International Journal of Innovative Computing, Information and Control
Volume **8**, Number **9**, September **2012**

ICIC International ©2012 ISSN 1349-4198
pp. 5999-6009

ADAPTIVE NONLINEAR CONTROL OF A BALL AND BEAM SYSTEM USING THE CENTRIFUGAL FORCE TERM

Min-Sung Koo[1], Ho-Lim Choi[2,*] and Jong-Tae Lim[1]

[1]Depatment of Electrical Engineering
Korea Advanced Institute of Science and Technology
373-1 Guseong-dong, Yuseong-gu, Daejeon 305-701, Korea
goose@kaist.ac.kr; jtlim@stcon.kaist.ac.kr

[2]Depatment of Electrical Engineering
Dong-A University
840, Hadan2-Dong, Saha-gu, Busan 604-714, Korea
*Corresponding author: hlchoi@dau.ac.kr

Received May 2011; revised September 2011

Abstract. *There have been several results on the nonlinear control of a ball and beam system. However, the existing methods often consider a simplified model, and particularly they neglect the centrifugal force term. In this paper, we propose a full-model based adaptive state feedback controller with dynamic gain in order to control the ball and beam system using the centrifugal force term. The dynamic gain calibrates the controller gain by monitoring the centrifugal force term in an on-line. We give a theoretical analysis of the proposed controller. We also undertake some experiments to show that the proposed controller which utilizes the centrifugal force term, improves the control performance compared with some of the existing methods.*

Keywords: Ball and beam system, Adaptive regulation, Centrifugal force term

1. **Introduction.** The ball and beam system is a well known nonlinear system and several researchers have investigated the problem of controlling the ball position of the system [2,8-11,17,18]. The relative degree of the ball and beam system is not well defined and thus the system is not fully input-output linearizable. To resolve this difficulty, [8,17] provide some methods for constructing the approximate input-output linearized system models. In [11], a state observer that utilizes a coordinate change that transforms the system into an approximated normal form is presented. For the Jacobian linearized ball and beam system model, a sliding mode controller is proposed in [9]. In [10], they consider the effects of parametric uncertainty on tracking performance. Even though the aforementioned methods demonstrate a certain degree of control performances with their own merits, they commonly neglect the particular high-order term in the system, i.e., the centrifugal force term. Thus, we can say they only consider simplified models in constructing their controllers.

A *non-simplified* ball and beam system model can be viewed in the class of perturbed feedforward systems, i.e., the major body of the system is a feedforward system and there is one or two non-feedforward terms. The stabilization or regulation of the feedforward systems has been thoroughly researched and there have been many results published related to either state or output feedback forms in very recent years [3,13,14,19,20]. However, in most of these results, the considered systems and control methods are naturally limited to a class of feedforward systems only. Thus, if the systems contain some additional 'non-feedforward' terms such as the centrifugal force term, the results of [3,14,19,20] are not applicable, and this is the case with the ball and beam system.

In this paper, we consider a full (*non-simplified*) ball and beam system model and design a state feedback controller with a dynamic gain for regulating the ball and beam system with the centrifugal force. The proposed dynamic gain involves appropriate powers of high-order nonlinearity and it is employed in the controller gain to deal with the effect of the centrifugal force term. Moreover, the dynamic gain has the adaptive feature such that the growth rate of the nonlinearity does not need to be known. Extending the theoretical background given in [4-6,12], we propose a control technique using a newly designed dynamic gain for regulating the ball and beam system. Compared with the dynamic gain developed in [13], our dynamic gain is continuous and differentiable. We give a theoretical analysis of the proposed controller and show that the control performance is indeed an improvement over the existing control methods. This improvement comes about by considering the effect of the centrifugal force term is clearly shown via the experimental results.

2. **Problem Statement and Preliminaries.**

We consider the ball and beam system shown in Figure 1. From [16], the modeling of the ball and beam system is given by

$$\begin{aligned} \ddot{r} &= \frac{m r_{arm} g R^2}{L_{beam}(mR^2 + J_b)} \sin\theta - \frac{m}{\frac{J_b}{R^2} + m} r\dot{\theta}^2 \\ \ddot{\theta} &= -\frac{1}{\tau}\dot{\theta} + \frac{K_1}{\tau} V_m \end{aligned} \tag{1}$$

where θ and r are the beam angle and the ball position, respectively. Also, K_1 is the steady-state gain, τ is the time-constant, L_{beam} is the length of the beam, m and J_b are the mass and moment of inertia of the ball, respectively. Moreover, R is the radius of the ball, g is the acceleration due to gravity, r_{arm} is the distance between screw and motor gear, and V_m is the input of the system.

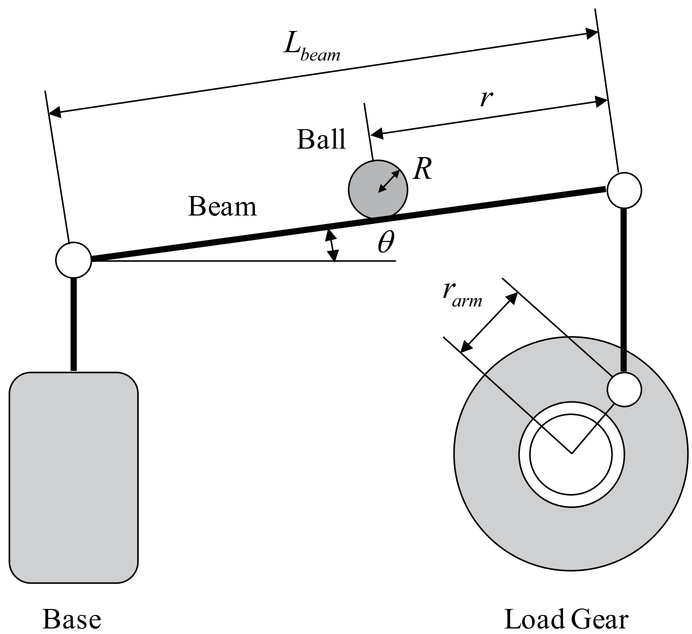

FIGURE 1. Ball and beam system

We define $K_{bb} = \frac{m r_{arm} g R^2}{L_{beam}(mR^2+J_b)}$ and $H = m/(J_b/R^2+m)$. Let $x = (x_1, x_2, x_3, x_4)^T = (r, \dot{r}, \theta, \dot{\theta})^T$. Then, we can obtain the following state space equations

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= K_{bb}\sin x_3 - Hx_1x_4^2 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -\frac{1}{\tau}x_4 + \frac{K_1}{\tau}V_m
\end{aligned} \tag{2}$$

where the centrifugal force term corresponds to $Hx_1x_4^2$.

