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DESIGN OF PROCESS FUZZY CONTROL FOR PROGRAMMABLE LOGIC CONTROLLERS

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Abstract. *Programmable logic controllers (PLCs) assist the wider application of process fuzzy logic controllers (FLCs). The fuzzy control algorithms, however, should be simple because of the bounded computational time and load of real time operation. The existing FLC algorithms and their design methods are complex, iterative and require qualified experts and specific software. This research aims at the development of an engineering design for simple PI-like FLCs, which ensures system robustness and can be performed by PLCs. The design is based on the derivation of tuning models employing the least square error method and an artificial neural networks approach. The tuning models relate the FLC' parameters and the parameters of the nominal plant model, the plant uncertainty, and the fuzzy unit. The engineering design is applied for the real time control of a laboratory dryer temperature.*

Keywords: Fuzzy PI-like controllers design, Robust performance, Tuning models, Artificial neural network, Programmable logic controllers, Temperature real time control

List of Acronyms

ANN – artificial neural networks
FAM – Fuzzy Associative Matrix
FLC – Fuzzy Logic Controller
FU – Fuzzy Unit
DAQ – Data Acquisition Board
LTI – Linear Time-Invariant
MF – Membership Function
PLC – Programmable Logic Controller
PSI FC – Position Single Input FLC
PWM – Pulse Width Modulator
TSK – Takagi-Sugeno-Kang
2I FC – Two-input FLC
SI FC – Single Input FLC
SSR – Solid State Relay

1. **Introduction.** Recent developments in fuzzy control follow two main trends – the model-free approach, which is based on a known linguistically described strategy for the control of the plant [1-3], and the model-based approach, where a Takagi-Sugeno-Kang

(TSK) plant model is first built using analytical relationships [4] or experimentally [5,6]. As the closed loop system is nonlinear and the plant changes with operating point, time, load or alternative variable, the fuzzy logic controller (FLC) design is based on stability and robustness requirements either in the frequency domain [7-10] or in the time domain [4,6,11]. Most FLC design approaches are rather specific and closely oriented to the plant considered. They often require complicated models, high computational load, expert knowledge and off-line software tools to solve high dimensional problems. Other drawbacks are the FLC complexity, its restricted tuning facilities, linked FLC design and structure. Any change involves restarting of the design. The design of the fuzzy control algorithm cannot be applied to other similar plant. Both the fuzzy control algorithms and their design are difficult to use with programmable logic controllers (PLCs) in an industrial environment. This applies for the real time control of most plants found in the engineering practice. PLCs with fuzzy supplement can perform only the simplest fuzzy control algorithms and design them under the restrictions of the PLC software [12-15].

Simplifications are needed for both the FLC structure and the FLC design. They have been suggested lately in few cases to enable the FLC industrial application with embedded technology or PLCs [16,17]. Till now the FLC has been designed off-line prior to the embedding of the control algorithm in the PLC for real time operation using modern scientific sophisticated FLC design methods. These methods, however, often remain in the research laboratory as rather difficult for the practicing engineers to use, taking a lot of time and resources. An advanced method can become useful in practice if the PLC, which performs the control algorithm, can first automatically compute the design and the tuning of the control algorithms. The PLCs operate with limited in number simple operations to meet real time restrictions. It is obvious from the above that there is a need to improve the design of FLC. The novelty we suggest here concludes in a method for adaptation of the design of FLC to the requirements that a PLC automatically tunes and retunes its parameters on-line.

The ideas, developed in this paper, are to concentrate, generalise and approximate the essence of an FLC design method into simple formulae or trained artificial neural networks (ANNs) relating the controllers' to the plants' parameters called tuning models. This implies multiple expert applications of sophisticated design approach using appropriate software tools for various plants followed by derivation of the corresponding relationships. Once the tuning model obtained, the FLC design will require only input data concerning the particular plant and no deep knowledge of the design method employed, of the derivation of the analytical and the ANNs models, or of the software tools used. These analytical and ANNs models can be embedded in a PLC program and can be used for a whole family of plants, for which they have been derived. This idea can be employed in order to simplify various design approaches. This enables the PLCs to perform the design by the help of the derived under certain restrictions easily computable models.

Here tuning models are developed to approximate the approach, suggested in [18-20]. It deals with the design of PI-like fuzzy logic controllers (FLCs) on the basis of modification and in combination of Popov stability criterion and Morari linear system robustness [21]. The approach is quite general and is also suitable for plants with time delay, which are found in the majority of industrial processes and need to be controlled. The FLCs structures are simple enough for the use in PLC implementation [14,22,23].

The aim of this research is to develop an engineering design for process fuzzy PI-like controllers. It implies derivation of tuning models for the design approach, described in [18]. This must ensure system robust performance and an easy implementation of the control and its automatic design in PLCs. The engineering design is applied and assessed for real time temperature control in a laboratory dryer.

The remainder of the paper is organized as follows. A brief summary of the frequency design approach from system robustness requirement is given in Section 2. Sections 3 and 4 cover the development of appropriate tuning models and engineering design method for three types of process fuzzy PI-like controllers. In Section 5 the tuning models and the engineering method are used to design FLCs for the real time temperature control in a laboratory dryer. The results obtained are discussed in Section 6.

2. **Problem Statement and Preliminaries.** The system considered in [18] is shown in Figure 1, where the difference between the reference y_r and the measured plant output y gives the system error $e = y_r - y$. It consists of a plant and a fuzzy PI-like controller with transfer function $C_{fPI}(s, e)$. The plant is smoothly nonlinear, with time delay, with bounded and slowly varying characteristics and with unknown mathematical model. It is represented by a linear time-invariant (LTI) model with transfer function $P(s)$, determined by $[P^o(s), l(s)]$. The transfer function $P^o(s)$ represents an approximate nominal Ziegler Nichols plant model with parameters $q^{oT} = [k^o \quad T^o.\tau^o] = [q_i]$, $i = 1 \div 3 - P^o(s) = k^o e^{-s.\tau o}.(T^o s + 1)^{-1}$. The plant model multiplicative uncertainty $|l(s)| = |P(s) - P^o(s)|/|P^o(s)|$ is defined by the worst conditions with respect to system stability variations $var = \frac{\Delta q_i}{q_i^o} = \max$, $\Delta q_i = q_i - q_i^o$ that lead to the greatest increase of k^o and τ^o and to the greatest decrease of T^o for all possible operating points and conditions. The plant uncertainty $l(s)$ reflects the time-variable and nonlinear plant properties as well as the modelling error in accepting a LTI Ziegler Nichols nominal plant model. Thus the plant description by the couple $[P^o(s), l(s)]$ is general, linear, and simple and applies to the majority of complex processes to be controlled.

Several PI-like FLCs are considered. They include an incremental two-input FLC (2I FC), an incremental single input (SI FC) or a position single input (PSI FC), which is given in Figure 2, Figure 3 and Figure 4 respectively.

