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ADAPTIVE OUTPUT FEEDBACK STABILIZATION USING MT-FILTERS FOR NONLINEAR SYSTEMS WITH INPUT AND OUTPUT TIME-DELAY

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Abstract. *This paper investigates the problem of adaptive output feedback stabilization using MT-filters and the backstepping design method for a class of nonlinear systems with unknown input and output time-delay. It is shown that all the signals in the closed-loop system are globally uniformly bounded, and the output can be regulated to zero.*
Keywords: Nonlinear systems, Input time-delay, Adaptive output feedback stabilization, MT-filters

1. **Introduction.** Since time-delay phenomena commonly exist in many practical systems such as biological reactors, rolling mills, economical systems, and the existence of time-delay is often a significant cause of instability and deteriorative performance, so the control design of nonlinear time-delay systems has been received much attention; see, e.g., [1, 2, 6, 10, 14, 17-19, 25] and the references therein. In the past decade, some results have been achieved when solving the stabilizing problem for nonlinear time-delay systems by using backstepping method. In [5], adaptive neural control cooperating with iterative backstepping was presented for strict-feedback nonlinear systems with unknown time-delay. The problem of robust output feedback backstepping control for strict-feedback nonlinear time-delay systems was considered by [7]. [8] investigated the robust output tracking control for nonlinear time-delay systems. In the newest two papers, [4, 13] considered state feedback and output feedback respectively for stochastic high-order nonlinear time-delay systems.

Up to now, however, most of the existing papers only consider nonlinear systems with state time-delay. In the only few papers on nonlinear systems with input time-delay, [20] considered adaptive control of linear systems with unknown input time-delay by using conventional pole placement adaptive scheme. The input delay compensation for forward complete and strict-feedforward nonlinear systems was solved by [11]. [21] considered the adaptive stabilization problem for feedforward nonlinear systems with time-delays. In [26], nonlinear systems with unknown input time-delay were considered by using K-filters and backstepping design method.

In the widely cited in-depth monograph [9] on the backstepping design method, Krstić et al. systematically studied two sets of filters, namely K-filters and MT-filters with different merits and demerits, and applied them respectively to the design of adaptive output feedback controllers. The design with MT-filters, which was firstly proposed by [15, 16], is motivated by the idea of using an adaptive observer for output feedback control.

Motivated by [26] and the advantages of MT-filters, the purpose of this paper is to further consider the similar problem as in [26] by using MT-filters and the backstepping design method. The contributions of this paper are as follows:

(i) Compared with [26], this paper considers more general nonlinear systems with unknown input time-delay and output time-delay.

(ii) Since the unmodeled dynamics appear in the output of the system, the adaptive laws obtained by using the conventional MT-filtered transformation eq.(8.156) in [9] are not available for measurement. In this paper, by introducing a new filtered transformation, we solve the important problem satisfactorily.

(iii) For this control scheme, we rigorously show that all the signals in the closed-loop system based on MT-filters are globally uniformly bounded, and the output is regulated to zero. The effectiveness of the scheme is demonstrated by a simulation example.

The paper is organized as follows. The problem is formulated in Section 2. An adaptive output feedback controller is designed and analyzed in Section 3 and Section 4, following a simulation example in Section 5. Section 6 concludes this paper.

2. **Problem Formulation.**

Consider the following nonlinear systems with input time-delay and output time-delay

$$\begin{aligned} y(t) = {} & \frac{B(s)}{A(s)}(u(t) + \mu_1\Delta_1(s)u(t-\tau) + \mu_2\Delta_2(s)u(t)) \\ & + \frac{D(s)}{A(s)}(f(y(t)) + \mu_1\Delta_1(s)f(y(t-\tau)) + \mu_2\Delta_2(s)f(y(t))) + \mu_3\Delta_3(s)y(t), \end{aligned} \quad (1)$$

where $u(t) \in R$, $y(t) \in R$ are the system input and output, respectively, s denotes the differential operator $\frac{d}{dt}$, τ is an unknown positive constant time-delay, $f(\cdot) \in R^n$ is a nonlinear function, $A(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_0$, $B(s) = b_m s^m + b_{m-1}s^{m-1} + \cdots + b_0$, $D(s) = (s^{n-1}, \cdots, s, 1)$, $\Delta_1(s)$, $\Delta_2(s)$ and $\Delta_3(s)$ are some rational functions of s, μ_1, μ_2 and μ_3 are positive constant scalars.

Remark 2.1. *$\mu_1\Delta_1(s)u(t-\tau)$ denotes the unmodeled dynamics from the system input with time-delay, $\mu_1\Delta_1(s)f(y(t-\tau))$ is the unmodeled dynamics from the nonlinear function with output time-delay, $\mu_2\Delta_2(s)u(t)$, $\mu_2\Delta_2(s)f(y(t))$ and $\mu_3\Delta_3(s)y(t)$ denote the unmodeled dynamics from the system input, nonlinear function and output, respectively. Obviously, such unmodeled dynamics are more general than those in* [26].

In this paper, we need the following assumptions:

Assumption 1: For system (1), a_i and b_j $(i = 0, \cdots, n-1,\ j = 0, \cdots, m)$ are unknown constants, $B(s)$ is a Hurwitz polynomial, the order n, the relative degree $\varrho = n - m$, and the sign of the high frequency gain b_m are known.

Assumption 2: $\Delta_1(s)$, $\Delta_2(s)$ and $\Delta_3(s)$ are stable and strictly proper with unity high frequency gains.

Remark 2.2. *Assumption 1 is a general assumption for the adaptive control design of nonlinear systems as in* [9]. *The purpose of Assumption 2 is to lead to $A_{\bar{f}}$, A_g, A_h in the realization (34)-(36) of $\Delta_1(s)$, $\Delta_2(s)$ and $\Delta_3(s)$ being Hurwitz.*

The objective of this paper is to design an adaptive output feedback controller for system (1) under Assumptions 1 and 2 such that all the signals in the closed-loop system are globally uniformly bounded, and the output is regulated to zero.

3. **The Design of Adaptive Controller Based on MT-Filters.** To simplify the procedure, we sometimes denote $X(t)$ by X for any variable $X(t)$.

