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NEW RESULTS ON MODEL REDUCTION FOR DISCRETE-TIME SWITCHED SYSTEMS WITH TIME DELAY

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Abstract. *This paper is concerned with the problem of delay-dependent exponential H_∞ model reduction for discrete-time switched delay systems under switching signals with average dwell time (ADT). The objective is to construct a reduced-order model, which ensures that the resulting error system under switching signal with ADT is exponentially stable with an H_∞ norm bound. A weighting factor α is introduced to construct a Lyapunov function for switched delay systems such that ADT approach is used with piecewise Lyapunov matrices instead of common Lyapunov function matrices. Furthermore, sufficient conditions for the solvability of this problem are obtained in terms of strict linear matrix inequalities (LMIs), which lessen the computation complexity. A numerical example is provided to show the effectiveness of the developed method.*
Keywords: Model reduction, Discrete-time switched systems, State delay, LMIs

1. **Introduction.** During the last decades, switched systems have been extensively studied [1-12]. On one hand, many real-world processes can be modeled as switched systems including networked control systems, robotic manufacture and so on. On the other hand, switching among different controllers can improve the system performance when no single controller is effective. Due to the hybrid nature of switched systems operations, it is very difficult to deal with them. The existence of a common Lyapunov function (CLF) for all subsystems is a sufficient and necessary condition for analysis and synthesis of switched systems under arbitrary switching signals (see [1] and references therein). In fact, most of switched systems are difficult to find or do not possess a CLF, but they may still be analyzed and synthesized under some constrained switching signals. Multiple Lyapunov function method (MLF) is proposed for finding such a switching signal, by which an individual decrescent Lyapunov function is constructed for each subsystem [2]. Switched Lyapunov function (SLF) method [5] and average dwell time (ADT) technique [6] are two special cases of Multiple Lyapunov function method. As time delays are the inherent features of many practical processes, recently, more and more attention is shifted to switched system with time delays. The behavior of switched delay systems is more complicated than that of switched systems or delay systems due to the simultaneous existence of switchings and time delays [8,13-16]. In this context, it has been recognized that ADT technique is a powerful and flexible tool for analysis and synthesis of such systems.

As is well known that many practical systems are often modeled as high-order models because it is straightforward to obtain the model formulation using many variables. However, this causes the great difficulties in analysis and synthesis of the systems. Thus, it is desirable to replace these high-order models with reduced-order ones for reducing the computational complexities in some given criteria without incurring much loss of performance or information, which has motivated the study of the model reduction problem. During the past few years, many criteria have been proposed including the L_2 model reduction [17-19] and the H_2 model reduction [20]. Recently, the problem of H_∞ model reduction is addressed by making use of LMIs technique for singular systems [21], time-delay system [22,23], Markovian jump systems [24] and switched systems [25]. It is worth mentioning that the criteria for the discrete-time context in [21,23,24] are formulated in terms of LMIs with some non-convex conditions, which make LMIs difficult to find the numerical solution. Although many numerical approaches have been proposed to overcome this difficulty, for example, the cone complementarity linearization method [26] and sequential linear programming matrix method [27], the computation is very heavy. Therefore, the strict LMI formulation is desired to avoid it. In [25], the existence conditions for H_∞ model reduction for discrete-time switched systems are derived in terms of strict LMIs by using SLF method. However, time delays are not taken into account in [25]. When ADT method is used to address the problem of H_∞ model reduction for discrete-time switched systems with time delays, it requires the existence of some common Lyapunov matrices for all subsystems [14,15]. It is desired to construct a piecewise Lyapunov function, i.e., to select the corresponding Lyapunov matrices for each subsystem. To the best of the authors' knowledge, however, no result with regard to this problem is reported up to date.

In this paper, we are interested in the problem of exponential H_∞ model reduction for discrete-time switched systems with time delay. When ADT method is used to deal with the analysis of switched delay systems, there exist some common Lyapunov matrices among all subsystems due to the existence of time delay [14,15]. Our objective is to remove such restriction, that is, a corresponding Lyapunov matrix is constructed for each subsystem. Sufficient conditions for the existence of the desired reduced-order model are derived and formulated in terms of strict LMIs, which lead to lessening the computational complexity. Finally, an example is shown to illustrate the effectiveness of the proposed techniques.

The remainder of this paper is organized as follows. The problem of exponential H_∞ model reduction for discrete-time switched systems with time delay under ADT switching signal is formulated. Section 3 presents sufficient conditions for the existence of desired reduced-order models. Section 4 provides an illustrative example and we conclude this paper in Section 5.

Notation: The notation used in this paper is fairly standard. The superscript 'T' stands for matrix transposition, R^n denotes the n dimensional Euclidean space. In addition, Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. $P < 0$ represents a negative definite matrix P. I and 0 denote, respectively, identity matrix and zero matrix. For simplicity and in the absence of confusion, the following notions are used: $x_k = x(k)$, $u_k = u(k)$, $d_k = d(k)$, $f_k = f(k)$.

