formula and (7) into (13) gives that the state error system model is

$$\dot{\tilde{x}} = [J(\tilde{x}) - R]D^{-1}\tilde{x} + J(x_0)D^{-1}\tilde{x} + J(\tilde{x})D^{-1}\tilde{x}_0 + g(\tilde{x} + x_0)\alpha(x) - g(x_0)u_0. \tag{15}$$

Let

$$\Psi = -[J_a - R_a - J(x_0)]D^{-1}x + J(x)D^{-1}x_0 + g(x)\alpha(x) - g(x_0)u_0, \tag{16}$$

then, according to (10) and (16), Equation (15) can be written as

$$\dot{\tilde{x}} = [J_d(\tilde{x}) - R_d]\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}} + \Psi. \tag{17}$$

Obviously, Equation (11) ensures that $\Psi = 0$. Thus Equation (12) holds.

Since $J_d(\tilde{x})$ is skew-symmetric matrix, thus

$$\left[\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}}\right]^T J_d(\tilde{x})\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}} = 0. \tag{18}$$

As $R_d(\tilde{x})$ is positive definite symmetric matrix, therefore, along the trajectories of the system (12), we have

$$\frac{dH_d(\tilde{x})}{dt} = \left[\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}}\right]^T \dot{\tilde{x}} = \left[\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}}\right]^T R_d(\tilde{x})\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}} < 0 \tag{19}$$

Moreover, the function $H_d(\tilde{x})$ is radically unbounded, since it tends to infinity as $\|x\| \to \infty$. Applying Lyapunov's stability theorem, the system (12) is globally asymptotically stable at equilibrium point $\tilde{x} = 0$, then $\tilde{x}$ tends to zero as t tends to infinity. Thus, the state x does converge to the desired equilibrium x_0.

Remark 2.1. *Proposition 2.1 shows that the system (3) can be expressed as the form of system (12) by $u = \alpha(x)$. Moreover, feedback control law $u = \alpha(x)$ can be solved from (11).*

Remark 2.2. *For Proposition 2.1, if the second formula of Equations (10) becomes $R_d(\tilde{x}) = R(\tilde{x}) + R_a = R_d^T(\tilde{x}) \geq 0$, then, the closed-loop system (12) will be globally asymptotically stable if, in addition, the largest invariant set under the closed-loop dynamics contained in $\left\{\tilde{x} \in R^n \middle| \left[\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}}\right]^T R_d(\tilde{x})\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}} = 0\right\}$ equals $\{x_0\}$. The globally asymptotic stability follows immediately invoking La Salle's invariance principle.*

2.2. **PCH controller design of IM.** The model of the induction motor can be described in a synchronously rotating $d-q$ reference frame, and then we get the electrical subsystem

$$\begin{aligned}\dot{\lambda}_s &= -R_s i_s - \omega_s J_2 \lambda_s + u_s \\ \dot{\lambda}_r &= -R_r i_r - (\omega_s - n_p\omega) J_2 \lambda_r\end{aligned} \tag{20}$$

and the mechanical subsystem

$$J_m \dot{\omega} = \tau - \tau_L - R_m \omega = n_p \lambda_r^T J_2 i_r - \tau_L - R_m \omega \tag{21}$$

where

$$\tau = n_p \lambda_r^T J_2 i_r = n_p \frac{L_m}{L_r} i_s^T J_2 \lambda_r \tag{22}$$

$$\lambda = Li \tag{23}$$

$$L = \begin{bmatrix} L_s I_2 & L_m I_2 \\ L_m I_2 & L_r I_2 \end{bmatrix}, \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad J_2 = -J_2^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{24}$$

and

$$\begin{aligned}\lambda &= \begin{bmatrix} \lambda_s^T & \lambda_r^T \end{bmatrix}^T = \begin{bmatrix} \lambda_{sd} & \lambda_{sq} & \lambda_{rd} & \lambda_{rq} \end{bmatrix}^T, \\ i &= \begin{bmatrix} i_s^T & i_r^T \end{bmatrix}^T = \begin{bmatrix} i_{sd} & i_{sq} & i_{rd} & i_{rq} \end{bmatrix}^T, \quad u_s = \begin{bmatrix} u_{sd} & u_{sq} \end{bmatrix}^T\end{aligned} \tag{25}$$

τ and τ_L are electromagnetic and load torque respectively, the subscripts s and r indicate the variables for the stator and the rotor respectively, J_m is the moment of inertia, L_s, L_r, L_m are the stator, rotor, mutual inductances respectively, R_s and R_r are the resistances for the stator and the rotor respectively, n_p is the number of pole pairs, ω_s is electrical angular speed of the stator (the rotating speed of the synchronously $d-q$ reference frame), ω is mechanical angular speed of the rotor, ω_r is electrical angular speed of the rotor, R_m is the friction coefficient of the rotor, and

$$\omega_r = n_p \omega. \tag{26}$$

We define the mechanical momentum, state vector and input vector as follows respectively

$$p = J_m \omega, \quad x = \begin{bmatrix} \lambda_s^T & \lambda_r^T & p \end{bmatrix}^T, \quad u = \begin{bmatrix} u_s^T & \omega_s & -\tau_L \end{bmatrix}^T \tag{27}$$

