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DESIGN OF AN ADAPTIVE FUZZY COMPENSATOR WITH DISTURBANCE OBSERVER USING THE SLIDING-MODE TECHNIQUE

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Abstract. *This paper presents an adaptive fuzzy control scheme using the sliding-mode technique for a class of nonlinear SISO systems whose input channel can have a gain function of full state variables. The proposed controller is composed of three parts: a nominal controller, a disturbance observer and an adaptive fuzzy compensator. The nominal controller specifies the desired closed-loop dynamics, while the disturbance observer and the fuzzy compensator compensate for the system perturbation, including parameter uncertainties and unexpected external disturbances. In contrast to existing adaptive fuzzy control schemes that indirectly extract perturbation information from the tracking error, the proposed scheme learns directly from a switching signal equivalent to the error of disturbance compensation; this accelerates the learning process. Moreover, the added disturbance observer enhances the performance robustness of the adaptive fuzzy system for exceptional disturbances that cannot be modeled by the fuzzy logic model. Stability analysis is provided based on Lyapunov stability theory. The experimental results concerning the tracking control of a nonlinear straight-line linkage system are also presented to illustrate the effectiveness of the proposed scheme.*
Keywords: Disturbance observer, Fuzzy control, Motion control, Sliding mode, Stability

1. **Introduction.** Variable-structure systems (VSS) theory offers great advantages over the traditional linear approach in terms of robustness and efficiency [1]. In particular, the popular sliding-mode control (SMC) is a robust nonlinear feedback control technique that utilizes discontinuous control actions to have a system state reach and thereafter stay within some predefined sliding regime. In sliding mode, all state trajectories are confined to the sliding regime, and system responses then completely depend on the characteristics of the sliding regime. However, before the sliding motion occurs, there usually exists a reaching phase, during which the invariance property of a sliding mode is not guaranteed. The existence of such a reaching phase deteriorates performance robustness. Moreover, the discontinuous control actions may excite unmodeled dynamics and lead to oscillations in the state vector at finite frequency. These oscillations, normally referred to as chatter, are known to result in low control accuracy, high heat loss in electric power circuits and

excessive wear of moving mechanical parts [2]. The chattering phenomenon is thus a serious implementation drawback.

With the advent of artificial intelligence systems, there have been increasing efforts to improve SMC performance by integrating fuzzy logic systems. This approach has emerged as a promising one for dealing with uncertain nonlinear systems and relieving SMC implementation difficulties [3]. To cope with perturbation sensitivity during the reaching phase of the SMC system, Ha et al. [4], Orowska-Kowalska et al. [5], Yagiz and Hacioglu [6], and Yorgancıoğlu and Kömürcügil [7] presented fuzzy tuning mechanisms that rotate or shift the sliding surface in such a direction that the reaching time and tracking error can be significantly reduced. To alleviate the chattering phenomenon, Choi and Kim [8], Fung et al. [9], and Abdelhameed [10] proposed fuzzy logic systems as tuning mechanisms to adjust the switching feedback gains of the SMC. However, the stability of these proposed fuzzy sliding-mode control (FSMC) systems [8-10] is not theoretically justified. In [11-13], the system to be controlled is approximately represented by a fuzzy model comprised of a set of linear models at various operating points, while an FSMC integrates SMCs designed based on those linearized models. Fung, Shaw and Wang [14] proposed a region-wise linear fuzzy sliding-mode controller for motor-mechanism systems. The FSMC proposed in [15] includes a fuzzy interpolator that combines two control laws in order to yield a variable-gain SMC. However, stability proofs for the proposed FSMC systems [14,15] are not provided. Wang [16] integrated an SMC with a proportional-integral (PI) controller by using a fuzzy logic system that schedules different control actions according to various operating conditions. A stability proof is given under the assumption that the functions describing the dynamics of the plant are precisely known. Barrero et al. [17] proposed a fuzzy reasoning inference system that combines an SMC and a PI-fuzzy logic-based controller, in which the SMC acts mainly in a transient state, while the PI-like fuzzy controller reduces the chattering phenomenon in the steady state. The proposed scheme [17] is designed specifically for a first-order linear-time-invariant system and requires the information on the time derivative of the plant's state variable. Moreover, the fuzzy logic control is assumed to be upper- and lower-bounded by functions proportional to the plant's state variable. The above-mentioned FMSC schemes for chatter alleviation either provide no rigorous proof of the system stability or require precise knowledge of the plant's model or strict assumptions. Moreover, the controller parameters are usually difficult to determine systematically; the design process may also be time-consuming.

When designing a controller, a designer relies on knowledge of the plants that have either uncertain or nearly unknown dynamics. For example, a designer might have to offer a user a motor drive with a position controller. However, the user could attach any linear/nonlinear mechanical load to the motor drive, which is a great challenge to the designer when designing the position controller. By integrating adaptation techniques into the FSMCs, Lu and Chen [18] and Huang and Lin [19] proposed schemes of adaptive fuzzy sliding-mode control (AFSMC) that adaptively tune consequent parameters of a fuzzy controller so that sliding motion is ensured. However, the input channel of a plant described in companion form can only have a gain function of partial state variables in [18]. In the AFSMC [19], the time derivative of the gain function is assumed to be finite, whereas, in [20], the time derivative of the gain function is assumed to be bounded by a function of plant's state variables. These assumptions, however, might not be valid since the time derivative of the gain function could contain the control input with several parameters adapted by integral laws. Huang and Huang [21] proposed an AFSMC in order to tackle system uncertainties. Erbatur and Kaynak [22] introduced a measure for chattering, and utilized an adaptive fuzzy system to tune an SMC parameter for chatter alleviation, in which an admissible chattering level needs to be assigned. Chang and

Yuan [23] and Chang [24] used a model matching technique to adjust a scaling factor of the AFSMC. However, there was no rigorous proof to guarantee the stability of the systems proposed in [21-25]. Guo and Woo [26], Tao et al. [27] and Wai [28] proposed AFSMC schemes that replace the switching control in conventional SMC with an adaptive fuzzy logic control. However, external disturbances were not dealt with in [26], input uncertainties associated with the gain functions of input channels were not studied in [27], and a lumped disturbance involving the control input was assumed to be bounded by a positive constant in [28]. References [29-31] proposed indirect AFSMC schemes, in which system functions are approximated by fuzzy models. However, the switching gain in [29,30] is difficult to determine in practice, and a lumped uncertainty containing the control input is assumed to be L_2-bounded in [31]. Sadati and Talasaz [32] proposed an AFSMC scheme which cannot guarantee the existence of a sliding mode. Efe [33] utilized fractional-order integration in parameter tuning, in which only the convergence of a switching function is validated. The AFSMC proposed in [34,35] assumed the plant's gain function to be a pure time function with known bounds. Although the AFSMC schemes [18-35] have several excellent features, such as their adaptability and chatter alleviation, convergence of tracking error can be slow due to the adaptation mechanisms, leading to poor transient responses. Moreover, their effectiveness in handling exceptional disturbances that cannot be modeled by a fuzzy logic system might be questionable.

Another approach to diminishing the chattering effect is to employ sliding-mode disturbance observers (SMDOs) [36,37] that use the sliding-mode technique to estimate a lumped disturbance including unknown disturbances and parametric uncertainties. The lumped disturbance is found to be equal to the equivalent value of a switching signal in the SMDO, and its estimate is obtained by feeding the switching signal through a low-pass filter whose cutoff frequency is sufficiently high to retain the equivalent part of the switching signal, yet low enough to attenuate its high-frequency components. In contrast to the SMC, the discontinuous switching action is on an artificially-introduced auxiliary process of an SMDO rather than on the plant; that is, there is no need for plant state to alter its phase velocity towards a switching hyperplane using switching control efforts. In this way, chattering can be alleviated since a continuous feedback control instead of SMC is applied [37]. However, in the previous SMDOs [36,37], switching gains must be greater than the bounds on uncertain disturbances, which leads to restricted chattering alleviation due to large switching gains. Recently, Lu and Chiu [38] and Lu [39] proposed SMDOs that relax the conventionally assumed upper-bounds restriction on the disturbance to the restriction on its estimation error, thus reducing the switching gain required for ensuring the existence of a sliding mode. This reduction in the switching gain further alleviates the chattering phenomenon.

A system perturbation can generally be divided into two parts: a modelable part that is usually due to the uncertainties associated with system functions, and an unmodelable part that includes exceptional external disturbances. An adaptive fuzzy logic system can be employed to model and compensate for the modelable perturbation, but it is inappropriate for dealing with unexpected external disturbances. On the other hand, an SMDO would yield phase lag in estimating a time-varying modelable perturbation, but is suitable for alleviating the effect of disturbances that cannot be modeled. This paper, therefore, proposes an AFSMC scheme augmented with an SMDO, referred to as the SMDO-AFSMC, combining the best features of an adaptive fuzzy logic system and an SMDO. The adaptive fuzzy logic system is gradually adjusted to model state-dependent system perturbations, while the SMDO compensates for the modeling error of the fuzzy logic system and exceptional disturbances that cannot be modeled. In the proposed scheme, precise knowledge of system functions is not required, and external disturbances and input uncertainties are handled. In addition to introducing the SMDO to the AFSMC system, another salient feature of the proposed scheme is the utilization of a switching signal in the adaptation of the AFSMC. Unlike existing AFSMCs [18-35] that extract perturbation information indirectly from the tracking error, the proposed AFSMC employs the switching signal shown to be equivalent to the error of perturbation compensation. This speeds up the learning process, while the active compensation by the SMDO further improves transient performance of the learning system. The stability of the proposed SMDO-AFSMC system, in which the input channel of the plant can have a gain function of full state variables, is theoretically verified using Lyapunov analysis. Experiments are conducted to demonstrate the effectiveness of the proposed scheme in practical applications.

2. Design of a Sliding-Mode Disturbance Observer-Based Adaptive Fuzzy Sliding-Mode Controller (SMDO-AFSMC).

2.1. Problem statement and controller structure.

Consider an nth-order nonlinear system

$$x^{(n)} = f(\mathbf{x},t) + b(\mathbf{x},t)\,(u+d) \tag{1}$$

in which the scalar x is the output of interest, the scalar u is the control input, $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_{n-1} & x_n \end{bmatrix}^T = \begin{bmatrix} x & \dot{x} & \cdots & x^{(n-2)} & x^{(n-1)} \end{bmatrix}^T$ is the state vector, $f(\mathbf{x},t)$ and $b(\mathbf{x},t)$ are uncertain system functions of state variables and time, and the scalar d denotes an unknown external disturbance. Without loss of generality, assume that $b(\mathbf{x},t)$ is strictly positive.

