International Journal of Innovative
Computing, Information and Control
Volume 8, Number 9, September 2012

ICIC International ©2012 ISSN 1349-4198
pp. 6317–6327

ADAPTIVE NONLINEAR CONTROL OF UNCERTAIN CHAOTIC GYROS SUBJECTED TO UNKNOWN PARAMETERS AND DISTURBANCES

Hamid Reza Koofigar[1] and Shahab Amelian[2]

[1]Department of Electrical Engineering
University of Isfahan
81746-73441, Isfahan, Iran
koofigar@eng.ui.ac.ir

[2]Department of Mechanical Engineering
Naein Branch, Islamic Azad University
Naein, Iran
amelian@naeiniau.ac.ir

Received June 2011; revised October 2011

Abstract. *This paper addresses the chaos control of uncertain chaotic nonlinear gyros with unknown parameters. In general, the underlying class of gyros may be perturbed by external disturbances with different sources. Depending on the characteristics of uncertain parameters and disturbances, two different adaptive-based control algorithms are proposed. In the first method, an adaptive H_∞ controller is designed when the external disturbances are square-integrable. The other proposed scheme is based on developing a pure adaptive controller, in which the parameters and disturbances can be time-varying with unknown bounds. In both methods, the prescribed robustness property is guaranteed and the stability analysis is presented, using the Lyapunov stability theorem. Simulation studies show that the goal of chaos control is achieved despite the system uncertainties and external disturbances.*

Keywords: Adaptive control, Chaotic gyros, Robustness, External disturbance, Time-varying parameters

1. **Introduction.** During the past years, gyroscopes have been studied by many researchers and applied in various scientific and engineering areas such as navigation, astronautics and optics [1]. The investigations can be classified into three main areas including gyro dynamics and nonlinear phenomena [2, 3, 4], chaos control [1, 5] and synchronization [6, 7]. From a historical viewpoint, researches in the field of dynamic behaviors of gyroscopes were first initiated about one hundred years ago [2], based upon which, the nonlinear dynamics of symmetric gyroscopes with linear damping coefficients were studied [4, 8]. Later, the assumption of a linear plus cubic form for dissipative force (damping term) was identified as a more realistic case and considered in many works [2, 7]. Investigations into field of gyro dynamics show that the system may exhibit regular or chaotic motions in different situations [6]. The chaotic behavior of gyros, first introduced in [9], caused to pay a considerable attention to chaos control and synchronization.

Dealing with the chaos phenomenon in chaotic nonlinear systems, a wide variety of approaches have been proposed such as adaptive-based methods [10, 11], active control [6, 12], sliding mode control [1, 13] and fuzzy logic [7, 14, 15]. Nevertheless, the existing methods suffer from at least one of the following drawbacks: (i) the perturbations are supposed to have slow variations with known bounds; (ii) the problem of tuning various numbers of adaptation mechanisms and large control efforts are not considered as the limitations of implementation; (iii) system uncertainties and external disturbances are not taken into account altogether; (iv) the analysis and the given proofs have some flaws. Hence, developing a chaos control algorithm with simplicity and versatility properties is highly desired. Among the reported algorithms, adaptive-based control techniques are powerful tools, especially when the variations of unknown parameters are slow enough [16, 17]. In fact, conventional adaptive algorithms, including adaptive control laws together with some parameter adjusting mechanisms may fail when the variations are fast. Meanwhile, these control methods do not guarantee the robustness properties against unstructured uncertainties and external disturbances, which inevitably affect the system performance. Hence, to tackle the both parametric uncertainties and external disturbances, the combination of tools from robust and adaptive approaches may yield better performance than those produced by each method alone.

This paper concerns with robust adaptive control for a general class of uncertain chaotic nonlinear gyros. To this end, two adaptive-based control algorithms are proposed, satisfying robustness with respect to system uncertainties and disturbances. Compared with some previous works, the specific properties of the proposed methods are (i) the unknown parameters and disturbances are required neither to be constant (slow time-varying) nor to have known bounds; (ii) the convergence of tracking error is analytically shown despite the perturbations; (iii) the number of adaptation mechanisms is reduced to minimum to provide simple implementation; (iv) the both universality and simplicity properties are met for implementation.

The organization of the paper is as follows. In Section 2, introducing the gyro dynamics, the control problem is formulated. Section 3 presents the proposed robust adaptive control techniques and their stability proofs. In Section 4, simulation studies are given to evaluate the performance of the developed control schemes, taking the system uncertainties and disturbances into account. Finally, the concluding remarks are given in Section 5.

Throughout the paper, for an $n \times 1$ vector V, $\|V\|_Q^2 := V^T Q V$ denotes the Euclidean vector norm with weighting matrix Q. Furthermore, $V \in L_2[0,T]$ if $\int_0^T \|V(t)\|^2 dt < \infty$, $T \in [0, \infty)$.

