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# PREDICTION OF SCOUR DEPTH AT CULVERT OUTLETS USING GENE-EXPRESSION PROGRAMMING

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Abstract. *The processes involved in local scour at culverts are so complex that make it difficult to establish a general empirical (regression) model to provide accurate estimation for scour. This study presents Gene-Expression Programming (GEP), which is an extension to Genetic Programming (GP), as an alternative approach to estimating the scour depth at culvert outlets. The data sets of laboratory measurements of scour depths at culverts were compiled from published literature and used to train the GEP network or evolve the program. The developed network and evolved programs were validated using a random subset of the scour observations that were not used in GEP training. The GEP was found to be more effective in predicting the scour depth at culvert outlets ($R^2$ = 0.989, RMSE = 0.0678), compared with the regression equations and artificial neural networks (ANN) modelling.*
**Keywords:** Local scour, Genetic programming, Gene-expression programming, Artificial neural networks, Radial basis function, Culverts

1. **Introduction.** Design of flow capacity is an essential feature in terms of designing drainage crossing over hydraulic structures such as culverts or storm drains [1]. Design flow and stagnation of eddy around the foundation of crossing over a structure are prime causing to failure of structure due to potential scouring. Accurate prediction of the dimensions of scour downstream from hydraulic structures is required to ensure that foundations are properly designed to minimize the structural damage due to undermining [2]. The estimation of scour characteristics at culvert outlets continues to be a concern for hydraulic engineers [3].

In order to estimate the equilibrium scour depth at culvert outlets, various empirical correlations have been developed by the previous researchers [1,4-11], as summarized in Table 1. A centre-line bed profiles downstream from a circular culvert at equilibrium scour condition is illustrated in Figure 1. However, these empirical relations did not model the actual scour processes, and were applicable only to a limited range of field conditions. Regression relations are commonly used to predict the culvert outlet scour; however, regression analysis has major drawbacks pertaining to idealization of the complex scour process, approximation and averaging the widely varying prototype conditions. Thus, the estimated scour depths using regression equations can have large uncertainties which can contribute to costly culvert failures. Apart from the complexity of the scour phenomenon involved, the limitations of regression analysis are (1) irrespective of the nature of the problem, it is difficult to model by a predefined equation, either linear or nonlinear and (2) the assumption of normality of residuals.

Table 1. Empirical formulae for estimating culvert scour depth [with permission from ASCE]

| Author | Equation |
|---|---|
| Lim [1] | $d_{se}/d_o = 0.45F_o$ |
| Liriano et al. [2] | $d_{se}/d_o = a\ln(F_o) + b$ where $a = 0.877(H/d_o)^{-0.37}$ and $b = 0.20\ln(H/d_o) - 0.24$ |
| Chiew and Lim [11] | $d_{se}/d_o = 0.21F_o$ |
| Abt et al. [31] | $d_{se}/d_o = -3.67(F_o^{0.57}d_{50}^{0.4}\sigma_g^{-0.4})$ |

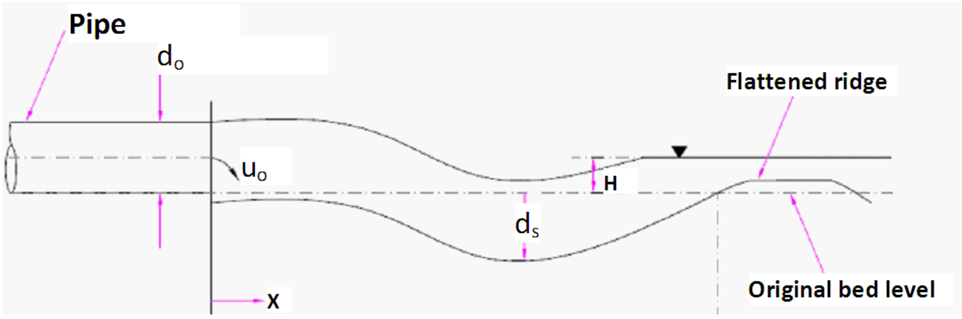


Figure 1. Typical centre-line bed profiles downstream from a circular culvert at equilibrium scour condition (after Lim [1]) [with permission from ASCE]