The incremental controllers use a differentiator with a transfer function $W_d(s) = K_d.T_d s/(T_d s + 1)$ in order to obtain the derivative-of-error signal $\dot{e}(de)$. The post-processing element is one-time-step memory or an integrating $W_2(s) = K_a/s$ with $K_a = K_{\Delta u}.K_i$ uniting the denormalisation factor $K_{\Delta u}$ and the integrator gain K_i. The post-processing unit in the position PI FLC – PSI FC is a classical position PI controller $C_{cPI}(s)$. The fuzzy unit (FU) has normalised input(s) and an output in the range $[-1, 1]$ using the factors K_e, K_{de}, K_{ds} and K_{FU} respectively. For the single input controllers the

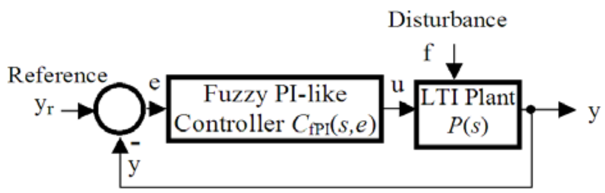


FIGURE 1. Control system with a fuzzy PI-like controller

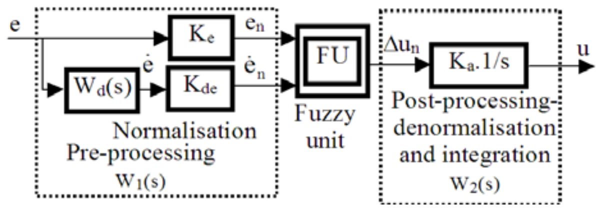


FIGURE 2. Incremental PI two-input fuzzy controller

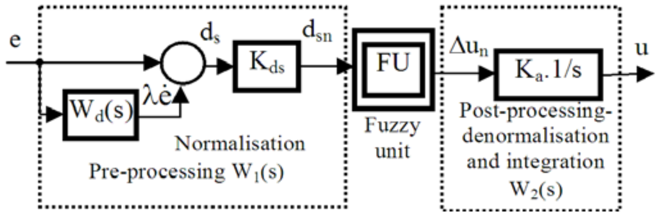


FIGURE 3. Incremental PI single input fuzzy controller

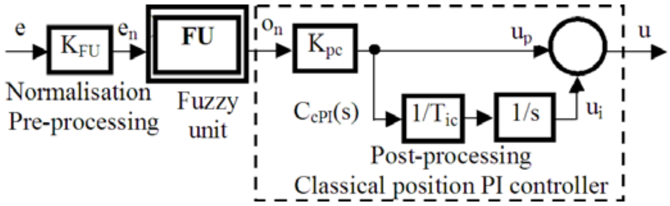


FIGURE 4. Position PI single input fuzzy controller

TABLE 1. 1-D FAM of SI FC

Input	LI_{-2}	LI_{-1}	LI_0	LI_1	LI_2
Output	LO_2	LO_1	LO_0	LO_{-1}	LO_{-2}

input to the FU is the normalised signed distance $x = d_{sn}(d_s = e + \lambda\dot{e})$ for SI FC, or the normalised error $x = e_n$ for PSI FC. Here the FU is a static sector-bounded nonlinearity with the gain $C(x) = \Psi(x)/x$ and the output $\Delta u_n = \Psi(d_{sn})$ for SI FC and $o_n = \Psi(e_n)$ – for PSI FC, which is uniquely determined from the requirement that output = – input. The rule base is a 1-D FAM as shown in Table 1. Here LI_k is the k-th linguistic value for the input (d_s or e_n) and LO_l is the l-th linguistic value for the output (Δu or o_n).

The product of the pre-processing unit $W_1(s) = K_{ds}[1+W_d(s)]$ and the post-processing unit $W_1(s)W_2(s) = C_{PI}(s) = K_p[1/(T_d s + 1) + 1/T_i s]$ in the incremental PI SI FC for small T_d yields a position PI controller with a gain of $K_p = K_d.T_d.K_{ds}.K_a$ and integral action time $T_i = K_d.T_d$. This makes the SI FC in Figure 3 equivalent to the PSI FC in Figure 4 – $C_{cPI}(s) = C_{PI}(s)$.

According to the procedure adopted for the design of the fuzzy controllers in [19] the FLC tuning parameters must be determined as functions of the parameters of the FU K and those of the plant $[P^o(s), l(s)]$ are $\mathbf{p} = [p_i]$. For PSI FC $\mathbf{p}_{PSIFC} = [T_{ic}\ K_{pc}]$, for SI FC $\mathbf{p}_{SIFC} = [T_d\ K_d\ K_a]$ and for 2I FC $\mathbf{p}_{2IFC} = [T_d\ K_d\ K_{a1}]$. The following is the design algorithm.

Case 1 – The FLC is an SI FC or a PSI FC

1. Now design the FU (the membership functions and the fuzzy rule base). We obtain from the control curve K and r_{max} as shown in Figure 5.

2. Now determine the significant for the system frequency range $D_\omega = [10^{-2}\omega_o, 10\omega_o]$ rad/s, where $\omega_o = 2\pi/T^o$ is the basic system frequency.

3. Calculate magnitude frequency response of the plant model uncertainty for $\omega \in D_\omega$ – $|l(j\omega)| = |P(j\omega) - P^o(j\omega)| / |P^o(j\omega)|$. Here the superscript "o" denotes that for nominal plant.

4. Then specify the ranges D_{pi} for the tuning parameters in $\mathbf{p}$. These ensure restricted closed loop system overshoot and settling time. They are calculated from the analogy with the linear PI controller. The general empirical tuning method used leads to the

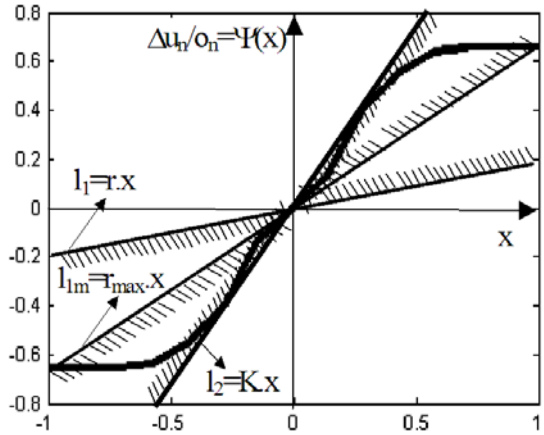


FIGURE 5. Sector bounded FU nonlinearity

following linear PI controller's parameters:

$$\left| \begin{array}{l} K_{\rm p}^{\rm lin} = (0.1 \div 2)T^{\rm o}/(k^{\rm o}\tau^{\rm o}) \\ T_{\rm i}^{\rm lin} = (0.1 \div 2)T^{\rm o} \end{array} \right. .$$

Thus for PSI FC since $K_{\rm p}^{\rm lin} = K_{\rm FU}.K.K_{\rm pc}$ and $T_{\rm i}^{\rm lin} = T_{\rm ic}$, can be written:

$$\left| \begin{array}{l} K_{\rm pc} = (0.1 \div 2)T^{\rm o}/(K_{\rm FU}.K.k^{\rm o}.\tau^{\rm o}) \\ T_{\rm ic} = (0.1 \div 2)T^{\rm o} \end{array} \right. ,$$

where $K_{\rm FU} = |e_{\rm max}|^{-1}$ depends on the maximal expected system error $|e_{\rm max}|$.

For SI FC $K_{\rm p}^{\rm lin} = K.K_{\rm a}.K_{\rm ds}.T_{\rm i}$ and $T_{\rm i}^{\rm lin} = K_{\rm d}.T_{\rm d}$. This leads to the following:

$$\left| \begin{array}{l} K_{\rm a} = (0.1 \div 2)T^{\rm o}/(K.K_{\rm ds}.T_{\rm i}.k^{\rm o}.\tau^{\rm o}) \\ K_{\rm d} = (0.1 \div 2)T^{\rm o}/T_{\rm d} \end{array} \right. ,$$

where $K_{\rm ds} = [(1 + K_{\rm d})|e_{\rm max}|]^{-1}$, $K_{\rm d} \geq 5T_{\rm d}$ and $T_{\rm d} = (1 \div 3)\Delta t$ for an efficient noise protected derivative $\dot{e}_{\rm n}$.

