International Journal of Innovative
Computing, Information and Control
Volume 8, Number 12, December 2012

ICIC International ©2012 ISSN 1349-4198
pp. 8077–8088

CONTINUOUS GAIN SCHEDULED H-INFINITY OBSERVER FOR UNCERTAIN NONLINEAR SYSTEM WITH TIME-DELAY AND ACTUATOR SATURATION

Yanqing Liu, Yanyan Yin and Fei Liu*

Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education)
Institute of Automation
Jiangnan University
No. 1800, Lihu Avenue, Wuxi 214122, P. R. China
yanqingliu2010@gmail.com; yinyanyan_2006@126.com
*Corresponding author: fliu@jiangnan.edu.cn

Received August 2011; revised March 2012

Abstract. *In this paper, we propose a method for designing continuous gain scheduled H-infinity observer for uncertain nonlinear continuous stirred-tank reactor system subject to time-delay and actuator saturation. Initially, gradient linearization procedure is applied to describe such nonlinear system into several linear systems. Next, a convex hull set is investigated in order to transform the actuator constraints into several linear constraints, and then, a set of H-infinity observers is designed for these linear models, which guarantees the system states belong to an ellipsoid invariant set. Finally, continuous gain-scheduled approach is employed to design continuous nonlinear observer on the entire uncertain nonlinear system. A simulation example is given to illustrate the effectiveness of developed techniques.*

Keywords: Continuous gain scheduling, Nonlinear system, Actuator saturation, H-infinity observer

1. **Introduction.** As well known, actuator saturation occurs commonly in many manufacture systems, and this is a very dangerous nonlinearity which cannot be avoided. It is also a well recognized fact that actuator nonlinearity degrades system performance and even leads a stable system to an instable one. In recent years, the problem of how to solve actuator saturation in complex industrial systems has received increasing attention and a number of results have been developed on linear systems subject to actuator saturation [1-6].

Actually, fast changes in parameters of systems are commonly encountered in lots of practical dynamical systems, and many systems are nonlinear ones, so the investigation of control problem on nonlinear systems may be more reasonable. This motivates us to challenge the robust observer design on nonlinear systems with actuator nonlinearity. In this paper, we will design H-infinity observer on nonlinear systems subject to actuator saturation, and we are more interested in getting an invariant set for such system, which guarantees the system state starting from it will remain in it.

In the past decades, a great deal of work has been devoted to time delay systems and some results on control problems for time delay systems have been reported (see, e.g., [7-10] and the references therein). However, to the best of our knowledge, there is little work done on time delay nonlinear systems with actuator nonlinearity. These motivate us to this study.

On another research front line, in order to design continuous nonlinear observer for the aforementioned system, and consider some prior work we have done regarding continuous gain scheduling approach [11-13], and we have not done much work on systems subject to actuation saturation; therefore, we are concerned with the continuous gain-scheduled robust H-infinity observer design approach for a class of nonlinear system with time delay and actuator saturation. The main advantages of our results in this paper are as follows: first, one can obtain a continuous observer in the concerned states interval of nonlinear systems, and second, the observer varies its parameters with the variation of the states. By using this method, nonlinear systems with fast parameters variation can be easily stabilized. Finally, by using Taylor fitting series method, the sufficient condition for the existence of continuous time-varying nonlinear observer is illustrated in terms of linear matrix inequalities.

2. **Problem Statement and Preliminaries.** The following nonlinear CSTR system (continuous stirred-tank reactor system) [14] with time delay and actuator saturation is considered in this paper:

$$\begin{cases} \dot{x}_1(t) = -(1+D_{a1})x_1(t) + 0.5x_1(t-h)\sigma(u(t)) + 0.25w(t) + f_1 \\ \dot{x}_2(t) = D_{a1}x_1(t) - x_2(t) - D_{a2}x_2^2(t) + 0.5x_2(t-h) + 0.22w(t) + f_2 \\ y(t) = 0.2x_1(t) + 0.3x_2(t) \\ z(t) = 0.2x_1(t) + 0.3x_2(t) \end{cases} \tag{1}$$