2. **Problem Statement.** Consider the equation governing the motion of a symmetric gyro, mounted on a vibrating base, as [2]

$$\ddot{\theta} + g(\theta) + h(\theta, \dot{\theta}) = f \sin wt \sin \theta \tag{1}$$

where the nutation angle θ is the angle which the spin axis of the gyro makes with the vertical axis, $f\sin wt$ is a parametric excitation that models the base excitation, g denotes a nonlinear resilience force described by

$$g(\theta) = \alpha^2 \frac{(1-\cos\theta)^2}{\sin^3\theta} - \beta \sin\theta \tag{2}$$

and h is a damping function, consisting of linear and nonlinear damping terms, given by

$$h(\theta, \dot{\theta}) = c_1 \dot{\theta} + c_2 \dot{\theta}^3 \tag{3}$$

In order to analyze the chaotic behavior of nonlinear gyros and also design adaptive chaos controllers, a state space parametric model is derived here. To this end, define $X = [x_1, x_2]^T = \left[\theta, \dot{\theta}\right]^T$ as the state vector, and

$$\eta = \left[\alpha^2 \ \beta \ c_1 \ c_2\right]^T$$

as the system parameter vector. Therefore, the nonlinear dynamics (1) can be written in a controlled form as

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= \phi^T(X)\eta + f \sin wt \sin x_1 + d(t) + u(t) \end{aligned} \tag{4}$$

where

$$\phi(X) = \left[-\frac{(1-\cos x_1)^2}{\sin^3 x_1}, \sin x_1, -x_2, -x_2^3 \right]^T$$

denotes the regression matrix, $u(t)$ is the control input, and $d(t)$ represents the external disturbance with unknown bound, i.e., $\|d(t)\| \leq D$, where $D > 0$ is unknown.

Researches on nonlinear behavior of gyros show that for the numerical values of $w = 2$, $f = 35.5$, and parameter vector $\eta = [100, 1, 0.5, 0.05]$, the symmetric gyro exhibits chaotic behavior [2]. The chaotic attractor, exhibited in the phase plane, and the irregular motion of the states for two different initial conditions $[x_1(0), x_2(0)] = [1, -1]$, and $[x_1(0), x_2(0)] = [-0.5, -1]$ are respectively illustrated in Figures 1 and 2.

The control objective is to design the control input $u(t)$ such that the states of gyro track the prescribed reference trajectories despite the system uncertainties and external disturbances. For a given desired state trajectory x_d, the tracking error vector is formed by $E = X - X_d = [e_1, e_2]^T$ with $X_d = [x_d, \dot{x_d}]^T$, and the error dynamic can be obtained as

$$\dot{E} = \Lambda E + B\left[-g(X) - h(X) + f \sin wt \sin x_1 + d(t) + u(t) - \ddot{x}_d\right] \tag{5}$$

where

$$\Lambda = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Choosing a gain vector as $K = [k_1, k_2]^T$ such that $A = \Lambda - BK^T$ is Hurwitz, the error dynamic (5) is rewritten as

$$\dot{E} = AE + B\left[K^T E + \phi^T(X)\eta + f \sin wt \sin x_1 + d(t) + u(t) - \ddot{x}_d\right] \tag{6}$$

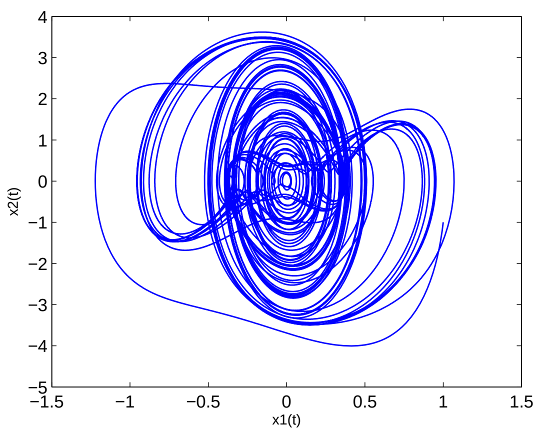

FIGURE 1. The phase plane trajectory x_1–x_2 of chaotic gyro

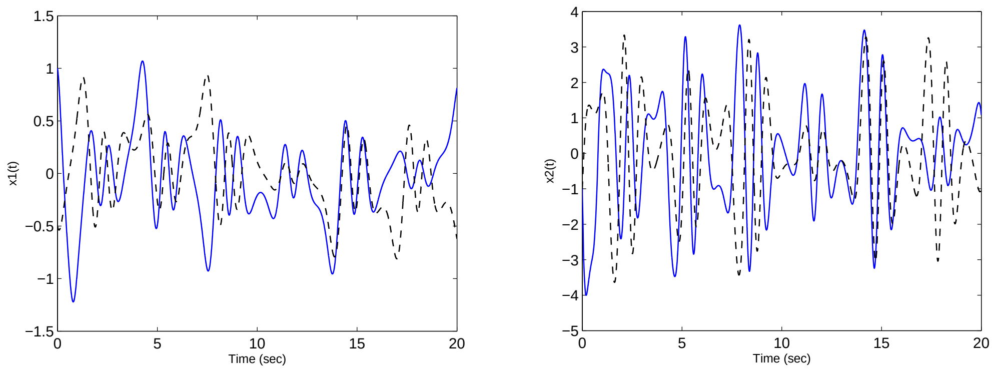

FIGURE 2. Demonstrating the irregular motion of system states for initial conditions $[x_1(0), x_2(0)] = [1, -1]$ (—), and $[x_1(0), x_2(0)] = [-0.5, -1]$ (- -)