For tasks involving the tracking of a desired output r, assumed to be n-times differentiable with respect to time, let the tracking error $e = x - r$. Moreover, define a filtered tracking error s to be

$$s = \left(p^{n-1} + c_{n-2}p^{n-2} + \cdots + c_1 p + c_0\right) e \tag{2}$$

where the differential operator $p = d/dt$, and the c_i's are constant parameters chosen so that the dynamics associated with $s = 0$ are asymptotically stable. Consider the control

$$u = u_{pa} + u_{do} + u_{fz} \tag{3}$$

where u_{pa} denotes the nominal control, and u_{do} and u_{fz} are the control components generated by a disturbance observer and a fuzzy compensator, respectively. Here, the nominal control, u_{pa}, designed based on the pole-assignment technique, is described by

$$u_{pa} = \hat{b}^{-1}(\mathbf{x},t)\left[-\hat{f}(\mathbf{x},t) + h\right] \tag{4}$$

where $\hat{f}(\mathbf{x},t)$ and $\hat{b}(\mathbf{x},t)$ are the estimates of functions $f(\mathbf{x},t)$ and $b(\mathbf{x},t)$, respectively, and

$$h = r^{(n)} - \left(c_{n-2}p^{n-1} + \cdots + c_1 p^2 + c_0 p\right) e - \lambda s \tag{5}$$

where λ is a positive constant parameter. Substituting (3) and (4) into (1) gives

$$\dot{x}_n = h + \left[f - b\hat{b}^{-1}\hat{f} + \left(b\hat{b}^{-1} - 1\right) h + b\left(u_{fz} + u_{do} + d\right)\right] \tag{6}$$

After rearrangement, (6) yields

$$\dot{x}_n = h + \hat{b}\left[b\hat{b}^{-1}u_{fz} + u_{do} + \hat{b}^{-1}\xi\right] \tag{7}$$

where the perturbation term ξ is described by

$$\xi\left(\mathbf{x}, \hat{b}^{-1}h + u_{do}, t\right) = f - b\hat{b}^{-1}\hat{f} + \left(b - \hat{b}\right)\left(\hat{b}^{-1}h + u_{do}\right) + bd \tag{8}$$

Substituting the definition of h into (7) gives

$$\dot{s} = -\lambda s + \hat{b}\left(b\hat{b}^{-1}u_{fz} + u_{do} + \hat{b}^{-1}\xi\right) \tag{9}$$

Without the SMDO-AFSMC, that is, $u_{do} = u_{fz} = 0$, one obtains $\dot{s} = -\lambda s + \xi$. Moreover, in the ideal situation when the perturbation term vanishes, i.e., $\xi = 0$; then one has $\dot{s} + \lambda s = 0$, implying asymptotic convergence of the tracking error. Thus, the nominal control u_{pa}, where the c_i's and λ are design parameters, is used to specify the desired closed-loop dynamics. In other words, provided that the external disturbance d vanishes, and the nominal functions $\hat{f}(\mathbf{x}, t)$ and $\hat{b}(\mathbf{x}, t)$ coincide with the actual functions $f(\mathbf{x}, t)$ and $b(\mathbf{x}, t)$, respectively, the closed-loop error dynamics without the SMDO-AFSMC can be characterized by

$$(p + \lambda)\left(p^{n-1} + c_{n-2}p^{n-2} + \cdots + c_1 p + c_0\right)e = 0 \tag{10}$$

which represents the desired error dynamics. In practice, however, perturbation often arises and causes the resultant error dynamics to deviate from the desired dynamics in an adverse way, such as an undesirable overshoot or, more severely, system instability.

2.2. **Sliding-mode disturbance observer (SMDO).** This paper presents the SMDO and the AFSMC in order to generate u_{do} and u_{fz}, respectively, so as to compensate for the perturbation ξ. Since the expression for the desired closed-loop dynamics is $\dot{s} + \lambda s = 0$, the ideal compensation for ξ is depicted by $b\hat{b}^{-1}u_{fz} + u_{do} + \hat{b}^{-1}\xi = 0$. In order to effect this, consider an artificially introduced auxiliary process described by

$$\dot{z} = h + \hat{b}\varphi \operatorname{sgn}(\sigma) \tag{11}$$

where z is the state variable of the auxiliary process, φ is a switching gain, $\operatorname{sgn}(\cdot)$ denotes the signum function, and the switching function σ is defined by

$$\sigma = x_n - z \tag{12}$$

Taking the derivative of (12) with respect to time, and substituting (7) and (11) into the resulting equation gives

$$\dot{\sigma} = \hat{b}\left[b\hat{b}^{-1}u_{fz} + u_{do} + \hat{b}^{-1}\xi - \varphi \operatorname{sgn}(\sigma)\right] \tag{13}$$

Provided the switching gain $\varphi > \left|b\hat{b}^{-1}u_{fz} + u_{do} + \hat{b}^{-1}\xi\right|$, then the sliding condition is satisfied, i.e., $\sigma\dot{\sigma} < 0$ if $\sigma \neq 0$. Assigning the initial condition of the auxiliary process such that $z(0) = x_n(0)$, one has $\sigma(0) = 0$. This, together with the satisfaction of the sliding condition, yields $\sigma(t) = 0$ for $t \geq 0$, meaning that the sliding mode $\sigma = 0$ exists throughout an entire response. Since $\sigma(t) = 0$, one has $\dot{\sigma}(t) = 0$, which gives, according to (13)

$$\varphi \operatorname{sgn}(\sigma) = b\hat{b}^{-1}u_{fz} + u_{do} + \hat{b}^{-1}\xi \tag{14}$$

in the sense of equivalent values [1]. Hence, the switching signal $\varphi \operatorname{sgn}(\sigma)$ represents the perturbation compensation error by the SMDO-AFSMC.

Consider the following integral law for disturbance compensation

$$\dot{u}_{do} = -k_{do}\varphi \operatorname{sgn}(\sigma) - k_{sv}\hat{b}s \tag{15}$$

where k_{do} and k_{sv} are arbitrary positive constant parameters. The unknown perturbation ξ defined in (8) can be divided into two parts: one that can be modeled and approximated by a fuzzy model, while the other part can be regarded as some exceptional, abrupt disturbances that cannot be modeled. In this paper, the AFSMC is designed to compensate for the modelable part of ξ. However, the control component u_{fz} produced by the AFSMC is initially zero and adaptively adjusted afterwards, implying that the AFSMC cannot effectively compensate for the modelable part of ξ during an initial period. The SMDO is proposed here to compensate for the unmodelable part of ξ, as well as the compensation error by the AFSMC. With this in mind, the switching signal $\varphi\mathrm{sgn}\,(\sigma)$ in (14) can also be interpreted as the compensation error by the SMDO. Hence, $\varphi\mathrm{sgn}\,(\sigma)$ is employed to update u_{do} in the integral law (15).

2.3. **Adaptive fuzzy sliding-mode controller (AFSMC).** Although the SMDO can compensate for the modelable part of ξ without the need to model it, the integral law (15) for the SMDO leads to a phase lag that would yield compensation error in the modelable part of ξ. As a remedy, the AFSMC is designed to adaptively compensate for the modelable part of ξ. Since ξ is a function of $\mathbf{x}$ and $(\hat{b}^{-1}h + u_{do})$, the AFSMC employs a T-S fuzzy model with $(n+1)$ inputs and m rules in its rule base, represented by $R_1, R_2, \cdots, R_m$. The general form of the jth rule is

$$R_j:\ \text{If } (x_1 \text{ is } A_1^j) \wedge (x_2 \text{ is } A_2^j) \wedge \cdots \wedge (x_n \text{ is } A_n^j) \wedge \left[(\hat{b}^{-1}h + u_{do}) \text{ is } A_{n+1}^j\right],\ \text{then } u_{fz}^j = p_j \tag{16}$$

where $\wedge$ denotes the AND intersection operation, A_i^j, $i = 1, 2, \cdots, n+1$, are fuzzy sets characterizing the corresponding variables of the premise in the jth rule, u_{fz}^j is the output from the jth implication, and p_j is the consequent parameter. Here, no exact membership functions are specified for the output linguistic variables. Since there are m rules to define the rule base of the fuzzy controller, and each rule can give a distinct value of the output, the weighted average of the individual output, u_{fz}^j, $j = 1, 2, \cdots, m$, is used to obtain the output, u_{fz}. The weighting assigned to each u_{fz}^j is the firing strength of the jth rule, designated as w_j, and is determined from

$$w_j = \min\left\{M_1^j(x_1), M_2^j(x_2), \cdots, M_n^j(x_n), M_{n+1}^j(\hat{b}^{-1}h + u_{do})\right\} \tag{17}$$

where $M_i^j(v_p)$ is the value of the membership function at v_p in the fuzzy set A_i^j. The final output of the fuzzy controller is inferred from

$$u_{fz} = \frac{\sum_{j=1}^m w_j u_{fz}^j}{\sum_{j=1}^m w_j} = \frac{\sum_{j=1}^m w_j p_j}{\sum_{j=1}^m w_j} \tag{18}$$

where the optimal value of the consequent parameter p_j is unknown and is to be obtained through learning.

The aim of the learning task is to modify the consequent parameter such that u_{fz} compensates for ξ. Rewrite (14) as

$$\hat{b}^{-1}\,(bu_{fz} + \xi) = \varphi\mathrm{sgn}\,(\sigma) - u_{do} \tag{19}$$

which can be considered to be the compensation error by u_{fz}. Thus, the consequent parameters of the fuzzy model are updated as

$$\dot{p}_j = \left[-\eta\,(\varphi\mathrm{sgn}(\sigma) - u_{do}) - k_{sv}\hat{b}s + k_{do}u_{do}\right]\frac{w_j}{\sum_{i=1}^m w_i} \tag{20}$$

where η is the learning gain, and the terms $(-k_{sv}\hat{b}s + k_{do}u_{do})$ are required to ensure system stability. Unlike previous learning laws that update parameters using the filtered tracking error only, the proposed learning law adjusts the consequent parameters and thus u_{fz}, based on the switching signal $(\varphi\mathrm{sgn}(\sigma) - u_{do})$ that is equivalent to the compensation error by u_{fz}. In this way, the learning process can be sped up since the fuzzy model is updated directly from its compensation error, rather than indirectly from the filtered tracking error s. Figure 1 shows the structure of the proposed SMDO-AFSMC system. It can be seen that the control is in an additive form and is composed of three components,

u_{pa}, u_{do} and u_{fz}, which are generated by the nominal controller, the SMDO and the AFSMC, respectively. The nominal control specifies the desired closed-loop dynamics, while the SMDO-AFSMC counteracts the perturbation. The auxiliary process, in which a sliding mode occurs, generates the switching signal $\varphi\text{sgn}(\sigma)$, revealing information on the compensation error due to the SMDO-AFSMC. The switching signal is then fed to both the SMDO and the AFSMC. Here, the SMDO compensates for the unmodelable part of the perturbation and the compensation error by the AFSMC, whereas the AFSMC is adaptively adjusted to compensate for the modelable part of the perturbation.

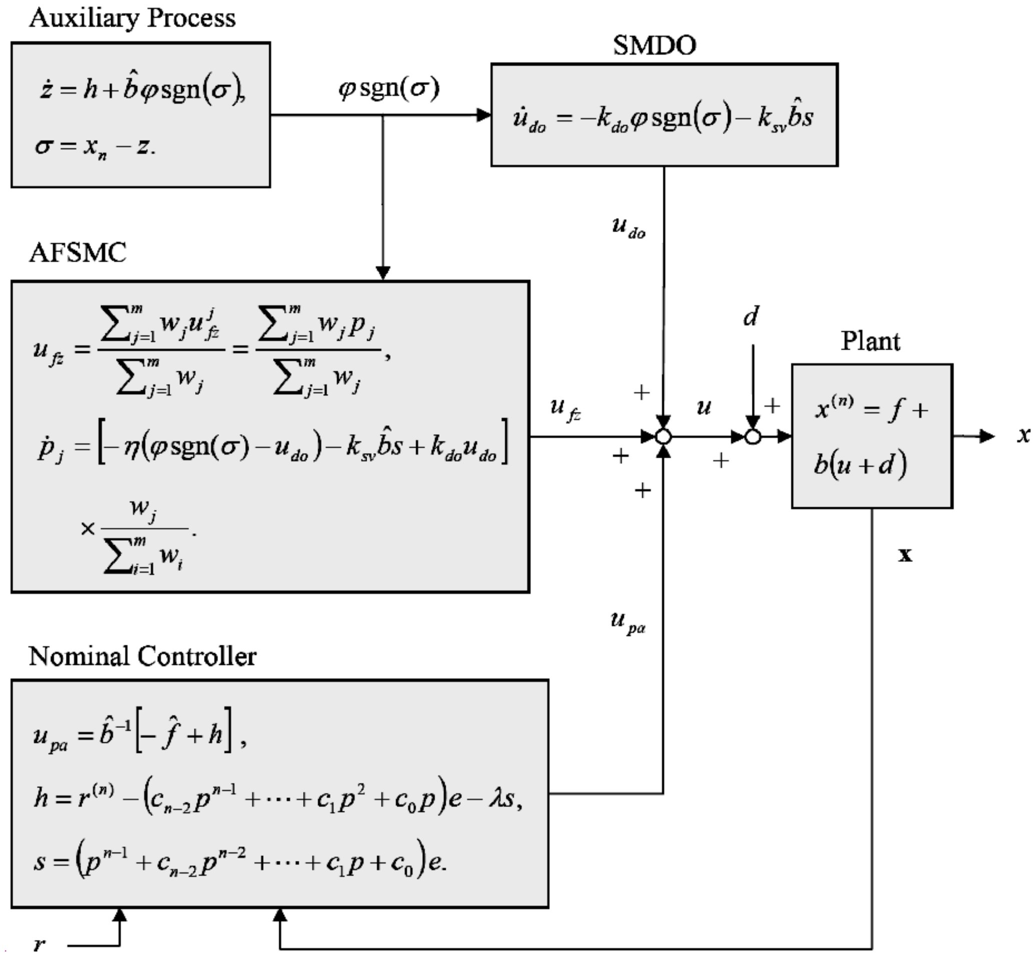

FIGURE 1. Structure of the SMDO-AFSMC system

2.4. **Simulation example.** Consider the control of a scotch yoke mechanism, in which the linear position of load is determined by the angular position of a crank driven by a rotational electric motor. The kinematic constraint on this mechanism can be described as $x = \ell(1 - \cos q)$, where x is the load position to be controlled, q is the angular position of the crank, and ℓ is the crank length. The dynamic model, including parasitic dynamics that represent the mismatch between nominal model dynamics and actual plant dynamics, is given by