By Appendix, system (1) can be transformed into the following state-space realization

$$\dot{x} = Ax + F^T(u, y)\theta + a\aleph + f(y),$$
$$y = x_1 + \aleph, \tag{2}$$

where

$$A = \begin{bmatrix} 0_{(n-1)\times 1} & I_{n-1} \\ 0 & 0_{1\times(n-1)} \end{bmatrix}, \quad F^T(u, y) = \left[\begin{bmatrix} 0_{(\varrho-1)\times(m+1)} \\ I_{m+1} \end{bmatrix} u, -I_n y\right],$$
$$\theta = [b^T, a^T]^T, \quad a = [a_{n-1}, \cdots, a_0]^T, \quad b = [b_m, \cdots, b_0]^T,$$
$$\aleph = \mu_1\Delta_1(s)x_1(t-\tau) + \mu_2\Delta_2(s)x_1 + \mu_3\Delta_3(s)y. \tag{3}$$

To estimate the system state, a MT-filter only using input and output is designed as

$$\dot{\xi} = A_l\xi + B_l f(y), \quad \xi \in R^{n-1},$$
$$\dot{\Omega}^T = A_l\Omega^T + B_l F^T(u, y), \quad \Omega^T \in R^{(n-1)\times(n+m+1)}, \tag{4}$$

where

$$A_l = \begin{bmatrix} -\bar{l} & \begin{matrix} I_{n-2} \\ 0_{1\times(n-2)} \end{matrix} \end{bmatrix}, \quad B_l = \left[-\bar{l}, I_{n-1}\right], \quad l = \left[1, \bar{l}_1, \cdots, \bar{l}_{n-1}\right]^T = \begin{bmatrix} 1 \\ \bar{l} \end{bmatrix}, \tag{5}$$

with $\bar{l}_1, \cdots, \bar{l}_{n-1}$ being the coefficients of any Hurwitz polynomial $L(s) = s^{n-1} + \bar{l}_1 s^{n-2} + \cdots + \bar{l}_{n-1}$. To reduce the order of filters, Ω^T is decomposed into $\Omega^T = [v_m, \cdots, v_0, \Xi]$, where $v_i \in R^{n-1}$ $(i = 0, \cdots, m)$ is the ith vector of v and $\Xi = [\delta_{n-1}, \cdots, \delta_0] \in R^{(n-1)\times n}$, $\delta_k \in R^{n-1}$ $(k = 0, \cdots, n-1)$ is the kth vector of δ. By using

$$(A_l)^i e_{n-1,n-1} = B_l e_{n,n-i}, \quad i = 0, 1, \cdots, n-1, \tag{6}$$
$$\dot{\lambda} = A_l\lambda + e_{n-1,n-1}u, \quad \lambda \in R^{n-1}, \tag{7}$$

one gets

$$v_i = (A_l)^i\lambda, \quad i = 0, \ldots, m, \tag{8}$$

where e_{ik} denotes the kth coordinate vector in R^i. From

$$\dot{\eta} = A_l\eta + e_{n-1,n-1}y, \quad \eta \in R^{n-1}, \tag{9}$$
$$\dot{\Xi} = A_l\Xi - B_l y, \tag{10}$$

it is easy to obtain that $\delta_k = -(A_l)^k\eta$, $k = 0, \ldots, n-1$.

Due to the presence of $\aleph$ in (2), we introduce the following filtered transformation

$$\chi = x - \begin{bmatrix} -\aleph \\ \xi + \Omega^T\theta \end{bmatrix}, \tag{11}$$

from which, and (2)-(5), a tedious but straightforward calculation leads to

$$\dot{\chi} = A\chi + l(\omega_0 + \omega^T\theta) + (a + se_{n1})\aleph,$$
$$y = \chi_1, \tag{12}$$

where $\omega_0 = \xi_1 + f_1$, $\omega^T = F_1^T + \Omega_1^T$, χ_1, ξ_1, f_1, F_1^T and Ω_1^T represent the first row of χ, ξ, f, F^T and Ω^T, respectively. Since θ is unknown, the adaptive observer for χ can be chosen as

$$\dot{\hat{\chi}} = A\hat{\chi} + K_0(y - \hat{\chi}_1) + l(\omega_0 + \omega^T\hat{\theta}), \tag{13}$$

where $\hat{\theta}$ is the estimate of θ, $K_0 = (A + c_0 I_n)l$, and c_0 is a positive constant. Defining

$$\varepsilon = \chi - \hat{\chi}, \tag{14}$$

and using $y-\hat{\chi}_1=\chi_1-\hat{\chi}_1=e_{n1}^T(\chi-\hat{\chi})$, by (12)-(14) and some computations, one gets

$$\dot{\varepsilon}=A_0\varepsilon+l\omega^T\tilde{\theta}+(a+se_{n1})\aleph, \tag{15}$$

where $\tilde{\theta}=\theta-\hat{\theta}$, $A_0=A-K_0e_{n1}^T$. Obviously, $e_{n1}^T(sI_n-A_0)^{-1}l=\frac{1}{(s+c_0)}$. From (3), and the definitions of Ω^T and ω^T, one leads to

$$\omega^T=F_1^T+\Omega_1^T=[v_{m1},\cdots,v_{01},\Xi_1-ye_{n1}^T], \tag{16}$$

where v_{i1} $(i=0,\cdots,m)$ denotes the first entry of v_i and Ξ_1 denotes the first row of Ξ. By (12), (14) and (16), one obtains

$$\begin{aligned}\dot{y}=\dot{\chi}_1&=\chi_2+\omega_0+\omega^T\theta+(s+a_{n-1})\aleph\\&=\hat{\chi}_2+\omega_0+\omega^T\theta+\varepsilon_2+(s+a_{n-1})\aleph\\&=b_mv_{m1}+\hat{\chi}_2+\omega_0+\bar{\omega}^T\theta+\varepsilon_2+(s+a_{n-1})\aleph,\end{aligned} \tag{17}$$

where $\bar{\omega}^T=[0,v_{m-1,1},\cdots,v_{01},\Xi_1-ye_{n1}^T]$. Now we replace (2) with the following new system, whose states depend on filters (4), (7) and (9), and thus are available for control design