3. **Robust Adaptive Control Algorithms for Uncertain Chaotic Gyros.** In this section, the design procedure of chaos control algorithms for uncertain chaotic gyros is focused. Depending on the characteristics of disturbance vector d and parameter vector η in (4), different robust adaptive techniques are developed based on a control input of the form

$$u = u_0 + u_a + u_r \tag{7}$$

where

$$u_0 = -K^T E - f \sin wt \sin x_1 + \ddot{x}_d$$

is called the nominal term, and u_a and u_r denote respectively the adaptive and robust terms. In fact, the unknown system parameters are tackled by the adaptive subcontroller u_a, and the system robustness with respect to disturbances is ensured by the robust subcontroller u_r.

3.1. ***H_∞*-based adaptive control.** Robust H_∞ control technique is incorporated into control input (7) to ensure the robustness property with respect to external disturbances. The disturbance signal $d(t)$ in (4) has unknown bound and is supposed to belong to $L_2[0, \infty]$. In the following, the H_∞-based adaptive controllers are developed for chaotic system (4) by two theorems, to deal with constant and time-varying parameters, respectively.

Theorem 3.1. *Consider the uncertain chaotic nonlinear gyros described by (4). Suppose there exists a positive definite symmetric matrix P, satisfying the Riccati-like inequality*

$$A^T P + PA + Q + PB\left(\frac{1}{\rho^2} I - \frac{1}{r} I\right) B^T P \leq 0 \tag{8}$$

where $\rho > 0$ is a prescribed attenuation gain, $Q > 0$ is a prescribed weighting matrix, and $r > 0$ is the H_∞ controller gain. The proposed control law (7) with

$$u_r = -\frac{1}{2r} B^T P E \tag{9}$$

$$u_a = -\phi^T(X)\hat{\eta} \tag{10}$$

and adaptation law

$$\dot{\hat{\eta}} = \Gamma \phi(X) B^T P E \tag{11}$$

where $\hat{\eta}$ denotes the estimate of η, and $\Gamma = \Gamma^T > 0$ is the adaptation gain matrix, ensures the convergence of tracking error, despite the system uncertainties and external disturbances.

Proof: Choose a Lyapunov function as

$$V(E, E_\eta) = \frac{1}{2}E^T P E + \frac{1}{2}E_\eta^T \Gamma^{-1} E_\eta \tag{12}$$

where $E_\eta = \eta - \hat{\eta} = \left[\alpha^2 - \hat{\alpha},\ \beta - \hat{\beta},\ c_1 - \hat{c}_1,\ c_2 - \hat{c}_2\right]^T$ denotes the estimation error vector. The time derivative of $V(E, E_\eta)$ along (6) is

$$\begin{aligned} \dot{V}(E, E_\eta) = & \frac{1}{2}E^T \left(A^T P + PA\right) E + \left(K^T E + f \sin wt \sin x_1 - \ddot{x}_d\right) B^T P E \\ & + \phi^T(X)\eta B^T P E + E^T P B d(t) + E^T P B u(t) + E_\eta^T \Gamma^{-1} \dot{E}_\eta \end{aligned} \tag{13}$$

Applying the control input (7) yields

$$\dot{V} = \frac{1}{2}E^T \left(A^T P + PA\right) E + \left(\phi^T(X)\eta + u_a\right) B^T P E + E^T P B \left(d + u_r\right) - E_\eta^T \Gamma^{-1} \dot{\hat{\eta}} \tag{14}$$

Using the inequality (8) and substituting subcontrollers u_r and u_a, respectively from (9) and (10), imply that

$$\begin{aligned} \dot{V} \le & -\frac{1}{2}E^T Q E + \phi^T(X) E_\eta B^T P E \\ & -\frac{1}{2}\left(\frac{1}{\rho}B^T P E - \rho d\right)^T \left(\frac{1}{\rho}B^T P E - \rho d\right) + \frac{1}{2}\rho^2 \|d\|^2 - E_\eta^T \Gamma^{-1} \dot{\hat{\eta}} \end{aligned} \tag{15}$$

Hence, by replacing the adaptation law (11), one can obtain

$$\dot{V}(E, E_\eta) \le -\frac{1}{2}E^T Q E + \frac{1}{2}\rho^2 \|d\|^2 \tag{16}$$

Now, integrating (16) from $t = 0$ to $t = T$ yields

$$\int_0^T \|E(t)\|_Q^2 dt \le 2V(E(0), E_\eta(0)) - 2V\left(E(T), E_\eta(T)\right) + \rho^2 \int_0^T \|d(t)\|^2 dt$$

for all $0 \le T < \infty$. This implies that $E(t)$ is square-integrable.