This is a CSTR model in which an isothermal, liquid-phase multi-component chemical reaction takes place. The chemical reaction system is $P \to Q \to R$, where P and Q are highly acidic, and R is neutral. $x(t) \in R^n$ is the state vector of the system, $u(t) \in R^m$ is the input vector of the system, $z(t) \in R^p$ is the controlled output vector of the system, $w(t) \in L_2^q[0,\infty]$ is the external disturbance vector of the system. f_1 and f_2 are time-varying and norm-bounded uncertainties. The objective of the controller is to keep the total concentration of P and Q at a constant value by adjusting the feed rate of P. C_p is the concentration of species P, and C_{p0} is the desired concentration of species P. $x_1 = C_p/C_{p0}$ is the ratio of concentration. State variable $x_2 = C_Q/C_{P0}$ is the ratio of the concentration C_Q and the desired concentration C_{P0}. $D_{a1} = k_1V/F$, $D_{a2} = k_2VC_{P0}/F$, control input is given by $u = N_{PF}/FC_{P0}$, where V is the volume of the tank, F is the volumetric feed rate and N_{PF} is the molar feed rate of the species P, k_1 and k_2 are known constants which represent the first and second-order rate respectively. We take the following values as the parameters: $D_{a1} = 1$, $D_{a2} = 1$. $\sigma(\cdot)$ is the standard saturation function with appropriate dimensions, $\sigma(u(t)) = \begin{bmatrix} \sigma(u_1(t)) & \sigma(u_2(t)) & \dots & \sigma(u_m(t)) \end{bmatrix}^{\mathrm{T}}$ and $\sigma(u_l(t)) = sign(u_l(t))\min\{1, |u_l(t)|\}$, $l = 1\dots m$.

We transform system (1) into the following form:

$$\begin{cases} \dot{x}(t) = f_3 \cdot x(t) + f_4 \cdot x(t-h) + g \cdot \sigma(u(t)) + d \cdot w(t) + f_5(x(t), x(t-h)) \\ x(t) = x(0), \quad t \in [-h, 0] \end{cases} \tag{2}$$

which equals to

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -(1+D_{a1})x_1(t) + 0.5x_1(t-h) \\ D_{a1}x_1(t) - x_2(t) - D_{a2}x_2^2(t) + 0.5x_2(t-h) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \sigma(u(t))$$

$$+ \begin{bmatrix} 0.25 \\ 0.22 \end{bmatrix} w(t) + f_5(x(t), x(t-h))$$

where $f_5(\cdot)$ is norm-bounded uncertainty represented by f_1 and f_2.

Assumption 2.1. *The norm-bounded uncertainty $f_5(\cdot)$ in (2) is assumed to satisfy*

$$f_5(x(t), x(t-h)) = \triangle A x(t) + \triangle A_d x(t-h)$$

and $\begin{bmatrix} \triangle A & \triangle A_d \end{bmatrix} = GF(t) \begin{bmatrix} E & E_d \end{bmatrix}$, *$G$, E and E_d are constant matrices with appropriate dimensions, $F(t)$ is an unknown matrix with Lebesgue measurable element satisfying $F(t) \leq 1$.*

In order to construct linear models of system (1) in the vicinity of selected operating states, gradient linearization procedure [15] is applied to the above nonlinear system. And then, based on gradient linear method, some selected working points are chosen as follows:

$$\begin{aligned} x_{2e} &= 1 + i/2, \quad i = 0, 1, 2, \cdots, 9 \\ x_{1e} &= x_{2e} + x_{2e}^2 \\ u_e &= (1 + D_{a1}) x_{1e} \end{aligned}$$

Subsequently, a series of linear models are obtained as follows:

$$\dot{x}(t) = (A + \triangle A)x(t) + (A_d + \triangle A_d)x(t-h) + B\sigma(u(t)) + Dw(t) \tag{3}$$

where $A = \begin{bmatrix} -2 & 0 \\ x_{1e}^2 + x_{2e}^2 + x_{1e}x_{2e}^2/x_{1e}^2 + x_{2e}^2 & -x_{1e}^2 - x_{2e}^2 - 2x_{1e}^2 x_{2e} + x_{2e}^3/x_{1e}^2 + x_{2e}^2 \end{bmatrix}$

$$A_d = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0.25 \\ 0.22 \end{bmatrix}$$

For simplicity, we denote $A(i)$, $A(di)$, $B(i)$, $D(i)$, $K(i)$ as coefficient matrices and gain matrices of the ith linear model of system (1).