Using (21) and (27) yields

$$\dot{p} = J_m \dot{\omega} = n_p \lambda_r^T J_2 i_r - R_m \omega - \tau_L. \tag{28}$$

The Hamiltonian (storage) function of the IM system is written as

$$H(x) = \frac{1}{2} x^T D^{-1} x = \frac{1}{2} \lambda^T L^{-1} \lambda + \frac{1}{2} \frac{p^2}{J_m} \tag{29}$$

$$D = \begin{bmatrix} L & 0 \\ 0 & J_m \end{bmatrix}. \tag{30}$$

(20) and (28) can be rewritten in (5), then the PCH model of IM is given by

$$\begin{aligned}\dot{x} = &\left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & n_p J_2 \lambda_r \\ 0 & n_p \lambda_r^T J_2 & 0 \end{pmatrix} - \begin{pmatrix} R_s I_2 & 0 & 0 \\ 0 & R_r I_2 & 0 \\ 0 & 0 & R_m \end{pmatrix} \right] \frac{\partial H(x)}{\partial x} \\ &+ \begin{bmatrix} I_2 & -J_2 \lambda_s & 0 \\ 0 & -J_2 \lambda_r & 0 \\ 0 & 0 & 1 \end{bmatrix} u + g_w(x) w\end{aligned} \tag{31}$$

where

$$w \doteq \tilde{\tau}_L = \tau_L - \tau_{L0}, \quad g_w(x) = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 \end{bmatrix}^T \tag{32}$$

τ_{L0} is the known and constant load torque.

$$\frac{\partial H(x)}{\partial x} = D^{-1}x = \begin{bmatrix} i_s \\ i_r \\ \omega \end{bmatrix}, \quad J(x) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & n_p J_2 \lambda_r \\ 0 & n_p \lambda_r^T J_2 & 0 \end{bmatrix}$$

$$R(x) = \begin{bmatrix} R_s I_2 & 0 & 0 \\ 0 & R_r I_2 & 0 \\ 0 & 0 & R_m \end{bmatrix}, \quad g(x) = \begin{bmatrix} I_2 & -J_2\lambda_s & 0 \\ 0 & -J_2\lambda_r & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{33}$$

When load torque is known and constant, $w = 0$ (that is $\tau_L = \tau_{L0}$). At equilibrium x_0, $\omega = \omega_0$. Following the idea of the IM rotor field orientation, given λ_{r0}, we have chosen the desired rotor vector to be aligned with the d-axis of the $d-q$ reference frame, then the desired equilibrium is

$$x_0 = \begin{bmatrix} x_{10} \\ x_{20} \\ x_{30} \end{bmatrix} = \begin{bmatrix} \lambda_{s0} \\ \lambda_{r0} \\ J_m\omega_0 \end{bmatrix}, \quad \lambda_{s0} = \begin{bmatrix} \lambda_{sd0} \\ \lambda_{sq0} \end{bmatrix}, \quad \lambda_{r0} = \begin{bmatrix} \mu \\ 0 \end{bmatrix}. \tag{34}$$

According to the principle of vector control, we have

$$\tau_0 = \tau_L + R_m\omega_0, \quad i_{sd0} = \frac{\mu}{L_m}, \quad i_{sq0} = \frac{L_r\tau_0}{L_m n_p \mu}, \quad i_{rd0} = 0. \tag{35}$$

$$i_{rq0} = -\frac{L_m}{L_r} i_{sq0} = -\frac{\tau_0}{n_p\mu}.$$

$$\omega_{s0} = n_p\omega_0 + \frac{R_r\tau_0}{n_p\mu^2}, \quad \lambda_{sd0} = L_s i_{sd0}, \quad \lambda_{sq0} = L_s i_{sq0} + L_m i_{rq0}.$$

From (11) we get

$$\begin{bmatrix} u_s - \omega_s J_2\lambda_s \\ -\omega_s J_2 \lambda_r \\ -\tau_L \end{bmatrix} = [J_a - R_a - J(x_0)] \begin{bmatrix} \tilde{i}_s \\ \tilde{i}_r \\ \tilde{\omega} \end{bmatrix} - \begin{bmatrix} 0 \\ n_p J_2 \tilde{\lambda}_r \omega_0 \\ n_p \tilde{\lambda}_r^T J_2 i_{r0} \end{bmatrix} + \begin{bmatrix} u_{s0} - \omega_{s0} J_2 \lambda_{s0} \\ -\omega_{s0} J_2 \lambda_{r0} \\ -\tau_L \end{bmatrix}. \tag{36}$$