Via a transformation $z = (z_1, z_2, z_3, z_4)^T = (x_1, x_2, K_{bb}\sin x_3, K_{bb}x_4\cos x_3)^T$ and the control input

$$V_m = \frac{\tau}{K_1K_{bb}\cos x_3}\left(\frac{1}{\tau}K_{bb}x_4\cos x_3 + K_{bb}x_4^2\sin x_3 + u\right) \tag{3}$$

with a new internal input u, the system (2) is transformed into

$$\dot{z} = Az + Bu + \delta(t, z, u) \tag{4}$$

The system matrices (A, B) are the Brunovsky canonical pair and the nonlinearity is $\delta(t, z, u) = [\delta_1(t, z, u), \delta_2(t, z, u), \delta_3(t, z, u), \delta_4(t, z, u)]^T$ where

$$\begin{aligned}
\delta_1(t, z, u) &= \delta_3(t, z, u) = \delta_4(t, z, u) = 0 \\
\delta_2(t, z, u) &= -\frac{Hz_1z_4^2}{K_{bb}^2\cos^2\left(\sin^{-1}\left(-\frac{z_3}{K_{bb}}\right)\right)}
\end{aligned} \tag{5}$$

The centrifugal force term is contained in $\delta_2(t, z, u)$.

Note that the physical operating ranges of the ball position r and beam angle θ are $r \in \{r \in \mathcal{R} : |r| \le L_{beam}/2\}$ and $\theta \in \{\theta \in \mathcal{R} : |\theta| \le \pi/4\}$. Then, by choosing a positive constant $\alpha < 1/3$ and using $|z_1| \le (L_{beam}/2)^{1-\alpha}|z_1|^\alpha$ and $\frac{1}{\cos^2(\sin^{-1}(-\frac{z_3}{K_{bb}}))} = \frac{1}{\cos^2 x_3} \le 2$, we obtain

$$|\delta_2(t, z, u)| \le \frac{H}{K_{bb}^2\cos^2\left(\sin^{-1}\left(-\frac{z_3}{K_{bb}}\right)\right)}|z_1||z_4|^2 \le L|z_1|^\alpha|z_4|^2 \tag{6}$$

where $L = 2(L_{beam}/2)^{1-\alpha}HK_{bb}^{-2}$.

Here, we note that the existing methods often neglect the centrifugal force term in their controller designs. For example, one well-known method outlined in [8] simply designs the controller based on the input-output linearized part of the system (4) ignoring $\delta_2(t, z, u)$. While such a method still shows the improved control results compared to ones using the Jacobian-linearization based methods, the absence of the centrifugal force term may degrade the control performance to some degree. Thus, for further improvement, we propose a new adaptive control method that accommodates the centrifugal force term in its dynamic gain.

3. **Design and Analysis of an Adaptive Controller with Dynamic Gain.**

To design a controller using the centrifugal force term, we introduce an adaptive controller with the dynamic gain as follows.

Controller:

$$u = K(\gamma(t))z \tag{7}$$

where $K(\gamma(t)) = [k_1/\gamma(t)^4, \cdots, k_n/\gamma(t)]$.

Dynamic gain:

$$\dot{\gamma}(t) = \gamma(t)^{c+3\alpha-1}\left(\left(\sum_{i=1}^{4}\gamma(t)^{-(4-i)}|z_i|\right)^{1+\alpha} + \left(\sum_{i=1}^{4}\gamma(t)^{-(n-i)}|z_i|\right)^{3+\alpha} + \left(\sum_{i=1}^{4}\gamma(t)^{-(4-i)}|z_i|\right)^{2}\right), \quad \gamma(0) = 1 \tag{8}$$

where the positive constant is $0 < c < 1 - 3\alpha$. For the existence of the positive constant c, the condition as $\alpha < 1/3$ is needed. The reason for the existence of c is explained in the proof of Theorem 3.1.

Here, we address some mathematical notations and setups. Define a matrix $E_{\gamma(t)} = \mathrm{diag}[\gamma(t)^{-3}, \gamma(t)^{-2}, \gamma(t)^{-1}, 1]$. Let $A_{K(\gamma(t))} = A + BK(\gamma(t))$. Then, we define $K = K(1)$ and $A_K = A_{K(1)}$. If it is given that A_K is Hurwitz, from [6], we can obtain the Lyapunov equation of $A^T_{K(\gamma(t))}P_{K(\gamma(t))} + P_{K(\gamma(t))}A_{K(\gamma(t))} = -\gamma(t)^{-1}E^2_{\gamma(t)}$ with $P_{K(\gamma(t))} = E_{\gamma(t)}P_K E_{\gamma(t)}$ from $A_K^T P_K + P_K A_K = -I$ where I denotes a 4×4 identity matrix.