The task of our work is to design a set of H-infinity observers for system (3), and then, by using Taylor fitting approach, a continuous gain-scheduled nonlinear observer will be obtained for the entire nonlinear system (1). One can determine a suitable state invariant set, meanwhile, state trajectory of system stays inside the domain of attraction under such observer. Before proceeding with the study, some concepts are presented as follows.

Definition 2.1. *For given matrices $P(i) > 0$, and ellipsoid sets $\varepsilon(P(i), 1) = \{x(t) \in R^n : x^{\mathrm{T}}(t)P(i)x(t) \leq 1\}$, one can denote the Lyapunov function for system (3) as $x^{\mathrm{T}}(t)P(i)x(t)$, such that if there exist $\dot{V} < 0$, then, $\varepsilon(P(i), 1)$ are said to be contractively invariant sets.*

Definition 2.2. *Given a matrix $F(i)$ for system (3), one can denote $f_q(i)$ as the qth row of matrix $F(i)$, subsequently, a symmetric polyhedron set is defined as follows:*

$$\Theta(F(i)) = \{x(t) \in R^n : |f_q(i)x(t)| \leq 1, q = 1, 2, \ldots, m\}$$

Definition 2.3. *Given a group of points t_1, t_2, $\ldots$, t_g, the convex hull of these points is defined as*

$$co\{t_l : l \in [1, g]\} := \left\{ \sum_{l=1}^{g} \beta_l t_l : \sum_{l=1}^{g} \beta_l = 1, \beta_l \geq 0 \right\}$$

Lemma 2.1. [4] *Given matrices $K(i) \in R^{m\times n}$ and $F(i) \in R^{m\times n}$, for system state $x(t) \in R^n$, if $x(t) \in \Theta(F(i))$, then, $\sigma(K(i)x(t)) = \sum_{t=1}^{2^m} \theta_t (D_t K(i) + D_t^- F(i))x(t)$, where $0 \leq \theta_t \leq 1$, $\sum_{t=1}^{2^m} \theta_t = 1$, D_t are $m \times m$ diagonal matrices whose diagonal elements are either 1 or 0, and $D_t^- = I - D_t$.*

Lemma 2.2. [16] *Consider L, R, W, T as real matrices of appropriate dimension, and W is assumed to satisfy $W^{\mathrm{T}}W \leq I$; then for a positive scalar $\alpha > 0$, it holds*

$$L + RWT + T^{\mathrm{T}}W^{\mathrm{T}}R^{\mathrm{T}} \leq L + \alpha^{-1}RR^{\mathrm{T}} + \alpha T^{\mathrm{T}}T$$

For system (3), the following observer is constructed

$$\begin{cases} \dot{\bar{x}}(t) = A(i)\bar{x}(t) + B(i)\sigma(u(t)) + H(i)(y(t) - \bar{y}(t)) \\ \bar{y}(t) = C(1i)\bar{x}(t) \\ u(t) = K(i)\bar{x}(t) \end{cases} \tag{4}$$

where $\bar{x}(t)$ and $\bar{y}(t)$ are the estimated state and output, $H(i)$ are gains of the designed observer, and $K(i)$ are gains of feedback controller.

Recalling Lemma 2.1, under condition $\bar{x}(t) \in \Theta(F(i))$, then,

$$\sigma(K(i)\bar{x}(t)) = \sum_{t=1}^{2^m} \theta_t(D_tK(i) + D_t^- F(i))\bar{x}(t)$$

subsequently, the following estimation error dynamic system can be obtained by combining systems (3) and (4).

$$\begin{cases} \dot{e}(t) = (A(i) - H(i)C(1i))e(t) + \Delta A(i)x(t) + (A_d(i) + \Delta A_d(i))x(t-h) + D(i)w(t) \\ \dot{x}(t) = -B(i)M(i)e(t) + (A(i) + \Delta A(i) + B(i)M(i))x(t) \\ \qquad\quad +(A_d(i) + \Delta A_d(i))x(t-h) + D(i)w(t) \\ z(t) = C(2i)x(t) \end{cases} \tag{5}$$

where $e(t) = x(t) - \bar{x}(t)$, $M(i) = \sum_{t=1}^{2^m} \theta_t(D_tK(i) + D_t^- F(i))$.

Remark 2.1. *It is easy to find that $\Theta(K(i))$ is a domain in which feedback control input $\sigma(u(t))$ is linear in $\bar{x}(t)$.*

3. Design of Gain-scheduled H_∞ Observer.

The first aim of our work is to design a set of H-infinity observers for system (3). The second aim of our work is to design a continuous gain scheduled nonlinear observer for the whole nonlinear system (1) by using Taylor fitting approach.