2. **Problem Statement and Preliminaries.**

Consider discrete-time switched systems with state delays described by:

$$\begin{aligned} x_{k+1} &= A_\sigma x_k + A_{d\sigma} x_{k-d} + B_\sigma u_k \\ y_k &= C_\sigma x_k + C_{d\sigma} x_{k-d} + D_\sigma u_k \\ x_\psi &= \phi_\psi, \quad \psi = -d, -d+1, \ldots, 1 \end{aligned} \tag{1}$$

where $x_k \in R^n$ is the state, $y_k \in R^m$ is the measured output, $u_k \in R^p$ is the input vector which belongs to $l_2\,[0, \infty)$. The piecewise constant function σ (denoting σ_k for simplicity): $[0,\ \infty) \to \mathcal{P} = \{1, \cdots, p\}$ is a switching signal to specify, at each time instant k, the index

$\sigma \in \mathcal{P}$ of the active subsystem, i.e., $\sigma = i$ means that the ith subsystem is activated. $p > 1$ is the number of subsystems. The ith subsystem is denoted by constant matrices A_i, A_{di}, B_i, C_i, C_{di} and D_i. $d > 0$ is a constant time delay.

In this paper, we are interested in constructing a reduced-order switched system described by

$$\begin{aligned} \hat{x}_{k+1} &= A_{ri}\hat{x}_k + A_{rdi}\hat{x}_{k-d} + B_{ri}u_k \\ \hat{y}_k &= C_{ri}\hat{x}_k + C_{rdi}\hat{x}_{k-d} + D_{ri}u_k \end{aligned} \tag{2}$$

where $\hat{x}_k \in R^q$ is the state vector of the reduced-order system with $q < n$. $\hat{y}_k \in R^m$ is the output of reduced-order system with the same dimension m as the original output y_k. A_{ri}, B_{ri}, C_{ri} and D_{ri}, $i \in \mathcal{P}$ are the matrices with compatible dimensions to be determined. Note that system (2) is assumed to be switched synchronously by switching signal σ in system (1).

Remark 2.1. *Note from (2) that the constant delay τ is assumed to be known a prior. Then, the conservatism can be reduced by extending the results to include the time-varying delays or the unknown delays. Another underlying assumption is to switch synchronously between the original systems and the reduced-order systems. A possible improvement to this point is to design an observer which allows the asynchronously switchings.*

Augmenting the model of system (1) to include the states of (2), we can obtain the following model error system:

$$\begin{aligned} \tilde{x}_{k+1} &= \tilde{A}_i\tilde{x}_k + \tilde{A}_{di}\tilde{x}_{k-d} + \tilde{B}_iu_k \\ \tilde{e}_k &= \tilde{C}_i\tilde{x}_k + \tilde{C}_{di}\tilde{x}_{k-d} + \tilde{D}_iu_k \end{aligned} \tag{3}$$

where

$$\begin{aligned} &\tilde{x}_k = \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix},\ \tilde{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & A_{ri} \end{bmatrix},\ \tilde{A}_{di} = \begin{bmatrix} A_{di} & 0 \\ 0 & A_{rdi} \end{bmatrix},\ \tilde{B}_i = \begin{bmatrix} B_i \\ B_{ri} \end{bmatrix} \\ &\tilde{C}_i = \begin{bmatrix} C_i & -C_{ri} \end{bmatrix},\ \tilde{C}_{di} = \begin{bmatrix} C_{di} & -C_{rdi} \end{bmatrix},\ \tilde{D}_i = D_i - D_{rdi},\ \tilde{e}_k = y_k - \hat{y}_k. \end{aligned}$$

To present the main objective of this paper, the following definitions are introduced.

Definition 2.1. [6]*: For any $k_v > k_s > 0$, let $N_\sigma(k_s, k_v)$ denote the switching numbers of σ during $[k_s, k_v]$. For given scalars $\tau^* > 0$ and $N_0 \geq 0$, we have*

$$N_\sigma(k_s,\ k_v) \leq N_0 + \frac{k_v - k_s}{\tau^*} \tag{4}$$

where τ^* and N_0 are called ADT and the chattering bound, respectively. Here we assume $N_0 = 0$ for simplicity as commonly used in literature.

Definition 2.2. [28]*: Exponential H_∞ performance: given a scalar $\gamma > 0$, system (3) is said to be exponentially stable with an H_∞ norm bound γ, if it is exponentially stable when $u_k = 0$ and under zero initial condition, $\|\tilde{e}_k\|_2 \leq \gamma\|u_k\|_2$ for all nonzero $u_k \in l_2[0, \infty)$.*

Our objective is to design a reduced-order system in the form of system (2), and find admissible switching signals with ADT such that the resulting model error system (3) under such switching signals is exponentially stable and guarantees an H_∞ performance index.

To obtain our main result, the following lemma is presented.

Lemma 2.1. [30]*: For any constant matrix $M \geq 0$, $\Phi_l \in R^n$, there exist positive integers $\beta_1 \geq \beta_2 \geq 1$ such that*

$$-(\beta_2 - \beta_1 + 1)\sum_{l=\beta_1}^{\beta_2} \Phi_l^T M \Phi_l \geq -\sum_{l=\beta_1}^{\beta_2} \Phi_l^T M \sum_{l=\beta_1}^{\beta_2} \Phi_l. \tag{5}$$

3. **Main Results.** In this section, the following theorem provides sufficient conditions for the existence of a reduced-order model (2) for system (1) based on Lemma 2.1.