Supposing

$$J_a(x) = \begin{bmatrix} 0 & 0 & J_{13} \\ 0 & 0 & J_{23} \\ -J_{13}^T & -J_{23}^T & 0 \end{bmatrix}, \quad R_a = \begin{bmatrix} r_s I_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{37}$$

where r_s is damping parameter, J_{13} and J_{23} are respectively assigned as

$$J_{13} = -n_p L_m J_2 i_{r0}, \quad J_{23} = n_p L_m J_2 i_{s0}.$$

From (36) we get the controller $\alpha(x) = \begin{bmatrix} u_{s\alpha} & \omega_{s\alpha} & -\tau_{L\alpha} \end{bmatrix}^T$ as follows:

$$\begin{aligned} u_{s\alpha} &= R_s i_{s0} - r_s(i_s - i_{s0}) - n_p L_m J_2 i_{r0}(\omega - \omega_0) + \omega_s J_2 \left[\left(L_s - \frac{L_m^2}{L_r} \right) i_s + \frac{L_m}{L_r}\lambda_r \right] \\ \omega_{s\alpha} &= n_p\omega_0 + \frac{\lambda_{rd}}{||\lambda_r||^2}\frac{R_r\tau_0}{n_p\mu} + \frac{n_p L_r(\omega - \omega_0)\lambda_{rq} i_{rq0}}{||\lambda_r||^2} \\ -\tau_{L\alpha} &= -\tau_{L0} \end{aligned} \tag{38}$$

Rotor flux λ_r in (38) is not directly measurable, so an observer is needed to estimate it. In order to get rid of rotor resistance changes, we use an open-loop rotor flux observer. From (20) and (23) we get the observer

$$\dot{\lambda}_s = u_s - R_s i_s - \omega_s J_2 \lambda_s \tag{39}$$

$$\lambda_r = \frac{L_r}{L_m}\lambda_s + \left(L_m - \frac{L_s L_r}{L_m}\right) i_s. \tag{40}$$

3. The L_2 Gain Disturbance Attenuation of IM.

3.1. The L_2 gain disturbance attenuation for PCH system. Considering system (5), there exists a load disturbance $\tilde{\tau}_L$ (that is $\tau_L \neq \tau_{L0}$) when $w \neq 0$, the form of controller is

$$u = \alpha(x) + \beta(x) \tag{41}$$

Meantime, the closed-loop system (12) turns to be

$$\dot{\tilde{x}} = [J_d(\tilde{x}) - R_d(\tilde{x})]\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}} + g(x)\beta(x) + g_w(x)w \tag{42}$$

Penalty signal is defined by

$$z = h(x)g^T(x)\left[\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}}\right]^T \tag{43}$$

where $h(x)$ is a weight matrix.

For system (42), the L_2 gain disturbance attenuation objective can be described as follows: given positive γ, penalty signal z, positive definite storage function $H_d(\tilde{x})$ and desired equilibrium point x_0, our aim is to find the state feedback control law

$$u' = \beta(x) \tag{44}$$

such that the γ-dissipative inequality

$$\dot{H}_d(\tilde{x}) + Q(x) \leq \frac{1}{2}\left(\gamma^2\|w\|^2 - \|z\|^2\right), \quad \forall w \tag{45}$$

holds along the trajectories of the closed-loop system (42), where $Q(x)$ is a given non-negative definite function.

The L_2 gain disturbance attenuation control principle of PCH system is given as follows [8, 9].

Proposition 3.1. *Consider system (42) and penalty signal (43). For any given positive γ, the L_2 disturbance attenuation objective is achieved by the state feedback*

$$\beta(x) = -\frac{1}{2}\left[\frac{1}{\gamma^2}I_5 + h^T(x)h(x)\right]g^T(x)\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}} \tag{46}$$

and

$$R_d + \frac{1}{2\gamma^2}g(x)g^T(x) - \frac{1}{2\gamma^2}g_w(x)g_w^T(x) \geq 0 \tag{47}$$

Proof: Let

$$Q(x) = \frac{\partial^T H_d(\tilde{x})}{\partial \tilde{x}}\left[R_d + \frac{1}{2\gamma^2}g(x)g^T(x) - \frac{1}{2\gamma^2}g_w(x)g_w^T(x)\right]\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}} \tag{48}$$