5. Specify the incremental changes for the tuning parameters **p** in the defined ranges $\mathrm{D_{pi}}$.

6. For all combinations of the tuning parameters in **p** check the fulfillment of the conditions for

– The stability of the plant $P_{\rm s}(s) = C_{\rm (c)PI}(s).P(s).[1 + r.C_{\rm (c)PI}(s).P(s)]^{-1}$

The marginally stable LTI dynamic part of the systems with the PI SI FC and the PSI FC is stabilized via the use of negative feedback with the minimal possible gain r, $0 < r \leq r_{\rm max}$. this is in order to reduce the additional restriction that this feedback puts on the location of the control curve. It makes the enclosing sector narrower as seen in Figure 5.

– The Popov stability of the fuzzy system with the nominal plant

The cross point of $P_{\rm 1m}^{\rm o}(j\omega) = {\rm Re}P_{\rm s}^{\rm o}(j\omega) + j\omega{\rm Im}P_{\rm s}^{\rm o}(j\omega)$ with the abscissa should be on the right from the cross point of the Popov line with the abscissa as shown in Figure 6 – $K_{\rm 1c} > K_1 = K - r > 0$.

– The robustness requirement for

(a) System robust stability – the Nyquist plot $P_{\rm 1m}^{\rm o}(j\omega)$ as shown in Figure 6 with all the uncertainty disks on it. It should be located below and to the right of the Popov line for all significant frequencies. This expresses the fulfilment of the requirement of the

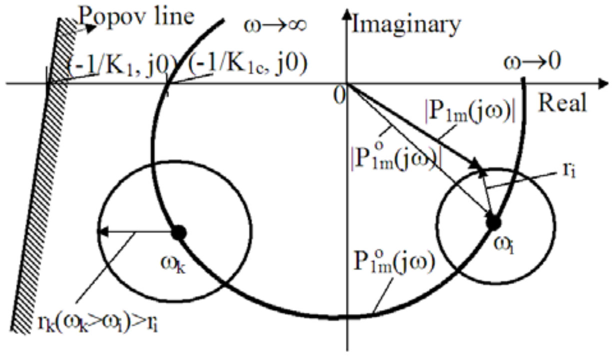


FIGURE 6. SI FC system robust stability

robust stability criterion:

$$|\Phi^{\mathrm{o}}_{1\mathrm{m}}(j\omega)|.|l_{\mathrm{m}}(j\omega)| < 1, \quad \forall\omega \geq 0 \tag{1}$$

where $|\Phi^{\mathrm{o}}_{1\mathrm{m}}(j\omega)| = |P^{\mathrm{o}}_{1\mathrm{m}}(j\omega).K_1|.|1 + P^{\mathrm{o}}_{1\mathrm{m}}(j\omega)K_1|^{-1}$ is the magnitude of the frequency response of the closed loop system with nominal modified plant $P^{\mathrm{o}}_{1\mathrm{m}}(s)$ and a linearised FU with $C(x) = K_1$. The modified plant model uncertainty $l_{\mathrm{m}}(j\omega)$ comes from the initial plant model uncertainty $l(j\omega)$. It is in the form of disks around the nominal modified plant Nyquist plot. The radiuses are $r_{\mathrm{i}}(\omega) = |P_{1\mathrm{m}}(j\omega)| - |P^{\mathrm{o}}_{1\mathrm{m}}(j\omega)| = |\Delta P_{1\mathrm{m}}(j\omega)| = |l_{\mathrm{m}}(j\omega).P^{\mathrm{o}}_{1\mathrm{m}}(j\omega)|$.

(b) System robust performance – the criterion is

$$|S^{\mathrm{o}}_{\mathrm{s}}(j\omega).W_{\mathrm{f}}(j\omega)| + |\Phi^{\mathrm{o}}_{\mathrm{slin}}(j\omega).l_{\mathrm{s}}(j\omega)| < 1, \quad \forall\omega \geq 0, \tag{2}$$

where $|\Phi^{\mathrm{o}}_{\mathrm{slin}}(j\omega)| = |P^{\mathrm{o}}_{\mathrm{s}}(j\omega).K_1|.|1 + P^{\mathrm{o}}_{\mathrm{s}}(j\omega).K_1|^{-1}$ is the magnitude of the frequency response of the closed loop system that consists of a nominal stabilised plant $P^{\mathrm{o}}_{\mathrm{s}}(s)$ and a linearised FU; $S^{\mathrm{o}}_{\mathrm{s}}(j\omega)$ is the system sensitivity function $S^{\mathrm{o}}_{\mathrm{s}}(s) = [1+P^{\mathrm{o}}_{\mathrm{s}}(s).K_1]^{-1}$ for $s = j\omega$ and y_{r}=0; $l_{\mathrm{s}}(j\omega) = |P_{\mathrm{s}}(j\omega) - P^{\mathrm{o}}_{\mathrm{s}}(j\omega)| \,/\, |P^{\mathrm{o}}_{\mathrm{s}}(j\omega)|$ is the multiplicative stabilising plant model uncertainty, induced by $l(j\omega)$. The term $|W_{\mathrm{f}}(j\omega)| = 0.3 \div 0.9$ is the disturbance shaping filter magnitude [20].

The second term in (2) represents the robust stability component. The robust performance condition sets stronger requirements and as more general is selected for tuning the fuzzy controllers' parameters $\mathbf{p}$. It ensures preservation of system stability and system performance for given plant model uncertainties.

7. Select the optimal vector $\mathbf{p}_{\mathrm{opt}}$ using the least tradeoff between desired nominal system performance and system robustness. This provides a fulfillment of the robustness requirement within the significant frequency range near below the boundary 1. For the optimal vector the ratio $K^{\mathrm{lin}}_{\mathrm{p}}/T_{\mathrm{i(c)}}$ is maximised.

Case 2 – The FLC is an incremental PI 2I FC

1. Design the FU for inputs e and $\dot{e}$ and output Δu normalised in the ranges $[-1, 1]$. Obtain K and δ from the $e - \Delta u$ projection (Figure 8) of the 2I FC control surface (Figure 7). The projection surface is enclosed within a sector except for a small area with a diameter δ around the origin. This defines an equivalent incremental PI SI FC FU with the control curve bounded in the same sector.

2. Tune the parameters $\mathbf{p}_{\mathrm{SIFC}}$ of the equivalent SI FC for K with a minimal possible r from the robust requirements.