The recent research initiatives on culvert scour modeling have been exploring ways to enhance the data collection efforts by collecting reliable field and laboratory data sets, and/or to improve the modeling tools used to fit empirical models to the available data sets. In particular, the development of modern data-driven modeling techniques such as those based on artificial intelligence (AI) techniques, is quite promising. Predictive approaches such as artificial neural networks (ANNs) [12-14] and adaptive neuro-fuzzy inference systems (ANFISs) [15] have been recently shown to yield effective estimates of scour around hydraulic structures. ANNs have been reported to provide reasonably good solutions for hydraulic-engineering problems, particularly for conditions having highly nonlinear and complex relationship among the input-output pairs in the corresponding data [16,17]. Recently, Weinert and Lopes [18] developed parallel rule induction system using Gene-Expression Programming (GEP) and Tsai [19] designed an intelligent novelty detection application for practical situations.

However, a model for the prediction of scour downstream from culverts that is generally applicable to all circumstances is not currently available. Accordingly an improved predictive model for estimating scour depth using GEP has been developed in the present study. The performance of the proposed model was compared with a standard Radial Basis Function (RBF) Neural Network and conventional regression-based equations. The explicit formulation of the GEP model is also presented.

2. **Analysis of Local Scour at Culvert Outlets.** The primary (or main) variables influencing the equilibrium scour depth ($d_s$) at culvert outlets are listed below [2].

$$d_{se} = f(\rho, \mu_0, u_0, d_0, H, W_0, g, \rho_s', d_{50}, \sigma_g) \tag{1}$$

where $d_{se}$ is the equilibrium depth of scour, $\rho$ is the density of water, $\mu_0$ is the dynamic viscosity of water, $u_0$ is the mean velocity at the outlet, $d_0$ is the pipe diameter for circular outlets and the outlet height for non-circular outlets, $H$ is the depth of water in the downstream receiving channel (tail-water depth), $W_0$ is the width of the outlet, $g$ is the acceleration due to gravity, $\rho_s$ is the density of the sediment bed material, $d_{50}$ is the median sediment size, $K_s$ is a shape factor of a culvert, and $\sigma_g$ is the geometric standard deviation of the sediment bed material and describes the gradation of sediments downstream from the culvert. Assuming that the viscous effect is not important and that the bed material consists of sand and gravel with constant $\rho_s$, a dimensional analysis of Equation (1) can yield a set of five non-dimensional parameters:

$$\frac{d_s}{d_0} = f\left(F_0, \frac{H}{d_0}, \frac{W_0}{d_0}, \frac{d_{50}}{d_0}, \sigma_g\right) \tag{2}$$

where $F_0$ is the densimetric Froude number $= u_0/[(S-1)gd_{50}]^{0.5}$, and $S$ is the specific gravity of the sediment $= \rho_s/\rho$. Experimental data containing 202 data sets were compiled from multiple sources [1,7,10,20-23].

During last two decades, researchers have noticed that the use of soft-computing techniques as alternative to conventional statistical (regression) methods based on controlled laboratory or field data yielded significantly better results. The ANN and Genetic Programming (GP) are the most widely used branches of soft computing in hydraulic engineering. Within the larger field of hydraulics, few researchers have dealt with the scour around and downstream of hydraulic structures using ANN [14-16].

3. **Overview of the Gene-Expression Programming.** Most recently a new technique called Gene-Expression Programming (GEP) was developed which is an extension of GP [24]. The GEP is a search technique that evolves computer programs (mathematical expressions, decision trees, and logical expressions). Recently this technique was found to give reasonably good prediction for sediment load [26]. Therefore, GEP has attracted the attention of researchers in the prediction of hydraulic characteristics. This study presents ANN and GEP as alternative tools in the prediction of scour depth downstream from a culvert. The computer programs of GEP are encoded in linear chromosomes, which are then expressed or translated into expression trees (ETs). ETs are sophisticated computer programs that are usually evolved to solve a particular problem, and are selected according to their fitness at solving that problem. From these trees, the corresponding empirical expressions can be derived. A population of ETs will discover traits, and therefore will adapt to the particular problem they are employed to solve. This means that, given enough time and setting the stage correctly, a good solution to the problem will be discovered [26,27].

The GEP is a full-fledged genotype/phenotype system, with the genotype totally separated from the phenotype, while in GP, genotype and phenotype are one entangled mess or more formally, a simple-replicator system. As a consequence of this, the full-fledged genotype/phenotype system of GEP surpasses the old GP system by a factor of 100-60,000 [26,27]. The functionality of each genetic operator included in GEP system was explained by Ferreira [26-28], and Guven and Aytek [29]; the latter provided an application for improving stage-discharge relationships.