On the other hand, the boundedness of disturbance signal d implies that there exists a $D > 0$ such that $\|d\| \le D$. By inequality (16), one can obtain $\dot{V} \le -\frac{1}{2}\lambda_Q \|E\|^2 + \frac{1}{2}\rho^2 D^2$, where λ_Q is the minimum eigenvalue of Q. Choosing $\lambda_Q > \frac{\rho^2 D^2}{\zeta^2}$ for any small $\zeta > 0$, there exists a $\kappa > 0$ such that $\dot{V} \le -\kappa \|E\|^2 < 0$ for all $\|E\| > \zeta$. Thus, there is a $T > 0$ such that $\|E\| \le \zeta$ for all $t \ge T$. This implies that the tracking error $E(t)$ is uniformly ultimately bounded [17], and all the closed-loop signals are bounded. The error dynamics (6) and the boundedness of all variables ensure that $\dot{E}(t)$ is also bounded. Hence, by Barbalat's lemma [17], the convergence of tracking error and the goal of chaos control are ensured. □

Remark 3.1. *There exists a trade-off between the value of sub-controller gain ρ and the magnitude of control input u. In other words, choosing a smaller $\rho > 0$ provides the system with faster time response at the expense of larger control effort.*

Remark 3.2. *Unlike some previous works* [1, 7, 14]*, the upper bound of perturbations is not required in controller design.*

Remark 3.3. *In general, the inequality (8) can be easily satisfied by various selections of r and P. As a special case, choosing $r = \rho^2$, one can obtain the Lyapunov equation $A^T P + PA = -Q$, known in nonlinear control literature.*

In practice, the system parameter vector η in (4), may be time-varying with unknown bound, as stated in the following property.

Property 3.1. *Parameter vector $\eta(t)$ may be a time-varying vector belonging to a compact set $\Omega = \{\eta(t) : \|\eta(t)\| \leq \delta\}$, in which $\delta > 0$ is an unknown constant parameter.*

Hence, a robust adaptive control scheme is developed for this case, in the sequel.

Theorem 3.2. *Consider the uncertain chaotic gyros described by (4) with Property 3.1. Suppose $P = P^T > 0$ exists, satisfying the inequality (8). The goal of chaos control is achieved by control input (7), with robust subcontroller (9) and adaptive law*

$$u_a = -\hat{\delta}^2 \frac{\phi^T(X)\phi(X)B^T PE}{\|\phi(X)B^T PE\|\hat{\delta} + \epsilon e^{-\sigma t}} \tag{17}$$

in which $\hat{\delta}$, the estimate of δ, is updated by the adaptation mechanism

$$\dot{\hat{\delta}} = \gamma\|\phi(X)B^T PE\|, \quad \gamma > 0 \tag{18}$$

where ϵ and σ are (small) positive constants and $\gamma > 0$ denotes the adaptation gain.

Proof: Choose the Lyapunov function candidate

$$V(E, \tilde{\delta}) = \frac{1}{2}E^T PE + \frac{1}{2\gamma}\tilde{\delta}^2 \tag{19}$$

where $\tilde{\delta} = \delta - \hat{\delta}$ represents the estimation error. Differentiating (19) along the error trajectory (6) and replacing the control law (7) with (17) yields

$$\begin{aligned} \dot{V} = & \frac{1}{2}E^T\left(A^T P + PA\right)E + \eta^T(t)\phi(X)B^T PE - \hat{\delta}^2 \frac{E^T PB\phi^T(X)\phi(X)B^T PE}{\|\phi(X)B^T PE\|\hat{\delta} + \epsilon e^{-\sigma t}} \\ & + E^T PB\left(d + u_r\right) + \frac{1}{\gamma}\tilde{\delta}\dot{\tilde{\delta}} \end{aligned} \tag{20}$$

Substituting u_r from (9) and using inequality (8) imply that

$$\begin{aligned} \dot{V} \leq & -\frac{1}{2}E^T QE + \delta\|\phi(X)B^T PE\| - \hat{\delta}\|\phi(X)B^T PE\| + \epsilon e^{-\sigma t} \\ & -\frac{1}{2}\left(\frac{1}{\rho}B^T PE - \rho d\right)^T\left(\frac{1}{\rho}B^T PE - \rho d\right) + \frac{1}{2}\rho^2\|d\|^2 - \frac{1}{\gamma}\tilde{\delta}\dot{\hat{\delta}} \end{aligned} \tag{21}$$