3. Tune the parameters $\mathbf{p}_{\mathrm{2IFC}}$ of the 2I FC, obtained from the relationship with the parameters $\mathbf{p}_{\mathrm{SIFC}}$ of the equivalent SI FC – the 2I FC and the SI FC tuning parameters K_{d} and T_{d} are the same. 2I FC has different values of $K_{\mathrm{a1}} = k.K_{\mathrm{a}}$, where $k = -0.067 +$

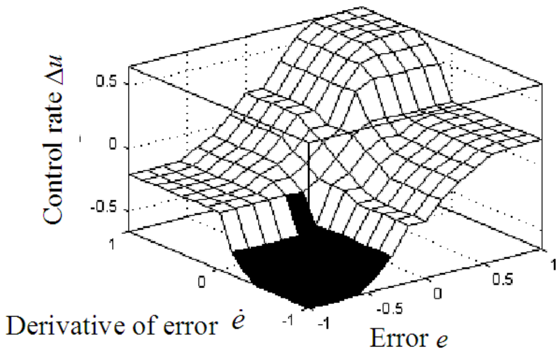


FIGURE 7. The FU control surface of 2I FC

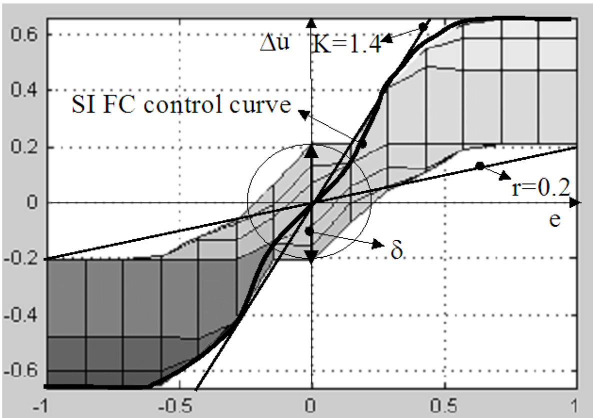


FIGURE 8. The $e - \Delta u$ projection of the 2I FC FU control surface

$0.134/\delta \in [0, 1]$ for $\delta = 0.01 \div 2$. The overall open loop FLC system gain varies inversely to the deflection of Δu from zero for an error close to zero. This compensates the δ-area violation of the condition for sector bounded nonlinearity in the $e - \Delta u$ projection. The 2I FC scaling factors are $K_{\rm e} = [|e_{\max}|]^{-1}$ and $K_{\rm de} = [K_{\rm d}.|e_{\max}|]^{-1}$.

The tuning of all investigated controllers is based on the tuning of SI FC. The PSI FC is equivalent to SI FC and the tuning of the 2I FC stems from the tuning of some equivalent SI FC.

The main problems that have to be solved in order to fulfill the stated aim – the development of an engineering design method, are the following.

1. Derivation of simple PLC applicable models of the relationship of fuzzy controllers tuning parameters **p** that ensure system robust performance and plant and FU parameters.

2. Development of an engineering design method, which is based on derived tuning models.

3. Assessment of the engineering method developed by its implementation in the FLC real time control of the air temperature in a laboratory dryer and comparison of processes in systems with model-based and precisely tuned FLCs.

3. System Robust Performance Tuning.

A MATLAB™ program has been developed to do the design procedure in [18] using input plant and FU parameters and delivering output – controller's tuning parameters. In order to find tuning models this program computes the controllers' parameters for various plants and FUs as input data. Then mathematical models are suggested to describe the relationships between the FLC parameters (the program output data) and the plants and FUs parameters. The models should be suitable for PLC programming.

Two types of simple and reliable models of these relationships are developed for PLC use. They are mathematical formulae and trained artificial neural networks (ANNs). The models can be applied for tuning of PI-like FLCs for plants with model parameters in the ranges, for which the tuning models are derived. Retuning must be possible for changed plant model, model uncertainty and FU parameters.

Substituting of the design procedure by simple engineering formulae or by ANNs reduces the expert knowledge on MATLAB^{TM} and on robustness theory and facilitates imbedding the FLCs and their design in PLCs. These tuning models allow robust process fuzzy control by PLCs industrial applications.

The tuning models are developed for a PSI FC because it has only two tuning parameters. Only two models need to be derived – that is for K_{pc} and T_{ic}. These models form the basis for computing of the parameters of the SIFC or the equivalent SI FC related to the corresponding 2I FC.

The main steps in derivation of the PSI FC tuning models are as follows.

1) Specification of input factors of influence

The number of the input factors of influence on the tuning parameters $\mathbf{p}_{PSIFC}$ determines the complexity of the model. The factors with impact on $\mathbf{p}_{PSIFC}$, which also are input parameters of the corresponding MATLAB^{TM} program, are

a) The nominal plant model parameters $[k^o\ T^o.\tau^o]$;

b) The worst variation related to the maximal increase of k^o and/or τ^o, and/or decrease of $T^o - var = \max(\frac{\Delta k}{k^o}, \frac{\Delta\tau}{\tau^o}, -\frac{\Delta o}{T^o})$. This allows describing the multiplicative plant model uncertainty $|l(j\omega)|$ using only one factor;

c) FU control curve parameter K – the FU parameter r is used temporarily to stabilise the LTI dynamic part of the system. It can be fixed at the most unfavourable with respect to the stability of $P_s(s)$ low value in order to keep as broad as possible the bounding sector of the control curve;

d) $|e_{max}|$ or $K_{FU} = K_e = [|e_{max}|]^{-1}$.

The vector of the input factors becomes $\boldsymbol{Q} = [k^o\ T^o.\tau^o\ var\ K\ K_e] = [Q_m]$. It takes various values for all of the possible combinations of values for the input factors. These determine the class of plants and FUs, which are considered.

The general methodology for the derivation of the tuning models is applied for a class of plants, which are specified by $k^o = [0.01\ 1\ 10]$, $T^o = [10\ 50\ 100]$, $\tau^o/T^o = [0.1\ 0.2\ 0.5\ 1]$, $var = [0.2\ 0.4\ 0.6\ 0.8]$, FU $K = [1\ 2\ 3]$ and $K_e = [0.1\ 0.2\ 1]$ (the maximal expected error $|e_{max}| = [10\ 5\ 1]$).

2) Design of computer experiments

The design of the computer experiments considers all possible M combinations of the defined input factors values. The MATLAB^{TM} program uses M different combinations of the values of $k^o\ T^o.\tau^o\ var\ K$ and K_e. For N combinations ($N \le M$), respectively N input vectors $\boldsymbol{Q}^j$, $j = 1 \div N$, a robust performance solution is found and the PSI FC tuning parameters $\mathbf{p}_{PSIFC} = [T^j_{ic}\ K^j_{pc}] = [p^j_{expi}]$, $i = 1,\ 2$ ($p^j_{exp\,1} = T^j_{ic}$, $p^j_{exp\,2} = K^j_{pc}$), computed according to the design procedure. The search of the parameters $[p^j_{exp\,i}]$ that satisfy the robust performance criterion is done for equally spaced values in the defined ranges 50 values for T_{ic} and K_{pc}, and 90 different frequencies.

3) Design of analytical tuning models

Preliminary analysis and graphical representations of the program input-output lead to various model structures of the relationships $T_{icm} = f_1(k^o, T^o, \tau^o, var, K, K_e) = p_{model1}$ and

$K_{\rm pcm} = {\rm f}_2(k^{\rm o}, T^{\rm o}, \tau^{\rm o}, var, K, K_{\rm e}) = p_{\rm model2}$. For each structure the model parameters – the coefficients in the function $\mathbf{f}_{\rm i}(.)$, $i = 1, 2$, are determined using the Least Square Error Method, minimizing a functional of the relative error. The relative error is defined as the difference between the controller's parameters, calculated from the model $\mathbf{f}_{\rm i}(.)$, and the obtained from the MATLAB$^{\rm TM}$ program for given input $\boldsymbol{Q}^{\rm j} - e_{\rm i}^{\rm j} = (p_{\rm modeli}^{\rm j} - p_{\rm expi}^{\rm j})/(p_{\rm expi}^{\rm j})$ (for $i = 1 - e_1^{\rm j} = [{\rm f}_1^{\rm j}(.) - T_{\rm ic}^{\rm j})]/(T_{\rm ic}^{\rm j})$ and for $i = 2 - e_2^{\rm j} = [{\rm f}_2^{\rm j}(.) - K_{\rm pc}^{\rm j})]/(K_{\rm pc}^{\rm j}))$. The functional is selected from among the following: 1) the sum of the relative square errors $I_{\rm i}^1 = \sum_{\rm j=1}^{N} (e_{\rm i}^{\rm j})^2$; 2) the maximal relative square error $I_{\rm i}^2 = \max_{\rm j}(e_{\rm i}^{\rm j})^2$; 3) the mean relative squared error $I_{\rm i}^3 = \frac{1}{N}\sum_{\rm j=1}^{N} (e_{\rm i}^{\rm j})^2$.