$$\begin{aligned} &\left(M_l + \frac{I_m}{\ell^2 \sin^2 q}\right)\ddot{x} - \frac{I_m \cos q}{\ell^3 \sin^4 q}\dot{x}^2 + \left(C_t + \frac{C_r}{\ell^2 \sin^2 q}\right)\dot{x} = \frac{v}{\ell \sin q}, \\ &\tau_p \dot{v} + v = u \end{aligned}$$

where u is the control input, v is the state variable of the first-order parasitic dynamics, τ_p is a small time constant, M_l is the load mass, I_m is the motor inertia, and C_t and C_r

are the viscous friction coefficients of the translational and rotational parts, respectively. Simulation parameters to be employed are $\ell = 1$, $M_l = 5$, $I_m = 10^{-3}$, $C_t = 5$, $C_r = 1$, $\tau_p = 1/300$, $x(0) = 0.2$, and $\dot{x}(0) = 0$.

For comparison studies, consider the AFSMC schemes presented in [19,21,34]. These existing control laws can be described by $u = u_{pa} + u_{fz}$. The nominal control u_{pa} is set to zero in [19,21], whereas the nominal control in [34] is determined by a function approximation technique based on the truncated Fourier series. The nominal control allows us to incorporate knowledge of the plant model into the controller design and improve transient performance. In the following comparison study, the nominal control is defined by (4), and different adaptation laws for u_{fz} are compared. In [19,21,34], u_{fz} is the same as in (18), and the adaptation law (neglecting the dead-zone and the so-called e-modification for comparison study) is given by

$$\dot{p}_j = -k_{sv} s \frac{w_j}{\sum_{i=1}^{m} w_i}$$

When compared with the proposed scheme, it can be seen that when u_{do}, η and k_{do} in (15) and (20) are set to zero, the proposed scheme becomes the previous schemes in [19,21,34]. Thus, the previous learning schemes are actually special cases of the proposed learning scheme.

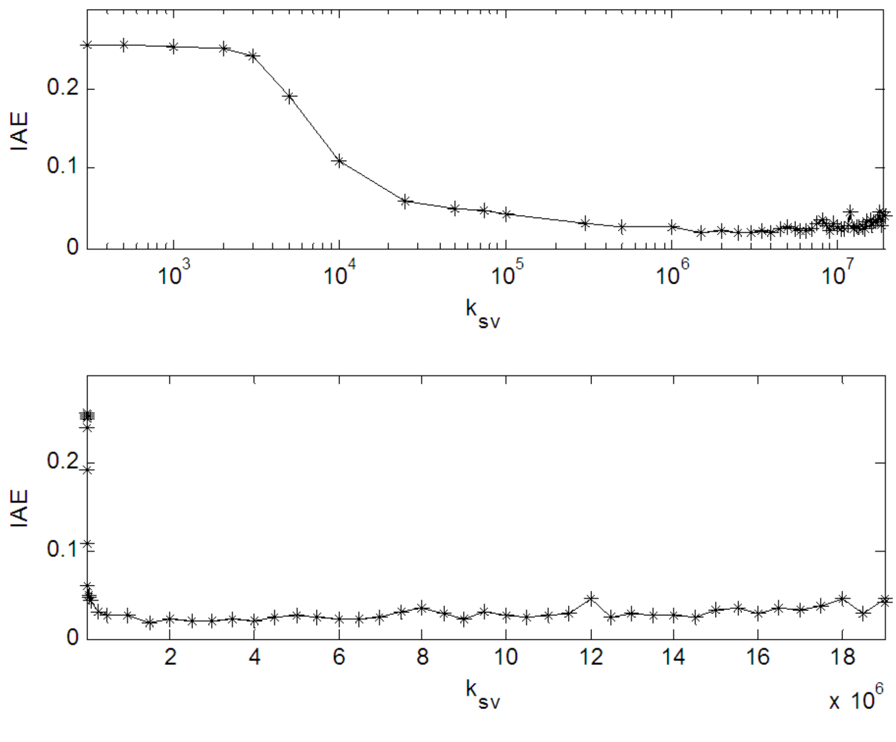

FIGURE 2. IAE by the previous scheme with various values of k_{sv}

For the controller design, the nonlinear scotch yoke mechanism is modeled simply as a pure double integrator described by: $\ddot{x} = u$; that is, $\hat{f} = 0$ and $\hat{b} = 1$. The controller parameters common to both schemes are: $c_0 = 50$ and $\lambda = 50$. Concerning the fuzzy model, the universe of discourse for each linguistic variable is assigned as $[0, 2]$, $[-25, 25]$ and $[-2000, 2000]$ for x, $\dot{x}$ and $(\hat{b}^{-1}h + u_{do})$, respectively, in which five linguistic values with equally spaced triangular membership functions are defined for each linguistic variable,

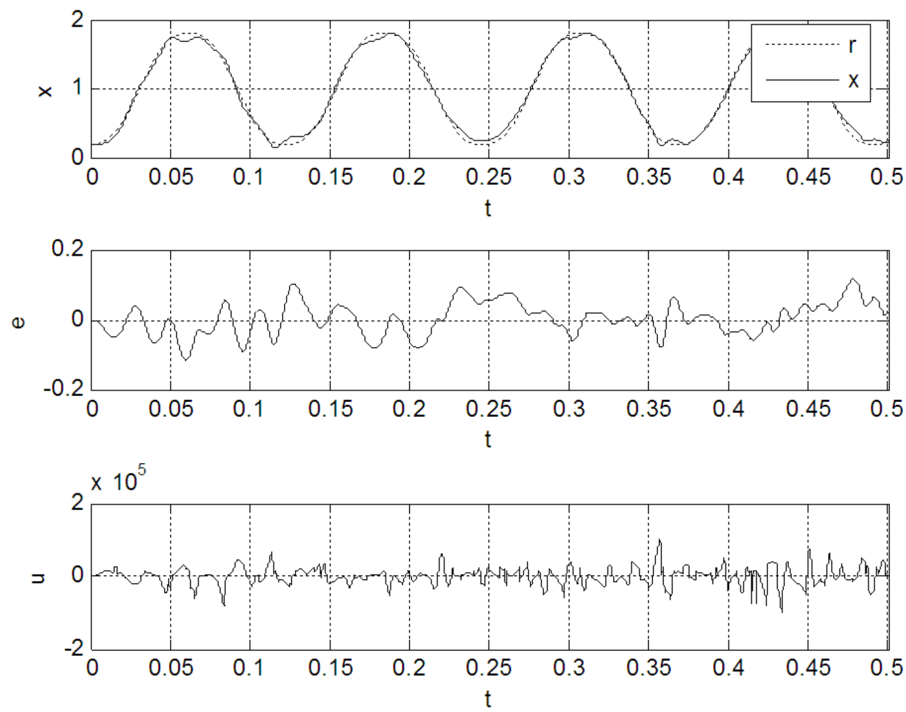

FIGURE 3. Response by the previous scheme with $k_{sv} = 1.5 \times 10^6$

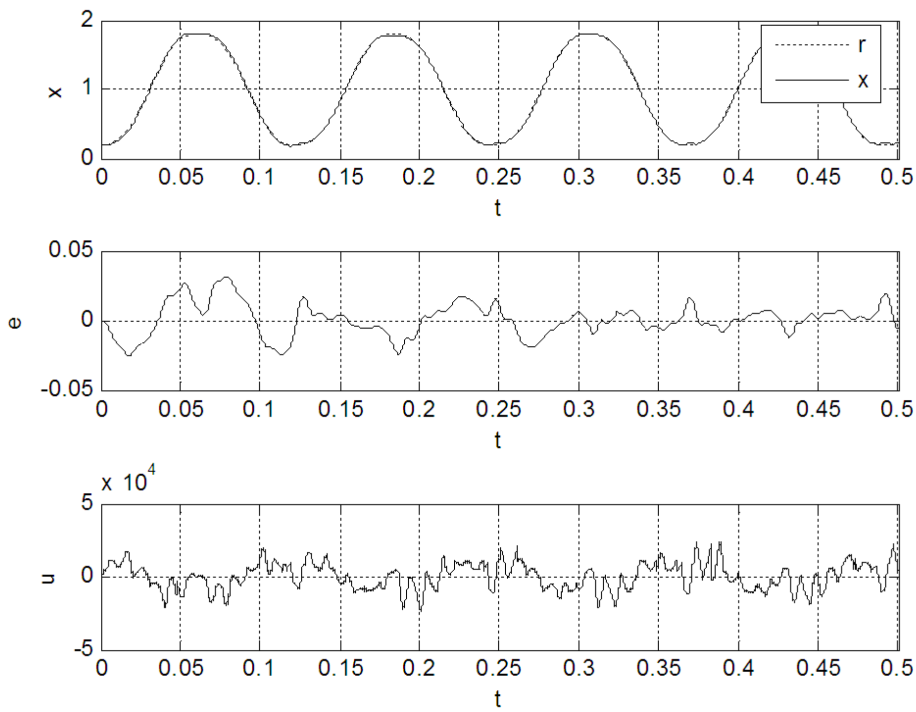

FIGURE 4. Response by the proposed scheme

giving 125 fuzzy rules in the rule base. The control task is to track a periodic reference, whose first cycle is described by

$$r(t) = 0.2 + 1.6 \left[\left(\frac{10}{\tau^3} t^3 - \frac{15}{\tau^4} t^4 + \frac{6}{\tau^5} t^5 \right) (H(t) - H(t - 0.061)) + \left(\frac{10}{\tau^3} (0.122 - t)^3 - \frac{15}{\tau^4} (0.122 - t)^4 + \frac{6}{\tau^5} (0.122 - t)^5 \right) H(t - 0.061) \right]$$

where $\tau = 0.061$, and $H(\cdot)$ denotes the unit-step function. For the simulation period $[0, 0.5]$, Figure 2 shows the integral absolute error (IAE) with respect to k_{sv} for the previous scheme in which, though the IAEs show no obvious trend towards instability with increasing k_{sv}, the control input becomes oscillatory and shows signs of instability as k_{sv} increases. Figure 3 shows the dynamic response with $k_{sv} = 1.5 \times 10^6$, which yields the minimum IAE of 0.019. For the proposed scheme, let $\eta = 10^4$, $k_{do} = 100$, $k_{sv}\hat{b} = 100$, and $\varphi = 2000$. Figure 4 shows the dynamic response with the proposed scheme, whose IAE is 0.004. Unlike the previous scheme, which gives the minimum IAE of 0.019, the proposed scheme yields much better performance in the sense of IAE. The major difference between the proposed and previous schemes is that the switching signal which directly reveals the compensation error by the fuzzy control is utilized in the learning process of the proposed scheme. By this means, the proposed scheme yields more effective learning than the previous scheme does by learning indirectly from the filtered tracking error s.