$$\begin{aligned}&\dot{y}=b_mv_{m1}+\hat{\chi}_2+\omega_0+\bar{\omega}^T\theta+\varepsilon_2+(s+a_{n-1})\aleph,\\&\dot{v}_{mi}=v_{m,i+1}-\bar{l}_iv_{m1},\quad i=1,\cdots,\varrho-2,\\&\dot{v}_{m,\varrho-1}=u+v_{m\varrho}-\bar{l}_{\varrho-1}v_{m1},\end{aligned} \tag{18}$$

where v_{mi} $(i=1,\cdots,\varrho)$ is the ith element of v_m. Define the change of coordinates

$$z_1=y,\quad z_i=v_{m,i-1}-\alpha_{i-1},\quad i=2,\dots,\varrho. \tag{19}$$

For (18), by using conventional backstepping design method, choosing the control law

$$\begin{aligned}&u=\alpha_\varrho-v_{m\varrho},\quad \alpha_1=\hat{\rho}\bar{\alpha}_1,\quad \bar{\alpha}_1=-(c_1+d_1)z_1-\hat{\chi}_2-\omega_0-\bar{\omega}^T\hat{\theta},\\&\alpha_2=-\hat{b}_mz_1-\left[c_2+d_2\left(\frac{\partial\alpha_1}{\partial y}\right)^2\right]z_2+\frac{\partial\alpha_1}{\partial\hat{\rho}}\dot{\hat{\rho}}+\frac{\partial\alpha_1}{\partial\hat{\theta}}\Gamma\tau_2+\beta_2,\\&\alpha_i=-z_{i-1}-\left[c_i+d_i\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2\right]z_i+\frac{\partial\alpha_{i-1}}{\partial\hat{\rho}}\dot{\hat{\rho}}+\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\Gamma\tau_i-\sum_{k=2}^{i-1}\sigma_{ki}z_k+\beta_i,\\&\beta_i=\frac{\partial\alpha_{i-1}}{\partial y}(\hat{\chi}_2+\omega_0+\omega^T\hat{\theta})+\frac{\partial\alpha_{i-1}}{\partial\xi}(A_l\xi+B_lf(y))+\sum_{k=1}^{m+i-2}\frac{\partial\alpha_{i-1}}{\partial\lambda_k}(-\bar{l}_k\lambda_1+\lambda_{k+1})\\&\qquad+\frac{\partial\alpha_{i-1}}{\partial\eta}(A_l\eta+e_{n-1,n-1}y)+\bar{l}_{i-1}v_{m1}+\frac{\partial\alpha_{i-1}}{\partial\hat{\chi}}\left[A\hat{\chi}+K_0(y-\hat{\chi}_1)+l(\omega_0+\omega^T\hat{\theta})\right],\\&\sigma_{ki}=\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma\frac{\partial\alpha_{i-1}}{\partial y}\omega,\quad i=2,\dots,\varrho,\end{aligned} \tag{20}$$

and the adaptive laws

$$\begin{aligned}&\tau_0=r_1\omega\varepsilon_1,\\&\tau_1=(\omega-\hat{\rho}\bar{\alpha}_1e_{n+m+1,1})z_1+\tau_0,\\&\tau_i=\tau_{i-1}-\frac{\partial\alpha_{i-1}}{\partial y}\omega z_i,\quad i=2,\cdots,\varrho,\\&\dot{\hat{\theta}}=\Gamma\tau_\varrho=\Gamma[W_\theta(z,t)z+r_1\omega\varepsilon_1],\\&\dot{\hat{\rho}}=-\gamma\mathrm{sgn}(b_m)\bar{\alpha}_1z_1,\end{aligned} \tag{21}$$

the error system (19) is compactly written as

$$\dot{z} = A_z(z,t)z + W_\theta^T(z,t)\tilde{\theta} - b_m\bar{\alpha}_1\tilde{\rho}e_{\varrho 1} + W_\varepsilon(z,t)[\varepsilon_2 + (s+a_{n-1})\aleph], \tag{22}$$

where $\hat{\rho}$ is the estimate of $\rho = \frac{1}{b_m}$, Γ, r_1, γ are some positive parameters,

$$A_z(z,t) = \begin{bmatrix} -c_1 - d_1 & \hat{b}_m & 0 & \cdots & 0 \\ -\hat{b}_m & -c_2 - d_2\left(\frac{\partial \alpha_1}{\partial y}\right)^2 & 1+\sigma_{23} & \cdots & \sigma_{2\varrho} \\ 0 & -1-\sigma_{23} & -c_3 - d_3\left(\frac{\partial \alpha_2}{\partial y}\right)^2 & \cdots & \sigma_{3\varrho} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -\sigma_{2\varrho} & -\sigma_{3\varrho} & \cdots & -c_\varrho - d_\varrho\left(\frac{\partial \alpha_{\varrho-1}}{\partial y}\right)^2 \end{bmatrix},$$

$$\begin{aligned} W_\varepsilon(z,t) &= \left[1, -\frac{\partial \alpha_1}{\partial y}, \cdots, -\frac{\partial \alpha_{\varrho-1}}{\partial y}\right]^T, \\ W_\theta^T(z,t) &= W_\varepsilon(z,t)\omega^T - \hat{\rho}\bar{\alpha}_1 e_{\varrho 1} e_{n+m+1,1}^T \in R^{\varrho\times(n+m+1)}. \end{aligned} \tag{23}$$

Remark 3.1. *As compared in* [9], *K-filters and MT-filters have different merits and demerits, that is, the reduced-order MT-filters are more simpler than the full-order K-filters, while the anti-disturbance ability of MT-filters is weaker than K-filters.*

Remark 3.2. *Let us discuss the implementation problem of the adaptive laws of two design methods in* [26] *and this paper. If we adopt the same design procedure as in* [26] *by using the K-filters, one obtains*

$$\dot{\hat{\theta}} = \Gamma\tau_\varrho = \Gamma\left(\tau_{\varrho-1} - \frac{\partial \alpha_{\varrho-1}}{\partial y}\omega z_\varrho\right) = \cdots = \Gamma\left((\omega - \hat{\rho}\bar{\alpha}_1 e_{n+m+1,1})\, z_1 - \sum_{i=2}^{\varrho}\frac{\partial \alpha_{i-1}}{\partial y}\omega z_i\right),$$

obviously, $\dot{\hat{\theta}}$ can be implemented. While for the controller design based on MT-filters, if we still adopt the conventional MT-filtered transformation $\chi = x - \begin{bmatrix} 0 \\ \xi + \Omega^T\theta \end{bmatrix}$ used in eq.(8.156) of [9], *then from (2), it follows that*

$$\begin{aligned} \dot{\chi} &= A\chi + l(\omega_0 + \omega^T\theta) + a\aleph, \\ y &= x_1 + \aleph = \chi_1 + \aleph. \end{aligned} \tag{24}$$