By adaptation law (18), one can obtain

$$\dot{V}(E, \tilde{\delta}) \leq -\frac{1}{2}E^T QE + \frac{1}{2}\rho^2\|d\|^2 + \epsilon e^{-\sigma t} \tag{22}$$

integrating the inequality (16) from $t = 0$ to $t = T$ implies that $E(t)$ is square-integrable. Moreover, by inequality (22), it can be concluded that $\dot{V} \leq -\frac{1}{2}\lambda_Q\|E\|^2 + \frac{1}{2}\rho^2 D^2 + \epsilon$, where λ_Q is the minimum eigenvalue of Q. Following a procedure, similar to the proof of Theorem 3.1, implies that the convergence of tracking error is achieved, despite the perturbations. □

Remark 3.4. *From a practical point of view, the exponential term in subcontroller u_a, formed by $\epsilon > 0$ and $\sigma > 0$, provides the smoothness of control law without violating the convergence property of tracking error.*

3.2. **Adaptive rejection of time-varying perturbations.** Theorems 3.1 and 3.2 show that the H_∞-based technique is restricted to those disturbances which belong to $L_2[0, \infty)$. This condition may be not satisfied for many disturbance signals. This fact motivates relaxing this assumption and developing a less conservative chaos control scheme to tackle such perturbations. To this end, define the augmented regressor vector $\phi_a(X)$ and the augmented time-varying vector $\eta_a(t)$ as

$$\begin{aligned} \phi_a(X) &= \left[\phi^T(X), 1\right]^T, \\ \eta_a(t) &= \left[\eta^T(t), d(t)\right]^T \end{aligned} \tag{23}$$

where $\eta_a(t)$ satisfies Property 3.1, i.e., $\|\eta_a(t)\| \leq \delta_a$, and δ_a is an unknown positive constant. Hence, the dynamical Equation (4) can be rewritten as

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= \phi_a^T(X)\eta_a(t) + f \sin wt \sin x_1 + u(t) \end{aligned} \tag{24}$$

Theorem 3.3. *The goal of chaos control for uncertain chaotic gyros (4) or equivalently (24) is ensured by control law (7) with $u_r = 0$, adaptive subcontroller*

$$u_a = \hat{\delta}_a^2 \frac{\phi_a(X)\phi_a^T(X)B^T PE}{\|\phi_a(X)B^T PE\|\hat{\delta}_a + \epsilon e^{-\sigma t}} \tag{25}$$

and adaptation law

$$\dot{\hat{\delta}}_a = \gamma_a \|\phi_a^T(X)B^T PE\| \tag{26}$$

where $\hat{\delta}_a$ represents the estimate of δ_a, and $\gamma_a > 0$ is the adaptation gain.

Proof: Take a Lyapunov function as

$$V(E, \tilde{\delta}_a) = \frac{1}{2}E^T PE + \frac{1}{2\gamma}\tilde{\delta}_a^2 \tag{27}$$

where $\tilde{\delta}_a = \delta - \hat{\delta}_a$ denotes the estimation error and $P = P^T > 0$ is the solution of the Lyapunov equation $A^T P + PA = -W$, for a given positive definite symmetric matrix W. Taking the time derivative of $V(E, \tilde{\delta}_a)$ along (6) and replacing control input (7) with adaptive law (25) imply that

$$\dot{V} = -\frac{1}{2}E^T WE + \eta_a^T(t)\phi_a(X)B^T PE - \hat{\delta}_a^2 \frac{E^T PB\phi_a^T(X)\phi_a(X)B^T PE}{\|\phi_a(X)B^T PE\|\hat{\delta}_a + \epsilon e^{-\sigma t}} - \frac{1}{\gamma_a}\tilde{\delta}_a\dot{\hat{\delta}}_a \tag{28}$$

Using the adaptation law (26) and some manipulations, one obtains

$$\dot{V}(E, \tilde{\delta}_a) \leq -\frac{1}{2}E^T WE + \epsilon e^{-\sigma t} \tag{29}$$

which $E \in L_2[0, \infty)$ is concluded by integrating. Besides, by inequality (29), $\dot{V}$ can be bounded as $\dot{V} \leq -\frac{1}{2}\lambda_W \|E\|^2 + \epsilon$, where λ_W is the minimum eigenvalue of W. Hence, following the procedure given for the proof of Theorem 3.1, completes the proof. □

Remark 3.5. *In various circumstances, the variation of unknown system parameters and external disturbances can violate the system stability. The stability analysis for the case of time-varying perturbations has not been presented in most of the existing methods* [1, 2, 14]*, or have some flaws* [6, 12]*. Such drawback is removed here by taking a general case for system parameters and external disturbances.*

4. **Simulation Study.** The dynamics under investigation are for a symmetrical gyro with nonlinear damping, which can be subjected to harmonic excitation [2], produced by white Gaussian noise. On the other hand, sinusoidal signal can model all periodic disturbances, occurred in some real world applications. Meanwhile, depending on environmental circumstances, the unknown system parameters may be constant or time-varying. In order to evaluate the performance of the developed algorithms, three cases are taken here to cover all the practical situations, depending on the characteristics of system parameters and external disturbances. Throughout the simulations, the system parameters are taken as $\eta_n = [100\ 1\ 0.5\ 0.05]^T$, $w = 2$, $f = 35.5$, and the initial state vector as $(x_1(0), x_2(0)) = (0.2, 0.9)$. Furthermore, the desired reference trajectory is specified by $x_d(t) = \sin(1.1t)$ [1], and the gain vector is taken as $K = [2\ 2]^T$. The external inputs including control law and disturbance signal are activated at $t = 5$s.