For a specified class of plants robust performance solution for PSI FC tuning parameters has been computed for $N = 891$ combinations of factors values. The best models derived with good compromise between simplicity and accuracy with minimal $I_{\rm i}^3 = 0.0207$ are

$$\left| \begin{array}{l} T_{\rm icm} = T^{\rm o}(0.3284 + 0.0978.var - 0.0046.k^{\rm o}.K) - 1.1627.(\tau^{\rm o}/T^{\rm o}) \\ K_{\rm pcm} = \dfrac{0.0589}{K_{\rm e}.K.k^{\rm o}.(\tau^{\rm o}/T^{\rm o}).var} + 0.2355.K + 0.0164 \end{array} \right. . \tag{3}$$

The models in (3) are validated with $I_{\rm i}^{3\rm validation} = 0.019 < 0.0207$ for a set of other 50 combinations of factors values which are different from those used for the modelling.

The accuracy of the analytical models can be assessed from Figure 9. There the exact values from the program (computer experiments) and the model values, obtained from (3), of the PSI FC parameters are also shown.

4) Design of ANN tuning models

In order to improve the accuracy of the analytical tuning models ANN tuning models are developed. Most modern PLCs have facilities for using ANNs [15,23].

For the defined class of plants, two two-layers backpropagation ANNs are designed. Each of them has 6 inputs $[K\ \ K_{\rm e}\ \ var\ \ k^{\rm o}\ \ T^{\rm o}\ \ \tau^{\rm o}/T^{\rm o}]$, five hidden layer neurons and one output and uses logistic sigmoid and linear activation functions in the two layers respectively. The two ANNs are trained on 891 input-output samples. The samples are collected from the designed computer experiments. After training 6 validation checks were done. Another 90 samples were used for testing. Both ANN tuning models are trained according to the Levenberg-Marquardt optimization learning algorithm [24].

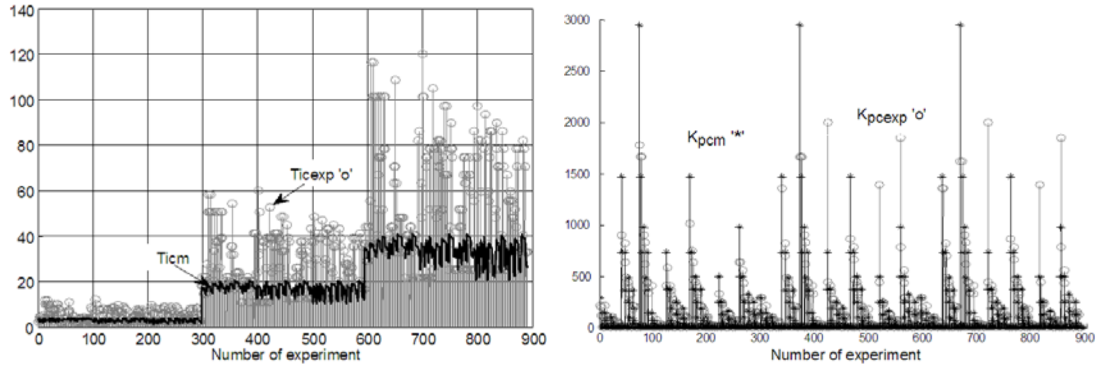
The first ANN has normalized target $T_{\rm ic}^{\rm j}/\max(T_{\rm ic}^{\rm j})$ in the range [0 1] with maximal value $\max(T_{\rm ic}^{\rm j}) = 120.2$s and output $T_{\rm icANNn}$. After 19 epochs the mean square error (MSE) reached is 0.01. In Figure 10(a) the experimental $T_{\rm icexp}$ and the ANN tuning model parameters are given only for the first 50 samples for illustration, where $T_{\rm icANN}$ is the denormalized ANN output $T_{\rm icANNn}$. The ANN has the following weights and biases:

$$\mathbf{W}_1 = \begin{bmatrix} -2.55 & -3.08 & 1.64 & 0.47 & 0.13 & -0.10 \\ 0.56 & 3.66 & 3.21 & -0.02 & 2.46 & -5.77 \\ 0.003 & 0.38 & -0.08 & -0.4 & 0.58 & 0.14 \\ 2.33 & 0.92 & 1.33 & 1.33 & -0.05 & 1.02 \\ 1.95 & 1.01 & 0.46 & 1.07 & 1.05 & 0.58 \end{bmatrix},$$

$$\mathbf{B}_1 = [\ 0.90\ \ 2.47\ \ -1.57\ \ -0.22\ \ 1.15\ ]$$

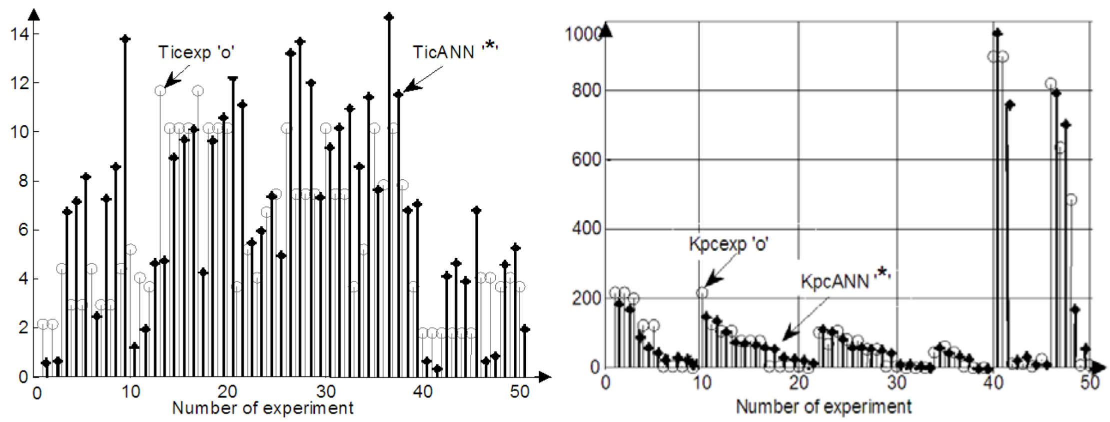
$$\mathbf{W}_2 = [\ 0.50\ \ 0.06\ \ 2.52\ \ -0.88\ \ 1.11\ ]$$

$$\mathbf{B}_2 = -1.70$$



(a) (b)

FIGURE 9. Experimental parameters and analytic tuning models for T_{icn} (a) and for K_{pc} (b)



(a) (b)

FIGURE 10. Experimental parameters and ANN tuning models for T_{icANN} (a) and for K_{pcANN} (b)