Proposition 3.1. *For given matrices D_t, D_t^- and $w(t) = 0$, the dynamic system (5) is stabilizable, if there exist a set of positive definite symmetric matrices $P_1(i)$ and $P_2(i)$, a definite symmetric matrix Q and a set of matrices $K(i)$ and $F(i)$ such that*

$$\Xi_1 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ * & a_{22} & a_{23} \\ * & * & a_{33} \end{bmatrix} < 0 \quad \forall t \in [1, 2^m] \tag{6}$$

$$\bar{x}(t) \in \Theta(F_k(i)) \tag{7}$$

where

$$\begin{aligned} a_{11} &= (A(i) - H(i)C(1i))^{\mathrm{T}}P_2(i) + P_2(i)(A(i) - H(i)C(1i)) \\ a_{12} &= -(D_tK(i) + D_t^- F(i))^{\mathrm{T}}B^{\mathrm{T}}(i)P_1(i) + P_2(i)\Delta A(i) \\ a_{22} &= (A(i) + \Delta A(i) + B(i)(D_tK(i) + D_t^- F(i)))^{\mathrm{T}}P_1(i) \\ &\quad + P_1(i)(A(i) + \Delta A(i) + B(i)(D_tK(i) + D_t^- F(i))) + Q \end{aligned}$$

$$\begin{aligned} a_{13} &= P_2(i)(A_d(i) + \Delta A_d(i)) \\ a_{23} &= P_1(i)(A_d(i) + \Delta A_d(i)) \\ a_{33} &= -Q \end{aligned}$$

Proof: The Lyapunov-Krasovskii function for system (5) is constructed by using symmetric positive definite matrices $P_1(i)$, $P_2(i)$ and Q:

$$V(x(t), e(t), i) = x^{\mathrm{T}}(t)P_1(i)x(t) + e^{\mathrm{T}}(t)P_2(i)e(t) + \int_{t-h}^{t} x^{\mathrm{T}}(\tau)Qx(\tau)d\tau$$

Under condition (7), recall Lemma 2.1, the time derivative of $V(x(t), e(t), i)$ for system (5) is

$$\begin{aligned} &\dot{V}(x(t), e(t), i) \\ =& x^{\mathrm{T}}(t)P_1(i)[(A(i) + \Delta A(i) + B(i)M(i))x(t) + (A_d(i) \\ &+ \Delta A_d(i))x(t-h) - B(i)M(i)e(t)] \\ &+ [(A(i) + \Delta A(i) + B(i)M(i))x(t) + (A_d(i) \\ &+ \Delta A_d(i))x(t-h) - B(i)M(i)e(t)]^{\mathrm{T}}P_1(i)x(t) \\ &+ e^{\mathrm{T}}(t)P_2(i)[(A(i) - H(i)C_1(i))e(t) + \Delta A(i)x(t) + (A_d(i) + \Delta A_d(i))x(t-h)] \\ &+ [(A(i) - H(i)C_1(i))e(t) + \Delta A(i)x(t) + (A_d(i) + \Delta A_d(i))x(t-h)]^{\mathrm{T}}P_2(i)e(t) \\ &+ x^{\mathrm{T}}(t)Qx(t) - x^{\mathrm{T}}(t-h)Qx(t-h) \end{aligned}$$

Thus, it follows that

$$\dot{V}(x(t), e(t), i) = \xi(t)\Xi\xi^{\mathrm{T}}(t)$$

where

$$\xi(t) = \begin{bmatrix} e^{\mathrm{T}}(t) & x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-h) \end{bmatrix}$$

$$\Xi = \begin{bmatrix} \Theta_1 & -M^{\mathrm{T}}(i)B^{\mathrm{T}}(i)P_1(i) + P_2(i)\Delta A(i) & P_2(i)(A_d(i) + \Delta A_d(i)) \\ * & \Theta_2 & P_1(i)(A_d(i) + \Delta A_d(i)) \\ * & * & -Q \end{bmatrix}$$

$$\Theta_1 = (A(i) - H(i)C_1(i))^{\mathrm{T}}P_2(i) + P_2(i)(A(i) - H(i)C_1(i))$$

$$\Theta_2 = (A(i) + \Delta A(i) + B(i)M(i))^{\mathrm{T}}P_1(i) + P_1(i)(A(i) + \Delta A(i) + B(i)M(i)) + Q$$

Obviously, a sufficiently stabilizable condition for system (5) is that all the vertex of the convex hull satisfy the desired stable requirements.