4. **Development of the Neural Network Model.** The ANN provides a random mapping between an input and an output vector, and typically consists of three layers of neuron namely, input, hidden and output, with each neuron acting as an independent computational element. Neural networks derive their strengths from the high degree of freedom associated with their architecture. Prior to application, the network is trained to the

observed data sets. This feeds the network with input and output pairs and determines the values of connection weights, bias or centers (Figure 2).

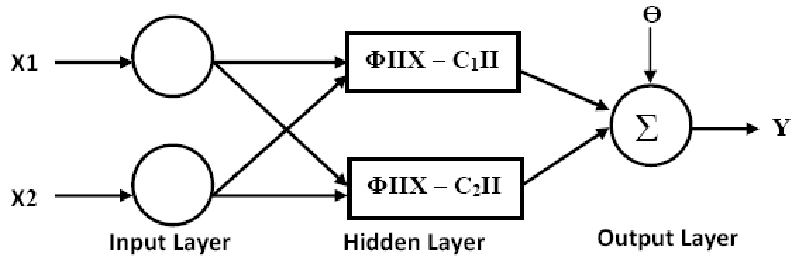


Figure 2. RBF neural network architecture

The training may require many epochs (presentation of complete data sets once to the network) being carried out until the training sum of squares error reaches a specified error goal. Concepts involved in these training schemes are outlined in the ASCE Task Committee [30]. A neural network toolbox contained within the MATLAB package was used in this study. The usual feed-forward type of network was trained using radial basis function (RBF). Out of the total of 202 input-output pairs, about 85% of data sets (151 sets) were selected randomly, and were used for model training, whereas the remaining 15% of data sets (31 sets) were employed for testing (model validation). As dictated by the use of a Gaussian function, all patterns were normalized within the range of $(0.0, 1.0)$ before their use. The RBF network architecture (5 inputs, 36 hidden neurons and 1 output as in Equation (2) was trained by using various values of spread ($\alpha$) between 0 and 1. A spread constant $\alpha$ for the radial-basis layer, and returns a network with weights and biases such that the outputs are exactly for given targets. The value of 0.01 was identified as providing the best performance for the training data.

5. **Development of the GEP Model.** The GEP model was developed using the same input variables used with the ANN-RBF model. Initially, the "training set" was selected from the whole data and the remaining data was used as the "testing set". Once the training set is selected, one could say that the learning environment of the system is defined. The next part of modeling consisted of five major steps. The first is to choose the fitness function. For scour downstream from a culvert, the fitness, $f_i$, of an individual program, $i$, was measured by

$$f_i = \sum_{j=1}^{C_t} \left(M - \left|C_{(i,j)} - T_j\right|\right) \tag{3}$$

where $M$ is the range of selection, $C_{(i,j)}$ is the value returned by the individual chromosome $i$ for fitness case $j$ (out of $C_t$ fitness cases) and $T_j$ is the target value for fitness case $j$. If $|C_{(i,j)} - T_j|$ (the precision) is less than or equal to 0.01, then the precision is equal to zero, and $f_i = f_{\max} = C_t M$. In this case, $M = 100$ was used; therefore, $f_{\max} = 1000$. The advantage of this fitness function is that the system identifies the optimal solution.

Secondly, the set of terminals $T$ and the set of functions $F$ were chosen to create the chromosomes. In this problem, the terminal set consists obviously of five independent variables, i.e., $T = F_0, \frac{H}{d_0}, \frac{W_0}{d_0}, \frac{d_{50}}{d_0}, \sigma_g$. The choice of the appropriate function set is not so obvious; however, a hydraulic background and experience is helpful for indentifying all the necessary functions. In this study, four basic arithmetic operators $(+, -, *, /)$ and some basic mathematical functions ($\sqrt{}$, power) were utilized.

The third major step was to choose the chromosomal architecture, i.e., the length of the head and the number of genes. Initially a single gene and 2 lengths of heads were used, and the number of genes and heads was increased one after another during each run, and the training and testing performance of each model was monitored. The number of genes more than 2 and length of heads more than 8 did not significantly increase the training and testing performance of the GEP models. Thus, length of the head, $l_h = 8$, and two genes per chromosome were employed for each GEP model in this study. The fourth step was to choose the linking function. In this study, we tried addition and multiplication as linking functions and observed that linking the sub-ETs by addition gave better fitness (Equation (3)) values. In the fifth step, the set of genetic operators that cause variation and their rates was chosen. A combination of all genetic operators (mutation, transposition and crossover) was used for this purpose.