In Cases 1 and 2, the tracking controllers are developed by choosing the weighting matrix $Q = 2I_{2\times2}$. For attenuation levels $\rho = 1$ and $\rho = 0.3$, the H_∞ controller gains are respectively computed as $r = 1$ and $r = 0.09$, and the Riccati-like inequality (8) gives

$$P = \begin{bmatrix} 20 & 2 \\ 2 & 6 \end{bmatrix}$$

Meanwhile, the adaptive controller u_a, in Cases 2 and 3, is constructed by $\epsilon = 0.5$, $\sigma = 0.1$, and adaptation gains $\gamma = \gamma_a = 0.5$.

Case 1. *Unknown constant parameters and square-integrable disturbances.* Consider a situation in which a Gaussian noise with mean 0 and variance 1 perturbs the system. To construct the control input developed by Theorem 3.1, take $\Gamma = diag(1, 0.5, 0.5, 0.5)$. Figures 3 and 4 show the effectiveness of control algorithm to make the system states track the reference trajectories. Demonstrating the role of attenuation gain ρ, Figure 5 illustrates the convergence of tracking error despite the perturbations.

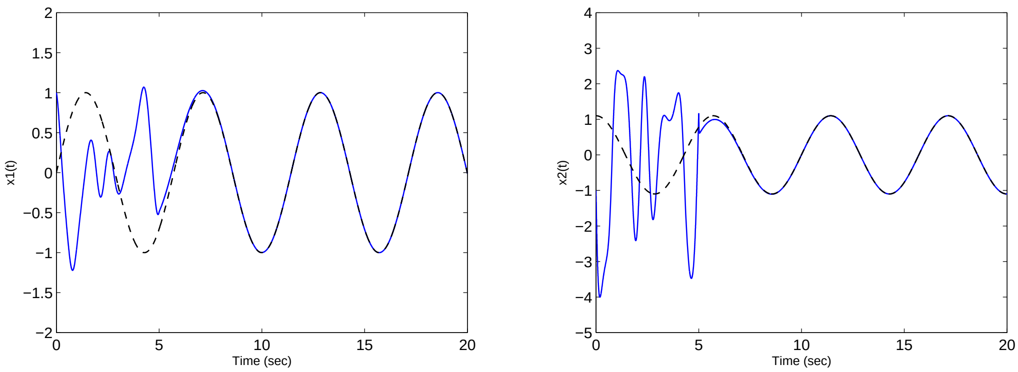

FIGURE 3. Time responses of controlled chaotic gyro in Case 1; system states (—), and reference trajectories (- -)

Case 2. *Time-varying parameters and square-integrable disturbances.* In this case, a time-varying term as $0.2\eta_n \sin(2t)$, is supposed to be added to the nominal vector η_n, in the presence of Gaussian noise as disturbance. Applying the control algorithm proposed by Theorem 3.2, the phase plane trajectory of tracking errors, plotted in Figure 6, shows that the error states are driven to zero.

Case 3. *Time-varying parameters and disturbances with unknown bound.* As a general case, the system parameters are taken similar to Case 2, whereas the system is perturbed by $d = 0.5 + 0.5 \sin t$, which do not belong to $L_2[0, \infty)$. By applying the robust adaptive controller, developed by Theorem 3.3, the phase portrait of tracking error is illustrated

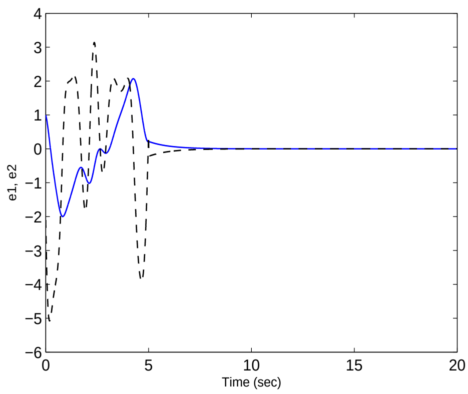

FIGURE 4. Tracking errors of chaotic gyro in case 1; $e_1(t)$ (—), and $e_2(t)$ (- -)