By (21), one obtains $\tau_\varrho = \tau_{\varrho-1} - \frac{\partial \alpha_{\varrho-1}}{\partial y}\omega z_\varrho = \cdots = r_1\omega\varepsilon_1 + (\omega - \hat{\rho}\bar{\alpha}_1 e_{n+m+1,1})z_1 - \sum_{i=2}^{\varrho}\frac{\partial \alpha_{i-1}}{\partial y}\omega z_i$, from which and (14), then

$$\varepsilon_1 = \chi_1 - \hat{\chi}_1 = y - \aleph - \hat{\chi}_1. \tag{25}$$

Since $\aleph$ is not available for measurement, by (25), it concludes that ε_1 and τ_ϱ are not available for measurement, hence $\dot{\hat{\theta}} = \Gamma\tau_\varrho$ is unable to be implemented. This is the main difference with the design using K-filters, and this important problem is easy to be neglected.

In this paper, similar to [12], *by adopting a new filtered transformation (11), one obtains $\varepsilon_1 \overset{(14)}{=} \chi_1 - \hat{\chi}_1 \overset{(12)}{=} y - \hat{\chi}_1$, thus $\dot{\hat{\theta}} = \Gamma\tau_\varrho$ can be implemented.*

4. **Main Result.** Introduce the following similarity transformations

$$\begin{bmatrix} \varepsilon_1 \\ \pi \end{bmatrix} =: \begin{bmatrix} \varepsilon_1 \\ T\varepsilon \end{bmatrix} = \begin{bmatrix} e_{n1}^T \\ T \end{bmatrix}\varepsilon, \tag{26}$$

$$\begin{bmatrix} \hat{\chi}_1 \\ \varphi \end{bmatrix} =: \begin{bmatrix} \hat{\chi}_1 \\ T\hat{\chi} \end{bmatrix} = \begin{bmatrix} e_{n1}^T \\ T \end{bmatrix} \hat{\chi}, \tag{27}$$

where $T = [A_l e_{n-1,1}, I_{n-1}] = [A_l, e_{n-1,n-1}]$. From (5), the definitions of K_0, A_0 and T, it follows that

$$Tl = 0, \quad TK_0 = A_l \bar{l}, \quad TA_0 = A_l T, \quad K_0 = c_0 l + \begin{bmatrix} \bar{l} \\ 0 \end{bmatrix}. \tag{28}$$

Combining (15), (26) with (28), one has

$$\begin{aligned} \dot{\pi} &= T\dot{\varepsilon} = A_l \pi + T(a + s e_{n1})\aleph \\ &= A_l \pi + T[\bar{a}\aleph + (s + a_{n-1}) e_{n1} \aleph], \end{aligned} \tag{29}$$

where $\bar{a} = (0, a_{n-2}, \cdots, a_0)^T$. By (26), one leads to

$$\varepsilon_2 - \bar{l}_1 \varepsilon_1 = \pi_1, \tag{30}$$

from which, (15), and the definitions of K_0 and A_0, it follows that

$$\begin{aligned} \dot{\varepsilon}_1 &= -(c_0 + \bar{l}_1)\varepsilon_1 + \varepsilon_2 + \omega^T \tilde{\theta} + (s + a_{n-1})\aleph \\ &= -c_0 \varepsilon_1 + \pi_1 + \omega^T \tilde{\theta} + (s + a_{n-1})\aleph. \end{aligned} \tag{31}$$

By the definition of A_0, (13) can be written as

$$\dot{\hat{\chi}} = A_0 \hat{\chi} + K_0 y + l(\omega_0 + \omega^T \hat{\theta}). \tag{32}$$

With the use of (27), (28) and (32), we have

$$\dot{\varphi} = T\dot{\hat{\chi}} = A_l \varphi + A_l \bar{l} y. \tag{33}$$

By Assumption 2, it is obvious that $\Delta_1(s)x_1$, $\Delta_2(s)x_1$ and $\Delta_3(s)y$ can be achieved by

$$\dot{\bar{f}} = A_{\bar{f}} \bar{f} + b_{\bar{f}} x_1, \quad \Delta_1(s)x_1 = (1, 0, \cdots, 0)\bar{f}, \tag{34}$$

$$\dot{g} = A_g g + b_g x_1, \quad \Delta_2(s)x_1 = (1, 0, \cdots, 0)g, \tag{35}$$

$$\dot{h} = A_h h + b_h y, \quad \Delta_3(s)y = (1, 0, \cdots, 0)h, \tag{36}$$

and $A_{\bar{f}}$, A_g and A_h are Hurwitz matrices.

Lemma 4.1. *The effects of the unmodeled dynamics are bounded by*

$$\begin{aligned} &|x_1|^2 \le 4(1 + 2\mu^2)|\Phi|^2 + 4\mu^2 |\Phi(t - \tau)|^2, \\ &|\aleph|^2 \le 6\mu^2 |\Phi|^2 + 3\mu^2 |\Phi(t - \tau)|^2, \\ &|(s + a_{n-1})\aleph|^2 \le 3\mu^2 (\bar{k}_1 |x_1(t - \tau)|^2 + (\bar{k}_2 + 4\tilde{k}_1 \mu^2)|\Phi(t - \tau)|^2 \\ &\qquad\qquad + (\hat{k}_1 + \tilde{k}_2 + 4\tilde{k}_1 + 8\tilde{k}_1 \mu^2)|\Phi|^2), \end{aligned}$$

where $\Phi = [z^T, \varepsilon_1, \pi^T, \bar{f}^T, g^T, h^T]^T$, $\mu =: \max\{\mu_1, \mu_2, \mu_3\}$, $\bar{k}_1$, $\bar{k}_2$, $\tilde{k}_1$, $\tilde{k}_2$ *and* $\hat{k}_1$ *are positive constants independent of* μ_1, μ_2 *and* μ_3.