The second ANN has normalized in the range [0 1] target $K^{\mathrm{j}}_{\mathrm{pc}}/\max(K^{\mathrm{j}}_{\mathrm{pc}})$ with maximal value $\max(K^{\mathrm{j}}_{\mathrm{pc}}) = 2000$ and output K_{pcANNn}. After 55 epochs the MSE is 0.0019. The experimental $K_{\mathrm{pc\,exp}}$ and the ANN tuning model parameters for the first 50 samples are given for illustration in Figure 10(b), where K_{pcANN} is the denormalised ANN output K_{pcANNn}. The ANN has the following weights and biases:

$$\mathbf{W}_1 = \begin{bmatrix} 2.44 & 3.05 & -8.08 & -2.99 & 1.72 & -2.46 \\ 9.79 & 2.14 & -1.28 & -7.86 & 0.80 & -1.29 \\ -0.69 & -4.20 & -1.30 & -19.97 & 0.16 & -0.23 \\ 0.81 & 2.83 & 2.13 & -11.22 & -0.08 & 5.04 \\ -1.93 & -3.41 & -7.15 & 15.32 & 0.10 & -0.20 \end{bmatrix}$$

$$\mathbf{B}_1 = \begin{bmatrix} -12.01 & -5.27 & -25.65 & -1.77 & 5.63 \end{bmatrix}$$

$$\mathbf{W}_2 = \begin{bmatrix} 0.21 & -0.08 & 2.56 & -0.54 & -0.62 \end{bmatrix}$$

$$\mathbf{B}_2 = -0.38$$

Given the plant (nominal plant and uncertainty), the FU parameters and the error scaling factor K_{e}, i.e., specific values for the ANN models inputs, the corresponding tuning parameters of the PSI FC are computed as denormalised ANN models outputs

$T_{\rm icANN}$ and $K_{\rm pcANN}$. The ANN tuning models demonstrate good accuracy and relative simplicity.

4. **Engineering Design Method for Process Fuzzy PI-like Controllers.** The engineering design method for process PI-like FLCs is based on the derived models of the relationships of the PSI FC parameters and the parameters of the plant and the FU. These tuning models can be either analytical formulae or ANNs, derived for any defined class of plants and FUs. Both of them are simple to facilitate the FLC implementation in PLCs, and accurate enough – the PSI FC parameters, obtained from the tuning models are close to the precise values, computed from robust performance requirement according to the design procedure. The development of such a method is necessary in order to employ the robust performance criterion in the practical design of FLCs when the FLC completion is via PLCs. Besides, an engineering approach will contribute to broadening of the industrial applications of FLCs.

The task of engineering design method is to compute in a simple way the tuning parameters $\mathbf{p}$ of a selected fuzzy controller among PSI FC, SIFC and 2I FC that will ensure system robust performance for given plant model uncertainties. The scaling factors $K_{\rm FU}$, $K_{\rm ds}$, $K_{\rm e}$ and $K_{\rm de}$ are calculated separately as functions of the tuning parameters and the maximal system error expected.

Input data are the nominal plant parameters, their maximal relative variation, the maximal expected system error, and the type of FLC – PSI FC, SI FC or 2I FC.

The design algorithm is built of the following steps.

Case 1 – PSI FC or SI FC

1. Design the FU of the PSI FC and obtain the control curve parameter K.

2. Compute the PSI FC parameters $\mathbf{p}_{\rm PSIFC} = [T_{\rm icm}\ K_{\rm pcm}]$ or $\mathbf{p}_{\rm PSIFC} = [T_{\rm icANN}\ K_{\rm pcANN}]$, using (3) or the ANN tuning model respectively.

The design ends if the controller is PSI FC.

3. Compute the parameters $\mathbf{p}_{\rm SIFC} = [T_{\rm d(m/ANN)}\ K_{\rm d(m/ANN)}\ K_{\rm am/ANN)}]$ of the SI FC from the PSI FC parameters $T_{\rm icm}$ (or $T_{\rm icANN}$) and $K_{\rm pcm}$ (or $K_{\rm pcANN}$), accounting for the equivalence between the PSI FC and the SI FC – $T_{\rm iSIFC} = K_{\rm d}.T_{\rm d} = T_{\rm iPSIFC} = T_{\rm ic(m/ANN)}$ and $K_{\rm pSIFC} = K_{\rm d}.T_{\rm d}.K_{\rm ds}.K_{\rm a}.K = K_{\rm pPSIFC} = K_{\rm pc(m/ANN)}.K_{\rm e}.K$, and the requirement for noise-free and close to ideal derivative-of-error SI FC input:

$$\left|\begin{array}{l} T_{\rm d} = (1 \div 3)\Delta t, \quad \Delta t = 0.1\,\mathbf{min}(\tau^{\rm o}, T^{\rm o}) \\ K_{\rm d(m/ANN)} = T_{\rm ic(m/ANN)}/T_{\rm d} \\ K_{\rm a(m/ANN)} = K_{\rm pc(m/ANN)}.K_{\rm e}/(K_{\rm d(m/ANN)}.T_{\rm d}.K_{\rm ds}) \\ \qquad\qquad\;\; = K_{\rm pc(m/ANN)}.(1 + K_{\rm d(m/ANN)})/T_{\rm ic(m/ANN)} \end{array}\right. \tag{4}$$

Case 2 – 2I FC

1. Design the FU of the 2I FC and obtain the control surface $e - \Delta u$ projection parameters K and δ that define the equivalent SI FC.

2. Compute the PSI FC parameters $\mathbf{p}_{\rm PSIFC}$ according to Step 1 for K.

3. Obtain the parameters $\mathbf{p}_{\rm SIFC}$ of the corresponding SI FC from (4) according to Step 2.

4. The parameters of the 2I FC $\mathbf{p}_{\rm 2IFC} = [T_{\rm d}\ K_{\rm d}\ K_{\rm a1}]$ differ from $\mathbf{p}_{\rm SIFC}$ only in $K_{\rm a1} = k.K_{\rm a}$ with $k = -0.067 + 0.134/\delta$.

The scaling factors needed are $K_{\rm FU} = K_{\rm e} = [|e_{\max}|]^{-1}$ and $K_{\rm ds} = [(1 + K_{\rm d})|e_{\max}|]^{-1}$.

5. **Engineering Design Method Assessment – Case Study.** The developed engineering method is assessed by its implementation for the MATLAB$^{\rm TM}$ real time control

[25] of the air temperature in a laboratory dryer and comparison of processes with tuning model-based and precisely tuned FLCs from robust performance requirement.

The laboratory dryer, used for the MATLAB^{TM} real time control, is depicted in Figure 11. It comprises a dryer with a fan, electrical heater, tube, temperature sensor Platinum 100 (Pt100) with transmitter, a Solid State Relay (SSR) for control of the heater, a Data Acquisition Board (DAQ) and a PC with MATLAB^{TM}.

The Simulink model developed consists of Analog Input, voltage-to-temperature converter, Step blocks for stepwise changes of system reference, fuzzy controller (PSI FC, SI FC or 2I FC), pulse-width-modulator (PWM), Digital Output for the control pulses that drive the SSR to connect the electrical heater to the net supply voltage during the pulses and Graph Scopes for the temperature, its reference, the analog and the pulse control action. For investigation of the plant the Simulink model is modified by disconnection of the controller and applying the input step changes to the PWM.