Theorem 3.1. *For given scalars, $0 < \alpha < 1$, $\gamma > 0$ and $\mu \geq 1$, system (3) is exponentially stable with an H_∞ norm bound γ under ADT switching signal σ, if there exist symmetric and positive definite matrices P_i, Q_i, Z_i and matrices M_{ci}, $c = 1, 2, 3, \forall i \in \mathcal{P}$, such that the following inequalities hold,*

$$\begin{bmatrix} \Xi_1 & \Xi_2 \\ * & \Xi_3 \end{bmatrix} < 0 \tag{6}$$

$$P_i \leq \mu P_j,\ Q_i \leq \mu Q_j,\ Z_i \leq \mu Z_j,\ \forall i, j \in \mathcal{P} \tag{7}$$

and average dwell time τ_a

$$\tau_a > \tau^* = -\ln \mu / \ln \alpha \tag{8}$$

where

$$\Xi_1 = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} \\ * & \Psi_{22} & \Psi_{23} \\ * & * & -\gamma^2 I \end{bmatrix},\ \Xi_2 = \begin{bmatrix} M_{1i} & \tilde{C}_i^T & \tilde{A}_i^T & A_i^T - I \\ M_{2i} & \tilde{C}_{di}^T & \tilde{A}_{di}^T & \tilde{A}_{di}^T \\ M_{3i} & \tilde{D}_i^T & \tilde{B}_i^T & \tilde{B}_i^T \end{bmatrix}$$

$$\Xi_3 = diag\left\{ -\alpha^d d^{-1} Z_i, -I, -P_i^{-1}, -(dZ_i)^{-1} \right\}$$

$$\Psi_{11} = Q_i - \alpha P_i + M_{1i}^T + M_{1i},\ \Psi_{12} = M_{2i}^T - M_{1i}$$

$$\Psi_{22} = -\alpha^{d_M} Q_i - M_{2i}^T - M_{2i},\ \Psi_{23} = M_{3i}^T,\ \Psi_{13} = M_{3i}^T.$$

Proof: Choosing the following Lyapunov-Krasovskii function as

$$V_i(k) = V_{1i}(k) + V_{2i}(k) + V_{3i}(k) \tag{9}$$

where

$$V_{1i}(k) = \tilde{x}_k^T P_i \tilde{x}_k,\ V_{2i}(k) = \sum_{l=k-d}^{k-1} \alpha^{k-l-1} \tilde{x}_l^T Q_i \tilde{x}_l$$

$$V_{3i}(k) = \sum_{j=-d}^{-1} \sum_{l=k+j}^{k-1} \alpha^{k-l-1} \eta_l^T Z_{1i} \eta_l,\ \eta_l = \tilde{x}_{l+1} - \tilde{x}_l$$

and P_i, Q_i and Z_i are symmetric and positive definite matrices.

In terms of $\Delta V_i(k) = V_i(k+1) - \alpha V_i(k)$, we have

$$\Delta V_{1i}(k) = \tilde{x}_{k+1}^T P_i \tilde{x}_{k+1} - \alpha \tilde{x}_k^T P_i \tilde{x}_k \tag{10}$$

$$\Delta V_{2i}(k) = \tilde{x}_k^T Q_i \tilde{x}_k - \alpha^d \tilde{x}_{k-d}^T Q_i \tilde{x}_{k-d} \tag{11}$$

$$\Delta V_{3i}(k) = d\eta_k^T Z_{1i} \eta_k - \sum_{l=k-d}^{k-1} \alpha^{k-l} \eta_l^T Z_{1i} \eta_l. \tag{12}$$

By means of Lemma 2.1, we have

$$\Delta V_{3i} \leq d\eta_k^T Z_{1i} \eta_k - \alpha^d d^{-1} \sum_{l=k-d}^{k-1} \eta_l^T Z_{1i} \sum_{l=k-d}^{k-1} \eta_l. \tag{13}$$

Define $\xi_k = \begin{bmatrix} \tilde{x}_k^T & \tilde{x}_{k-d}^T & w_k^T \end{bmatrix}^T$, due to

$$2\xi_k^T M_i \left[x_k - x_{k-d} - \sum_{l=k-d}^{k-1} \eta_l \right] = 0,$$

one gets

$$\begin{aligned}
&\Delta V_i(k) + \tilde{e}_k^T\tilde{e}_k - \gamma^2 w_k^T w_k + 2\xi_k^T M_i\left[x_k - x_{k-d_k} - \sum_{l=k-d_k}^{k-1}\eta_l\right] \\
&\le \xi_k^T\left(\Xi_1 + M_i\alpha^{-d}dZ_i^{-1}M_i^T\right)\xi_k + \tilde{z}_k^T\tilde{z}_k + \tilde{x}_{k+1}^T P_i\tilde{x}_{k+1} + \eta_k^T dZ_i\eta_k \\
&\quad - \left[\xi_k^T M_i + \sum_{l=k-d}^{k-1}\alpha^d d^{-1}\eta_l^T Z_{1i}\right](\alpha^d d^{-1}Z_{1i})^{-1}\times\left[M_i^T\xi_k + \sum_{l=k-d}^{k-1}\alpha^d d^{-1}Z_{1i}\eta_l\right].
\end{aligned} \tag{14}$$

Due to $Z_i > 0$, the last term is non-positive. By Schur Complement, further, from (6), we have

$$\Delta V_i(k) + \tilde{e}_k^T\tilde{e}_k - \gamma^2 w_k^T w_k \le 0. \tag{15}$$

Furthermore, one can obtain from (15) that

$$V_i(k+1) \le \alpha V_i(k) - \Gamma(k), \tag{16}$$

where $\Gamma(k) = \tilde{e}_k^T\tilde{e}_k - \gamma^2 w_k^T w_k$. On the other hand, it follows from (7) and (9) that

$$V_i\left(k\right) \le \mu V_j\left(k\right). \tag{17}$$

Constructing the finite switching time instants $k_1 < \cdots < k_{s-1} < k_s$, $s = 1, 2, \cdots, N$ of the switching signal σ during $[0, k)$. Then, let $N_\sigma\left[0, k\right)$ denote the switching numbers of σ during $[0, k)$.