Theorem 3.1. *Select K such that A_K is Hurwitz. Then, the controller V_m in (3) with the internal controller (7) and the dynamic gain (8) regulates the ball and beam system (2).*

Proof: We only need to show that by using the internal controller (7) and the dynamic gain (8), the system (4) is regulated. Then, the regulation of system (1) naturally follows. With controller (7), we obtain a the closed-loop system described by

$$\dot{z} = A_{K(\gamma(t))}z + \delta(t, z, u) \tag{9}$$

Since A_K is Hurwitz, there exists $P_K = P_K^T > 0$ such that $A_K^T P_K + P_K A_K = -I$ and $\pi_1 I \le P_K D + DP_K \le \pi_2 I$ where $D = \frac{1}{2}\mathrm{diag}[7, 5, 3, 1]$, $\pi_1, \pi_2 > 0$. With this, we set the Lyapunov function $V(z) = \gamma(t)^{-1}z^T P_{K(\gamma(t))}z$. Then, we have

$$\gamma(t)^{-1}\lambda_1\|E_{\gamma(t)}z\|^2 \le V(z) \le \gamma(t)^{-1}\lambda_2\|E_{\gamma(t)}z\|^2 \tag{10}$$

where $\lambda_1 = \lambda_{\min}(P_K)$ and $\lambda_2 = \lambda_{\max}(P_K)$. Then, along the trajectory of (9), we obtain

$$\begin{aligned}\dot{V}(z) &= \gamma(t)^{-1}\left(\dot{z}^T P_{K(\gamma(t))}z + z^T P_{K(\gamma(t))}\dot{z} + z^T \dot{P}_{K(\gamma(t))}z\right) \\ &\quad -\dot{\gamma}(t)\gamma(t)^{-2}z^T P_{K(\gamma(t))}z \\ &= -\gamma(t)^{-2}\|E_{\gamma(t)}z\|^2 + 2\gamma(t)^{-1}z^T P_{K(\gamma(t))}\delta(t, z, u) \\ &\quad -\dot{\gamma}(t)\gamma(t)^{-2}z^T E_{\gamma(t)}(P_K\bar{D} + \bar{D}P_K)E_{\gamma(t)}z \\ &\quad -\dot{\gamma}(t)\gamma(t)^{-2}z^T P_{K(\gamma(t))}z\end{aligned} \tag{11}$$

where $\bar{D} = \mathrm{diag}[3, 2, 1, 0]$.

Note that $u = \gamma(t)^{-1}KE_{\gamma(t)}z$ and $z^T E_{\gamma(t)}(P_K\bar{D} + \bar{D}P_K)E_{\gamma(t)}z + z^T P_{K(\gamma(t))}z = z^T E_{\gamma(t)}(P_K D + DP_K)E_{\gamma(t)}z$. Using the inequalities from (11), we have

$$\begin{aligned}\dot{V}(z) &\le -\gamma(t)^{-2}\|E_{\gamma(t)}z\|^2 + 2\gamma(t)^{-1}\|P_K\|\|E_{\gamma(t)}z\|\|E_{\gamma(t)}\delta(t, z, u)\|_1 \\ &\quad -\pi_1\dot{\gamma}(t)\gamma(t)^{-2}\|E_{\gamma(t)}z\|^2\end{aligned} \tag{12}$$

Regarding the term $\|E_{\gamma(t)}\delta(t, z, u)\|_1$ of (12), we have

$$\|E_{\gamma(t)}\delta(t, z, u)\|_1 \le L\gamma(t)^{-2}|z_1|^\alpha|z_4|\left(|z_4|\right) \le 2L\gamma(t)^{-2}|z_1|^\alpha|z_4|\|E_{\gamma(t)}z\| \tag{13}$$

The upper bound of the term $|z_1|^\alpha|z_4|$ is obtained as

$$|z_1|^\alpha|z_4| \le \gamma(t)^{3\alpha}\|E_{\gamma(t)}z\|^{1+\alpha} \tag{14}$$

From (12)-(14), we have

$$\dot{V}(z) \le -\gamma(t)^{-2}\|E_{\gamma(t)}z\|^2 + \sigma\gamma(t)^{3\alpha-3}\|E_{\gamma(t)}z\|^{1+\alpha}\|E_{\gamma(t)}z\|^2 - \pi_1\dot{\gamma}(t)\gamma(t)^{-2}\|E_{\gamma(t)}z\|^2 \tag{15}$$

where $\sigma = 4L\|P_K\|$.

From (8), it is clear that $\gamma(t)^{3\alpha+c-1}\|E_{\gamma(t)}z\|^{1+\alpha} \le \dot{\gamma}(t)$. Substituting (8) into (15), we obtain

$$\dot{V}(z) \le -\gamma(t)^{-2}\|E_{\gamma(t)}z\|^2 - \pi_1\gamma(t)^{-3+3\alpha}\left(\gamma(t)^c - \pi_1^{-1}\sigma\right)\|E_{\gamma(t)}z\|^{3+\alpha} \tag{16}$$

Looking at the term $\gamma(t)^c - \pi_1^{-1}\sigma$ in (16), we can see that the proposed controller is robust against an unknown growth rate of nonlinearity because the constant c is positive. The closed-loop system (9) has a unique solution $(z(t), \gamma(t))$ on $[0, T_f)$ for some $T_f \in (0, \infty]$. We first show that $\gamma(t)$ cannot escape at $t = T_f$. To prove this, suppose that $\lim_{t\to\infty} \gamma(t) = +\infty$. Since $\gamma(t)$ is monotonically nondecreasing and c is a positive constant, there exists a finite time $t^* \in (0, T_f)$, such that

$$\gamma(t) \ge (\pi_1^{-1}\sigma)^{1/c} \tag{17}$$

for $t \in [t^*, T_f)$. From (16) and (17), it follows that

$$\dot{V}(z) \le -\gamma(t)^{-2}\|E_{\gamma(t)}z\|^2 \tag{18}$$

From (10) and (18), we obtain, for $t \in [t^*, T_f)$

$$\|E_{\gamma(t)}z\| \le \sqrt{\frac{\lambda_2}{\lambda_1}}\|E_{\gamma(t^*)}z(t^*)\|e^{-\frac{1}{2\lambda_2}\int_{t^*}^{t}\gamma(s)^{-1}ds} \tag{19}$$