Proof: See the Appendix.

We state the main result in this paper.

Theorem 4.1. *Consider the adaptive control systems consisting of the system (1), MT-filters (4), (7), (10), and the adaptive controller (20), (21). Under Assumptions 1 and 2, there always exists a positive constant* μ^* *such that for any* $\mu \in [0, \mu^*)$ *and all initial values, all the signals in the closed-loop system are globally uniformly bounded and* $\lim_{t\to\infty} |y(t)| = 0$, *where* μ *is defined as in Lemma 4.1.*

Proof: Consider the following Lyapunov function

$$\begin{aligned}\bar{V} =& \frac{1}{2}\left(|z|^2 + \tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + |b_m|\gamma^{-1}\tilde{\rho}^2 + r_1\varepsilon_1^2\right) + r_2\pi^T P_l\pi \\ &+ l_{\bar{f}}\bar{f}^T P_{\bar{f}}\bar{f} + l_g g^T P_g g + l_h h^T P_h h,\end{aligned} \tag{37}$$

where r_1, r_2, $l_{\bar{f}}$, l_g, l_h are some parameters to be determined, $\tilde{\theta} = \theta - \hat{\theta}$, $\tilde{\rho} = \rho - \hat{\rho}$, and P_l, $P_{\bar{f}}$, P_g and P_h satisfy

$$\begin{aligned}&A_l^T P_l + P_l A_l = -I, \quad A_{\bar{f}}^T P_{\bar{f}} + P_{\bar{f}} A_{\bar{f}} = -I, \\ &A_g^T P_g + P_g A_g = -I, \quad A_h^T P_h + P_h A_h = -I.\end{aligned} \tag{38}$$

The time derivative of $\bar{V}$ along (21)-(23), (29), (31) and (34)-(36) is given by

$$\begin{aligned}\dot{\bar{V}} =& -\sum_{i=1}^{\varrho} c_i z_i^2 - d_1 z_1^2 - \sum_{i=2}^{\varrho} d_i\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_i^2 + [\varepsilon_2 + (s + a_{n-1})\aleph]z_1 \\ &- [\varepsilon_2 + (s + a_{n-1})\aleph]\sum_{i=2}^{\varrho}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right) z_i - c_0 r_1\varepsilon_1^2 + r_1\varepsilon_1[\pi_1 + (s + a_{n-1})\aleph] \\ &- r_2|\pi|^2 + 2r_2\pi^T P_l T[\bar{a}\aleph + (s + a_{n-1})\aleph e_{n1}] - l_{\bar{f}}|\bar{f}|^2 + 2l_{\bar{f}}\bar{f}^T P_{\bar{f}} b_{\bar{f}} x_1 \\ &- l_g|g|^2 + 2l_g g^T P_g b_g x_1 - l_h|h|^2 + 2l_h h^T P_h b_h z_1 \\ =& -d_1 z_1^2 + [\varepsilon_2 + (s + a_{n-1})\aleph]z_1 - \sum_{i=2}^{\varrho} d_i\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 z_i^2 - [\varepsilon_2 + (s + a_{n-1})\aleph] \\ &\cdot\sum_{i=2}^{\varrho}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right) z_i - \frac{1}{2}c_0 r_1\varepsilon_1^2 + r_1\varepsilon_1[\pi_1 + (s + a_{n-1})\aleph] - \frac{1}{2}r_2|\pi|^2 + 2r_2\pi^T P_l T \\ &\cdot[\bar{a}\aleph + (s + a_{n-1})\aleph e_{n1}] - \frac{1}{4}l_{\bar{f}}|\bar{f}|^2 + 2l_{\bar{f}}\bar{f}^T P_{\bar{f}} b_{\bar{f}} z_1 - \frac{1}{8}c_1 z_1^2 - \frac{1}{4}l_{\bar{f}}|\bar{f}|^2 - 2l_{\bar{f}}\bar{f}^T P_{\bar{f}} b_{\bar{f}}\aleph \\ &- \frac{1}{4}l_g|g|^2 + 2l_g g^T P_g b_g z_1 - \frac{1}{8}c_1 z_1^2 - \frac{1}{4}l_g|g|^2 - 2l_g g^T P_g b_g\aleph - \frac{1}{2}l_h|h|^2 + 2l_h h^T P_h b_h z_1 \\ &- \frac{1}{4}c_1 z_1^2 - \sum_{i=2}^{\varrho} c_i z_i^2 - \frac{1}{2}c_0 r_1\varepsilon_1^2 - \frac{1}{2}r_2|\pi|^2 - \frac{1}{2}l_{\bar{f}}|\bar{f}|^2 - \frac{1}{2}l_g|g|^2 - \frac{1}{2}l_h|h|^2 - \frac{1}{2}c_1 z_1^2.\end{aligned} \tag{39}$$

Choosing $\frac{1}{d_0} = \sum_{i=1}^{\varrho}\frac{1}{d_i}$, $l_{\bar{f}} \leq \frac{c_1}{32|P_{\bar{f}} b_{\bar{f}}|^2}$, $l_g \leq \frac{c_1}{32|P_g b_g|^2}$, $l_h \leq \frac{c_1}{8|P_h b_h|^2}$, and using the complete square inequality, one gets

$$\begin{aligned}\dot{\bar{V}} \leq& \frac{1}{4d_0}[\varepsilon_2 + (s + a_{n-1})\aleph]^2 + \frac{r_1}{2c_0}[\pi_1 + (s + a_{n-1})\aleph]^2 + 2r_2|P_l T|^2[\bar{a}\aleph + (s + a_{n-1})\aleph e_{n1}]^2 \\ &+ 4l_{\bar{f}}|P_{\bar{f}} b_{\bar{f}}|^2|\aleph|^2 + 4l_g|P_g b_g|^2|\aleph|^2 - \frac{1}{2}c_1 z_1^2 - \sum_{i=2}^{\varrho} c_i z_i^2 - \frac{1}{2}c_0 r_1\varepsilon_1^2 - \frac{1}{2}r_2|\pi|^2 \\ &- \frac{1}{2}l_{\bar{f}}|\bar{f}|^2 - \frac{1}{2}l_g|g|^2 - \frac{1}{2}l_h|h|^2 \\ \leq& \frac{1}{2d_0}\varepsilon_2^2 + k_a|(s + a_{n-1})\aleph|^2 + \frac{r_1}{c_0}\pi_1^2 + k_b|\aleph|^2 - \frac{1}{2}c_1 z_1^2 - \sum_{i=2}^{\varrho} c_i z_i^2 - \frac{1}{2}c_0 r_1\varepsilon_1^2 \\ &- \frac{1}{2}r_2|\pi|^2 - \frac{1}{2}l_{\bar{f}}|\bar{f}|^2 - \frac{1}{2}l_g|g|^2 - \frac{1}{2}l_h|h|^2,\end{aligned} \tag{40}$$