3. Stability Analysis.

To analyze system stability, the following assumptions are made:

Assumption 3.1. [18, 19, 26, 30, 31] *There exist optimal values for the consequent parameters of rules* $\mathbf{p}^* = \begin{bmatrix} p_1^* & p_2^* & \cdots & p_m^* \end{bmatrix}^T$ *such that*

$$\left| b u_{fz}^* + \xi \right| < \varepsilon \tag{21}$$

where ε *is the minimum positive parameter for all possible values of the consequent parameters, and* u_{fz}^* *is the inferred output of the fuzzy rule base with the vector of consequent parameters* $\mathbf{p} = \begin{bmatrix} p_1 & p_2 & \cdots & p_m \end{bmatrix}^T$ *equal to* $\mathbf{p}^*$.

Assumption 3.2. [29, 40] *The gain function is upper- and lower-bounded; that is, there exist* $b_{\max}(\mathbf{x}, t)$ *and* $b_{\min}(\mathbf{x}, t)$ *such that* $b_{\max} > b > b_{\min} > 0$.

Assumption 3.3. *There exists positive* υ *such that*

$$\lambda > \upsilon + \frac{1}{2} \max \left\{ \left(b_{\max} - \hat{b} \right)^2, \left(b_{\min} - \hat{b} \right)^2 \right\}$$

Assumption 3.4. *There exist* $\mu \in (0, 2)$ *and positive* υ, *such that*

$$\eta > \upsilon + \hat{b} b_{\min}^{-1} \left[\frac{k_{sv}}{2} + \frac{k_{do}}{2\mu} \left(1 + b_{\max}^2 \hat{b}^{-2} \right) \right]$$

Remark 3.1. *The validity of Assumptions 3.3 and 3.4 requires the parameter* λ *in the nominal controller and the learning rate* η *in the AFSMC, respectively, to be large enough.*

Theorem 3.1. *If these assumptions are valid, boundedness of the tracking error is guaranteed, and overall system stability is maintained by employing the control law, (3), (4), (15) and (18) with the learning law (20), on system (1). Moreover, provided the optimal approximation error vanishes, i.e.,* $\varepsilon = 0$, *the tracking error converges asymptotically.*

To prove this, consider the following Lyapunov candidate V

$$2V = k_{sv}s^2 + u_{do}^2 + (\mathbf{p} - \mathbf{p}^*)^T (\mathbf{p} - \mathbf{p}^*) \tag{22}$$

Its derivative with respect to time along any system trajectory is

$$\dot{V} = k_{sv}s\dot{s} + u_{do}\dot{u}_{do} + (\mathbf{p} - \mathbf{p}^*)^T \dot{\mathbf{p}} \tag{23}$$

Substituting (9), (15) and (20) into (23), and using (18) yields

$$\begin{aligned}\dot{V} = &- \lambda k_{sv}s^2 + k_{sv}s\hat{b}\left(b\hat{b}^{-1}u_{fz} + u_{do} + \hat{b}^{-1}\xi\right) - k_{do}u_{do}^2 - k_{do}u_{do}\hat{b}^{-1}(bu_{fz} + \xi) \\ &- k_{sv}\hat{b}su_{do} - \eta\hat{b}^{-1}(bu_{fz} + \xi)(u_{fz} - u_{fz}^*) - k_{sv}\hat{b}s(u_{fz} - u_{fz}^*) + k_{do}u_{do}(u_{fz} - u_{fz}^*)\end{aligned} \tag{24}$$

Replacing $(bu_{fz} + \xi)$ with $\left[b(u_{fz} - u_{fz}^*) + (bu_{fz}^* + \xi)\right]$ in (24) yields

$$\begin{aligned}\dot{V} = &- \lambda k_{sv}s^2 + k_{sv}s\left(bu_{fz}^* + \xi\right) + k_{sv}sb\left(u_{fz} - u_{fz}^*\right) \\ &- k_{do}u_{do}^2 - k_{do}\hat{b}^{-1}u_{do}\left(bu_{fz}^* + \xi\right) - k_{do}\hat{b}^{-1}u_{do}b\left(u_{fz} - u_{fz}^*\right) \\ &- \eta\hat{b}^{-1}b(u_{fz} - u_{fz}^*)^2 - \eta\hat{b}^{-1}(bu_{fz}^* + \xi)(u_{fz} - u_{fz}^*) \\ &- k_{sv}\hat{b}s(u_{fz} - u_{fz}^*) + k_{do}u_{do}(u_{fz} - u_{fz}^*)\end{aligned} \tag{25}$$

Let $\lambda = \gamma_{s0} + \gamma_{s1}$, where $\gamma_{s0} > 0$ and $\gamma_{s1} > 0$. Rewrite some terms in (25):

$$-\lambda k_{sv}s^2 + k_{sv}s\left(bu_{fz}^* + \xi\right) = -\gamma_{s0}k_{sv}s^2 + k_{sv}\left[-\gamma_{s1}s^2 + s\left(bu_{fz}^* + \xi\right)\right] \tag{26}$$

The fact that $-\left[\gamma_{s1}s - \frac{1}{2}(bu_{fz}^* + \xi)\right]^2 \leq 0$ indicates

$$-\gamma_{s1}^2 s^2 + \gamma_{s1}s(bu_{fz}^* + \xi) - \frac{1}{4}(bu_{fz}^* + \xi)^2 \leq 0 \tag{27}$$

Combined with assumption (21) this gives

$$-\gamma_{s1}s^2 + s(bu_{fz}^* + \xi) \leq \frac{1}{4\gamma_{s1}}(bu_{fz}^* + \xi)^2 \leq \frac{\varepsilon^2}{4\gamma_{s1}} \tag{28}$$

Substituting (28) into (26) yields

$$-\lambda k_{sv}s^2 + k_{sv}s[bu_{fz}^* + \xi] \leq -\gamma_{s0}k_{sv}s^2 + \frac{k_{sv}\varepsilon^2}{4\gamma_{s1}} \tag{29}$$

Likewise, let $k_{do} = (\gamma_{d0} + \gamma_{d1})k_{do}$, where $\gamma_{d0} > 0$, $\gamma_{d1} > 0$, and $\gamma_{d0} + \gamma_{d1} = 1$. Rewrite some terms in (25)

$$-k_{do}u_{do}^2 - k_{do}\hat{b}^{-1}u_{do}(bu_{fz}^* + \xi) = -\gamma_{d0}k_{do}u_{do}^2 - k_{do}\left[\gamma_{d1}u_{do}^2 + \hat{b}^{-1}u_{do}(bu_{fz}^* + \xi)\right] \tag{30}$$

The fact that $-\left[\gamma_{d1}u_{do} + \frac{1}{2}\hat{b}^{-1}(bu_{fz}^* + \xi)\right]^2 \leq 0$ yields

$$-\gamma_{d1}^2 u_{do}^2 - \gamma_{d1}u_{do}\hat{b}^{-1}(bu_{fz}^* + \xi) - \frac{1}{4}\hat{b}^{-2}(bu_{fz}^* + \xi)^2 \leq 0 \tag{31}$$

which, in combination with assumption (21), gives

$$-\gamma_{d1}u_{do}^2 - u_{do}\hat{b}^{-1}(bu_{fz}^* + \xi) \leq \frac{(bu_{fz}^* + \xi)^2}{4\hat{b}^2\gamma_{d1}} \leq \frac{\varepsilon^2}{4\gamma_{d1}\hat{b}^2} \tag{32}$$

Substituting (32) into (30) gives the relation

$$-k_{do}u_{do}^2 - k_{do}\hat{b}^{-1}u_{do}(bu_{fz}^* + \xi) \leq -\gamma_{d0}k_{do}u_{do}^2 + \frac{k_{do}\varepsilon^2}{4\gamma_{d1}\hat{b}^2} \tag{33}$$

Furthermore, let $\eta = (\gamma_{f0}+\gamma_{f1})\eta$, where $\gamma_{f0} > 0$, $\gamma_{f1} > 0$, and $\gamma_{f0}+\gamma_{f1} = 1$. Rearranging some terms in (25) yields

$$\begin{aligned} & -\eta\hat{b}^{-1}b(u_{fz}-u_{fz}^*)^2 - \eta\hat{b}^{-1}(u_{fz}-u_{fz}^*)(bu_{fz}^*+\xi) \\ = & -\gamma_{f0}\eta\hat{b}^{-1}b(u_{fz}-u_{fz}^*)^2 - \eta\hat{b}^{-1}\left[\gamma_{f1}b(u_{fz}-u_{fz}^*)^2 + (u_{fz}-u_{fz}^*)(bu_{fz}^*+\xi)\right] \end{aligned} \tag{34}$$

Using assumption (21) and the fact that $-\left[\gamma_{f1}b(u_{fz}-u_{fz}^*) + \frac{1}{2}(bu_{fz}^*+\xi)\right]^2 \leq 0$ leads to

$$-\gamma_{f1}b(u_{fz}-u_{fz}^*)^2 - (u_{fz}-u_{fz}^*)(bu_{fz}^*+\xi) \leq \frac{(bu_{fz}^*+\xi)^2}{4\gamma_{f1}b} \leq \frac{\varepsilon^2}{4\gamma_{f1}b} \tag{35}$$

Substituting (35) into (34) yields the relation

$$-\eta\hat{b}^{-1}(u_{fz}-u_{fz}^*)^2 - \eta\hat{b}^{-1}(u_{fz}-u_{fz}^*)(bu_{fz}^*+\xi) \leq -\gamma_{f0}\eta\hat{b}^{-1}b(u_{fz}-u_{fz}^*)^2 + \frac{\eta\hat{b}^{-1}\varepsilon^2}{4\gamma_{f1}b} \tag{36}$$

Subsequently, substituting the three relations (29), (33) and (36) into (25) gives

$$\begin{aligned} \dot{V} \leq & -\gamma_{s0}k_{sv}s^2 - \gamma_{d0}k_{do}u_{do}^2 - \gamma_{f0}\eta\hat{b}^{-1}b(u_{fz}-u_{fz}^*)^2 + \delta \\ & + k_{sv}s(b-\hat{b})(u_{fz}-u_{fz}^*) + k_{do}u_{do}(1-\hat{b}^{-1}b)(u_{fz}-u_{fz}^*) \end{aligned} \tag{37}$$

where $\delta = \frac{\varepsilon^2}{4}\left(\frac{k_{sv}}{\gamma_{s1}} + \frac{k_{do}}{\gamma_{d1}\hat{b}^2} + \frac{\eta\hat{b}^{-1}}{\gamma_{f1}b}\right)$. Rearranging the following relation

$$\begin{aligned} & -\frac{k_{sv}}{2}[(b-\hat{b})s]^2 + k_{sv}(b-\hat{b})s(u_{fz}-u_{fz}^*) - \frac{k_{sv}}{2}(u_{fz}-u_{fz}^*) \\ = & -\frac{k_{sv}}{2}[(b-\hat{b})s - (u_{fz}-u_{fz}^*)]^2 \end{aligned} \tag{38}$$

gives

$$k_{sv}(b-\hat{b})s(u_{fz}-u_{fz}^*) = \frac{k_{sv}}{2}[(b-\hat{b})s]^2 + \frac{k_{sv}}{2}(u_{fz}-u_{fz}^*)^2 - \frac{k_{sv}}{2}[(b-\hat{b})s-(u_{fz}-u_{fz}^*)]^2 \tag{39}$$