By combining (15) with (16) during $[0, k)$, based on (9), one can get

$$\begin{aligned}
V_\sigma(k) &\le \alpha^{k-k_s}V_\sigma(k_s) - \sum_{l=k_s}^{k-1}\alpha^{k-l-1}\Gamma(l) \le \alpha^{k-k_s}\mu V_\sigma(k_s) - \sum_{l=k_s}^{k-1}\alpha^{k-l-1}\Gamma(l) \\
&\le \alpha^{k-k_{s-1}}\mu V_\sigma(k_{s-1}) - \mu\sum_{l=k_{s-1}}^{k_s-1}\alpha^{k-l-1}\Gamma(l) - \sum_{l=k_s}^{k-1}\alpha^{k-l-1}\Gamma(l) \\
&\le \alpha^k\mu^{N_\sigma(0,k)}V_\sigma(0) - \mu^{N(0,k)}\sum_{l=0}^{k_1-1}\alpha^{k-l-1}\Gamma(l) \\
&\quad -\mu^{N(k_1,k)}\sum_{l=k_1}^{k_2-1}\alpha^{k-l-1}\Gamma(l) - \cdots - \sum_{l=k_s}^{k-1}\alpha^{k-l-1}\Gamma(l) \\
&\le \alpha^k\mu^{N_\sigma(0,k)}V_\sigma(0) - \sum_{l=0}^{k-1}\mu^{N_\sigma(l,k)}\alpha^{k-l-1}\Gamma(l).
\end{aligned} \tag{18}$$

Considering (4), (8) and (18), we have

$$V_\sigma(k) \le e^{\lambda k}V_\sigma(0) - \sum_{l=0}^{k-1}e^{\lambda(k-l)}\alpha^{-1}\Gamma(l) \tag{19}$$

where $\lambda = \ln\alpha + \ln\mu/\tau_a$. It follows from (8) that $\lambda < 0$.

Assuming the zero disturbance input $w_k = 0$ to the state equation of system (3), it follows from (19) that $V_\sigma(k) \le e^{\lambda k}V_\sigma(0)$. In addition, there exist some constants $a_\sigma > 0$ and $b_\sigma > 0$ such that $a_\sigma\left\|\tilde{x}_k\right\|^2 \le V_\sigma(k)$, $V_\sigma(0) \le b_\sigma\left\|\tilde{x}_0\right\|^2$. Then, we have $\left\|\tilde{x}_k\right\| \le \sqrt{b_\sigma/a_\sigma}e^{\lambda k/2}\left\|\tilde{x}_0\right\|$. Therefore, system (3) is exponentially stable.

Now, we consider the following performance index:

$$J = \sum_{k=0}^{\infty} \left[\tilde{e}_k^T \tilde{e}_k - \gamma^2 w_k^T w_k \right].$$

For any nonzero $w_k \in l_2[0,\infty)$ and under zero-initial condition, one has $\sum_{l=0}^{k-1} e^{\lambda(k-l)}\alpha^{-1}\Gamma(l) \leq 0$ from (19). Note in the above inequality that $\sum_{l=0}^{k-1} e^{\lambda(k-l)}\alpha^{-1}\tilde{e}_l^T\tilde{e}_l$ is summable from $k = 1$ to ∞ since $\sum_{l=0}^{k-1} e^{\lambda(k-l)}\alpha^{-1} w_l^T w_l$ is summable for any $w_l \in l_2[0,\infty)$ in the same interval. Then, from $k = 1$ to ∞, we obtain that $\sum_{k=1}^{\infty}\sum_{l=0}^{k-1} e^{\lambda(k-l)}\alpha^{-1}\Gamma(l) \leq 0$. Exchanging the double-sum region yields

$$\sum_{l=0}^{\infty}\Gamma(l)\sum_{k=l+1}^{\infty} e^{\lambda(k-l)}\alpha^{-1} = \frac{e^{\lambda}\alpha^{-1}}{1-e^{\lambda}}\sum_{l=0}^{\infty}\Gamma(l) \leq 0$$

which means that $J \leq 0$. Then, one has $\|\tilde{z}_k\|_2 \leq \gamma \|w_k\|_2$ for nonzero $w_k \in l_2[0,\infty)$. The proof is completed.

The stability analysis is given above; in the sequel, the parameters of the reduced-order systems are designed by the following theorem.