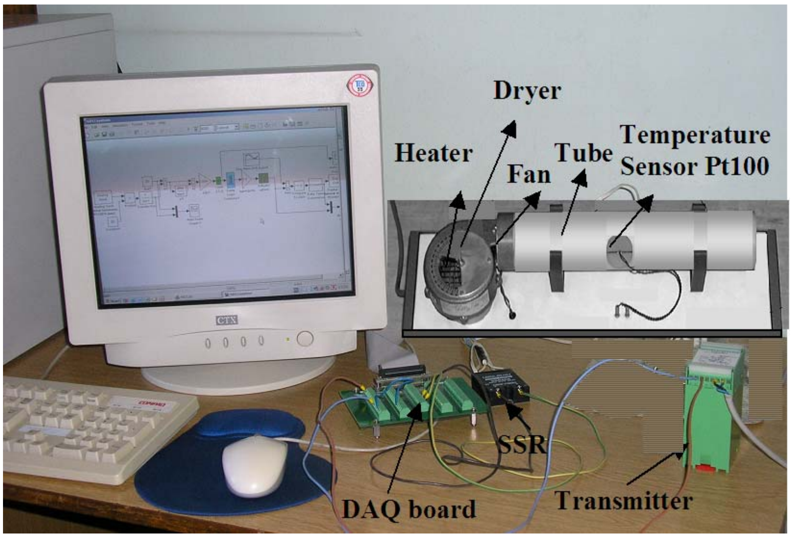


FIGURE 11. Laboratory dryer

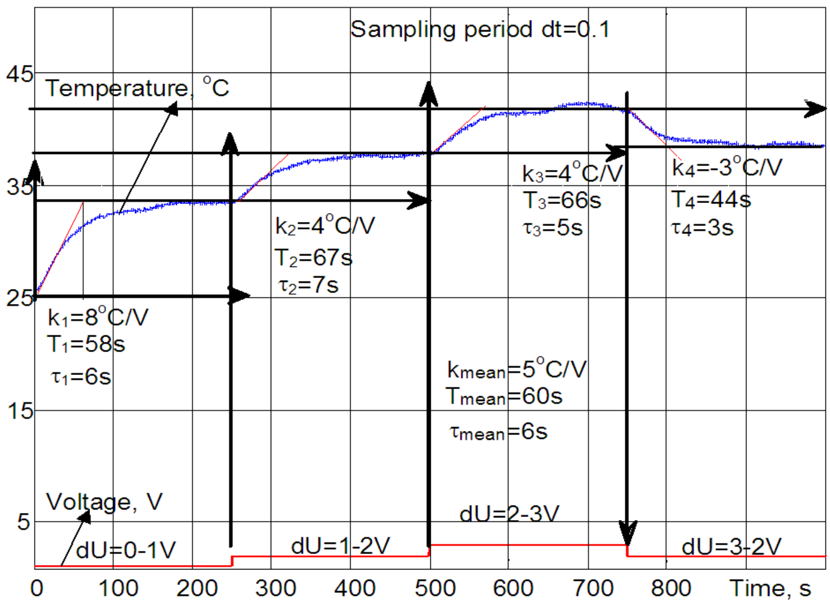


FIGURE 12. Laboratory dryer temperature step responses

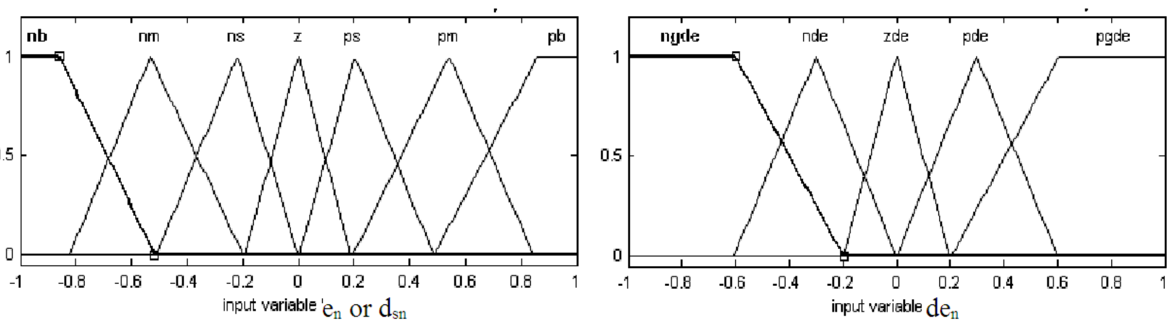


Figure 13. Membership functions of FLCs' FUs

1. If (error is nb) and (de is ngde) then (du is nb_u) (1)
2. If (error is nm) and (de is ngde) then (du is nb_u) (1)
3. If (error is ns) and (de is ngde) then (du is nm_u) (1)
4. If (error is z) and (de is ngde) then (du is nm_u) (1)
5. If (error is ps) and (de is ngde) then (du is ns_u) (1)
6. If (error is pm) and (de is ngde) then (du is ns_u) (1)
7. If (error is pb) and (de is ngde) then (du is z_u) (1)
8. If (error is nb) and (de is nde) then (du is nb_u) (1)
9. If (error is nm) and (de is nde) then (du is nm_u) (1)
10. If (error is ns) and (de is nde) then (du is nm_u) (1)
11. If (error is z) and (de is nde) then (du is ns_u) (1)
12. If (error is ps) and (de is nde) then (du is z_u) (1)
13. If (error is pm) and (de is nde) then (du is ps_u) (1)
14. If (error is pb) and (de is nde) then (du is ps_u) (1)
15. If (error is nb) and (de is zde) then (du is nm_u) (1)
16. If (error is nm) and (de is zde) then (du is nm_u) (1)
17. If (error is ns) and (de is zde) then (du is ns_u) (1)
18. If (error is z) and (de is zde) then (du is z_u) (1)
19. If (error is ps) and (de is zde) then (du is ps_u) (1)
20. If (error is pm) and (de is zde) then (du is pm_u) (1)
21. If (error is pb) and (de is zde) then (du is pm_u) (1)
22. If (error is nb) and (de is pde) then (du is ns_u) (1)
23. If (error is nm) and (de is pde) then (du is ns_u) (1)
24. If (error is ns) and (de is pde) then (du is z_u) (1)
25. If (error is z) and (de is pde) then (du is ps_u) (1)
26. If (error is ps) and (de is pde) then (du is pm_u) (1)
27. If (error is pm) and (de is pde) then (du is pm_u) (1)
28. If (error is pb) and (de is pde) then (du is pb_u) (1)
29. If (error is nb) and (de is pgde) then (du is z_u) (1)
30. If (error is nm) and (de is pgde) then (du is ps_u) (1)
31. If (error is ns) and (de is pgde) then (du is ps_u) (1)
32. If (error is z) and (de is pgde) then (du is pm_u) (1)
33. If (error is ps) and (de is pgde) then (du is pm_u) (1)
34. If (error is pm) and (de is pgde) then (du is pb_u) (1)
35. If (error is pb) and (de is pgde) then (du is pb_u) (1)

Figure 14. Fuzzy rules of 2I FC

The dynamic behaviour of the nonlinear plant is studied in different operating point at successive step changes of input voltage dU of 1V. The experimental plant step responses are given in Figure 12. The estimated Ziegler-Nichols models in the different operating points have different parameters as expected for a nonlinear plant. The plant average parameters $k^{\mathrm{o}} = 5^{\mathrm{o}}$C/V, $T^{\mathrm{o}} = 60$s, $\tau^{\mathrm{o}} = 6$s are accepted for nominal. The plant worst parameters are $k = 8^{\mathrm{o}}$C/V, $T = 44$s, $\tau = 7$s, which gives the worst variation $var = 0.6$. The plant belongs to the defined class, for which the model (3) and the ANN tuning models in Section 3 are developed.