Moreover, consider the following identity relation

$$\begin{aligned} & -\frac{r_{d2}k_{do}}{2}u_{do}^2 + \gamma_{d2}k_{do}u_{do}\left[\gamma_{d2}^{-1}(1-b\hat{b}^{-1})(u_{fz}-u_{fz}^*)\right] \\ = & -\frac{\gamma_{d2}k_{do}}{2}\left[\gamma_{d2}^{-1}(1-b\hat{b}^{-1})(u_{fz}-u_{fz}^*)\right]^2 - \frac{\gamma_{d2}k_{do}}{2}\left[u_{do} - \gamma_{d2}^{-1}(1-b\hat{b}^{-1})(u_{fz}-u_{fz}^*)\right]^2 \end{aligned} \tag{40}$$

where positive γ_{d2} is constrained to fulfill $2\gamma_{d0} > \gamma_{d2} > 0$, and the term $\gamma_{d2}k_{do}u_{do}\left[\gamma_{d2}^{-1}(1-\hat{b}^{-1}b)(u_{fz}-u_{fz}^*)\right]$ is equal to $k_{do}u_{do}(1-\hat{b}^{-1}b)(u_{fz}-u_{fz}^*)$. Rearranging this equality then gives

$$\begin{aligned} k_{do}u_{do}(1-\hat{b}^{-1}b)(u_{fz}-u_{fz}^*) = & \frac{\gamma_{d2}k_{do}}{2}u_{do}^2 + \frac{\gamma_{d2}k_{do}}{2}\left[\gamma_{d2}^{-1}(1-b\hat{b}^{-1})(u_{fz}-u_{fz}^*)\right]^2 \\ & - \frac{\gamma_{d2}k_{do}}{2}\left[u_{do} - \gamma_{d2}^{-1}(1-b\hat{b}^{-1})(u_{fz}-u_{fz}^*)\right]^2 \end{aligned} \tag{41}$$

Subsequently, replacing the last two terms in (37) with (39) and (41) gives

$$\begin{aligned}\dot{V} \leq & -\left[\gamma_{s0} - \frac{(b-\hat{b})^2}{2}\right] k_{sv} s^2 - \left(\gamma_{d0} - \frac{\gamma_{d2}}{2}\right) k_{do} u_{do}^2 \\ & - \left\{\gamma_{f0}\eta\hat{b}^{-1}b - \frac{k_{sv}}{2} - \frac{\gamma_{d2}k_{do}}{2}\left[\gamma_{d2}^{-1}(1-b\hat{b}^{-1})\right]^2\right\}(u_{fz}-u_{fz}^*)^2 + \delta \\ & - \frac{k_{sv}}{2}\left[(b-\hat{b})s - (u_{fz}-u_{fz}^*)\right]^2 - \frac{\gamma_{d2}k_{do}}{2}\left[u_{do} - \gamma_{d2}^{-1}(1-b\hat{b}^{-1})(u_{fz}-u_{fz}^*)\right]^2\end{aligned} \tag{42}$$

Based on Assumption 3.3, one can have positive γ_{s2} satisfying

$$\gamma_{s2} = \gamma_{s0} - \frac{1}{2}\max\left\{(b_{\max}-\hat{b})^2, (b_{\min}-\hat{b})^2\right\} > 0 \tag{43}$$

Define a constant $\gamma_{d3} = \gamma_{d0} - \gamma_{d2}/2$. Since γ_{d2} satisfies $\gamma_{d0} > \gamma_{d2}/2 > 0$, one has

$$\gamma_{d3} = \gamma_{d0} - \frac{\gamma_{d2}}{2} > 0 \tag{44}$$

According to Assumption 3.4, there exists positive γ_{f2} satisfying

$$\gamma_{f2}\eta = \gamma_{f0}\eta - \hat{b}b_{\min}^{-1}\left[\frac{k_{sv}}{2} + \frac{k_{do}}{2\gamma_{d2}}\left(1 + b_{\max}^2\hat{b}^{-2}\right)\right] > 0 \tag{45}$$

Substituting (43)-(45) into (42) gives

$$\dot{V} \leq -\gamma_{s2}k_{sv}s^2 - \gamma_{d3}k_{do}u_{do}^2 - \gamma_{f2}\eta\hat{b}^{-1}b(u_{fz}-u_{fz}^*)^2 + \bar{\delta} \tag{46}$$

where $\bar{\delta} = \frac{\varepsilon^2}{4}\left(\frac{k_{sv}}{\gamma_{s1}} + \frac{k_{do}}{\gamma_{d1}\hat{b}^2} + \frac{\eta}{\gamma_{f1}\hat{b}b_{\min}}\right)$. Define the following compact sets $\Omega_s = \left\{s \,\middle|\, |s| < \sqrt{\frac{\bar{\delta}}{\gamma_{s2}k_{sv}}}\right\}$, $\Omega_d = \left\{u_{do} \,\middle|\, |u_{do}| < \sqrt{\frac{\bar{\delta}}{\gamma_{d3}k_{do}}}\right\}$, and $\Omega_p = \left\{(\mathbf{p}-\mathbf{p}^*) \,\middle|\, |u_{fz}-u_{fz}^*| < \sqrt{\frac{\hat{b}\bar{\delta}}{\gamma_{f2}\eta b_{\min}}}\right\}$. The relation (46) implies that

$$\dot{V} < 0 \text{ when } s \notin \Omega_s,\ u_{do} \notin \Omega_d,\ \text{or } (\mathbf{p}-\mathbf{p}^*) \notin \Omega_p \tag{47}$$

This shows the boundedness of the tracking error as well as the stability of the overall system. When the approximation ability of the fuzzy model is enhanced and the unmodelable part of the perturbation ξ vanishes, then ε and thus $\bar{\delta}$ approach zero. In this case, $\dot{V}$ is negative definite along any system trajectory, and the tracking error e converges asymptotically. Therefore, provided the fuzzy model can perfectly model the perturbation ξ, the tracking error asymptotically converges to zero and the SMDO-AFSMC completely rejects the unknown perturbation ξ.

Remark 3.2. *Assumptions 3.1-3.4 are required to obtain Theorem 3.1. Although the fuzzy model is considered as a universal approximator, it yields nonzero approximation error in practice. Assumption 3.1 supposes that the approximation error is bounded by a positive parameter. Assumptions 3.2-3.4 are relevant to the gain function of the input channel, $b(\mathbf{x},t)$. To the best of the authors' knowledge, the problem of ensuring the stability of AFSMC systems with uncertain gain functions of full state variables has yet to be thoroughly solved. A contribution of this paper is the stability analysis of the proposed AFSMC system, in which the gain function of the plant's input channel can be an uncertain function of full state variables. Assumption 3.2 can also be found in [29,40]. Assumption 3.3 supposes that feedback gain in the nominal control is high enough, while Assumption 3.4 requires the learning gain in the fuzzy control to be sufficiently large. Although the theoretical result presented might be conservative, Theorem 3.1 is developed to qualitatively demonstrate the stability of the proposed AFSMC system. Note that when* *there is no uncertainty associated with the gain function; i.e., as $\hat{b} = b$, then Assumptions 3.2-3.4 are no longer required.*

4. Experimental Study.

4.1. Plant description. Consider the position control of the Hoekens straight-line linkage shown in Figure 5. This planar four-bar mechanism is comprised of the fixed link (link 1), the shorter pivoting link (link 2), the coupler or connecting arm (link 3), and the longer pivoting link (link 4). A permanent-magnet ac servomotor fixed to the frame drives link 2, which in turn steers link 3, while link 4 is pivoted on the middle of link 3 and then connected to the frame by a pin joint. Especially, the point P on link 3 will move in an approximately straight line. The four-bar-linkage system has the following specifications:

$$\ell_1 = 105.20 \text{ [mm]}, \quad \ell_2 = 47.83 \text{ [mm]}, \quad \ell_3 = 267.86 \text{ [mm]}, \quad \ell_4 = 133.93 \text{ [mm]},$$
$$M_2 = 58.44 \text{ [g]}, \quad M_3 = 251.19 \text{ [g]}, \quad M_4 = 180.74 \text{ [g]},$$
$$I_2 = 1.55 \times 10^{-2} \text{ [gm}^2\text{]}, \quad I_3 = 1.53 \text{ [gm}^2\text{]}, \quad I_4 = 2.92 \times 10^{-1} \text{ [gm}^2\text{]}$$

in which ℓ_i, M_i and I_i denote the length, the mass and the moment of inertia, respectively, of link i. Since link 1 is fixed, information on M_1 and I_1 is not required. The four-bar linkage under study can be described by

$$J_g(x)\ddot{x} + C_g(x)\dot{x}^2 = K_t K_{vc} u + \tau_d \tag{48}$$

where the angular position of the second link, x, is the output, u denotes the control input, τ_d represents the disturbance torque, K_t is the torque parameter of the motor, and K_{vc} is the current gain of the power amplifier. The coefficient terms, J_g and C_g, denote the generalized inertia and the centrifugal and Coriolis term, respectively. Note that J_g and C_g are periodic in x, and that the dynamics of this planar linkage are highly nonlinear.

Figure 6 shows the configuration of the servo system, in which the ac servomotor driven by a regulated current converter is Mitsubishi Electric model HC-KFS73. The shaft encoder mounted to the ac servomotor has a resolution of 8,000 lines, which yields

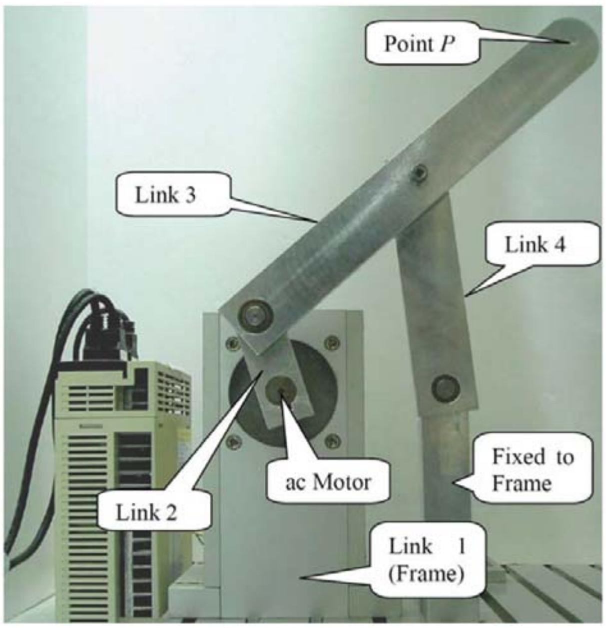

FIGURE 5. Photo of the straight-line mechanism

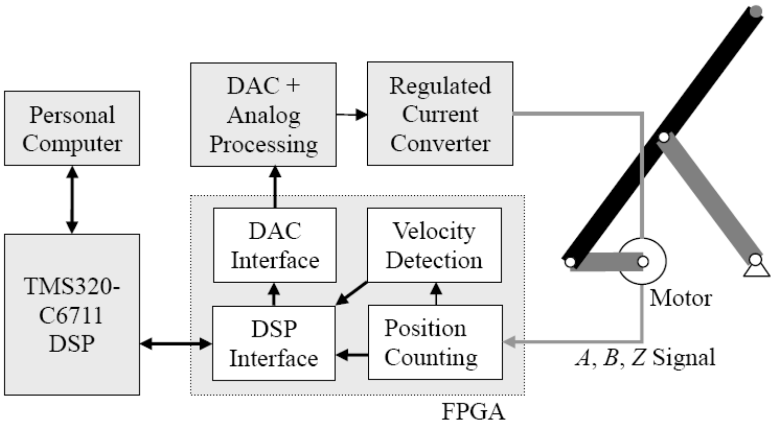

FIGURE 6. Hardware configuration of the experimental system

a resolution of 32,000 pulses/rev after the A and B signals from the encoder have been processed by the field-programmable gate array (FPGA), model XCV50PQ240-C6 from Xilinx, Inc. Designed with VHDL, both position counting and velocity detection are implemented in the FPGA. The controller core is a floating-point TMS320C6711 digital signal processor (DSP), which obtains position and velocity information from the FPGA, calculates the control algorithm, and sends the control effort to the regulated current converter through a 12-bit digital-to-analog converter (DAC). A sampling rate of 12.2 kHz was chosen, and the integration in the control algorithm is made discrete by the trapezoidal method.

4.2. **Plant model and controller design.** For the controller design, this nonlinear straight-line linkage system is modeled simply as a linear double integrator with a lumped term consisting of unmodeled nonlinear functions and unknown external disturbances:

$$\ddot{x} = \hat{b}\,(u + d) \tag{49}$$

where d denotes the lumped perturbation, and $\hat{b} = K_t K_{vc}/(I_m + G)$, where I_m is the inertia constant of the motor, and $G = M_2\ell_2^2/4 + I_2 + M_3\ell_2^2 = 6.24\times10^{-4}$ [kgm^2] represents a rough estimate of the linkage inertia. The values of the drive's parameters are found to be $K_t K_{vc} = 7.83$ [Nm/V] and $I_m = 1.51\times10^{-4}$ [kgm^2], giving the parameter $\hat{b} = 1.01\times10^4$.