The best generation individual had 30 chromosomes and a fitness of 840.94. The explicit and analytical form of the GEP for relative scour depth is given by:

$$\frac{d_{se}}{d_0} = \left( \frac{-6.62 + F_0}{\left( \sqrt{F_o}/\frac{H}{d_o} \right) + \frac{9.65}{\sigma_g}} \right) + \left\langle \frac{d_{50}}{d_o} - \frac{H/d_o}{e^{\left(2.34 + \frac{d_{50}}{d_o}\right)} + e^{\sigma_g}} \right\rangle + \left\{ F_o \sqrt{\sigma_g \frac{H}{d_o}} \right\}^{1/2} \tag{4}$$

Figure 3 shows the expression trees of the above formulation. Table 2 shows the range of the compiled culvert-scour data and its parameters. Table 3 shows the training and testing data set. The functional set and operational parameters used in the present GEP modeling are summarized in Table 4. The sample computation is given in Appendix I.

Table 2. Range of data used [with permission from ASCE]

| Variable | Range of data |
|---|---|
| Outlet shape<br>Culvert Shape | Circular and box<br>Rectangular<br>Circular<br>Square |
| Outlet diameter, $d_0$ (m) | 0.0254-0.146 |
| $\sigma_g$ geometric standard deviation of the sediment bed material | 0.97-4.78 |
| Sediment size, $d_{50}/d_0$ | 0.00082-1.35 |
| $W_0/d_0$ | 5.0-66.7 |
| Relative tail-water depth, $H/d_0$ | 0.5-25 |
| Exit velocity, $u_o$ (m/s) | 0.747-11.176 |
| $F_0$ | 1.04-29.34 |
| $d_{se}/d_0$ | 0.81-24.2 |

## 6. **Results and Discussion.**

6.1. **Training and testing results.** The performance of GEP in training and testing sets was evaluated and compared in terms of four common statistical measures $R^2$ (coefficient of determination), RMSE (root mean square error), MAE (mean average error),