Subsequently, under condition (7), for dynamic system (5), condition (6) implies

$$\dot{V}(x(t), e(t), i) < 0 \quad \forall t \in [1, 2^m]$$

Therefore, the dynamic error system (5) is stabilizable with $w(t) = 0$, and this concludes the proof.

Remark 3.1. *In order to minimize the influences of the disturbances, we will design H-infinity performance index for system (5) subject to all admissible disturbances, such that the dynamic system (5) is stable.*

Condition (8) is investigated in order to decrease the influences of the disturbances and design the matrices $K(i)$ and $H(i)$ subject to H-infinity performance index λ, such that the dynamic system (5) is stable.

$$\int_0^{\infty} z^{\mathrm{T}}(t)z(t)dt \le \lambda^2 \int_0^{\infty} w^{\mathrm{T}}(t)w(t)dt \tag{8}$$

Theorem 3.1. *For given matrices D_t and D_t^-, the dynamic system is stabilizable in the region $\varepsilon(P(i),1)$, and it also satisfies condition (8), if there exist a set of positive definite symmetric matrices $P_1(i)$ and $P_2(i)$, a definite symmetric matrix Q and a set of matrices $K(i)$ and $F(i)$ such that*

$$\Xi_2 = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ * & b_{22} & b_{23} & b_{24} \\ * & * & b_{33} & b_{34} \\ * & * & * & b_{44} \end{bmatrix} < 0 \quad \forall t \in [1, 2^m] \tag{9}$$

$$\bar{x}(t) \in \Theta(F(i)) \tag{10}$$

where

$$\begin{aligned}
&b_{11} = (A(i) - H(i)C(1i))^{\mathrm{T}} P_2(i) + P_2(i)(A(i) - H(i)C(1i)) \\
&b_{12} = -(D_t K(i) + D_t^- F(i))^{\mathrm{T}} B^{\mathrm{T}}(i) P_1(i) + P_2(i)\Delta A(i) \\
&b_{13} = P_2(i)(A_d(i) + \Delta A_d(i)) \\
&b_{14} = P_2(i)D(i), \quad b_{22} = a_{22} + C_2^{\mathrm{T}}(i)C_2(i) \\
&b_{23} = P_1(i)(A_d(i) + \Delta A_d(i)), \quad b_{24} = P_1(i)D(i) \\
&b_{33} = -Q, \quad b_{34} = 0, \quad b_{44} = -\lambda^2 I \\
&\tilde{M}(i) = (D_t K(i) + D_t^- F(i))
\end{aligned}$$

Proof: Introduce the following cost function for system (5) as $T > 0$

$$J(T) = \int_0^T z^{\mathrm{T}}(t)z(t)dt - \lambda^2 \int_0^T w^{\mathrm{T}}(t)w(t)dt \tag{11}$$

Under zero initial condition, index $J(T)$ can be rewritten as

$$J(T) = \int_0^T \left[z^{\mathrm{T}}(t)z(t) - \lambda^2 w^{\mathrm{T}}(t)w(t) + \dot{V}(x(t), e(t), i) \right] - V(x(t), e(t), i) \tag{12}$$

Recalling Proposition 3.1, under condition (10), for each vertex of convex hull, it follows that

$$\begin{aligned}
J(T) = \int_0^T &\{x^{\mathrm{T}}(t)C^{\mathrm{T}}(2i)C(2i)x(t) - \lambda^2 w^{\mathrm{T}}(t)w(t) \\
&+ x^{\mathrm{T}}(t)P_1(i)[(A(i) + \Delta A(i) + B(i)\tilde{M}(i))x(t) + (A_d(i) + \Delta A_d(i))x(t-h) \\
&+ D(i)w(t) - B(i)\tilde{M}(i)e(t)] + [(A(i) + \Delta A(i) + B(i)\tilde{M}(i))x(t) + (A_d(i) \\
&+ \Delta A_d(i))x(t-h) - B(i)\tilde{M}(i)e(t) + D(i)w(t)]^{\mathrm{T}} P_1(i)x(t) + e^{\mathrm{T}}(t)P_2(i)[(A(i) \\
&- H(i)C_1(i))e(t) + \Delta A(i)x(t) + (A_d(i) + \Delta A_d(i))x(t-h) + D(i)w(t)] \\
&+ [(A(i) - H(i)C(1i))e(t) + \Delta A(i)x(t) + (A_d(i) + \Delta A_d(i))x(t-h) \\
&+ D(i)w(t)]^{\mathrm{T}} P_2(i)e(t) + x^{\mathrm{T}}(t)Qx(t) - x^{\mathrm{T}}(t-h)Qx(t-h)\}dt - V(x(t), e(t), i)
\end{aligned}$$