Note that, from (8),

$$\begin{aligned} &\gamma(t)^{\rho} - \gamma(t^*)^{\rho} \\ &= \rho\int_{t^*}^{t}\dot{\gamma}(s)\gamma(s)^{1-3\alpha-c}ds \\ &= \rho\int_{t^*}^{t}\left(\|E_{\gamma(s)}z(s)\|_1^{1+\alpha} + \|E_{\gamma(s)}z(s)\|_1^{3+\alpha} + \|E_{\gamma(s)}z(s)\|_1^{2}\right)ds \end{aligned} \tag{20}$$

where $\rho = 2 - 3\alpha - c > 1$. From (19), $\|E_{\gamma(t)}z\|_1 \le 2\|E_{\gamma(t)}z\| \le \sqrt{\frac{4\lambda_2}{\lambda_1}}\|E_{\gamma(t^*)}z(t^*)\|$. With this and (20), we have

$$\gamma(t) \le (\rho_1(t-t^*) + \gamma(t^*)^{\rho})^{\frac{1}{\rho}} \tag{21}$$

where $\rho_1 = \left(\frac{4\lambda_2}{\lambda_1}\right)^{(1+\alpha)/2}\|E_{\gamma(t^*)}z(t^*)\|^{1+\alpha} + \left(\frac{4\lambda_2}{\lambda_1}\right)^{(3+\alpha)/2}\|E_{\gamma(t^*)}z(t^*)\|^{3+\alpha} + \left(\frac{4\lambda_2}{\lambda_1}\right)\|E_{\gamma(t^*)} z(t^*)\|^2$. Then, from (21), we obtain

$$\int_{t^*}^{t}\gamma(s)^{-1}ds \ge \rho_2\left((\rho_1(t-t^*) + \gamma(t^*)^{\rho})^{1-\frac{1}{\rho}} - \gamma(t^*)^{\rho-1}\right) \tag{22}$$

where $\rho_2 = \rho_1^{-1}\left(1 - \frac{1}{\rho}\right)^{-1}$. Using (19) and (22), we obtain

$$\begin{aligned} &\int_{t^*}^{t}\left(\|E_{\gamma(s)}z(s)\|_1^{1+\alpha} + \|E_{\gamma(s)}z(s)\|_1^{3+\alpha} + \|E_{\gamma(s)}z(s)\|_1^{2}\right)ds \\ &\le \int_{t^*}^{t}\rho_1 e^{-\frac{\rho_3\rho_2}{2\lambda_2}\left((\rho_1(s-t^*)+\gamma(t^*))^{1-\frac{1}{\rho}} - \gamma(t^*)^{\rho-1}\right)}ds \end{aligned} \tag{23}$$

where $\rho_3 = 1+\alpha$. To help understand this, the boundedness of the right-hand side in (23) can be considered as the boundedness of $\int_0^t \eta e^{-\varepsilon s^\theta} ds$ where $\eta \geq 0$, $\varepsilon > 0$, and $0 < \theta < 1$ are constants.

Now, we investigate the boundedness of $\int_0^t \eta e^{-\varepsilon s^\theta} ds < +\infty$. Let $s^\theta = k$.

$$\int_0^t \eta e^{-\varepsilon s^\theta} ds \leq \int_0^{t^\theta} \frac{\eta}{\theta} k^{\frac{1-\theta}{\theta}} e^{-\varepsilon k} dk \tag{24}$$

Let ω be the minimum integer such that $\omega \geq \frac{1-\theta}{\theta}$. Note that $k^{\frac{1-\theta}{\theta}} \leq 1 + k^\omega$ for $k \geq 0$ and $\int k^\omega e^{-\varepsilon k} dk = e^{-\varepsilon k} \sum_{j=0}^{\omega} (-1)^j \frac{\omega! k^{\omega-j}}{(\omega-j)!(-\varepsilon)^{j+1}}$ from [21]. From these inequalities, we get

$$\begin{aligned}
&\int_0^{t^\theta} \frac{\eta}{\theta} k^{\frac{1-\theta}{\theta}} e^{-\varepsilon k} dk \\
&\leq \int_0^{t^\theta} \frac{\eta}{\theta}(1+k^\omega) e^{-\varepsilon k} dk = -\frac{\eta}{\varepsilon\theta} e^{-\varepsilon k}\Big|_{k=0}^{t^\theta} + \int_0^{t^\theta} \frac{m_1}{\theta} k^\omega e^{-\varepsilon k} dk \\
&= -\frac{\eta}{\varepsilon\theta} e^{-\varepsilon k}\Big|_{k=0}^{t^\theta} + \frac{\eta}{\theta} e^{-\varepsilon k} \sum_{j=0}^{\omega} (-1)^j \frac{\omega! k^{\omega-j}}{(\omega-j)!(-\varepsilon)^{j+1}}\Big|_{k=0}^{t^\theta}
\end{aligned} \tag{25}$$

Then, from (24) and (25) and $\varepsilon > 0$, we have

$$\int_0^t \eta e^{-\varepsilon s^\theta} ds < +\infty \tag{26}$$

From (20), (23), and (26), we have

$$\begin{aligned}
+\infty &= \gamma(T_f)^\rho - \gamma(t^*)^\rho \\
&\leq \rho \int_{t^*}^t \left(\|E_{\gamma(s)} z(s)\|_1^{1+\alpha} + \|E_{\gamma(s)} z(s)\|_1^{3+\alpha} + \|E_{\gamma(s)} z(s)\|_1^2 \right) ds \\
&\leq \rho\sqrt{n} \int_{t^*}^t \left(|E_{\gamma(s)} z(s)\|^{1+\alpha} + \|E_{\gamma(s)} z(s)\|^{3+\alpha} + \|E_{\gamma(s)} z(s)\|^2 \right) ds \\
&< +\infty
\end{aligned} \tag{27}$$

this leads to a contradiction. Thus, the dynamic gain $\gamma(t)$ is well defined and bounded on $[0, T_f)$.