Theorem 3.2. *Consider system (1) and let $0 < \alpha < 1$, $\gamma > 0$ and $\mu \geq 1$ be given constants, if there exist symmetric and positive definite matrices $P_i \in R^{m\times m}$, $Q_i \in R^{m\times m}$, $Z_i \in R^{m\times m}$, and matrices $X_i \in R^{n\times n}$, $Y_i \in R^{q\times q}$, $M_{si} \in R^{m\times m}$, $m = n + q$, $s = 1, 2, 3$, $\forall i \in \mathcal{P}$ such that (7), ADT satisfies (8) and the following inequalities hold*

$$\begin{bmatrix} \Xi_1 & \tilde{\Xi}_4 \\ * & \tilde{\Xi}_5 \end{bmatrix} < 0 \tag{20}$$

where

$$\tilde{\Xi}_4 = \begin{bmatrix} M_{1i} & \tilde{\Psi}_{15}^T & \tilde{\Psi}_{16}^T & \tilde{\Psi}_{17}^T \\ M_{2i} & \tilde{\Psi}_{25}^T & \tilde{\Psi}_{26}^T & \tilde{\Psi}_{26}^T \\ M_{3i} & \tilde{\Psi}_{35}^T & \tilde{\Psi}_{36}^T & \tilde{\Psi}_{36}^T \end{bmatrix}, \ \Omega_i = \begin{bmatrix} X_i & 0 \\ Y_i^T E & Y_i \end{bmatrix}$$

$$\tilde{\Xi}_5 = diag\left\{ -\alpha^d d^{-1} Z_i \quad -I \quad \Psi_{66} \quad \Psi_{77} \right\}, \ \tilde{\Psi}_{17} = \tilde{\Psi}_{16} - \Omega_i$$

$$\Psi_{66} = P_i - \left(\Omega_i + \Omega_i^T\right), \ \Psi_{77} = dZ_i - \left(\Omega_i + \Omega_i^T\right),$$

$$\tilde{\Psi}_{15} = \begin{bmatrix} C_i & -\bar{S}_i \end{bmatrix}, \ \tilde{\Psi}_{25} = \begin{bmatrix} C_{di} & -\bar{N}_i \end{bmatrix}, \ \tilde{\Psi}_{35} = D_i - \bar{R}_i, \ E = \begin{bmatrix} I & 0 \end{bmatrix}$$

$$\tilde{\Psi}_{16} = \begin{bmatrix} X_i A_i & 0 \\ Y_i^T E A_i & L_i \end{bmatrix}, \ \tilde{\Psi}_{26} = \begin{bmatrix} X_i A_{di} & 0 \\ Y_i^T E A_{di} & H_i \end{bmatrix}, \ \tilde{\Psi}_{36} = \begin{bmatrix} X_i B_i \\ Y_i^T E B_i + T_i \end{bmatrix}.$$

Then, there exists an admissible reduced-order model (2) such that system (3) is exponentially stable with H_∞ norm bound γ under switching signals with ADT satisfying (8). In addition, if (7) and (20) have a feasible solution, switching signals with ADT satisfying (8) are found and the parameters of an admissible reduced-order model can be constructed by

$$\begin{bmatrix} A_{ri} & A_{rdi} & B_{ri} \\ C_{ri} & C_{rdi} & D_{ri} \end{bmatrix} = \begin{bmatrix} Y_i^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} L_i & H_i & T_i \\ \bar{S}_i & \bar{N}_i & \bar{R}_i \end{bmatrix}. \tag{21}$$

Proof: By Theorem 3.1, system (3) under switching signals with ADT satisfying (8) is exponentially stable with H_∞ norm bound γ, if LMIs (6) and (7) hold.

For $P_i > 0$ and the nonsingular matrix Ω_i, from the fact that $(P_i - \Omega_i) P_i^{-1} (P_i - \Omega_i)^T \geq 0$, we have $-\Omega_i P_i^{-1} \Omega_i^T \leq P_i - \left(\Omega_i + \Omega_i^T\right)$. Similarly, one has $-\Omega_i (dZ_i)^{-1} \Omega_i^T \leq dZ_i - \left(\Omega_i + \Omega_i^T\right)$. Therefore, if the following inequality holds,

$$\begin{bmatrix} \Xi_1 & \Xi_4 \\ * & \tilde{\Xi}_5 \end{bmatrix} < 0 \tag{22}$$

where

$$\Xi_4 = \begin{bmatrix} M_{1i} & \tilde{C}_i^T & \tilde{A}_i^T \Omega_i^T & \Psi_{17} \\ M_{2i} & \tilde{C}_{di}^T & \tilde{A}_{di}^T \Omega_i^T & \tilde{A}_{di}^T \Omega_i^T \\ M_{3i} & \tilde{D}_i^T & \tilde{B}_i^T \Omega_i^T & \tilde{B}_i^T \Omega_i^T \end{bmatrix}, \ \Psi_{17} = (\tilde{A}_i^T - I)\Omega_i^T$$

one can infer

$$\begin{bmatrix} \Xi_1 & \Xi_4 \\ * & \Xi_6 \end{bmatrix} < 0 \tag{23}$$

where

$$\Xi_6 = diag\left\{ -\alpha^d d^{-1} Z_i \quad -I \quad \hat{\Psi}_{66} \quad \hat{\Psi}_{77} \right\}, \ \hat{\Psi}_{66} = -\Omega_i P_i^{-1} \Omega_i^T, \ \hat{\Psi}_{77} = -\Omega_i (dZ_i)^{-1} \Omega_i^T.$$

Performing a congruence transformation to (23) via $diag\left\{I, I, I, I, I, \Omega_i^{-T}, \Omega_i^{-T}\right\}$ yields (6). In what follows, we will show (20) ensures that (22) holds.