where $k_a = \frac{1}{2d_0} + \frac{r_1}{c_0} + 4r_2|P_lT|^2$, $k_b = 4r_2|P_lT|^2|\bar{a}|^2 + 4l_{\bar{f}}|P_{\bar{f}}b_{\bar{f}}|^2 + 4l_g|P_gb_g|^2$. By (30), (40), and choosing $r_1 \geq \frac{4\bar{l}_1^2}{c_0d_0}$, $r_2 \geq \frac{4r_1}{c_0} + \frac{4}{d_0}$, one has

$$\dot{\bar{V}} \leq k_a|(s+a_{n-1})\aleph|^2 + k_b|\aleph|^2 - \frac{r_1c_0}{4}\varepsilon_1^2 - \frac{r_2}{4}\pi^2 - \frac{1}{2}c_1z_1^2 - \sum_{i=2}^{\varrho} c_iz_i^2 \\ -\frac{1}{2}l_{\bar{f}}|\bar{f}|^2 - \frac{1}{2}l_g|g|^2 - \frac{1}{2}l_h|h|^2. \quad (41)$$

Defining $q = \min\left\{\frac{c_1}{4}, c_2, \cdots, c_\varrho, \frac{r_1c_0}{4}, \frac{r_2}{4}, \frac{l_{\bar{f}}}{2}, \frac{l_g}{2}, \frac{l_h}{2}\right\}$, from Lemma 4.1, it follows that

$$\dot{\bar{V}} \leq -q|\Phi|^2 - \frac{1}{4}c_1z_1^2 + 3k_a\bar{k}_1\mu^2|x_1(t-\tau)|^2 + ((3k_a\bar{k}_2 + 3k_b)\mu^2 + 12k_a\tilde{k}_1\mu^4)|\Phi(t-\tau)|^2 \\ +((3k_a\hat{k}_1 + 3k_a\tilde{k}_2 + 12k_a\tilde{k}_1 + 6k_b)\mu^2 + 24k_a\tilde{k}_1\mu^4)|\Phi|^2. \quad (42)$$

Considering the following Lyapunov function for the total system

$$V = \bar{V} + 3k_a\bar{k}_1\mu^2\int_{t-\tau}^{t}|x_1(\sigma)|^2d\sigma + ((3k_a\bar{k}_2 + 3k_b)\mu^2 \\ +12k_a(\tilde{k}_1 + \bar{k}_1)\mu^4)\int_{t-\tau}^{t}|\Phi(\sigma)|^2d\sigma, \quad (43)$$

and using (42) and Lemma 4.1, one obtains

$$\dot{V} \leq -q|\Phi|^2 - \frac{1}{4}c_1z_1^2 + 3k_a\bar{k}_1\mu^2|x_1|^2 + ((3k_a\hat{k}_1 + 3k_a\tilde{k}_2 + 12k_a\tilde{k}_1 + 6k_b)\mu^2 + 24k_a\tilde{k}_1\mu^4) \\ \cdot|\Phi|^2 + ((3k_a\bar{k}_2 + 3k_b)\mu^2 + 12k_a(\tilde{k}_1 + \bar{k}_1)\mu^4)|\Phi|^2 - 12k_a\bar{k}_1\mu^4|\Phi(t-\tau)|^2 \\ \leq -(q - \kappa_2\mu^2 - \kappa_1\mu^4)|\Phi|^2 - \frac{1}{4}c_1z_1^2, \quad (44)$$

where $\kappa_1 = 36k_a(\tilde{k}_1 + \bar{k}_1)$, $\kappa_2 = 3k_a\hat{k}_1 + 3k_a\tilde{k}_2 + 12k_a\tilde{k}_1 + 6k_b + 3k_a\bar{k}_2 + 3k_b + 12k_a\bar{k}_1$. Since q, κ_1 and κ_2 are some constants independent of μ, there exists a constant $\mu^* = \sqrt{\frac{1}{2\kappa_1}\sqrt{\kappa_2^2 + 4\kappa_1q} - \frac{\kappa_2}{2\kappa_1}}$, such that for any $\mu \in [0, \mu^*)$ and all initial values,

$$\dot{V} \leq -\frac{1}{4}c_1z_1^2, \quad (45)$$

from which and (43), we conclude that all the signals in the closed-loop system are globally uniformly bounded, and $\lim_{t\to\infty}|y(t)| = 0$ by Barbălat lemma in [9].

5. **A Simulation Example.** Consider the following nonlinear time-delay systems

$$y(t) = \frac{b}{s^2 + a_1s + a_0}\left(u(t) + \frac{\mu_1}{s+1}u(t-\tau) + \frac{\mu_2}{s+1}u(t)\right) \\ +\frac{[s,1]}{s^2 + a_1s + a_0}\left(\left[\begin{array}{c} 0 \\ y(t)\sin y(t) \end{array}\right] + \frac{\mu_1}{s+1}\left[\begin{array}{c} 0 \\ y(t-\tau)\sin y(t-\tau) \end{array}\right]\right. \\ \left.+\frac{\mu_2}{s+1}\left[\begin{array}{c} 0 \\ y(t)\sin y(t) \end{array}\right]\right) + \frac{\mu_3}{s+1}y(t). \quad (46)$$

(46) can be transformed into the following state-space realization

$$\dot{x} = Ax - \left[\begin{array}{c} a_1 \\ a_0 \end{array}\right]x_1 + \left[\begin{array}{c} 0 \\ b \end{array}\right]u + f(y) \\ y = x_1 + \aleph, \quad (47)$$

where $x = \left[\begin{array}{c} x_1 \\ x_2 \end{array}\right]$, $A = \left[\begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array}\right]$, $f(y) = \left[\begin{array}{c} 0 \\ y\sin y \end{array}\right]$, $\aleph = \frac{\mu_1}{s+1}x_1(t-\tau) + \frac{\mu_2}{s+1}x_1 + \frac{\mu_3}{s+1}y$.