Next, we claim that z is well defined and bounded on the interval $[0, T_f)$. From (16), we have

$$\begin{aligned}
&V(z) - V(z(0)) \\
&\leq -\int_0^t \gamma(T_f)^{-2} \|E_{\gamma(s)} z(s)\|^2 ds \\
&\quad - \int_0^t \pi_1 \gamma(T_f)^{-3+3\alpha} \left(\gamma(T_f)^c - \pi_1^{-1}\sigma \right) \|E_{\gamma(s)} z(s)\|^{3+\alpha} ds
\end{aligned} \tag{28}$$

The boundedness of $\gamma(t)$ and (20) implies that $\int_{t^*}^t \|E_{\gamma(s)} z(s)\|^{3+\alpha} < +\infty$ and $\int_{t^*}^t \|E_{\gamma(s)} z(s)\|^2 < +\infty$ on $[0, T_f)$. Using these inequalities and $\frac{\lambda_1}{\gamma(T_f)} \|E_{\gamma(t)} z\|^2 \leq V(z)$, from (28), we obtain $\|E_{\gamma(t)} z\|^2 < +\infty$ on $[0, T_f)$. This, with the boundedness of $\gamma(t)$, implies that z is well defined and bounded on the interval $[0, T_f)$.

In summary, we have shown that $\gamma(t)$ and $\|E_{\gamma(t)} z\|$ are well defined and bounded on the maximally extended interval $[0, T_f)$. From the boundedness of $\gamma(t)$ and $\|E_{\gamma(t)} z\|^2$ on

$[0, T_f)$ together with (13) and (14), we obtain, for $t \in [0, T_f)$,

$$\begin{aligned} \left\| \frac{d(E_{\gamma(t)} z)}{dt} \right\| \leq & \ \gamma(t)^{-1} \|A_K\| \|E_{\gamma(t)} z\| + \|E_{\gamma(t)} \delta(t, z, u)\|_1 \\ & + \dot{\gamma}(t)\gamma(t)^{-1} \|\bar{D}\| \|E_{\gamma(t)} z\| < +\infty \end{aligned} \tag{29}$$

Letting $T_f \to +\infty$, we get $\gamma(t) < +\infty$, $\|E_{\gamma(t)} z\| < +\infty$, $\int_0^t \|E_{\gamma(s)} z(s)\|^2 ds < +\infty$, and $\left\| \frac{d(E_{\gamma(t)} z)}{dt} \right\| < +\infty$ on $[0, +\infty)$. This yields $z \to 0$ as $t \to +\infty$ by Lemma 7 [7] and the boundedness of $\gamma(t)$. Therefore, the regulation of system (4) is achieved. Trivially, the regulation of the system (1) follows by V_m in (3) with (7) and (8).

Remark 3.1. *In implementing the proposed controller for our experiment, there may be a case where a small measurement error can drive $\gamma(t) \to \infty$ as $t \to \infty$, which results in $K(\gamma(t)) \to 0$ as $t \to \infty$. Let $e(t) = r(t) - x_1(t)$ where $r(t)$ is the reference signal. The following modified dynamic gain may provide robustness against the measurement noise issue outlined above.*

$$\begin{aligned} \dot{\gamma}(t) &= \beta(\gamma(t), z) q(\|e(t)\| - \epsilon) \\ q(\|e(t)\| - \epsilon) &= \begin{cases} 1, & \text{if } \|e(t)\| - \epsilon > 0 \\ 0, & \text{if } \|e(t)\| - \epsilon \leq 0 \end{cases} \end{aligned} \tag{30}$$

where

$$\begin{aligned} \beta(\gamma(t), z) = \gamma(t)^{c+3\alpha-1} \Bigg(& \left(\sum_{i=1}^n \gamma(t)^{-(n-i)} |z_i| \right)^{1+\alpha} + \left(\sum_{i=1}^n \gamma(t)^{-(n-i)} |z_i| \right)^{3+\alpha} \\ & + \left(\sum_{i=1}^n \gamma(t)^{-(n-i)} |z_i| \right)^2 \Bigg). \end{aligned}$$

Here, $\epsilon > 0$ is a pre-specified value that sets the tolerance for measurement errors. Under this modified dynamic gain, once the system is regulated, any measurement error smaller than ϵ will not cause further switching. The proposed controller, in the main part, consists of a pole-placement part and a dynamic gain part. The pole-placement part is in the typical form of a linear controller and is very easy to implement. The dynamic gain part involves only one integrator. As a whole, our proposed controller is simple in structure and as such does not require any heavy computations. As shown in the next section, our controller performs well in real-time experiments.

Remark 3.2. *We consider a cart-pole system with small length with strong gravity effects* [15]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 + \kappa \frac{x_3 x_4^2}{(1 + x_3^2)^{\frac{3}{2}}} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= u \end{aligned} \tag{31}$$

As discussed in [15]*, this is an underactuated system. With $\delta_2(t, x, u) = \kappa \frac{x_3 x_4^2}{(1+x_3^2)^{\frac{3}{2}}}$, we have $|\delta_2(t, x, u)| \leq \kappa |x_4|^2$. Compared with (6), we see that α can be set as 0. Thus, with the proposed controller, the dynamic gains with appropriate powers of the nonlinearity can be implemented, similarly. Our control scheme is widely applicable to various practical systems.*