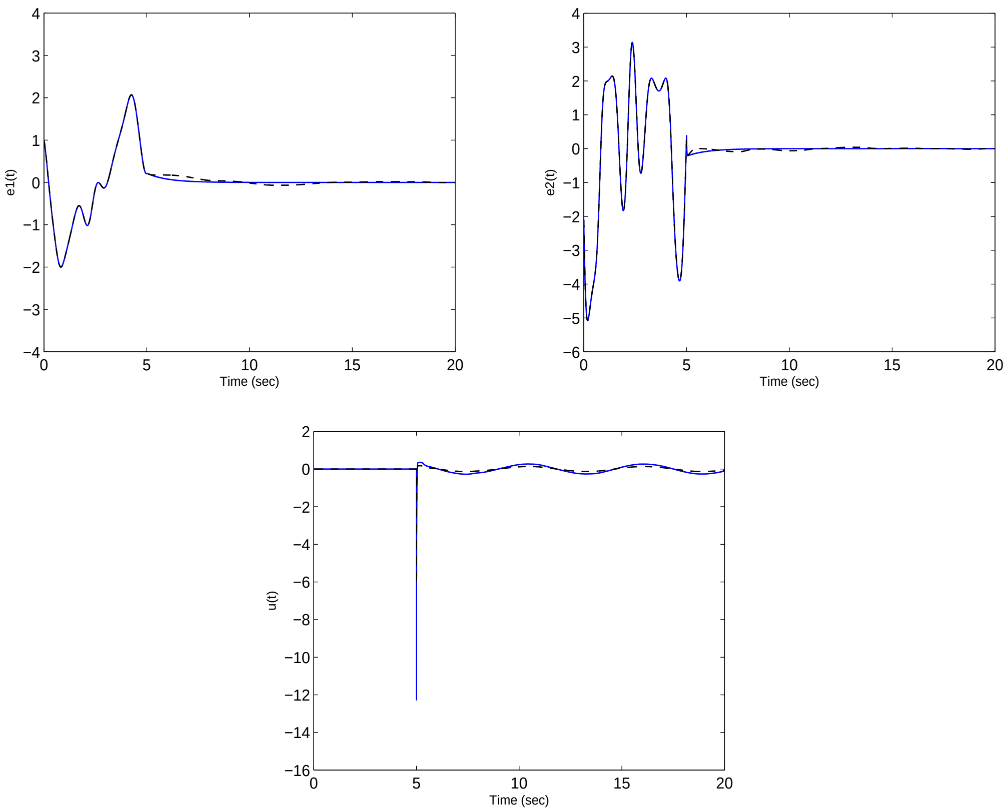

FIGURE 5. Time responses of tracking errors and control input in case 1, when $\rho = 0.3$ (—), and $\rho = 1$ (- -)

in Figure 7. As demonstrated in Figure 8, comparing the performance of the control algorithms proposed by Theorems 3.2 and 3.3 shows the effectiveness and necessity of the latter method to tackle non-square integrable disturbances with unknown bounds.

5. **Conclusions.** Two robust adaptive algorithms are proposed for chaos control of uncertain chaotic nonlinear gyros, based on H_∞ control and adaptive control strategies. The bounds of perturbations need not to be known in the design procedure. The performance of the proposed robust adaptive control schemes are demonstrated by various simulations.