The FUs of the FLCs are designed with membership functions (MFs) for the input $e_{\mathrm{n}}(d_{\mathrm{sn}})$ and $\mathrm{d}e_{\mathrm{n}}$, shown in Figure 13. Singleton MFs for the output o_{n} (Δu_{n}) are accepted at $[-1 \ \ -0.8 \ \ -0.4 \ 0 \ 0.4 \ 0.8 \ 1]$ to be feasible in SIMATIC PLC (SIMATIC S7 2002; Yordanova et al. 2009). The terms for $e_{\mathrm{n}}(d_{\mathrm{sn}})$ [**nb, nm, ns, z, ps, pm, pb**] and for o_{n} (Δu_{n}) [**nb_u nm_u ns_u z_u ps_u pm_u pb_u**] correspond to "negative big", "negative medium", "negative small", "zero", "positive small", "positive medium" and "positive big" respectively, and the terms for $\mathrm{d}e_{\mathrm{n}}$ [**ngde, nde, zde, pde, pgde**] are "negative great", "negative", "zero", "positive" and "positive great".

The rules for the 2I FC are presented in Figure 14. The rules of the single input controllers are of the type

IF input (e_{n} or d_{sn}) is **nb THEN** output (o_{n} or Δu_{n}) is **nb_u**

and so on for all the 7 rules for the 7 input MFs. The MFs and the rule bases are designed employing expert knowledge and empirical rules.

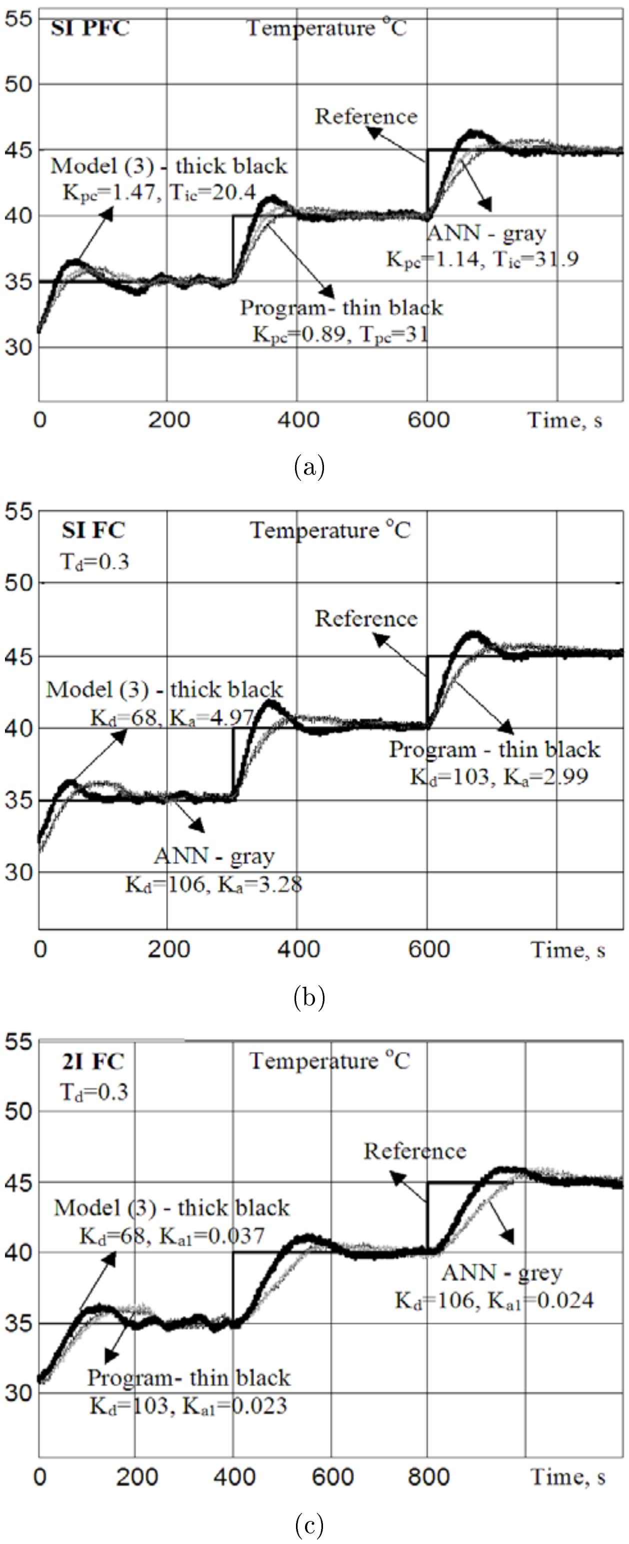


FIGURE 15. Temperature step responses of fuzzy control systems to reference changes

The control surface $e - \Delta u$ projection for the 2I FC and the control curve for the SI controllers are of the type, shown in Figure 8 and Figure 5 respectively but with $K = 2$, $r_{\max} = 1$ and $\delta = 1.8$ and hence $k = -0.067 + 0.134/\delta = 0.0074$. The LTI dynamic part is stabilized by a negative feedback with $r = 0.2$.

The FLCs parameters are tuned from system robust performance requirement using the developed: 1) MATLAB^{TM} program; 2) analytical tuning models (3); 3) ANN tuning models. The parameters are shown together with step responses in the real time control in Figure 15. Three step responses are observed in order to see the impact on system performance of the plant nonlinearity, expressed in changes of the plant parameters with the operating point. As expected for FLCs designed to ensure closed loop system robust performance, the overshoot and the settling time do not significantly vary with the operating point.

It has been established experimentally that the reduction of the scaling factor K_{de} in the 2I FC to K_{e} by merely limiting the derivative within $[-1\ 1]$ improves the performance of the closed loop system, because the derivative takes co-measurable values with the error input.

The comparison shows close responses of the systems with the precisely tuned FLCs on the basis of the MATLAB^{TM} program, and with the FLCs, tuned, using the analytical or the ANN tuning models. The overshoot is a little greater for the tuned by analytical tuning models FLCs. The responses of the systems with the FLCs, tuned by the MATLAB^{TM} program and the ANN tuning models almost coincide for the three types of FLCs. The dynamic behaviour of the PSI FC and the SI FC systems is very close as expected due to the controllers' equivalence. The PSI FC is superior as it escapes the problems of computation of derivative-of-error input and substitutes the pre- and post-processing blocks with a standard position PI controller, which makes easier the PLC completion and design of the FLC.

6. **Conclusions and Future Work.** The main results of this investigation conclude in the following.

1. An engineering method is developed for the design of three types of PI-like FLCs out of robust performance considerations. It is based on the approximation of the results from a precise tuning procedure of PSI FC by simple engineering tuning models. The tuning of SI FC and 2I FC is further accomplished on their established relationship with the PSI FC. The FLC and its tuning model is easy to employ in a PLC.

2. Two types of engineering tuning models – analytical, based on the application of the least square error approach, and ANN, are derived from full-factor experiments for computation of the precise PSI FC tuning parameters. The experiments design considers all combinations of values for the identified input factors and a given class of plant and FUs. The models relate FLC tuning parameters, ensuring system robust performance, and plant and FU parameters and are both simple and relatively accurate.

3. The engineering method is experimented and assessed in the real time control of the temperature in a laboratory dryer.

The system closed loop responses for precisely tuned FLCs and FLCs tuned using the engineering approach with the two types of tuning models are very close. The model-based tuned controllers require no expert knowledge on MATLAB^{TM} and robustness theory. The tuning is simple and fast, can be imbedded in the PLC algorithm that realizes the FLC, and applied to a great number of industrial plants.

Future research will concentrate on the SIMATIC PLC implementation of both the developed PI-like FLCs and their tuning models.

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