It is easy to know from (47) and (48) that

$$\begin{aligned}\dot{H}_d(\tilde{x}) = \frac{\partial^T H_d(\tilde{x})}{\partial \tilde{x}}\dot{\tilde{x}} &= -\frac{\partial^T H_d(\tilde{x})}{\partial \tilde{x}}R_d\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}} + \frac{\partial^T H_d(\tilde{x})}{\partial \tilde{x}}[g(x)\beta(x) + g_w(x)w] \\ &= -\frac{\partial^T H_d(\tilde{x})}{\partial \tilde{x}}R_d\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}} + \frac{\partial^T H_d(\tilde{x})}{\partial \tilde{x}}g_w(x)w \\ &\quad -\frac{1}{2}\frac{\partial^T H_d(\tilde{x})}{\partial \tilde{x}}g(x)\left[\frac{1}{\gamma^2}I_5 + h^T(x)h(x)\right]g^T(x)\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}}\end{aligned}$$

$$= -\frac{\partial^T H_d(\tilde{x})}{\partial \tilde{x}} \left[R_d + \frac{1}{2\gamma^2} g(x) g^T(x) - \frac{1}{2\gamma^2} g_w(x) g_w^T(x) \right] \frac{\partial H_d(\tilde{x})}{\partial \tilde{x}} + \frac{1}{2}\left(\gamma^2 \|w\|^2 - \|z\|^2\right) - \frac{1}{2}\left\| \gamma w - \frac{1}{\gamma} g_w^T(x) \frac{\partial H_d(\tilde{x})}{\partial \tilde{x}} \right\|^2$$

we have

$$\dot{H}_d(\tilde{x}) + Q(x) = \frac{1}{2}\left(\gamma^2 \|w\|^2 - \|z\|^2\right) - \frac{1}{2}\left\| \gamma w - \frac{1}{\gamma} g_w^T(x) \frac{\partial H_d(\tilde{x})}{\partial \tilde{x}} \right\|^2, \tag{49}$$

i.e., (45) holds.

Remark 3.1. *Proposition 3.1 indicates that if the additional damping*

$$R_d \to R_d + \frac{1}{2\gamma^2} g(x) g^T(x) - \frac{1}{2\gamma^2} g_w(x) g_w^T(x) \tag{50}$$

which provided by additional feedback $\beta(x)$ is injected to the closed-loop system (42), then finite L_2 disturbance attenuation can be achieved for closed-loop systems.

3.2. **The L_2 gain controller design.** For the case $h(x) = I_5$, according to (43),

$$z = h(x) g^T(x) \left[\frac{\partial H_d(\tilde{x})}{\partial \tilde{x}}\right]^T = \begin{bmatrix} i_s - i_{s0} \\ \lambda_s^T J_2 (i_s - i_{s0}) + \lambda_r^T J_2 (i_r - i_{r0}) \\ \omega - \omega_0 \end{bmatrix} \tag{51}$$

From (46), we can get controller

$$\beta(x) = \begin{bmatrix} u_{s\beta} \\ \omega_{s\beta} \\ \tau_{L\beta} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\left(\frac{1}{\gamma^2}+1\right)(i_s - i_{s0}) \\ -\frac{1}{2}\left(\frac{1}{\gamma^2}+1\right)\left[\lambda_s^T J_2 (i_s - i_{s0}) + \lambda_r^T J_2 (i_r - i_{r0})\right] \\ -\frac{1}{2}\left(\frac{1}{\gamma^2}+1\right)[\omega - \omega_0] \end{bmatrix} \tag{52}$$

Moreover, from (41), we obtain

$$\begin{aligned} u_s &= u_{s\alpha} + u_{s\beta} \\ \omega_s &= \omega_{s\alpha} + \omega_{s\beta} \\ \tau_L &= \tau_{L\alpha} + \tau_{L\beta} \end{aligned} \tag{53}$$

When disturbance is added to the system, τ_L of (35) can be replaced by $(\tau_{L\alpha} + \tau_{L\beta})$. Then the new desired equilibrium becomes

$$\hat{x}_0 = \begin{bmatrix} \hat{x}_{10} & \hat{x}_{20} & \hat{x}_{30} \end{bmatrix}^T = \begin{bmatrix} \hat{\lambda}_{s0} & \hat{\lambda}_{r0} & J_m \omega_0 \end{bmatrix}^T \tag{54}$$

Consequently, the entire system controller turns to be

$$\begin{aligned} u_s = {} & R_s \hat{i}_{s0} - r_s (i_s - \hat{i}_{s0}) - n_p L_m J_2 \hat{i}_{r0} (\omega - \omega_0) \\ & + \omega_s J_2 \left[\left(L_s - \frac{L_m^2}{L_r}\right) i_s + \frac{L_m}{L_r} \lambda_r\right] - \frac{1}{2}\left(\frac{1}{\gamma^2}+1\right)(i_s - \hat{i}_{s0}). \end{aligned} \tag{55}$$

$$\begin{aligned} \omega_s = {} & n_p \omega_0 + \frac{\lambda_{rd}}{||\lambda_r||^2} \frac{R_r \tau_0}{n_p \beta} + \frac{n_p L_r (\omega - \omega_0) \lambda_{rq} \hat{i}_{rq0}}{||\lambda_r||^2} \\ & - \frac{1}{2}\left(\frac{1}{\gamma^2}+1\right)\left[\lambda_s^T J_2 (i_s - \hat{i}_{s0}) + \lambda_r^T J_2 (i_r - \hat{i}_{r0})\right] \end{aligned} \tag{56}$$

$$\tau_L = \tau_{L0} - \frac{1}{2}\left(\frac{1}{\gamma^2}+1\right)[\omega - \omega_0] \tag{57}$$