Note that from (20), we have

$$\Omega_i + \Omega_i^T = \begin{bmatrix} X_i + X_i^T & E^T Y_i \\ * & Y_i + Y_i^T \end{bmatrix} > 0 \tag{24}$$

which means that X_i and Y_i are nonsingular. And thus, by using system (3), the following definitions are introduced,

$$L_i \triangleq Y_i A_{ri}, \ H_i \triangleq Y_i A_{rdi}, \ T_i \triangleq Y_i B_{ri}, \ \bar{S}_i \triangleq C_{ri}, \ \bar{N}_i \triangleq C_{rdi}, \bar{R}_i \triangleq D_{ri}. \tag{25}$$

By using (3) and (25), we can get

$$\Omega_i \tilde{A}_i = \tilde{\Psi}_{110}, \ \Omega_i \tilde{A}_{di} = \tilde{\Psi}_{210}, \ \Omega_i \tilde{B}_i = \tilde{\Psi}_{310}, \ \tilde{C}_i = \tilde{\Psi}_{19}, \ \tilde{C}_{di} = \tilde{\Psi}_{29}, \tag{26}$$

and substitute them into (22) to have (20). This means that if (20) holds, (6) is true, which implies that the error system (3) is exponentially stable with an H_∞ performance index. Meanwhile, from (25) that the parameters of a reduced-order model are given by (21). The proof is completed.

Remark 3.1. *It should be pointed out that the Lyapunov function (9) is general with comparison to the existing Lyapunov function. When $\alpha = 1$ in (9), the Lyapunov function (9) reduces to those in* [23] *for non-switched delay systems. When $\alpha = 0$, the Lyapunov function (9) is capable of dealing with switched systems* [25]*. It is obvious to see from Ξ_2 and Ξ_3 in (6) that it is difficult to deal with the problem of model reduction due to the existence of P_i^{-1}, which leads to some product terms between P_i and $\tilde{A}_i$, $\tilde{A}_{di}$ and $\tilde{B}_i$. There exists the same case between Z_i and $\tilde{A}_i$, $\tilde{A}_{di}$ and $\tilde{B}_i$. To overcome the difficulties, motivated by the work* [29]*, an auxiliary slack matrix Ω_i is introduced in the proof of Theorem 3.1 such that these product terms are decoupled. That is, P_i and Z_i are not involved in any product with $\tilde{A}_i$, $\tilde{A}_{di}$ and $\tilde{B}_i$ in (22). This makes it feasible to construct a reduced-order model. On the other hand, the introduction of the auxiliary slack matrices leads to the use of so much resources such as computer storage space. Therefore, it is the future topic to make the proper tradeoff between them.*

Remark 3.2. *Note that many existing results on model reduction in the discrete-time context are formulated in terms of LMIs with inverse constraints or other non-convex conditions, (see, for instance,* [21,23] *and references therein), which is difficult to obtain the numerical solutions. An LMI-based solution is obtained for the model reduction problem of switched systems in* [25]*, but the time delay is not taken into account. The main reason is that the existence of time delay will lead to the common Lyapunov matrices, which is the conservatism* [15]*. To reduce the conservatism, a factor α is introduced such that all Lyapunov matrices are piecewise.*

4. **Numerical Example.** In this section, an example is provided to illustrate the effectiveness of the proposed method.

Consider discrete-time switched system (1) consisting of two subsystems, with parameters from [23,25]:

$$A_1 = \rho\begin{bmatrix} 1.3 & 2.2 & -1.3 & 0.8 \\ 0.5 & -0.3 & 1.9 & 0.6 \\ -0.7 & -0.5 & -0.4 & -1.2 \\ -1.7 & 2.1 & 0.3 & 2.8 \end{bmatrix}, \quad A_2 = \rho\begin{bmatrix} 1.1 & 2.2 & -1.3 & 0.8 \\ 0.5 & -0.3 & 1.5 & 0.6 \\ -0.7 & -0.3 & -0.4 & -1.2 \\ -1.7 & 2.1 & 0.3 & 2.0 \end{bmatrix}$$
$$A_{d1} = A_{d2} = \rho\begin{bmatrix} 0.2 & 0.1 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.2 & 0.1 \\ 0 & 0 & 0 & 0.2 \end{bmatrix}, \quad B_1 = \rho\begin{bmatrix} 1.9 \\ -1.8 \\ 1.6 \\ -0.8 \end{bmatrix}, \quad B_2 = \rho\begin{bmatrix} 2.3 \\ -1.3 \\ 1.6 \\ -0.4 \end{bmatrix}$$
$$C_1 = \rho\begin{bmatrix} 12 & 5 & 0.3 & 2.8 \end{bmatrix}, \qquad C_2 = \rho\begin{bmatrix} 12 & 5 & 0.3 & 2.8 \end{bmatrix}$$
$$C_{d1} = C_{d2} = \rho\begin{bmatrix} 0.2 & 0.5 & 0.1 & 0.9 \end{bmatrix}, \quad D_1 = D_2 = \rho, \quad d = 2.$$

The parameter A_{di} of time delay term stems from those of [23] while others are from those in [25] since the latter is concerned with switched delay-free systems. Here, we are interested in designing a q-order ($q < 4$) system (2) and find out an admissible ADT switching signals such that the model error system (3) is exponentially stable with H_∞ norm bound γ. Given $\rho = 0.1$, $\alpha = 0.9$, $\mu = 1.2$ and $\gamma = 2.0$, by utilizing the LMI Toolbox, it follows from Theorem 3.2 that the following reduced-order models can be given:

Third order model:

$$A_{r1} = \begin{bmatrix} 0.2610 & 0.0527 & -0.0216 \\ 0.0475 & 0.2262 & 0.0234 \\ -0.0099 & -0.0128 & 0.2653 \end{bmatrix}, \quad A_{dr1} = \begin{bmatrix} 0.0008 & 0.0026 & -0.0021 \\ 0.0059 & -0.0013 & 0.0037 \\ 0.0008 & -0.0007 & 0.0047 \end{bmatrix}$$
$$B_{r1} = \begin{bmatrix} -0.3226 & 0.5332 & -0.3734 \end{bmatrix}^T, \quad C_{r1} = \begin{bmatrix} -0.0618 & 0.0461 & -0.0118 \end{bmatrix}$$
$$C_{dr1} = \begin{bmatrix} -0.0125 & 0.0126 & -0.0113 \end{bmatrix}, \quad D_{r1} = -0.7156$$
$$A_{r2} = \begin{bmatrix} 0.2554 & 0.0497 & -0.0243 \\ 0.0322 & 0.2466 & 0.0058 \\ -0.0117 & -0.0147 & 0.2678 \end{bmatrix}, \quad A_{dr2} = \begin{bmatrix} 0.0006 & 0.0028 & -0.0021 \\ 0.0058 & -0.0012 & 0.0039 \\ -0.0011 & 0.0014 & 0.0027 \end{bmatrix}$$
$$B_{r2} = \begin{bmatrix} -0.6795 & 0.1368 & -0.5570 \end{bmatrix}^T, \quad C_{r2} = \begin{bmatrix} -0.0452 & 0.0055 & -0.0121 \end{bmatrix}$$
$$C_{dr2} = \begin{bmatrix} -0.0039 & 0.0071 & -0.0053 \end{bmatrix}, \quad D_{r2} = -0.3804.$$

Second order model:

$$\left[\begin{array}{c|c|c} A_{r1} & A_{rd1} & B_{r1} \\ \hline C_{r1} & C_{rd1} & D_{r1} \end{array}\right] = \left[\begin{array}{cc|cc|c} 0.2719 & 0.0399 & 0.0000 & 0.0032 & -0.5025 \\ 0.0292 & 0.2509 & 0.0043 & 0.0005 & 0.6031 \\ \hline -0.0661 & 0.0372 & -0.0083 & 0.0112 & -0.6833 \end{array}\right]$$

$$\left[\begin{array}{c|c|c} A_{r2} & A_{rd2} & B_{r2} \\ \hline C_{r2} & C_{rd2} & D_{r2} \end{array}\right] = \left[\begin{array}{cc|cc|c} 0.2711 & 0.0380 & 0.0007 & 0.0027 & -0.7842 \\ 0.0242 & 0.2563 & 0.0035 & 0.0009 & 0.4300 \\ \hline -0.0600 & -0.0140 & -0.0004 & 0.0056 & -0.2979 \end{array}\right].$$

First order model:

$$\left[\begin{array}{c|c|c} A_{r1} & A_{rd1} & B_{r1} \\ \hline C_{r1} & C_{rd1} & D_{r1} \end{array}\right] = \left[\begin{array}{c|c|c} 0.2532 & -0.0024 & -0.7170 \\ \hline -0.0600 & -0.0086 & -0.6759 \end{array}\right]$$

$$\left[\begin{array}{c|c|c} A_{r2} & A_{rd2} & B_{r2} \\ \hline C_{r2} & C_{rd2} & D_{r2} \end{array}\right] = \left[\begin{array}{c|c|c} 0.2502 & -0.0018 & -1.0418 \\ \hline -0.0496 & 0.0006 & -0.3038 \end{array}\right].$$

In addition, the minimal ADT $\tau^* = 1.7305$ is obtained by (8). Thus, the reduced-order switched model under ADT switching signals is constructed by the proposed approach.

Remark 4.1. *When non-switched delay systems are considered, the model reduction problem of such systems is addressed in* [23]*. By making use of Finsler's Lemma, sufficient condition with a non-convex constraint is established, which is difficult to find the numerical solution. Although many numerical approaches have been proposed to overcome this difficulty, for example, the cone complementarity linearization method* [26] *and sequential linear programming matrix method* [27]*, the computation is very heavy. In the present paper,* Ω *are introduced to obtained the strict LMI-based condition. The computation is obviously light. Thus, the proposed approach is less conservative from the computational point of view.*

5. **Conclusions.** In this paper, we have studied the problem of the exponential H_∞ model reduction for discrete-time switched systems under ADT switching signals. Time delay under consideration is interval time-varying. Based on H_∞ performance analysis and new linearization technique, sufficient conditions for the solvability of this problem have been established in terms of strict LMIs, which avoid the numerous work of the calculation and decrease the computation complexity. An example has been given to show the effectiveness of the proposed methods.

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REFERENCES

[1] Z. Sun and S. S. Ge, *Switched Linear Systems: Control and Design*, Springer-verlag, New York, 2005.

[2] M. S. Branicky, Multiple Lyapunov functions and other analysis tools for switched and hybrid systems, *IEEE Trans. Automat. Control*, vol.43, no.4, pp.475-482, 1998.

[3] J. Lian and J. Zhao, Output feedback variable structure control for a class of uncertain switched systems, *Asian J. Control*, vol.11, no.1, pp.31-39, 2009.

[4] J. Lian, J. Zhao and G. M. Dimirovski, Integral sliding mode control for a class of uncertain switched nonlinear systems, *European Journal of Control*, vol.16, no.1, pp.16-22, 2010.

[5] J. Daafouz, P. Riedinger and C. Iung, Stability analysis and control synthesis for switched systems: A switched Lyapunov function approach, *IEEE Trans. Automat. Control*, vol.47, no.11, pp.1883-1887, 2002.