The control task is to have this nonlinear system track a periodic reference trajectory, whose first cycle is described by the following equation in radians

$$\begin{aligned} r(t) = &-2\left[\left(\frac{10}{\tau^3}t^3 - \frac{15}{\tau^4}t^4 + \frac{6}{\tau^5}t^5\right)(H(t) - H(t-0.209))\right. \\ &\left. + \left(\frac{10}{\tau^3}(0.418-t)^3 - \frac{15}{\tau^4}(0.418-t)^4 + \frac{6}{\tau^5}(0.418-t)^5\right)H(t-0.209)\right] \end{aligned} \tag{50}$$

where $\tau = 0.209$. According to (4), the pole-placement design with both desired poles at -50 gives: $u_{pa} = \hat{b}^{-1}h$, where $h = \ddot{r} - c_0\dot{e} - \lambda s$, $s = \dot{e} + c_0 e$, $c_0 = 50$ and $\lambda = 50$. The auxiliary process is described by: $\dot{z} = h + \hat{b}\varphi\,\mathrm{sgn}\,(\sigma)$, with the switching gain $\varphi = 1.5$. For the SMDO, the parameter k_{do} is set to 150, and another parameter $k_{sv} = 12/\hat{b}$ is chosen. For the second-order plant (48), the input variables of the fuzzy model are: x, $\dot{x}$ and $(\hat{b}^{-1}h + u_{do})$. According to the operation range of the system, the universe of discourse for each linguistic variable is assigned as $[-0.1, 2.1]$, $[-25, 25]$ and $[-4, 4]$ for x, $\dot{x}$ and $(\hat{b}^{-1}h + u_{do})$, respectively, in which there are five linguistic values defined for each linguistic variable. Equally spaced triangular membership functions are used to describe these linguistic values. Thus, the total number of regular fuzzy rules in the rule base is $5 \times 5 \times 5 = 125$, with all consequent parameters initially set to zero. In the AFSMC, the learning gain is set to 50. Concerning computation issues, the computational requirement of a control algorithm is often a problem in the real-time implementation of a complex control system. To realize the proposed scheme, most of the computational burden is on the AFSMC since the controller needs to determine each rule's firing strength and tune its consequent parameter. In the experiments, there are three inputs to the fuzzy model. Due to the use of equally spaced triangular membership functions for each linguistic variable, there are only eight rules fired at any instant. That is, the DSP only needs to deal with eight rules in an interrupt interval, making the real-time implementation feasible.

4.3. **Experimental evaluation.** To examine the performance of the SMDO, the nominal controller with the SMDO (i.e., $u = u_{pa} + u_{do}$) is compared with the nominal controller without the SMDO (i.e., $u = u_{pa}$). Figure 7 shows the tracking performance by the nominal controller with and without the SMDO, demonstrating that the SMDO can improve tracking precision without introducing much chatter. However, since the SMDO is designed to on-line estimate an unknown perturbation that is obviously time-varying in this case, its dynamics with limited bandwidth cause inevitable estimation error, and there is still some perceptible tracking error with the compensation by the SMDO, as shown in Figure 7. Subsequently, the AFSMC, with all consequent parameters being initially zero, is incorporated. Figure 8 shows the tracking responses by the proposed controller (i.e., $u = u_{pa} + u_{do} + u_{fz}$) and the SMDO-augmented nominal controller (i.e., $u = u_{pa} + u_{do}$), demonstrating that the AFSMC adaptively and stably improves tracking precision without introducing much control chatter. It can also be observed that the proposed controller outperforms the SMDO-augmented nominal controller. Figure 9 shows the time evolution of the proposed control and its two components, u_{do} and u_{fz}. During the initial period, the AFSMC, whose consequent parameters are initially zero, has not been well trained and cannot effectively compensate for system perturbation, and the SMDO takes primary control. Subsequently, as the AFSMC continually adapts itself to compensate for system perturbation, the output of the SMDO diminishes. Because the AFSMC has been trained to compensate for most system perturbations, the SMDO becomes less indispensable in the steady state. That is, the AFSMC gradually replaces the SMDO and further improves the tracking performance. The AFSMC compensates for system perturbation better than the SMDO does in steady state because, whereas the SMDO estimates system perturbation in real time without the learning capability, the AFSMC learns and stores the information on system perturbation in the fuzzy model; this lookup-table-like compensation structure can rapidly and effectively counteract strongly time-varying perturbations. Figure 10 shows the dynamic responses in two experiments by the proposed control: the experiment with all consequent parameters being initially zero is considered as the first trial, while in the second trial the initial values of consequent parameters are obtained from the tuning result of the first trial. That is, the initial consequent parameters of the second trial are assigned by reloading the final values of consequent parameters of the first trial. It can be seen that, although both trials have similar steady-state performance, the previous learning experience of AFSMC in the first trial helps to improve the transient performance in the second trial. This reveals that useful information on system perturbation is continually updated and stored in the fuzzy model for efficient perturbation compensation.

We now define an artificially introduced input disturbance $H(t - 1.675)$ V. Figure 11 shows the dynamic responses with and without the additional disturbance by the proposed scheme. It can be seen that immediately after the instant of the unit-step jump in

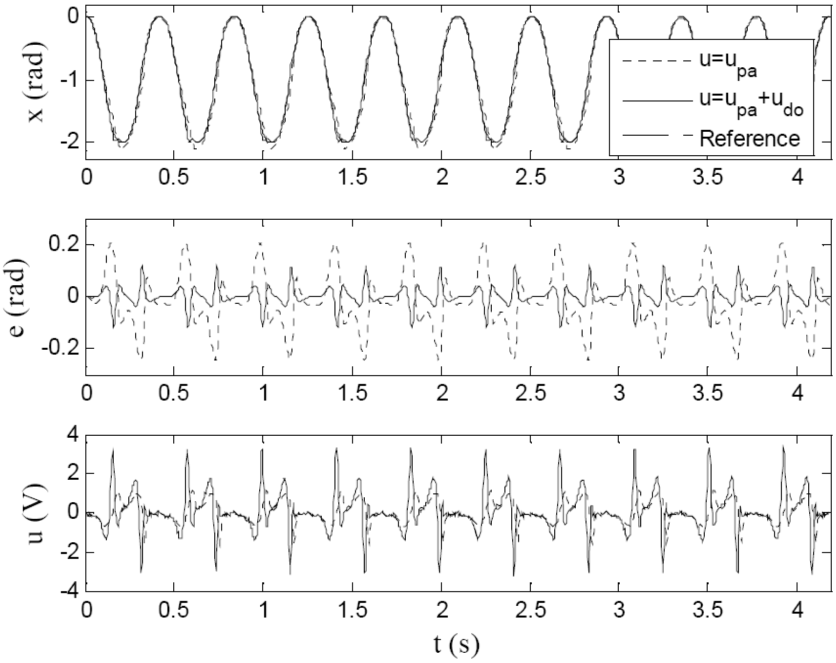

FIGURE 7. Tracking responses by the nominal control with and without the SMDO

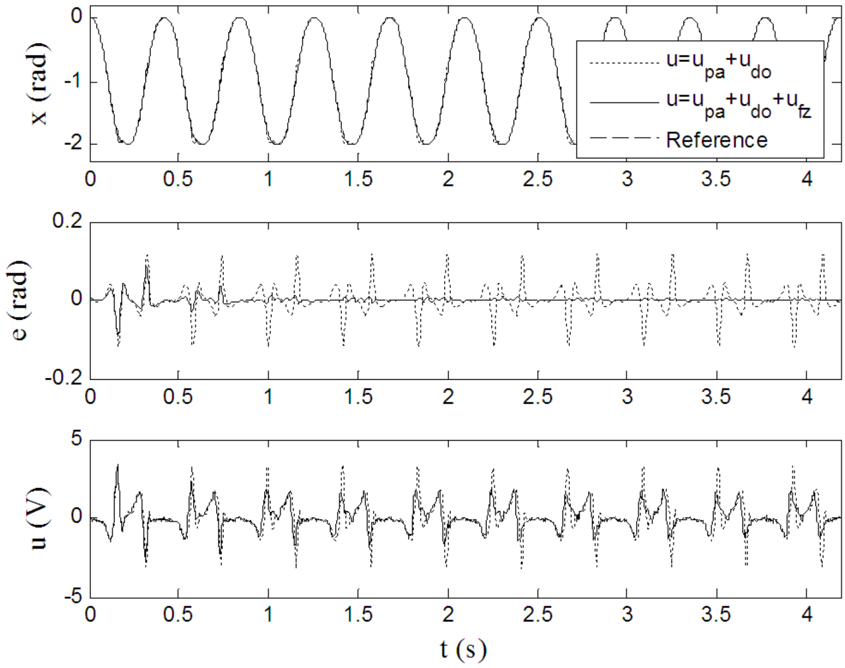

FIGURE 8. Tracking responses by the proposed controller and the SMDO-augmented nominal controller

the additional disturbance, the proposed scheme quickly develops a control against this additional disturbance, and the degradation in the tracking performance is hereafter little affected by this additional disturbance. Figure 12 shows the detailed time evolution of two components of the proposed control with and without the additional disturbance,

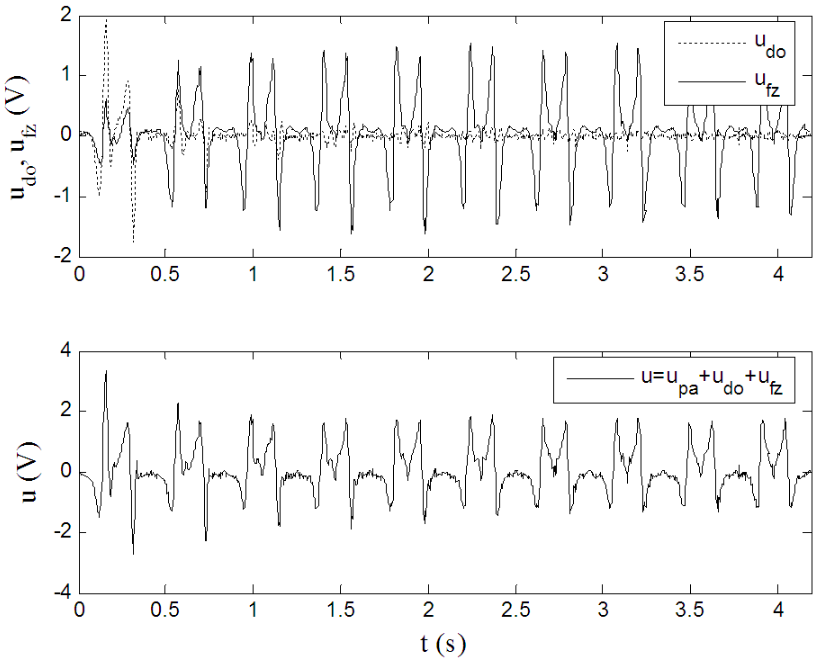

FIGURE 9. Time history of the proposed control and its components

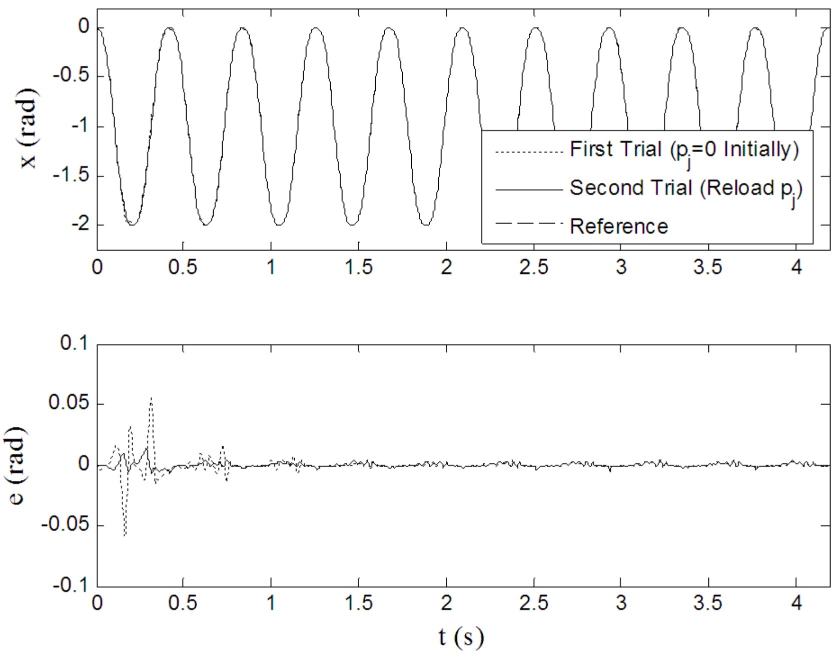

FIGURE 10. Dynamic responses by the proposed control in two consecutive trials

demonstrating that the SMDO generates a counter control immediately after the occurrence of the unit-step jump, alleviating the influence of this additional disturbance on the tracking performance. In addition, the AFSMC gradually models the disturbance and compensates for it, replacing the role of the SMDO and reducing the SMDO's output. Therefore, the purpose of introducing the SMDO is two-fold: one is to improve the