3.3. **Application of PI control.** In order to track the changes of the load torque and eliminate speed steady error of IM drives, the PI control of speed error is added to the system. The integral separation principle is used to avoid integral saturation. The load torque estimator is designed as follows.

$$\triangle\hat{\tau}_L = \begin{cases} -k_p(\omega-\omega_0), & |\omega-\omega_0| > \rho, \\ -k_p(\omega-\omega_0) - k_i\int_{t_0}^{t}(\omega-\omega_0)\,dt, & |\omega-\omega_0| \le \rho, \end{cases} \tag{58}$$

where ρ is integral separation threshold value. Then, the expression (57) becomes

$$\tau_L = \tau_{L0} - \frac{1}{2}\left(\frac{1}{\gamma^2}+1\right)[\omega-\omega_0] + \triangle\hat{\tau}_L \tag{59}$$

Thus, τ_L of (35) can be replaced by τ_L above, and new equilibrium can be obtained.

4. **The Comparative Studies and Simulation Results.** The state error PCH control of IM is shown in Figure 1. The IM is connected to the a three-phase DC/AC voltage-source inverter. The outputs of the inverter are connected to the three-phase stator windings of IM which are Wye-connected. Feedback speed and currents are considered as the inputs of the PCH controller. Then, applying SVPWM signal transformation principle, we can get six pulses which are used to control the inverter to provide the desired three-phase voltages. As a result, the speed tracking control of IM can be implemented.

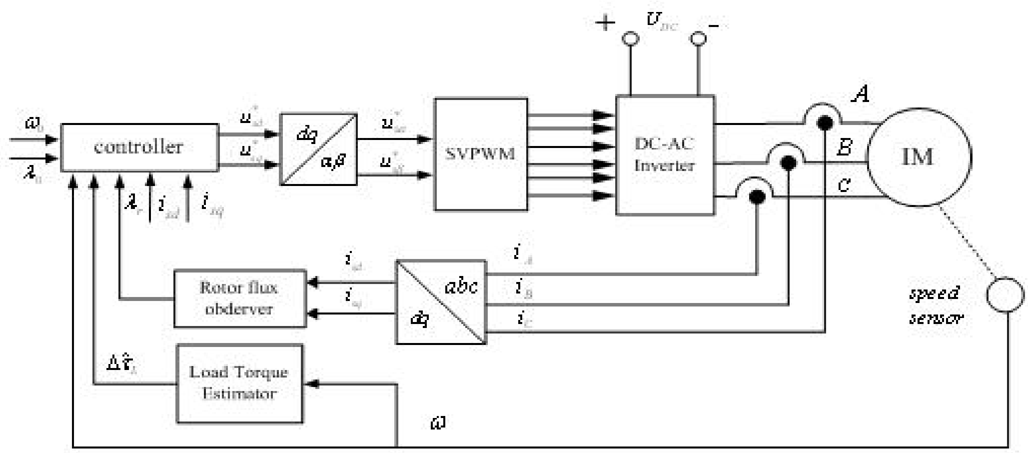

FIGURE 1. The state error PCH control of the IM

The IM parameters are $R_s = 0.687\Omega$, $R_r = 0.642\Omega$, $n_p = 2$, $L_s = 0.084$H, $L_r = 0.0852$H, $L_m = 0.0813$H, $J_m = 0.3$kg·m^2, $R_m = 0.001$kg·m^2/s.

At startup, the load torque τ_L is set to 3Nm. The desired speed ω_0 is 60 rad/s. $U_{DC} = 300$V, $\lambda_{rd0} = \mu = 1$Wb, $\rho = 2$. From (35) and (36), we get the equilibriums as follows: $i_{sd0} = 12.3001$A, $i_{sq0} = 1.572$A, $i_{rd0} = 0$A, $i_{rq0} = -1.5$A.

Figures 2-4 show the performance of the controller (38). Figure 2 gives the speed responses of different damping parameters ($r_s = 5$, $r_s = 10$ and $r_s = 20$) when load torque is constant and known. We can see that speed response is the fastest and no steady state error when r_s is 5. Figure 3 shows the rotor flux response. It seems that the

IM rotor flux tracks given value very well. Figure 4 shows the speed curve when a load torque disturbance (3Nm) is added to the system at t = 1s. It can be seen that rotor speed becomes slow and the system appears steady state error.