Thus,

$$J(T) \leq \int_0^T \{S \cdot \Xi_2 \cdot S^{\mathrm{T}}\}dt \tag{13}$$

where

$$S = \begin{bmatrix} e^{\mathrm{T}}(t) & x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-h) & w^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$$

It is clear that under the condition (10), $\Xi_2 < 0$ can be reduced to inequality (6) by denoting $w(t) = 0$, so the dynamic error system (5) is stabilizable in the proposed region. On the other hand, for $T \to \infty$, $\Xi_2 < 0$ results in $J(\infty) < -V(\infty) < 0$, that is

$$\int_0^\infty z^{\mathrm{T}}(t)z(t)dt \leq \lambda^2 \int_0^\infty w^{\mathrm{T}}(t)w(t)dt \tag{14}$$

Now we are ready to present the following result.

Theorem 3.2. *For given matrices D_t, D_t^-, and initial state x_0, $\bar{x}_0$, system (5) is stabilizable in the region $\varepsilon(P(i), 1)$ under such observer, and it also satisfies condition (14), if there exist a set of positive definite symmetric matrices $P(i)$, a definite symmetric matrix Q, and a set of matrices $K(i)$ and $F(i)$ such that*

$$\Xi_3 = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix} < 0 \tag{15}$$

$$f_q^{\mathrm{T}}(i)f_q(i) \leq P(i) \tag{16}$$

$$\hat{x}_0^{\mathrm{T}}(t)\hat{P}(i)\hat{x}_0(t) \leq 1 \tag{17}$$

where $f_q(i)$ is the qth row of matrix $F(i)$, $q = 1, 2, \ldots, m$

$$\hat{P}(i) = \begin{bmatrix} P(i) & 0 \\ 0 & P(i) \end{bmatrix}, \quad \hat{x}_0(i) = \begin{bmatrix} x_0(i) \\ \bar{x}_0(i) \end{bmatrix}$$

and

$$c_1 = \begin{bmatrix} c_{11} & c_{12} & A_d(i) & D(i) & X(i)C^{\mathrm{T}}(1i) \\ * & c_{22} & A_d(i) & D_k(i) & 0 \\ * & * & -Q & 0 & 0 \\ * & * & * & -\lambda^2 I & 0 \\ * & * & * & * & -I/2 \end{bmatrix}$$

$$c_2 = \begin{bmatrix} 0 & 0 \\ X(i)C^{\mathrm{T}}(2i) & X(i)E^{\mathrm{T}}(i) \\ 0 & E_d^{\mathrm{T}}(i) \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned}
c_4 &= -\,diag\left\{ \; I \quad \alpha I \; \right\}, \quad c_3 = c_2^{\mathrm{T}} \\
c_{11} &= X(i)A^{\mathrm{T}}(i) + A(i)X(i) + \alpha G(i)G^{\mathrm{T}}(i) \\
c_{12} &= -\,(\hat{K}^{\mathrm{T}}(i)D_t^{\mathrm{T}} + \hat{F}^{\mathrm{T}}(i)(D_t^-)^{\mathrm{T}})B^{\mathrm{T}}(i) + \alpha G(i)G^{\mathrm{T}}(i) \\
c_{22} &= (A(i)X(i) + B(i)D_t\hat{K}(i) + B(i)D_t^-\hat{F}(i))^{\mathrm{T}} \\
&\quad + \alpha G(i)G^{\mathrm{T}}(i) + A(i)X(i) + B(i)D_t\hat{K}(i) + B(i)D_t^-\hat{F}(i) + \hat{Q} \\
\hat{K}(i) &= K_k(i)X(i), \quad H(i) = -X(i)C_d^{\mathrm{T}}(i), \quad \hat{F}(i) = F(i)X(i)
\end{aligned}$$