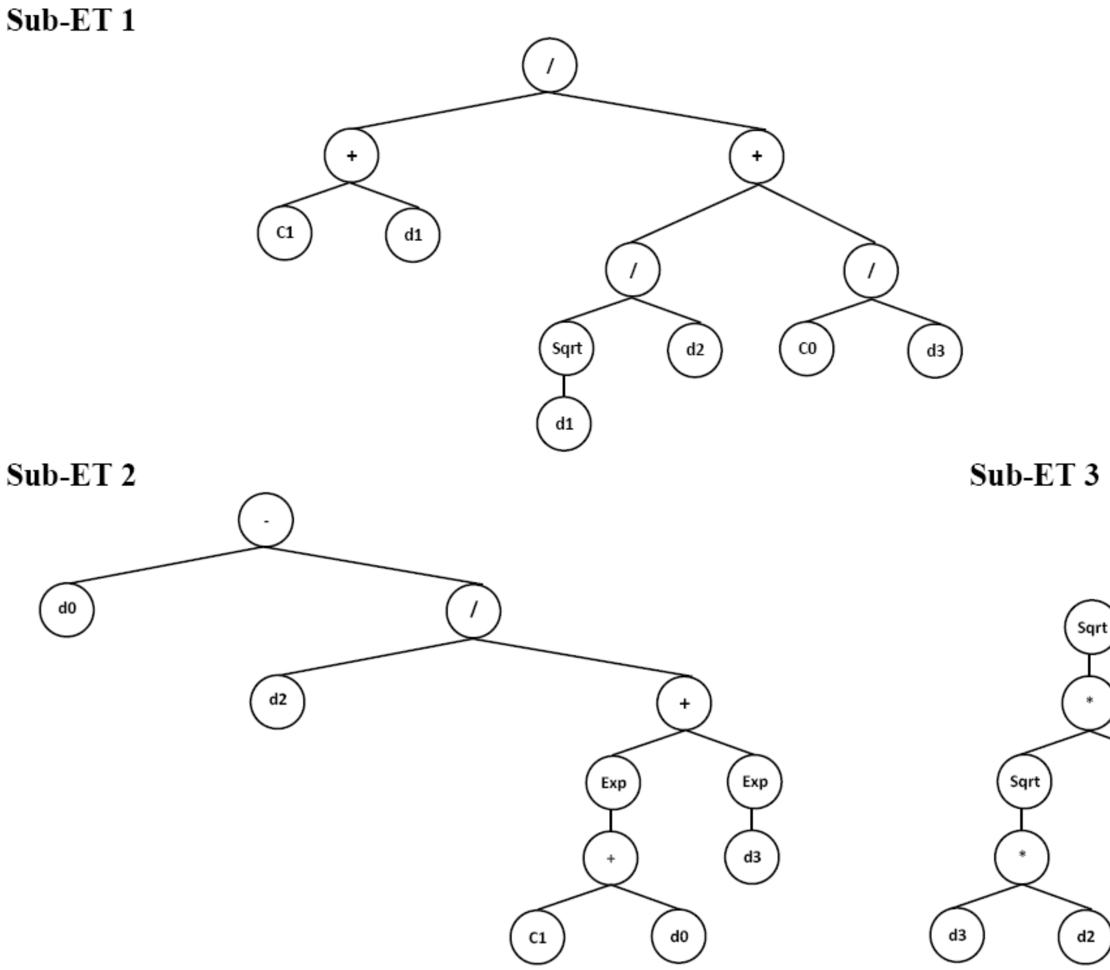


FIGURE 3. Expression tree (ET) for GEP formulation for culvert scour depth (where $d_{50}/d_0 = d1$; $F_0 = d2$; $H/d_0 = d3$, $\sigma_g = d4$; $W_0/d_0 = d5$)

TABLE 3. The minimum and maximum values of the training and testing data parameters

| Parameters | Training Data Set | | Testing Data Set | |
|---|---|---|---|---|
| | Minimum | Maximum | Minimum | Maximum |
| $F_o$ | 1.32 | 29.34 | 6.19 | 17.29 |
| $d_{50}/d_o$ | 0.00082 | 1.3500 | 0.11 | 0.99 |
| $H/d_o$ | 0.3 | 60.0 | 0.55 | 21.47 |
| $W_o/d_o$ | 5 | 66.7 | 9.4 | 22.85 |
| $\sigma_g$ | 0.97 | 4.78 | 1.25 | 2.02 |
| $d_{se}/d_o$ | 0.81 | 24.2 | 3.2 | 11.85 |

and $\delta$ (average absolute deviation). These four performance statistics are listed below:

$$R^2 = 1 - \frac{\sum_{i=1}^{N} (o_i - t_i)^2}{\sum_{i=1}^{N} (o_i - \bar{o}_i)^2} \tag{5}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (o_i - t_i)^2}{N}} \tag{6}$$

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|o_i - t_i| \tag{7}$$

$$\delta = \frac{\sum|(o_i - t_i)|}{\sum o_i} * 100 \tag{8}$$

where $t_i$ denotes the target values of relative scour depth, $o_i$ and $\bar{o}_i$ denote the observed and average observed values of relative scour depth, respectively, and $N$ is the number of data points. First, an attempt was made to assess the significance or influence of each input parameter on the estimated $d_{se}/d_o$ values.

Table 4. Genetic operators used in GEP modeling

| Parameter | Definition | Value |
|---|---|---|
| $p_1$ | Mutation rate | 0.044 |
| $p_2$ | Inversion rate | 0.1 |
| $p_3$ | One-point recombination rate | 30% |
| $p_4$ | Two-point recombination rate | 30% |
| $p_5$ | Gene recombination rate | 0.1 |
| $p_6$ | Gene transposition rate | 0.1 |

Table 5. Sensitivity analysis for independent parameters for the testing

| Set Model | RMSE | MAE | $R^2$ |
|---|---|---|---|
| $d_{se}/d_0 = f(F_0, \frac{H}{d_0}, \frac{W_o}{d_0}, \frac{d_{50}}{d_0}, \sigma_g)$ | 0.087 | 0.68 | 0.9721 |
| $d_{se}/d_0 = f(F_0, \frac{H}{d_0}, \frac{W_o}{d_0}, \frac{d_{50}}{d_0})$ | 0.097 | 0.89 | 0.86 |
| $d_{se}/d_0 = f(F_0, \frac{H}{d_0}, \frac{W_o}{d_0}, \sigma_g)$ | 0.094 | 0.96 | 0.79 |
| $d_{se}/d_0 = f(F_0, \frac{H}{d_0}, \frac{d_{50}}{d_0}, \sigma_g)$ | 0.109 | 0.89 | 0.76 |
| $d_{se}/d_0 = f(F_0, \frac{W_o}{d_0}, \frac{d_{50}}{d_0}, \sigma_g)$ | 0.342 | 0.93 | 0.78 |