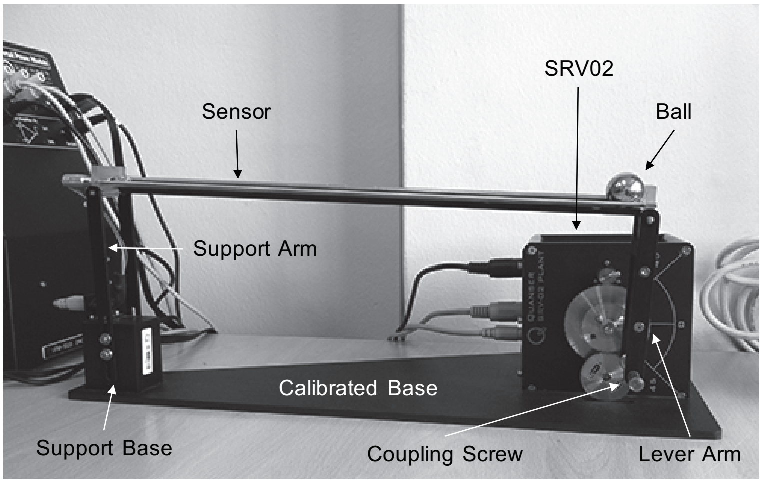

FIGURE 2. Experimental setup

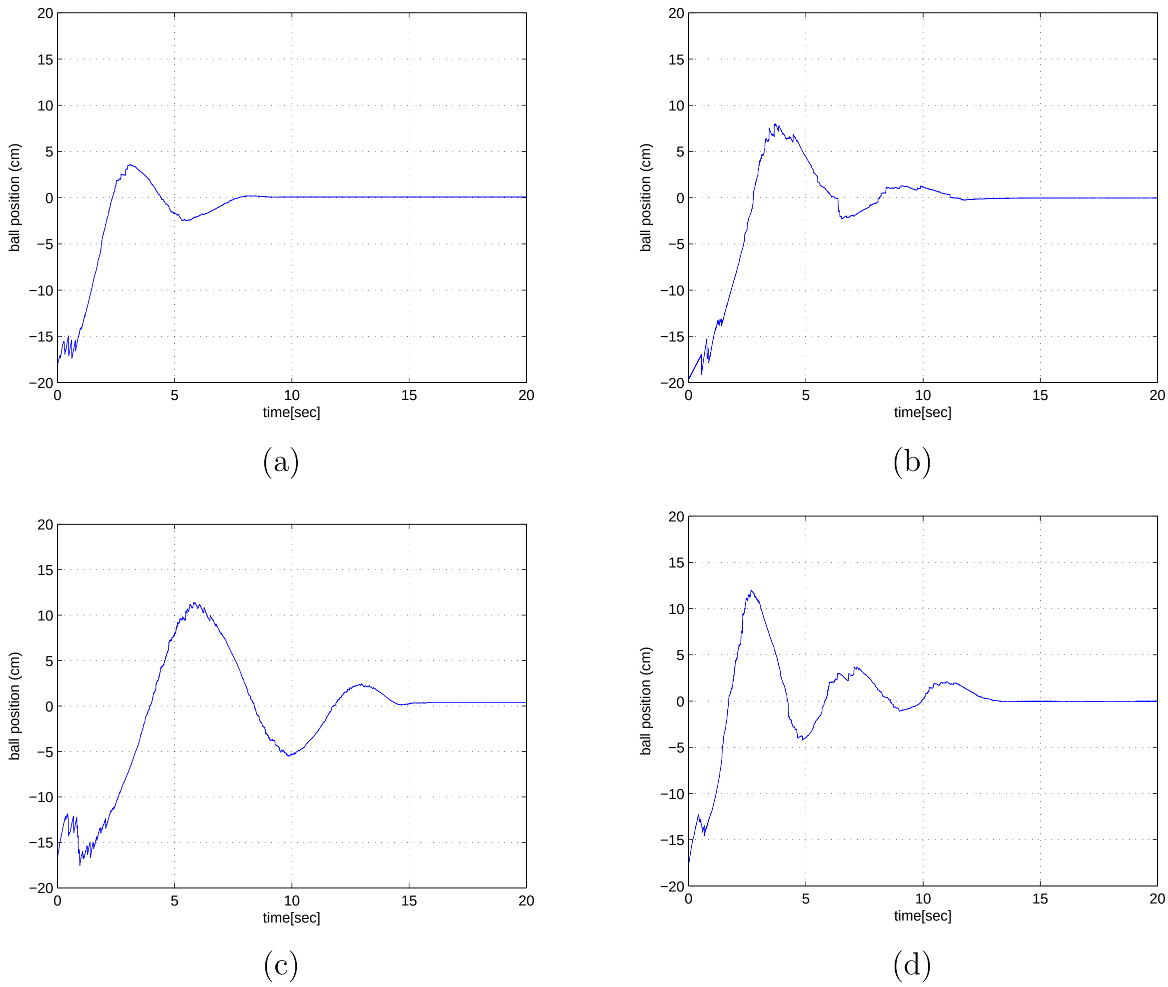

FIGURE 3. Ball position trajectories: (a) proposed controller, (b) controller from [8], (c) controller from [1], and (d) PID controller

4. Experiment Results.

4.1. Experiment setup. The experiment setup is shown in Figure 2. Our proposed controller and other existing controllers are engaged on the Quanser ball and beam system in which the beam is actuated with a DC servomotor. A P4 2.40GHz computer with Quanser Q4-PCI-DAQ is used to process feedback signals and derive the control input for the system. There is also a power op-amp module between the DAQ and DC servo providing input signals for the motor. The power module used is the Quanser UPM1503 with ±10V and 3A output. The data acquisition board used is a Q8 controlPaQ-FW and the rotary servo plant is SRV02. This model is equipped with a Vishary Spectrol model 132 potentiometer and tachometer. The potentiometer is a single turn 10kΩ sensor with no physical stops and has an electrical range of 352 degrees. The tachometer prevents any latencies in the timing of the response and ensures that the speed of the motor is accurately measured. The mechanical system parameters used were $L_{beam} = 42.55$cm, $r_{arm} = 2.54$cm, $R = 1.27$cm, $m = 0.064$kg, $g = 9.81$m/s^2, $J_b = 4.1290 \times 10^{-6}$kg·m^2, $K_1 = 1.76$rad/sv, and $\tau = 0.0285$s.