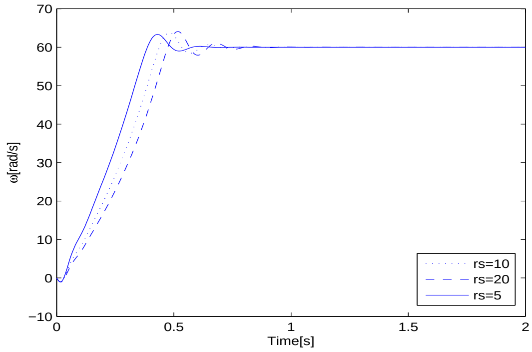

FIGURE 2. Rotor speed curve

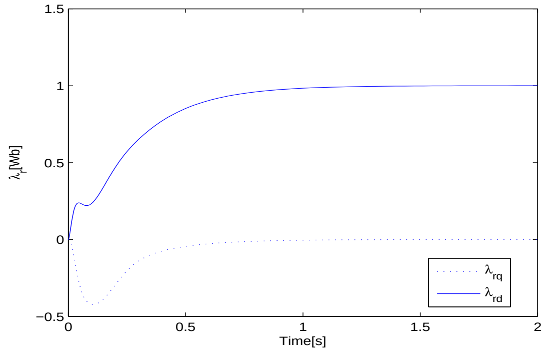

FIGURE 3. Rotor flux curve

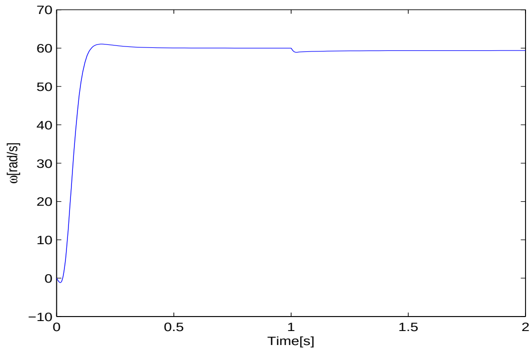

FIGURE 4. Speed curve without L_2 control

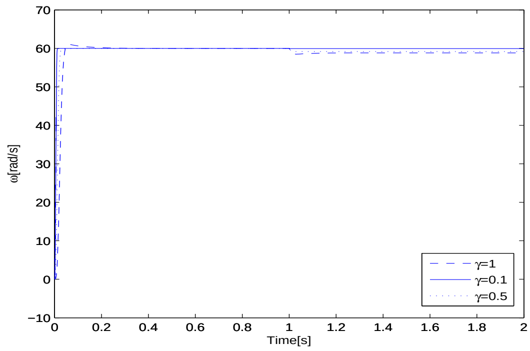

FIGURE 5. Speed curve with L_2 control

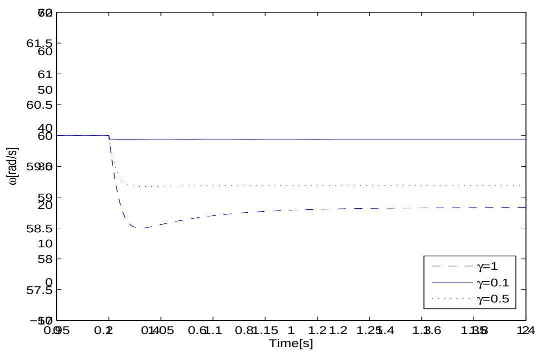

FIGURE 6. Local enlarged curve of Figure 5

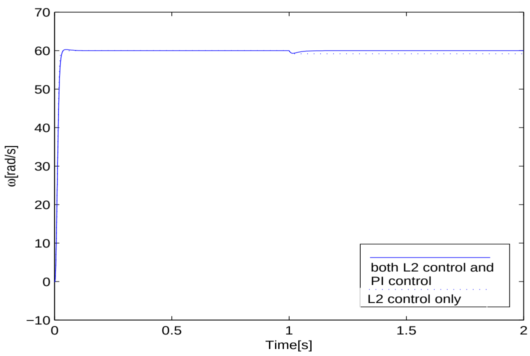

FIGURE 7. Speed curve with L_2 and PI

Figures 5 and 6 present the speed curves with L_2 disturbance attenuation (load torque disturbance 3Nm is added to the system at t = 1s) when γ = 0.1, 0.5, 1 respectively. From the simulation results, we can see that L_2 gain control has good attenuation effect to external disturbance.

Although the Figures 5 and 6 show that the effect of disturbances attenuation is very good, but speed steady state error still exists. Thus, the PI control is added to the motor system. Figures 7-11 present the simulation results of the controllers (55, 56 and 59). Figures 7 and 8 show the speed curves with L_2 disturbance attenuation ($\gamma = 0.6$) and PI control. The estimator parameters are $k_p = 0.1$ and $k_i = 90$. The simulation results show that speed steady state error is almost eliminated. Figures 9-11 show the A-phase current response, A-phase voltage response and electromagnetic torque response respectively.