MT-filters are chosen as

$$\dot{\xi} = -l\xi + y\sin y, \quad \dot{\eta} = -l\eta + y, \quad \dot{\lambda} = -l\lambda + u. \tag{48}$$

The change of coordinates are $z_1 = y$, $z_2 = \lambda - \alpha_1$. The observer is given by

$$\dot{\hat{\chi}} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\hat{\chi} + \begin{bmatrix} c_0 + l \\ c_0 l \end{bmatrix}(y - \hat{\chi}_1) + \begin{bmatrix} 1 \\ l \end{bmatrix}(\xi + \omega^T\hat{\theta}), \tag{49}$$

where $\omega^T = [\lambda, l\eta - y, -\eta]$. The control law is

$$\begin{aligned} \alpha_1 &= \hat{\rho}\bar{\alpha}_1, \quad \bar{\alpha}_1 = -(c_1 + d_1)z_1 - \hat{\chi}_2 - \xi - \bar{\omega}^T\hat{\theta}, \\ u &= -\hat{\theta}_1 z_1 - \left[c_2 + d_2\left(\frac{\partial\alpha_1}{\partial y}\right)^2\right] z_2 + \frac{\partial\alpha_1}{\partial\hat{\rho}}\dot{\hat{\rho}} + \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} \\ &\quad + \frac{\partial\alpha_1}{\partial y}(\hat{\chi}_2 + \xi + \omega^T\theta) + \frac{\partial\alpha_1}{\partial\eta}\dot{\eta} + \frac{\partial\alpha_1}{\partial\xi}\dot{\xi} + \frac{\partial\alpha_1}{\partial\hat{\chi}_2}\dot{\hat{\chi}}_2 + l\lambda. \end{aligned} \tag{50}$$

The parameter adaptive laws are chosen as

$$\begin{aligned} \dot{\hat{\theta}} &= \Gamma\left(z_1 + r_1 z_1 - r_1\hat{\chi}_1 - \frac{\partial\alpha_1}{\partial y}z_2\right)\omega - \Gamma[\hat{\rho}\bar{\alpha}_1 z_1, 0, 0]^T, \\ \dot{\hat{\rho}} &= -\gamma\mathrm{sgn}(b)\bar{\alpha}_1 z_1, \end{aligned} \tag{51}$$

where $\hat{\theta} = [\hat{b}, \hat{a}_1, \hat{a}_0]^T$ and $\hat{\rho}$ are the estimates of $\theta = [b, a_1, a_0]^T$ and $\rho = \frac{1}{b}$, respectively.

In simulation, we choose $\tau = 1s$, the system parameters $a_1 = 1$, $a_0 = 2$, $b = 1$, the design parameters $c_0 = 0.8$, $l = 2$, $c_1 = 2$, $c_2 = 1$, $d_1 = 0.2$, $d_2 = 0.3$, $\mu_1 = 0.3$, $\mu_2 = 0.2$, $\mu_3 = 0.4$, $r_1 = 0.5$, $\gamma = 0.6$, $\Gamma = 1$, and the initial values $x_1(0) = 1$, $x_2(0) = -1$, $\hat{\chi}_1(0) = 0.4$, $\hat{\chi}_2(0) = 0.3$, $\lambda(0) = \eta(0) = 0$, $\xi(0) = 1$, $\hat{\rho}(0) = 0.1$, $\hat{\theta}(0) = [0.8, 0.6, 0.5]^T$. Figure 1 gives the responses of the closed-loop system with MT-filters.

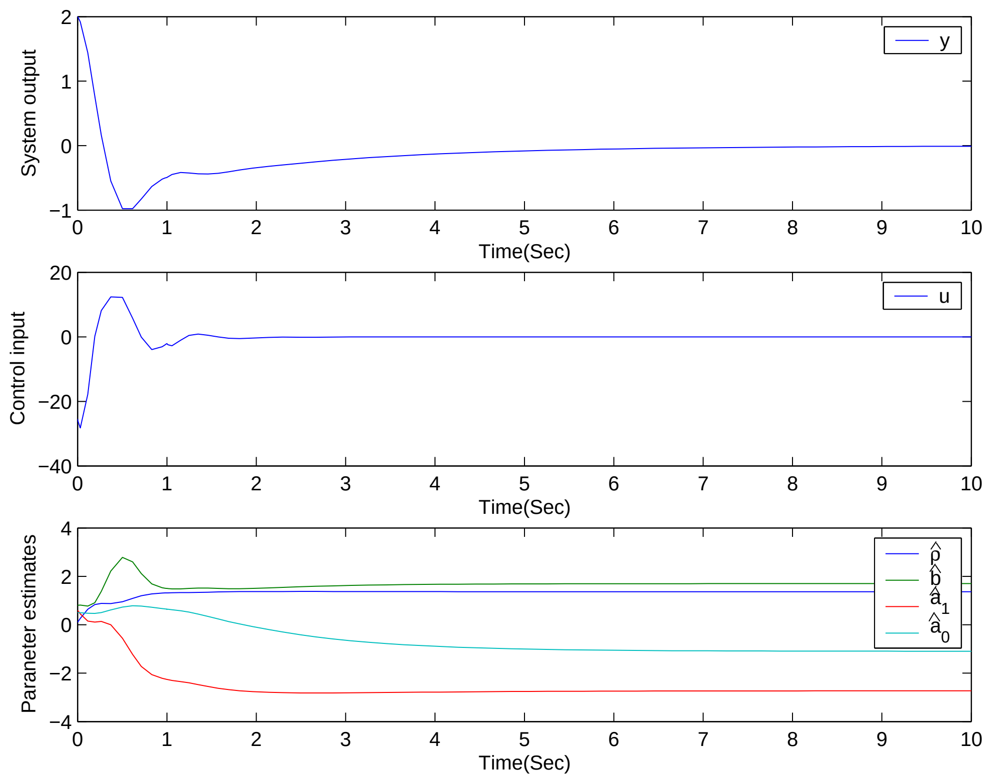

Figure 1. The responses of the closed-loop system

6. **Conclusions.** This paper investigates adaptive output feedback problem using MT-filters and the backstepping design method for nonlinear systems with unknown input and output time-delay.