4.2. Experiment results. Figures 3 and 4 show the ball position and input trajectories generated by the proposed controller and various existing controllers. While the existing

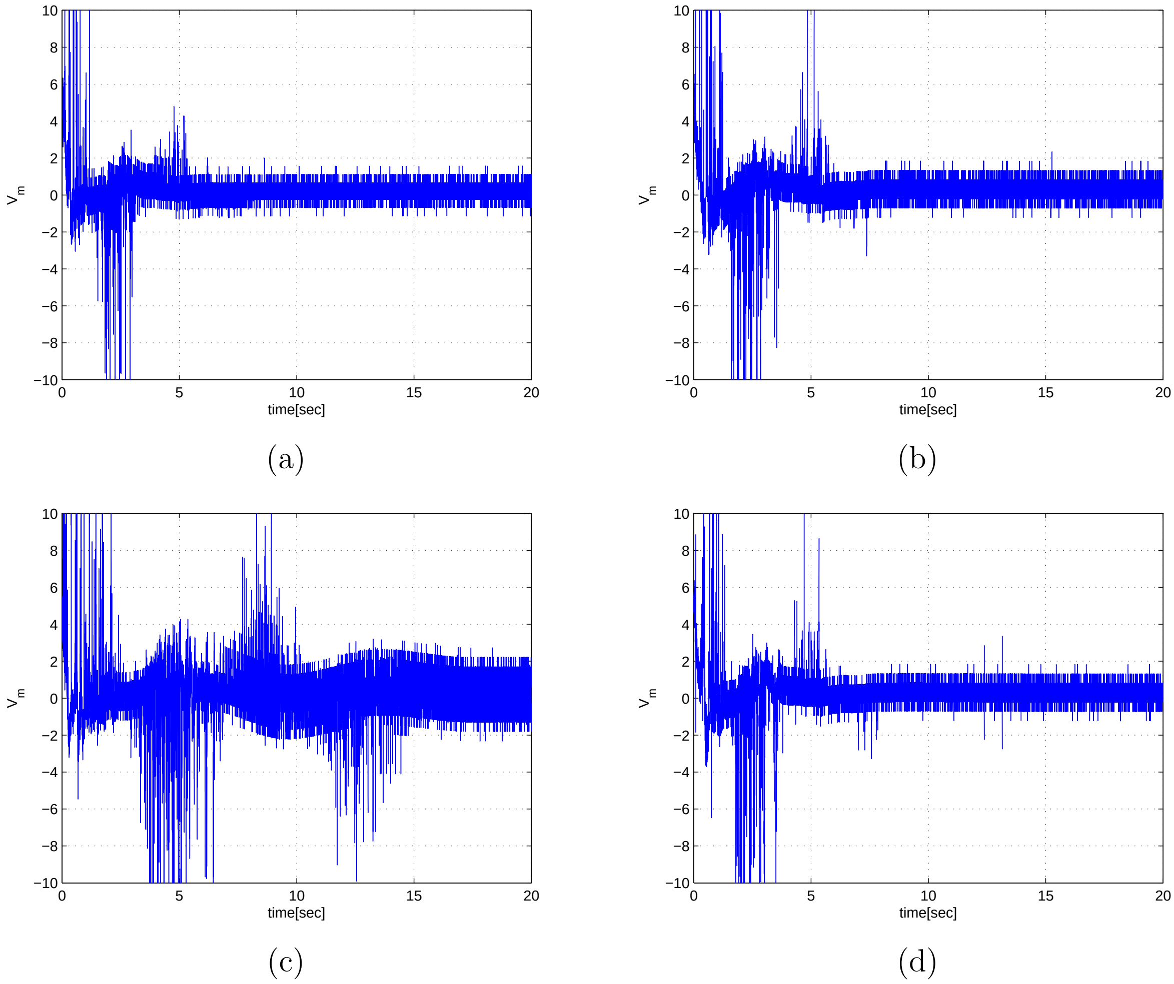

FIGURE 4. Input trajectories: (a) proposed controller, (b) controller from [8], (c) controller from [1], and (d) PID controller

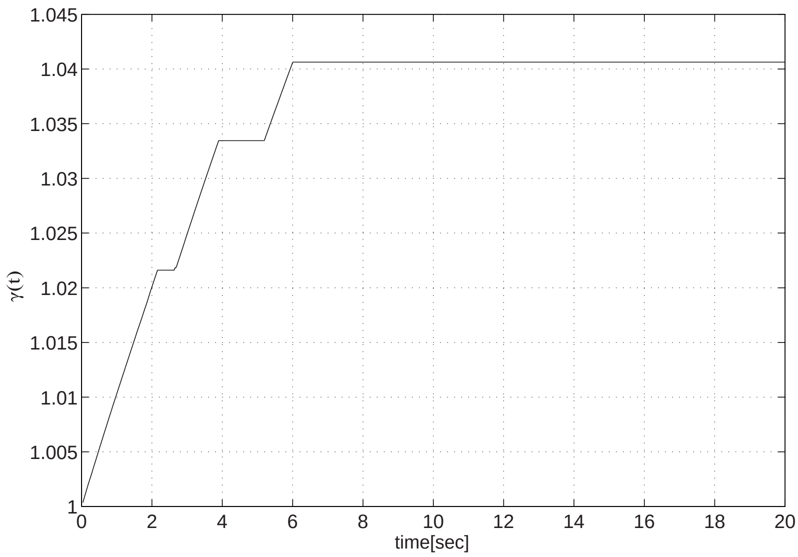

FIGURE 5. Evolution of $\gamma(t)$

controllers from [1,8] and PID controller are designed without considering the centrifugal force term, our proposed controller is designed by considering the powers of high-order nonlinearity in the centrifugal force term and adaptively regulates the ball and beam system without the knowledge of the growth rate in the nonlinearity. Note that there is noticeable high overshoot and slow convergence in the ball position response in Figures 3(c) and 3(d) by the controller [1] and the PID controller. In Figure 3(b), the controller based on approximate input-output linearization in [8] exhibits better results than those of [1] and the PID controller. As explained in [8], the controller based on approximate input-output linearization generates better performance compared to the controller designed based on Jacobian linearization. Now, with the additional dynamic gain compensating for the centrifugal force term, our controller further improves on the control performance of [8] as shown in Figures 3(a) and 3(b). For a fair comparison, we use the same eigenvalues of -5 for both our controller and the one from [8]. The evolution of the implemented dynamic gain is shown in Figure 5.