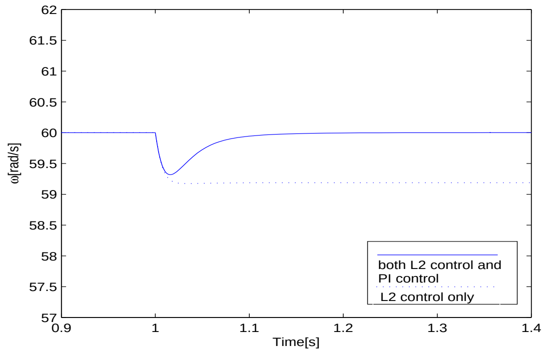

FIGURE 8. Local enlarged curve of Figure 7

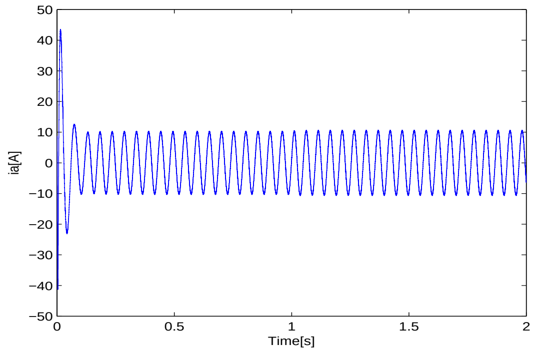

FIGURE 9. A-phase current with L_2 and PI

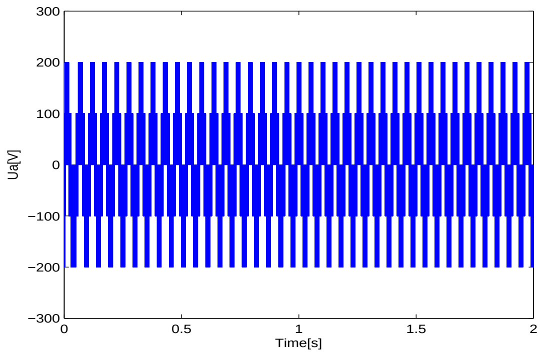

FIGURE 10. A-phase voltage with L_2 and PI

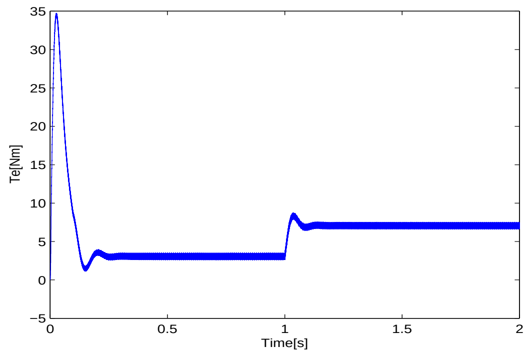

FIGURE 11. T_e curve with L_2 and PI

In order to compare the proposed control algorithm and the existing vector control (VC) [1], the vector control simulation experiments have been carried out. The vector control of IM is shown in Figure 12. The IM parameters are the same as Figure 1. The G1, G2, G3 and G4 are PI regulators, and parameters are $k_{p1} = 2$, $k_{i1} = 50$; $k_{p2} = 5$, $k_{i2} = 2$; $k_{p3} = 1$, $k_{i3} = 2$; $k_{p4} = 2$, $k_{i4} = 10$.

Figures 13-17 give the simulation results of the vector control. The dynamic performances of state error PCH control with L_2 and PI regulation are better than ones of the vector control. Figures 13 and 14 show that the speed fluctuation of the vector control is bigger in the presence of load torque disturbance. From Figures 15 and 16, we can see that the A-phase current and voltage become larger for the vector control when the load torque disturbance is added to the system at $t = 1$s. Figure 17 shows that the dynamic performance of starting electromagnetic torque deteriorates for the vector control.