There are still two remaining problems to be investigated: One is to extend the method to more general systems, such as stochastic nonlinear time-delay systems with SiISS inverse dynamics in [22-24], stochastic high-order nonlinear systems [13] with input time-delay. The other is to find a practical example on system (1).

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Appendix.

Proof of (2) and (3): Define $x_1(t) = \frac{B(s)}{A(s)}u(t) + \frac{D(s)}{A(s)}f(y(t))$, $\aleph(t) = \mu_1\Delta_1(s)x_1(t-\tau) + \mu_2\Delta_2(s)x_1(t) + \mu_3\Delta_3(s)y(t)$. By (1), one has

$$y(t) = x_1(t) + \mu_1\Delta_1(s)x_1(t-\tau) + \mu_2\Delta_2(s)x_1(t) + \mu_3\Delta_3(s)y(t) = x_1(t) + \aleph(t). \tag{52}$$

With the help of $A(s)$, $B(s)$ and $D(s)$ in (1), there exist minimal realization matrices $\bar{A} = \left[-a \; \begin{matrix} I_{n-1} \\ 0_{1\times(n-1)} \end{matrix}\right]$, $\bar{b} = \left[\begin{matrix} 0 \\ b \end{matrix}\right]$, $c^T = [1, 0, \cdots, 0]$, $a = [a_{n-1}, \cdots, a_0]^T$, $b = [b_m, \cdots, b_0]^T$ such that $c^T(sI - \bar{A})^{-1}\bar{b} = \frac{B(s)}{A(s)}$, $c^T(sI - \bar{A})^{-1} = \frac{D(s)}{A(s)}$. By [3], obviously, x_1 can be achieved by $\dot{x} = \bar{A}x + \bar{b}u + f(y)$, $x_1 = c^T x$. From (3), it follows that

$$\begin{aligned} \dot{x} &= \bar{A}x + \bar{b}u + f(y) \\ &= Ax - ax_1 + \bar{b}u + f(y) \\ &= Ax - ay + a\aleph + \bar{b}u + f(y) \\ &= Ax + F^T(u, y)\theta + a\aleph + f(y). \end{aligned}$$

Proof of Lemma 4.1: By the definition of Φ, (34)-(36), it is easy to conclude that

$$\begin{aligned} &|\Delta_1(s)x_1(t-\tau)|^2 \le |\Phi(t-\tau)|^2, \quad |\Delta_2(s)x_1(t)|^2 \le |\Phi(t)|^2, \\ &|\Delta_3(s)y(t)|^2 \le |\Phi(t)|^2. \end{aligned} \tag{53}$$

By (34) and (53), one has

$$\begin{aligned} &|(s + a_{n-1})\Delta_1(s)x_1(t-\tau)|^2 \\ &= |(1, 0, \cdots, 0)(A_{\bar{f}}\bar{f}(t-\tau) + b_{\bar{f}}x_1(t-\tau)) + a_{n-1}\Delta_1(s)x_1(t-\tau)|^2 \\ &\le \bar{k}_1|x_1(t-\tau)|^2 + \bar{k}_2|\Phi(t-\tau)|^2, \end{aligned} \tag{54}$$

where $\bar{k}_1$ and $\bar{k}_2$ are positive constants independent of μ_1, μ_2 and μ_3. Similar to (54), one obtains

$$\begin{aligned} &|(s + a_{n-1})\Delta_2(s)x_1(t)|^2 \le \tilde{k}_1|x_1(t)|^2 + \tilde{k}_2|\Phi(t)|^2, \\ &|(s + a_{n-1})\Delta_3(s)y(t)|^2 \le \hat{k}_1|\Phi(t)|^2. \end{aligned} \tag{55}$$

By (2) and (53), one gets

$$\begin{aligned} |x_1(t)|^2 &= |y(t) - \mu_1\Delta_1(s)x_1(t-\tau) - \mu_2\Delta_2(s)x_1(t) - \mu_3\Delta_3(s)y(t)|^2 \\ &\le 4(|y(t)|^2 + |\mu_1\Delta_1(s)x_1(t-\tau)|^2 + |\mu_2\Delta_2(s)x_1(t)|^2 + |\mu_3\Delta_3(s)y(t)|^2) \\ &\le 4(|\Phi(t)|^2 + \mu_1^2|\Phi(t-\tau)|^2 + \mu_2^2|\Phi(t)|^2 + \mu_3^2|\Phi(t)|^2) \\ &\le 4(1 + 2\mu^2)|\Phi(t)|^2 + 4\mu^2|\Phi(t-\tau)|^2, \end{aligned} \tag{56}$$

where $\mu = \max\{\mu_1, \mu_2, \mu_3\}$. From (3) and (53)-(55), it follows that

$$\begin{aligned}|\aleph(t)|^2 &\le 3(|\mu_1\Delta_1(s)x_1(t-\tau)|^2 + |\mu_2\Delta_2(s)x_1(t)|^2 + |\mu_3\Delta_3(s)y(t)|^2)\\ &\le 3(\mu_1^2|\Phi(t-\tau)|^2 + \mu_2^2|\Phi(t)|^2 + \mu_3^2|\Phi(t)|^2)\\ &\le 6\mu^2|\Phi(t)|^2 + 3\mu^2|\Phi(t-\tau)|^2,\end{aligned} \tag{57}$$

and

$$\begin{aligned}|(s+a_{n-1})\aleph(t)|^2 &\le 3(|\mu_1(s+a_{n-1})\Delta_1(s)x_1(t-\tau)|^2 + |\mu_2(s+a_{n-1})\Delta_2(s)x_1(t)|^2\\ &\quad +|\mu_3(s+a_{n-1})\Delta_3(s)y(t)|^2)\\ &\le 3\mu^2(\bar{k}_1|x_1(t-\tau)|^2 + (\bar{k}_2 + 4\tilde{k}_1\mu^2)|\Phi(t-\tau)|^2\\ &\quad +(\hat{k}_1 + \tilde{k}_2 + 4\tilde{k}_1 + 8\tilde{k}_1\mu^2)|\Phi(t)|^2).\end{aligned} \tag{58}$$