Table 5 compares the GEP models, with one of the independent parameters removed in each case, and deleting any independent parameter from the input set that yielded larger RMSE and lower $R^2$ values. These five independent parameters have influence on $d_{se}/d_o$; so the functional relationship given in Equation (2) was used in this study. The GEP model resulted in a highly nonlinear relationship between $d_{se}/d_o$ and the input parameters, and showed the highest accuracy and lowest error (Table 5). The testing performance of the proposed GEP model revealed a high generalization capacity with $R^2 = 0.97$, RMSE = 0.87, MAE = 0.68%, and $\delta = 9.9$.

6.2. **Performance and validation.** In this culvert scour study, grouped variables (non-dimensional data) of input data were explored to assess their influence on scour-depth processes (Table 5). The GEP model was developed and tested for predicting scour depth at culvert outlets. Dimensional analysis was used to determine the parameters for scour at culvert outlets. The sensitivity analysis of a non-dimensional parameter in Equation (2) shows that the dimensionless values of $\sigma_g$ and $d_{50}/d_0$, have the most and the least effects respectively, on the normalized scour depth, $d_{se}/d_o$. The observed equilibrium scour depth values were plotted against the predicted ones and the robustness of the proposed GEP model is demonstrated well. The capability of GEP in estimation of scour depth values is evaluated based on the observed scour depth values.

Figure 4 illustrates the results with the performance indices between predicted and observed data for the validating (testing) data sets using dimensional parameters. Traditional scour-depth predictors such as Chiew and Lim's [11] equation, Lim's [1] equation

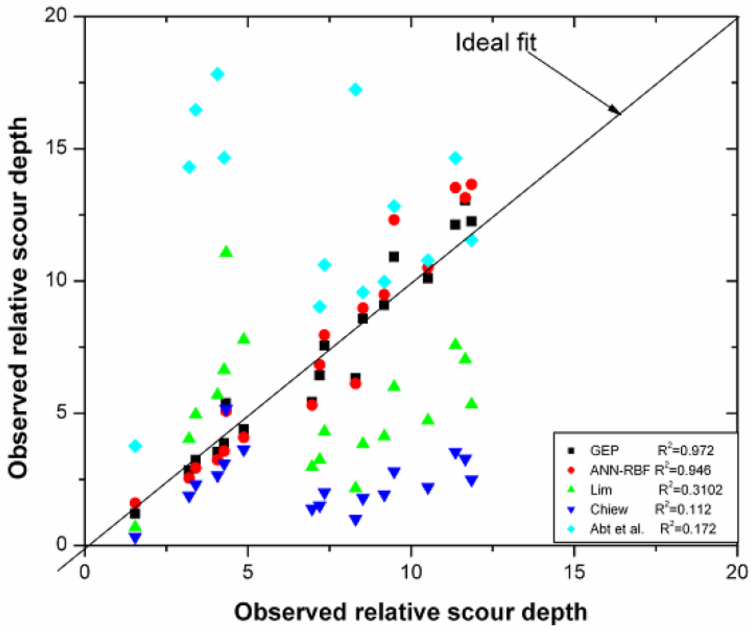


FIGURE 4. Observed versus predicted scour depth-validation (testing)

and the equation of Abt et al. [31] yielded low values for the coefficient of determination ($R^2$ = 0.112, 0.3102 and 0.1723 respectively), for the testing data set. However the proposed GEP model shows a high $R^2$ (= 0.989) and low RMSE (= 0.0678), and the ANN-RBF has $R^2$ = 0.946 and RMSE = 0.988 in training (Table 6).