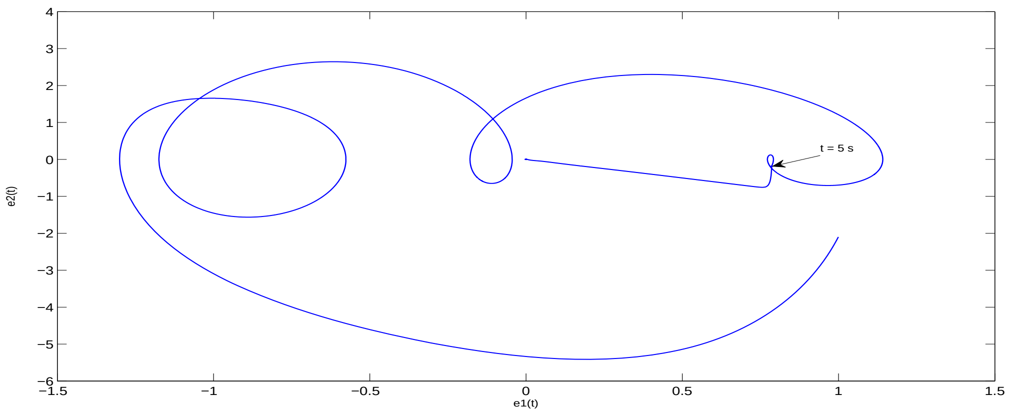

FIGURE 6. Phase plane trajectory of tracking errors e_1–e_2 in case 2

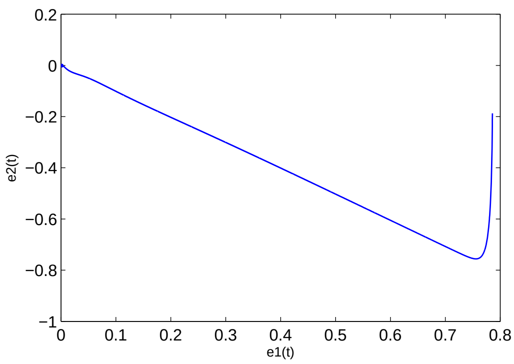

FIGURE 7. The phase portrait e_1–e_2 in case 3, for $t \geq 5$ s

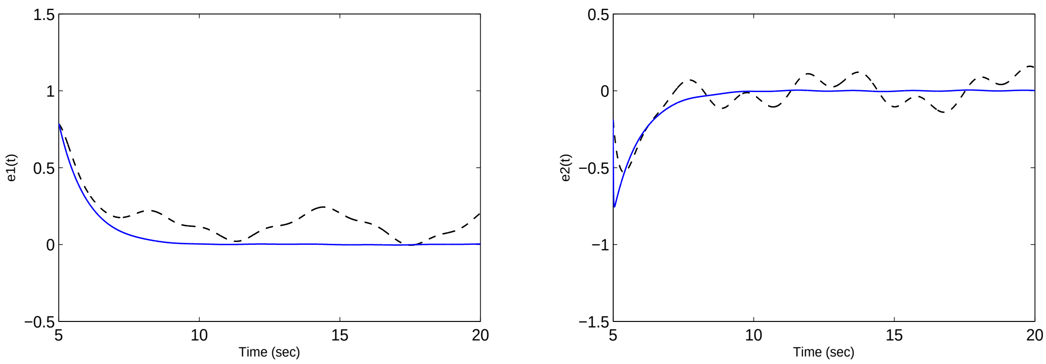

FIGURE 8. Time responses of tracking errors in case 3, applying the control law developed by Theorem 3.2 (- -), and Theorem 3.3 (—)

REFERENCES

[1] J. J. Yan, M. L. Hung, J. S. Lin and T. L. Liao, Controlling chaos of chaotic nonlinear gyro using variable structure control, *Mechanical Systems and Signal Processing*, vol.21, pp.2515-2522, 2007.

[2] H. K. Chen, Chaos and chaos synchronization of a symmetric gyro with linear-plus-cubic damping, *Journal of Sound and Vibration*, vol.255, pp.719-740, 2002.

[3] Y. W. Kim and H. H. Yoo, Design of a vibrating MEMS gyroscope considering design variable uncertainties, *Journal of Mechanical Science and Technology*, vol.24, no.11, pp.2175-2180, 2010.

[4] X. Tong and N. Mard, Chaotic motion of a symmetric gyro subjected to a harmonic base excitation, *Journal of Applied Mechanics*, vol.68, pp.681-684, 2001.

[5] N. Tsai and B. Wu, Nonlinear dynamics and control for single-axis gyroscope systems, *Nonlinear Dynamics*, vol.51, pp.355-364, 2008.

[6] Y. Lei, W. Xu and H. Zheng, Synchronization of two chaotic nonlinear gyros using active control, *Physics Letters A*, vol.343, pp.153-158, 2005.

[7] H. Yau, Nonlinear rule-based controller for chaos synchronization of two gyros with linear-plus-cubic damping, *Chaos Solitons Fractals*, vol.34, pp.1357-1365, 2007.

[8] Z. M. Ge and H. K. Chen, Stability and chaotic motions of a symmetric heavy gyroscope, *Japanese Journal of Applied Physics*, vol.35, pp.1954-1965, 1996.

[9] R. B. Leipnik and T. A. Newton, Double strange attractors rigid body motion with linear feedback control, *Physics Letters A*, vol.86, pp.63-67, 1981.

[10] W. Xiang, F. Zhu and Y. Huangfu, An adaptive sliding mode controller for uncertain chaotic systems with mismatched perturbation, *ICIC Express Letters*, vol.5, no.9(A), pp.2983-2988, 2011.

[11] A. Poursamad and A. H. D. Markazi, Adaptive fuzzy sliding-mode control for multi-input multi-output chaotic systems, *Chaos Solitons Fractals*, vol.42, pp.3100-3109, 2009.

[12] H. Salarieh, Comment on: Synchronization of two chaotic nonlinear gyros using active control, *Physics Letters A*, vol.372, pp.2539-2540, 2008.

[13] W. Xiang and Y. Huangpu, Sliding mode control for a new hyperchaotic dynamical system, *ICIC Express Letters*, vol.4, no.2, pp.547-552, 2010.

[14] H. T. Yau, Chaos synchronization of two uncertain chaotic nonlinear gyros using fuzzy sliding mode control, *Mechanical Systems and Signal Processing*, vol.22, pp.408-418, 2008.

[15] J. Yu, A. Jiang, Y. Ma, B. Chen and H. Yu, Adaptive fuzzy control of chaos in the PMSM via backstepping, *ICIC Express Letters*, vol.5, no.8(B), pp.2725-2730, 2011.

[16] K. J. Astrom and B. Wittenmark, *Adaptive Control*, 2nd Edition, Addision-Wesley, New York, 1994.

[17] M. Krstic, I. Kanellakopoulos and P. Kokotovic, *Nonlinear and Adaptive Control Design*, Wiley, New York, 1995.