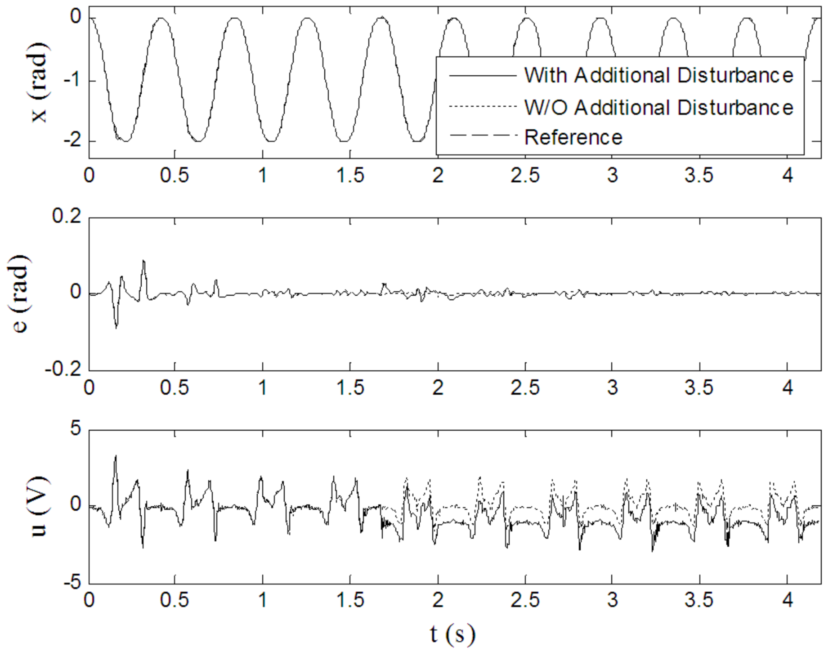

FIGURE 11. Responses by the proposed control with and without the additional disturbance

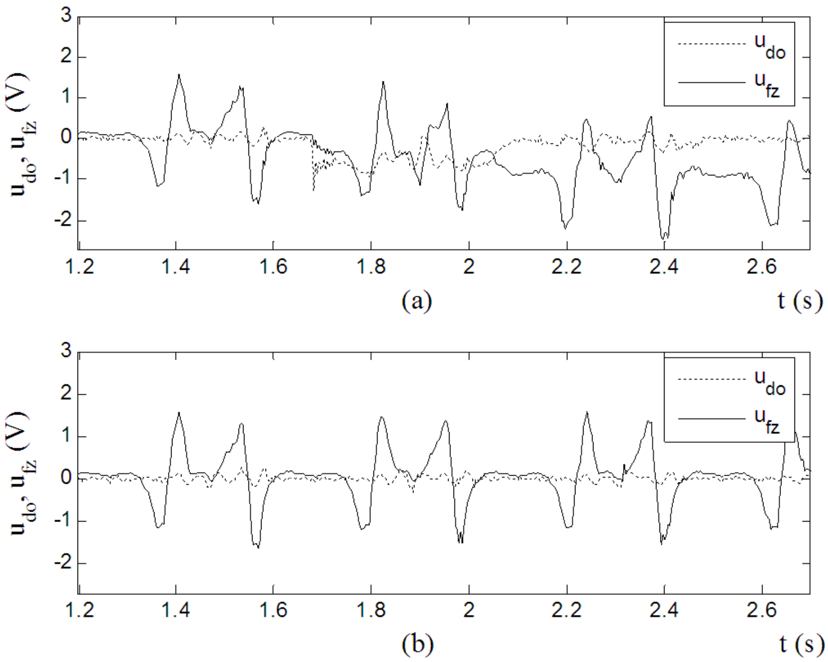

FIGURE 12. Time evolution of two components of the proposed control: (a) with the additional disturbance, (b) without the additional disturbance

transient performance of the adaptive system, and the other is to compensate for system perturbation that cannot be modeled by the AFSMC.

In contrast to the existing AFSMC learning laws that update consequent parameters entirely according to the filtered tracking error s, the proposed learning law (20) contains

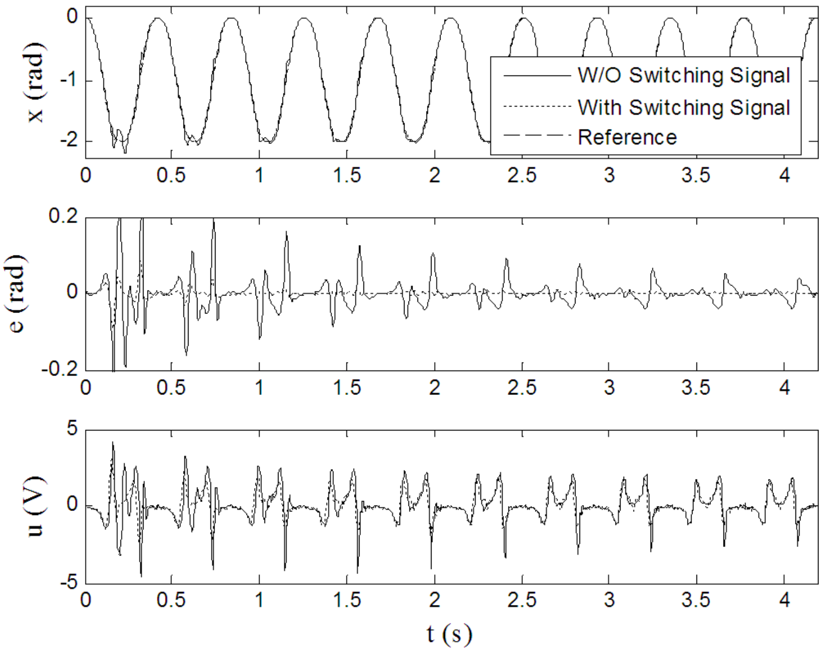

FIGURE 13. Comparisons of the performance with and without the switching signal

the switching signal $\varphi\text{sgn}(\sigma)$, which is equivalent to the perturbation compensation error by the SMDO-AFSMC. To investigate the effect of this switching signal on the learning performance, the proposed control law is simplified to approximate the conventional control law [19,21,34] by setting two design parameters: k_{do} and η in (15) and (20), to zero so that the switching signal has no influence on the control. The performance of the control law without the switching signal, where $k_{do} = \eta = 0$, is compared with that of the original one, as shown in Figure 13. It can be seen that the control without the switching signal leads to a slow convergence of the tracking error because this control learns the system perturbation information indirectly from the filtered tracking error. Conversely, the proposed control that learns from the switching signal equivalent to the compensation error of the SMDO-AFSMC speeds up the learning process.

The switching signal is available only when the sliding mode $\sigma = 0$ exists. However, the existence of the sliding mode depends on the appropriate choice of the switching gain, φ, whose value is required to be greater than the magnitude of the compensation error of the SMDO-AFSMC; that is, the inequality $\varphi > \left|b\hat{b}^{-1}u_{fz} + u_{do} + \hat{b}^{-1}\xi\right|$ should be fulfilled to ensure the existence of the sliding mode. However, the bound on the compensation error of the SMDO-AFSMC is sometimes difficult to determine. To eliminate this requirement of manually tuning the switching gain, the switching-gain adaptation law proposed in [39] is employed here. This law ensures that the sliding mode occurs in finite time; it is given by

$$\dot{\varphi} = \begin{cases} -\beta, & \text{if } \left|\hat{b}^{-1}\dot{\sigma} + \varphi\text{sgn}(\sigma)\right| + \mu < \varphi \\ \alpha\hat{b}\,|\sigma| + \beta, & \text{otherwise} \end{cases} \tag{51}$$

where μ, α and β are design constants set to 0.1, $100/\hat{b}$ and 1.5, respectively. Please refer to [39] for more details on the meaning of these design constants. Figure 14 presents the dynamic responses subject to the artificially-introduced input disturbance by the

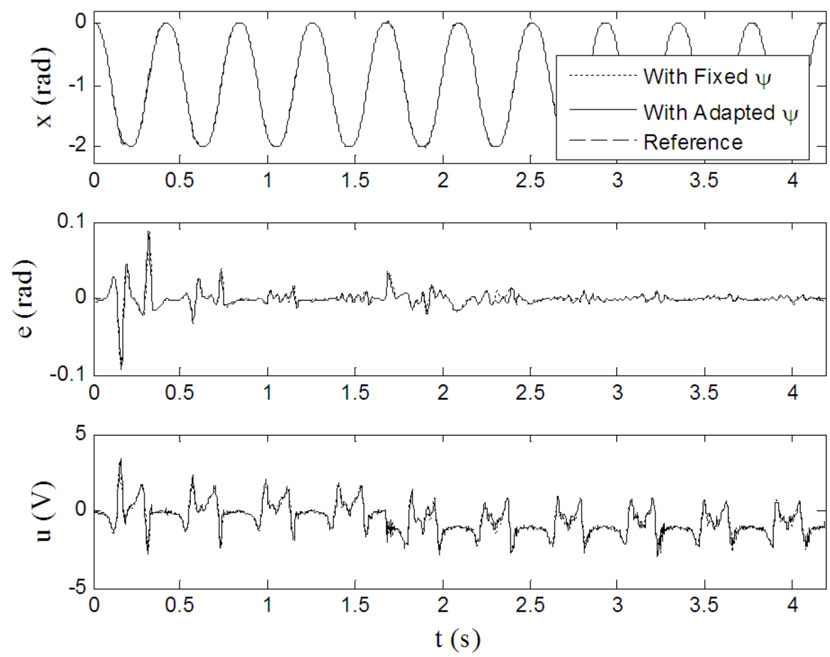

FIGURE 14. Dynamic responses with and without switching-gain adaptation

proposed scheme with and without the switching-gain adaptation. In the experiment, the switching gain to be adapted is initially set to zero. It can be seen from Figure 14 that while the tracking performance with the switching-gain adaptation is almost identical to that with the fixed switching gain, the designer is not required to determine the switching gain in advance. Figure 15 shows the time history of the adapted switching gain, in which the switching gain is automatically adjusted from zero to an average value, rapidly and stably. Since the disturbance varies with time, the adapted switching gain is also time-varying and is reduced whenever possible. The adaptation result confirms that the previously selected constant switching gain is not conservative. Moreover, just after the step jump of the additional input disturbance occurs, the switching gain increases, but a decrease in the switching gain follows. This decrease is due to the reduced compensation error of the SMDO-AFSMC, and the switching-gain adaptation mechanism can adapt the SMDO-AFSMC to different operating conditions without the need to manually readjust the switching gain.