TABLE 6. Comparison of the GEP and ANN-RBF models

| Model | $R^2$ | | RMSE | | MAE | | $\delta$ | |
|---|---|---|---|---|---|---|---|---|
| | Training | Validation | Training | Validation | Training | Validation | Training | Validation |
| GEP | 0.989 | 0.972 | 0.0678 | 0.87 | 0.516 | 0.68 | 4.78 | 9.56 |
| ANN-RBF | 0.946 | 0.883 | 0.988 | 1.27 | 0.956 | 1.065 | 12.76 | 14.23 |
| Chiew and Lim's [11] | 0.112 | 0.099 | 25.567 | 28.888 | 30.23 | 34.54 | 50.67 | 65.34 |
| Lim [1] | 0.3102 | 0.267 | 16.78 | 19.84 | 22.36 | 26.94 | 35.67 | 40.23 |
| Abt et al. [31] | 0.1723 | 0.245 | 23.45 | 26.34 | 28.67 | 31.23 | 45.36 | 61.23 |

The ANN-RBF model yields biased results (underestimating at lower relative scour depths and overestimating at higher values of relative scour depth). The results of the ANN based approach for prediction of scour depth reported by Liriano and Day [2], Azamathulla and Ghani's [17] ANFIS scour model, were also interesting, but they could not produce any general purpose expression like Equation (4). All these findings exhibit a successful performance of the GEP models for estimating scour depth, both in training and testing stages. The ANN-RBF network was trained in a significantly less number of epochs and in a fraction of the time compared with GEP.

## 7. Conclusion.

The application of relatively new soft-computing approach of genetic programming to predict the local scour depth at culvert outlets was demonstrated. GEP and ANN-RBF models were developed to predict the values of relative scour depth from laboratory culvert-scour measurements. This new approach was developed to estimate the equilibrium depth scour at a culvert outlet from optimum data sets by using the GEP and ANNs modelling techniques. The application of the GEP in this study is an important contribution to scour-depth estimation methodologies downstream culverts. The dimensionless values of $\sigma_g$ and $d_{50}/d_0$, were found to have the most and the least effects respectively, on the normalized scour depth. The present study indicated that employing the original data set yielded a network that could predict measured depth scour at culvert outlets more accurately than standard regression analysis. The overall performance of GEP model was superior to the ANN model. Development of the general purpose equation like Equation (4), for the prediction of scour depth was also unique in the current study. The largest culvert diameter available in the database from Opie's (1967) work could be deemed closest to the filed condition. Further work is required to provide a complete data set to train the network and validate its usefulness.

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**Notation**

$d_{50}$ = particle mean diameter,
$\dot{g}$ = gravitational acceleration,
$u_0$ = mean flow velocity,
RMSE = root mean squared error,
$\rho$ = fluid density,
$\mu$ = fluid dynamic viscosity,
$\delta$ = average absolute deviation.
$d_{se}$ = equilibrium scour depth,
$F_0$ = the densimetric Froude number,
$\mathrm{R}^2$ = coefficient of determination,
MAE = mean average error,
$\rho_s'$= buoyant sediment density,
$\alpha$ = spread,

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**Appendix I.** Estimating Scour at Culvert Outlets

$F_o = 1.32$; $d_{50}/d_o = 0.00082$; $H/d_o = 0.3$; $W_o/d_o = 5$; $\sigma_g = 0.97$ and $d_{se}/d_o = 0.41$.
Substituted in Equation (4)

$$\frac{d_{se}}{d_0} = \left(\frac{-6.62 + F_0}{\left(\sqrt{F_o}\Big/\frac{H}{d_o}\right) + \frac{9.65}{\sigma_g}}\right) + \left\langle \frac{d_{50}}{d_o} - \frac{H/d_o}{e^{(2.34 + \frac{d_{50}}{d_o})} + e^{\sigma_g}} \right\rangle + \left\{ F_o \sqrt{\sigma_g \frac{H}{d_o}} \right\}^{1/2}$$

$$\frac{d_{se}}{d_0} = \left(\frac{-6.62 + 1.32}{\left(\sqrt{1.32}/0.3\right) + \frac{9.65}{0.97}}\right) + \left\langle 0.00082 - \frac{0.3}{e^{(2.34+0.00082)} + e^{0.97}} \right\rangle + \left\{ 1.32\sqrt{0.97 * 0.3} \right\}^{1/2}$$

$$\frac{d_{se}}{d_0} = \left(\frac{-5.01}{(13.77)}\right) + \left\langle 0.00082 - \frac{0.3}{13.027} \right\rangle + \{0.8438\}$$

the relative equilibrium scour depth, $\frac{d_{se}}{d_0} = 0.802$.