[6] J. P. Hespanha and A. S. Morse, Stability of switched systems with average dwell-time, *Proc. of the 38th Conf. Decision Control*, pp.2655-2660, 1999.

[7] J. Lian and K. Zhang, Exponential stability analysis for switched Cohen-Grossberg neural networks with average dwell time, *Nonlinear Dynamics*, vol.63, no.3, pp.331-343, 2011.

[8] J. Lian and J. Zhao, Robust H_∞ sliding mode control for a class of uncertain switched delay systems, *International Journal of Systems Science*, vol.40, no.8, pp.855-866, 2009.

[9] Y. Wang, W. Wang and D. Wang, LMI approach to design fault detection filter for discrete-time switched systems with state delays, *International Journal of Innovative Computing, Information and Control*, vol.6, no.1, pp.387-397, 2010.

[10] L. Zhang and B. Jiang, Stability of a class of switched linear systems with uncertainties and average dwell time switching, *International Journal of Innovative Computing, Information and Control*, vol.6, no.2, pp.667-676, 2010.

[11] C. Cui, F. Long and C. Li, Disturbance attenuation for switched system with continuous-time and discrete-time subsystems: State feedback case, *ICIC Express Letters*, vol.4, no.1, pp.205-212, 2010.

[12] Z. Liu, H. Zhang, Q. Sun and D. Yang, Output feedback switching controller design for state-delayed linear systems with input quantization and disturbances, *ICIC Express Letters*, vol.4, no.5(B), pp.1791-1797, 2010.

[13] J. Lian, Z. Feng and P. Shi, Observer design for switched recurrent neural networks: An average dwell time approach, *IEEE Trans. Neural Networks*, vol.22, no.10, pp.1547-1556, 2011.

[14] L. Zhang, P. Shi and M. Basin, Robust stability and stabilisation of uncertain switched linear discrete time-delay systems, *IET Control Theory Appl.*, vol.2, no.7, pp.606-614, 2008.

[15] L. Zhang, P. Shi, E. K. Boukas and C. Wang, Robust L_2-L_∞ filtering for switched linear discrete time-delay systems with polytopic uncertainties, *IET Control Theory Appl.*, vol.1, no.3, pp.722-730, 2007.

[16] D. Wang, W. Wang and P. Shi, Exponential H_∞ filtering for switched linear systems with interval time-varying delay, *Int. J. Robust and Nonlinear Control*, vol.19, no.5, pp.532-551, 2009.

[17] Y. Li, J. Lam and X. Luo, Hankel norm model reduction of uncertain neutral stochastic time-delay systems, *International Journal of Innovative Computing, Information and Control*, vol.5, no.9, pp.2819-2828, 2009.

[18] J. T. Spanos, M. H. Milman and D. L. Mingori, A new algorithm for L_2 optimal model reduction, *Automatica*, vol.28, no.5, pp.897-909, 1992.

[19] F. Tjärnström and L. Ljung, L_2 model reduction and variance reduction, *Automatica*, vol.38, no.9, pp.1517-1530, 2002.

[20] W. Y. Yan and J. Lam, An approximate approach to H_2 optimal model reduction, *IEEE Trans. Automat. Control*, vol.44, no.7, pp.1341-1358, 1999.

[21] S. Xu and J. Lam, H_∞ model reduction for discrete-time singular systems, *Syst. Control Lett.*, vol.48, no.2, pp.121-133, 2003.

[22] J. Lam, H. Gao and C. Wang, H_∞ model reduction of linear systems with distributed delay, *IEE Control Theory Appl.*, vol.152, no.6, pp.662-674, 2005.

[23] H. Gao, J. Lam, C. Wang and S. Xu, H_∞ model reduction for discrete time-delay systems: Delay-independent and dependent approaches, *Int. J. Control*, vol.77, no.4, pp.321-335, 2004.

[24] L. Zhang, B. Huang and J. Lam, H_∞ model reduction of Markovian jump linear systems, *Syst. Control Lett.*, vol.50, no.2, pp.103-118, 2003.

[25] L. X. Zhang, P. Shi, E. K. Boukas and C. H. Wang, H_∞ model reduction for uncertain switched linear discrete-time systems, *Automatica*, vol.44, no.11, pp.2944-2949, 2008.

[26] L. El Ghaoui, F. Oustry and M. Aitrami, A cone complementarity linearization algorithm for static outputfeedback and related problems, *IEEE Trans. Automat. Control*, vol.42, no.8, pp.1171-1176, 1997.

[27] F. Leibfritz, An LMI-based algorithm for designing suboptimal static H_2/H_∞ output feedback controllers, *SIAM J. Control Optim.*, vol.57, no.6, pp.1711-1735, 2001.

[28] S. Xu and J. Lam, Exponential H_∞ filter design for uncertain Takagi-Sugeno fuzzy systems with time delay, *Eng. Appl. Artificial Intelligence*, vol.17, no.6, pp.645-659, 2004.

[29] M. C. De Oliveira, J. Bernussou and J. C. Geromel, A new discrete-time robust stability condition, *Syst. Control Lett.*, vol.37, no.4, pp.261-265, 1999.

[30] X. F. Jiang, Q. L. Han and X. H. Yu, Stability criteria for linear discrete-time systems with interval-like time-varying delay, *Proc. of the American Control Conference*, Portland Oregon, USA, pp.2817-2822, 2005.