5. **Conclusions.** This paper presents the design of an SMDO-AFSMC scheme for a class of nonlinear systems. The proposed control is composed of three components: the nominal control, the SMDO and the AFSMC. The nominal control specifies the desired closed-loop dynamics, while both the SMDO and the AFSMC counteract unknown system perturbation. The SMDO compensates for the unmodelable part of the system perturbation and improves transient performance of the learning system, while the AFSMC is adaptively adjusted to compensate for the modelable part of system perturbation. Since the SMDO, which has dynamics of limited bandwidth, yields estimation errors to time-varying perturbations, the AFSMC is ideal for compensating for the modelable part of the system perturbation, which can be strongly time-varying. The stability of the proposed SMDO-AFSMC system, in which the plant's input channel can have a gain function of full state variables, has been verified through Lyapunov analysis. In contrast to the existing AFSMC schemes, the proposed scheme both improves the transient performance by

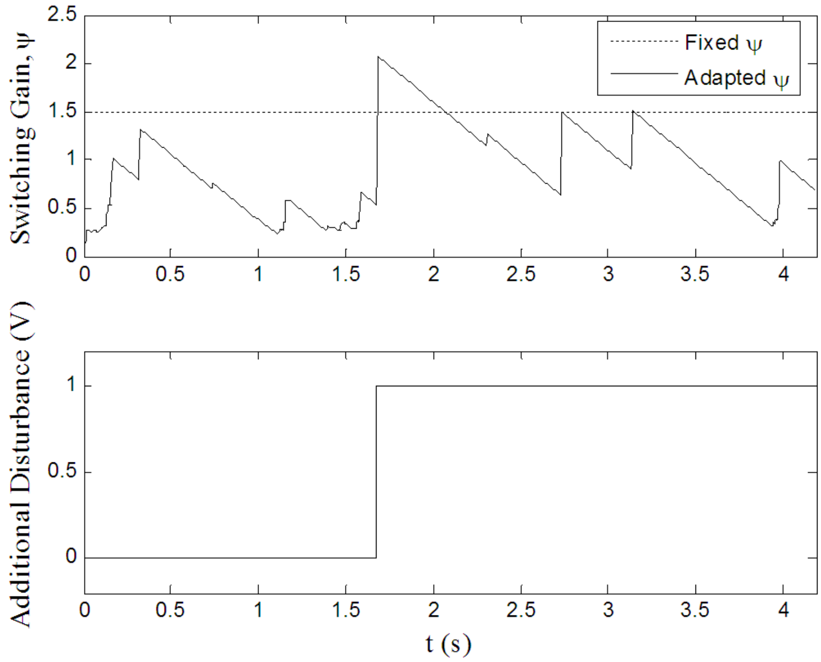

FIGURE 15. Time history of switching gains and the additional disturbance

introducing the SMDO, and speeds up the learning process by utilizing a switching signal that is equivalent to the compensation error of the SMDO-AFSMC. Simulation and experimental studies have been conducted on nonlinear four-bar linkage systems; the results confirm the effectiveness of the proposed scheme in improving the tracking performance of the system.

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REFERENCES

[1] V. I. Utkin, J. Guldner and J. Shi, *Sliding Mode Control in Electromechanical Systems*, Taylor & Francis, PA, 1999.
[2] V. I. Utkin, Sliding mode control design: Principles and applications to electric drives, *IEEE Trans. Industrial Electronics*, vol.40, no.1, pp.23-36, 1993.
[3] O. Kaynak, K. Erbatur and M. Ertugrul, The fusion of computationally intelligent methodologies and sliding mode control – A survey, *IEEE Trans. Industrial Electronics*, vol.48, no.1, pp.4-17, 2001.
[4] Q. P. Ha, D. C. Rye and H. F. D. Whyte, Fuzzy moving sliding mode control with application to robotic manipulators, *Automatica*, vol.35, no.4, pp.607-616, 1999.
[5] T. Orowska-Kowalska, M. Kaminski and K. Szabat, Implementation of a sliding-mode controller with an integral function and fuzzy gain value for the electrical drive with an elastic joint, *IEEE Trans. Industrial Electronics*, vol.57, no.4, pp.1309-1317, 2010.
[6] N. Yagiz and Y. Hacioglu, Robust control of a spatial robot using fuzzy sliding modes, *Mathematical and Computer Modelling*, vol.49, no.1-2, pp.114-127, 2009.
[7] F. Yorgancıoğlu and H. Kömürcügil, Single-input fuzzy-like moving sliding surface approach to the sliding mode control, *Electrical Engineering*, vol.90, no.3, pp.199-207, 2008.
[8] S. B. Choi and J. S. Kim, A fuzzy-sliding mode controller for robust tracking of robotic manipulators, *Mechatronics*, vol.7, no.2, pp.199-216, 1997.
[9] R. F. Fung, K. W. Chen and J. Y. Yeh, Fuzzy sliding mode controlled slider-crank mechanism using a PM synchronous servo motor drive, *Int. J. Mechanical Sci.*, vol.41, no.3, pp.337-355, 1999.

[10] M. M. Abdelhameed, Enhancement of sliding mode controller by fuzzy logic with application to robotic manipulators, *Mechatronics*, vol.15, no.4, pp.439-458, 2005.
[11] Z. Xi, G. Feng and T. Hesketh, Piecewise integral sliding-mode control for T-S fuzzy systems, *IEEE Trans. Fuzzy Systems*, vol.19, no.1, pp.65-74, 2011.
[12] H. H. Choi, Robust stabilization of uncertain fuzzy systems using variable structure system approach, *IEEE Trans. Fuzzy Systems*, vol.16, no.3, pp.715-724, 2008.
[13] Y. W. Liang, S.-D. Xu, D.-C. Liaw and C.-C. Chen, A study of T-S model-based SMC scheme with application to robot control, *IEEE Trans. Industrial Electronics*, vol.55, no.11, pp.3964-3971, 2008.
[14] R. F. Fung, C. C. Saw and A. P. Wang, Region-wise linear fuzzy sliding mode control of the motor-mechanism systems, *J. Sound Vibr.*, vol.234, no.3, pp.471-489, 2000.
[15] Z. H. Akpolat, G. M. Asher and J. C. Clare, A practical approach to the design of robust speed controllers for machine drives, *IEEE Trans. Industrial Electronics*, vol.47, no.2, pp.315-324, 2000.
[16] L. K. Wang, H. F. Leung and K. S. Tam, A fuzzy sliding controller for nonlinear system, *IEEE Trans. Industrial Electronics*, vol.48, no.1, pp.32-37, 2001.
[17] F. Barrero, A. Gonzalez, A. Torralba, E. Galvan and L. G. Franquelo, Speed control of induction motors using a novel fuzzy sliding-mode structure, *IEEE Trans. Fuzzy Systems*, vol.10, no.3, pp.375-383, 2002.
[18] Y. S. Lu and J. S. Chen, A self-organizing fuzzy sliding mode controller design for a class of nonlinear servo systems, *IEEE Trans. Industrial Electronics*, vol.41, no.5, pp.492-502, 1994.
[19] S.-J. Huang and W.-C. Lin, Adaptive fuzzy controller with sliding surface for vehicle suspension control, *IEEE Trans. Fuzzy Systems*, vol.11, no.4, pp.550-559, 2003.
[20] C. Li, S. Tong and Y. Wang, Fuzzy adaptive fault tolerant sliding mode control for SISO nonlinear systems, *International Journal of Innovative Computing, Information and Control*, vol.4, no.12, pp.3375-3383, 2008.
[21] S.-J. Huang and K.-S. Huang, An adaptive fuzzy sliding mode controller for servomechanism disturbance rejection, *IEEE Trans. Industrial Electronics*, vol.48, no.4, pp.845-852, 2001.
[22] K. Erbatur and O. Kaynak, Use of adaptive fuzzy systems in parameter tuning of sliding-mode controllers, *IEEE/ASME Trans. Mechatronics*, vol.6, no.4, pp.474-482, 2001.
[23] M.-K. Chang and T.-H. Yuan, Experimental implementations of adaptive self-organizing fuzzy sliding mode control to 3-DOF rehabilitation robot, *International Journal of Innovative Computing, Information and Control*, vol.5, no.10(B), pp.3391-3404, 2009.
[24] M.-K. Chang, An adaptive self-organizing fuzzy sliding mode controller for a 2-DOF rehabilitation robot actuated by pneumatic muscle actuators, *Control Engineering Practice*, vol.18, no.1, pp.13-22, 2010.
[25] R.-J. Lian and B.-F. Lin, New modifying self-organizing fuzzy controller for robotic motion control, *Int. J. Adv. Manuf. Technol.*, vol.29, no.9-10, pp.1074-1087, 2006.
[26] Y. Guo and P.-Y. Woo, An adaptive fuzzy sliding mode controller for robotic manipulator, *IEEE Trans. Systems, Man and Cybernetics, Part A*, vol.33, no.2, pp.149-159, 2003.
[27] C. Tao, J. S. Taut and M. Chan, Adaptive fuzzy terminal sliding mode controller for linear systems with mismatched time-varying uncertainties, *IEEE Trans. Systems, Man and Cybernetics, Part B*, vol.34, no.1, pp.255-262, 2004.
[28] R.-J. Wai, Fuzzy sliding-mode control using adaptive tuning technique, *IEEE Trans. Industrial Electronics*, vol.54, no.1, pp.586-594, 2007.
[29] C.-C. Kung, T.-H. Chen and L.-H. Kung, Modified adaptive fuzzy sliding mode controller for uncertain nonlinear systems, *IEICE Trans. Fundamentals*, vol.E88-A, no.5, pp.1328-1334, 2005.
[30] E. Sebastián and M. A. Sotelo, Adaptive fuzzy sliding mode controller for the kinematic variables of an underwater vehicle, *J. Intell. Robot. Syst.*, vol.49, no.2, pp.189-215, 2007.
[31] C. C. Kung and T.-H. Chen, H_∞ tracking-based adaptive fuzzy sliding mode controller design for nonlinear systems, *IET Control Theory & Applications*, vol.1, no.1, pp.82-89, 2007.
[32] N. Sadati and A. Talasaz, Variable structure control with adaptive fuzzy sliding surface, *J. Vibration and Control*, vol.12, no.11, pp.1251-1270, 2006.
[33] M. Ö. Efe, Fractional fuzzy adaptive sliding-mode control of a 2-DOF direct-drive robot arm, *IEEE Trans. Systems, Man, and Cybernetics, Part B*, vol.38, no.6, pp.1561-1570, 2008.
[34] S.-J. Huang and H.-Y. Chen, Adaptive sliding controller with self-tuning fuzzy compensation for vehicle suspension control, *Mechatronics*, vol.16, no.10, pp.607-622, 2006.
[35] H.-Y. Chen and J.-W. Liang, Adaptive sliding control with self-tuning fuzzy compensation for a piezoelectrically actuated X-Y table, *IET Control Theory & Applications*, vol.4, no.11, pp.2516-2526, 2010.

[36] Y. S. Lu and J. S. Chen, Design of a perturbation estimator using the theory of variable-structure systems and its application to magnetic suspension systems, *IEEE Trans. Industrial Electronics*, vol.42, no.3, pp.281-289, 1995.
[37] K. D. Young, V. I. Utkin and Ü. Özgüner, A control engineer's guide to sliding mode control, *IEEE Trans. Contr. Syst. Tech.*, vol.7, no.3, pp.328-342, 1999.
[38] Y. S. Lu and C. W. Chiu, A stability-guaranteed integral sliding disturbance observer for systems suffering from disturbances with bounded first time derivatives, *International Journal of Control, Automation, and Systems*, vol.9, no.2, pp.402-409, 2011.
[39] Y. S. Lu, Sliding-mode disturbance observer with switching-gain adaptation and its application to optical disk drives, *IEEE Trans. Industrial Electronics*, vol.54, no.9, pp.3743-3750, 2009.
[40] C. H. Wang, H. L. Liu and T. C. Lin, Direct adaptive fuzzy-neural control with state observer and supervisory controller for unknown nonlinear dynamical systems, *IEEE Trans. Fuzzy Syst.*, vol.10, no.1, pp.39-49, 2002.
[41] R. L. Norton, *Design of Machinery*, McGraw-Hill, New York, 1992.