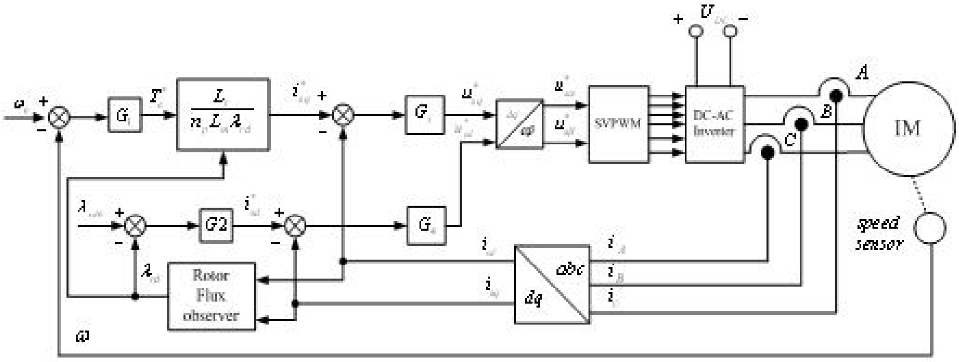

FIGURE 12. The vector control of the IM

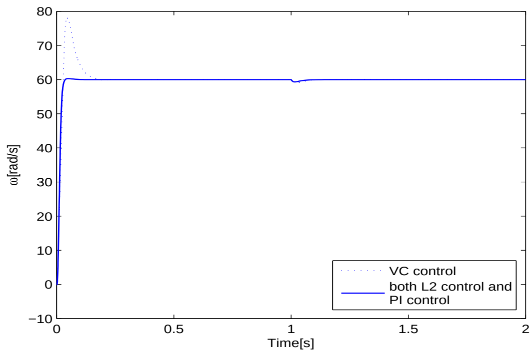

FIGURE 13. Speed curve of the VC control

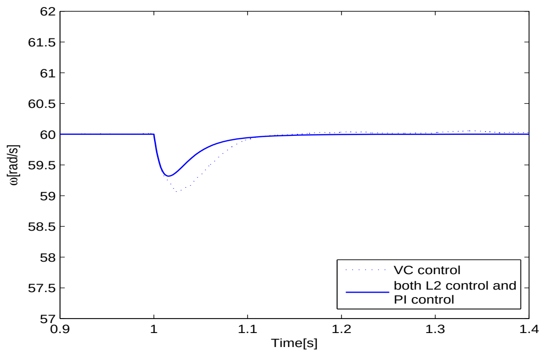

FIGURE 14. Local enlarged curve of Figure 13

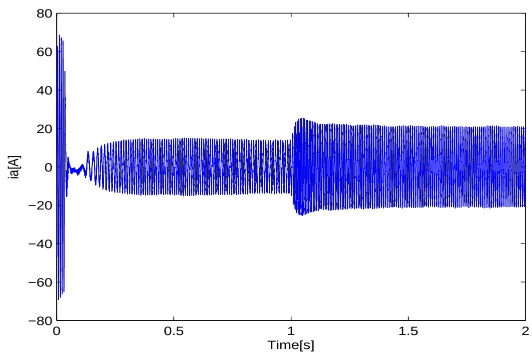

FIGURE 15. A-phase current of the VC control

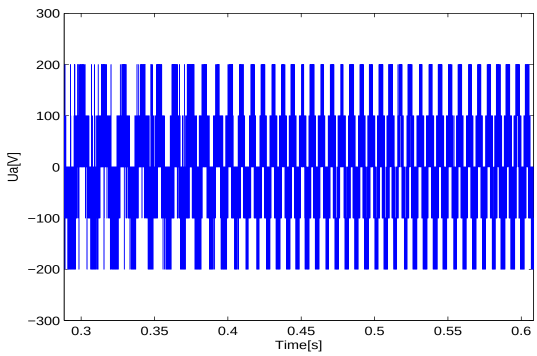

FIGURE 16. A-phase voltage of the VC control

5. **Conclusions.** In this paper, the speed tracking control and disturbance attenuation of IM are presented based on the state error PCH systems method and L_2 gain technique. Using the interconnection and damping assignment and ES, the desired state error PCH structure is assigned to the closed-loop IM drive system. The equilibrium stability of the closed-loop system is also verified according to the Lyapunov's stability theorem. The L_2

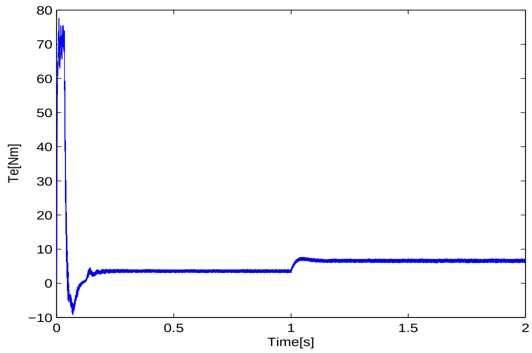

FIGURE 17. T_e curve of the VC control

gain control problem is discussed for the state error PCH system. The L_2 gain from load torque disturbance to a penalty signal can be reduced to any level by injection of additional damping, if the penalty signal is defined properly. Moreover, the PI control is added to the system, so as to track the changes of the load torque and eliminate the speed steady error of IM system. Compared with the classical vector control, the proposed scheme has good control performance. The simulation results illustrate the effectiveness of the developed controller. The deficiency of the research results is that the load torque disturbance attenuation is developed only in this paper. Moreover, induction motor parameters disturbances, such as the stator resistance, rotor resistance and inductance disturbances can be considered for the future research.

Acknowledgments. This work is partially supported by the Natural Science Foundation of China (61174131, 61104076), the Natural Science Foundation of Shandong Province (ZR2009GM034, ZR2011FQ012), the Science and Technology Project of College and University in Shandong Province (J11LG04) and Shandong Province Excellent Young Teacher Domestic Visiting Scholar Project.

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