5. **Conclusions.** We presented a state feedback controller that considers the centrifugal force term in a ball and beam system. The proposed controller is based on a fully descriptive ball and beam system model and adaptive dynamic gain that takes account of the centrifugal force term is employed. We carried out the system analysis with the proposed controller and illustrated its improved control performance compared to previous controllers via experiment. The experimental results show that our proposed controller provides the better ball position control than the several existing methods. One deficiency of our control scheme is that only a fractional power of the centrifugal force term is utilized ($|z_1||z_4|^2 \to |z_1|^\alpha |z_4|^2$, $\alpha < 1/3$). In future work, we will further generalize our current results so that the controller can utilize the full power of nonlinearity.

Acknowledgement. This work was supported by Brain Korea 21 Project, BK Electronics and Communications Technology Division, KAIST in 2011.

REFERENCES

[1] N. B. Almutairi and M. Zribi, On the sliding mode control of a ball on a beam system, *Nonlinear Dynamics*, vol.59, no.1-2, pp.221-238, 2010.

[2] T. Aoustin and A. M. Formal'skii, Beam-and-ball system under limited control: Stabilization with large basin of attraction, *Proc. of American Control Conference*, Hyatt Regency Riverfront, St. Louis, Mo, USA, 2009.

[3] T. Chen and J. Huang, Global robust output regulation by state feedback for strict feedforward systems, *IEEE Trans. Automat. Contr.*, vol.54, no.9, pp.2157-2163, 2009.

[4] H.-L. Choi and J.-T. Lim, Stabilization of a class of nonlinear systems by adaptive output feedback, *Automatica*, vol.41, no.6, pp.1091-1097, 2005.

[5] H.-L. Choi and J.-T. Lim, Stabilisation of non-linear systems with unknown growth rate by adaptive output feedback, *Int. J. of Systems Science*, vol.41, no.6, pp.673-678, 2010.

[6] H.-L. Choi and J.-T. Lim, Asymptotic stabilization of an input-delayed chain of integrators with nonlinearity, *Syst. Control Lett.*, vol.59, no.6, pp.374-379, 2010.

[7] F. A. C. C. Fontes, A general framework to design stabilizing nonlinear model predictive controllers, *Syst. Control Lett.*, vol.42, pp.127-143, 2001.

[8] J. Hauser, S. Sastry and P. Kokotovic, Nonlinear control via approximate input-output linearization: The ball and beam example, *IEEE Trans. on Automat. Contr.*, vol.37, no.3, pp.392-398, 1992.

[9] R. M. Hirschorn, Incremental sliding mode control of the ball and beam, *IEEE Trans. on Automat. Contr.*, vol.47, no.10, pp.1696-1700, 2002.

[10] J. Huang and C. F. Lin, Robust nonlinear control of the ball and beam system, *Proc. of American Control Conference*, Seattle, Washington, 1995.

[11] N.-H. Jo and J.-H. Seo, A state observer for nonlinear systems and its application to ball and beam system, *IEEE Trans. on Automat. Contr.*, vol.45, no.5, pp.968-973, 2000.

[12] M.-S. Koo, H.-L. Choi and J.-T. Lim, Global regulation of a class of uncertain nonlinear systems by switching adaptive controller, *IEEE Trans. Automat. Contr.* vol.55, no.12, pp.2822-2827, 2010.

[13] P. Krishnamurthy and F. Khorrami, Generalized state scaling and applications to feedback, feedforward, and nontriangular nonlinear systems, *IEEE Trans. Automat. Contr.*, vol.52, no.1, pp.102-108, 2007.

[14] M. Krstic, Feedback linearizability and explicit integrator forwarding controllers for classes of feedforward systems, *IEEE Trans. Automat. Contr.*, vol.49, no.10, pp.1668-11682, 2002.

[15] R. Olfari-Saber, *Nonlinear Control of Underactuated Mechanical Systems with Application to Robotics and Aerospace Vehicles*, Ph.D. Thesis, MIT, 2001.

[16] Quanser, *Ball and Beam User Manuals*, 2008.

[17] A. R. Teel, Semi-global stabilization of the ball and beam using output feeback, *Proc. of American Control Conference*, San Francisco, CA, 1983.

[18] T. Turker, H. Gorgun, E. Zergeroglu and G. Canserver, Exact model knowledge and direct adaptive controllers on ball and beam, *Proc. of International Conference on Mechatronics*, Kumamoto, Japan, 2007.

[19] X. Ye, Adaptive output feedback control of nonlinear systems with unknown nonlinearities, *Automatica*, vol.41, no.8, pp.1367-1374, 2005.

[20] X. Ye and H. Unbehauen, Global adaptive stabilization for a class of feedforward nonlinear systems, *IEEE Trans. on Automat. Contr.*, vol.49, no.5, pp.786-792, 2004.

[21] D. Zwillinger, *Standard Mathematical Tables and Formulae*, 31st Edition, A CRC Press, 2003.

[22] M. Mahmoud and P. Shi, *Methodologies for Control of Jump Time-delay Systems*, Kluwer Academic Publishers, Boston, 2003.

[23] P. Shi, Limited Hamilton-Jacobi-Isaacs equations for singularly perturbed zero-sum dynamic (discrete time) games, *SIAM J. Control and Optimization*, vol.41, no.3, pp.826-850, 2002.

[24] S. K. Nguang and P. Shi, Fuzzy H-infinity output feedback control of nonlinear systems under sampled measurements, *Automatica*, vol.39, no.12, pp.2169-2174, 2003.

[25] E. K. Boukas, Z. Liu and P. Shi, Delay-dependent stability and output feedback stabilization of Markov jump systems with time-delay, *IEE-Part D, Control Theory and Applications*, vol.149, no.5, pp.379-386, 2002.

[26] P. Shi, E. K. Boukas and R. K. Agarwal, H_∞ control of discrete-time linear uncertain systems with delayed-state, *Proc. of the 37th IEEE Conf. on Decision & Control*, Tampa, FL, pp.4551-4552, 1998.