Proof: Recalling Theorem 3.1, one can obtain the following equation:

$$\Xi_2 = \Xi_4 + M(i)\Upsilon_k(i)N(i) + N^{\mathrm{T}}(i)\Upsilon^{\mathrm{T}}(i)M^{\mathrm{T}}(i) \tag{18}$$

where

$$\Xi_4 = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ * & d_{22} & d_{23} & d_{24} \\ * & * & d_{33} & d_{34} \\ * & * & * & d_{44} \end{bmatrix}$$

$$\begin{aligned} d_{11} &= b_{11}, \quad d_{12} = -(D_t K(i) + D_t^- F(i))^{\mathrm{T}} B^{\mathrm{T}}(i) P_1(i), \\ d_{13} &= P_2(i) A_d(i), \quad d_{14} = b_{14}, \\ d_{22} &= (A(i) + B(i)(D_t K(i) + D_t^- F(i)))^{\mathrm{T}} P_1(i) \\ &\quad + P_1(i)(A(i) + B(i)(D_t K(i) + D_t^- F(i))) + Q + C^{\mathrm{T}}(2i) C(2i) \\ d_{23} &= P_1(i) A_d(i), \quad d_{24} = b_{24}, \quad d_{33} = b_{33}, \quad d_{34} = 0, \quad d_{44} = b_{44} \end{aligned}$$

and

$$M(i) = \begin{bmatrix} P_2(i)G(i) \\ P_1(i)G(i) \\ 0 \\ 0 \end{bmatrix}, \quad N(i) = \begin{bmatrix} 0 & E(i) & E_d(i) & 0 \end{bmatrix}$$

From Lemma 2.2, it is easy to find that $\Xi_2 < 0$ equals to $\Xi_5 < 0$

$$\Xi_5 = \begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{16} & e_{17} \\ * & e_{22} & e_{23} & e_{24} & e_{25} & e_{26} & e_{27} \\ * & * & e_{33} & e_{34} & e_{35} & e_{36} & e_{37} \\ * & * & * & e_{44} & e_{45} & e_{46} & e_{47} \\ * & * & * & * & e_{55} & e_{56} & e_{57} \\ * & * & * & * & * & e_{66} & e_{67} \\ * & * & * & * & * & * & e_{77} \end{bmatrix} < 0$$

where

$$\begin{aligned} e_{11} &= A^{\mathrm{T}}(i) P_2(i) + P_2(i) A(i) + \alpha P_2(i) G(i) G^{\mathrm{T}}(i) P_2(i) \\ e_{12} &= (D_t K(i) + D_t^- F(i))^{\mathrm{T}} B^{\mathrm{T}}(i) P_1(i) + \alpha P_2(i) G(i) G^{\mathrm{T}}(i) P_1(i) \\ e_{13} &= P_2(i) A_d(i), \quad e_{14} = b_{14}, \quad e_{15} = C^{\mathrm{T}}(1i), \quad e_{16} = 0, \quad e_{17} = 0 \\ e_{22} &= (A(i) + B(i)(D_t K(i) + D_t^- F(i)))^{\mathrm{T}} P_1(i) \\ &\quad + P_1(i)(A(i) + B(i)(D_t K(i) + D_t^- F(i))) + Q + \alpha P_1(i) G(i) G^{\mathrm{T}}(i) P_1(i) \\ e_{23} &= P_1(i) A_d(i), \quad e_{24} = b_{14}, \quad e_{25} = 0, \quad e_{26} = C^{\mathrm{T}}(2i), \quad e_{27} = E^{\mathrm{T}}(i) \\ e_{33} &= -Q, \quad e_{34} = 0, \quad e_{35} = 0, \quad e_{36} = 0, \quad e_{37} = E_d^{\mathrm{T}}(i) \\ e_{44} &= -\lambda^2 I, \quad e_{45} = 0, \quad e_{46} = 0, \quad e_{47} = 0 \\ e_{55} &= -I/2, \quad e_{56} = 0, \quad e_{57} = 0 \\ e_{66} &= -I, \quad e_{67} = 0, \quad e_{77} = -\alpha I \end{aligned}$$

In order to bring convenience, we denote $P_1(i) = P_2(i) = P(i)$, subsequently, one can pre- and post-multiply Ξ_5 by $diag\left\{ P^{-1}(i) \quad P^{-1}(i) \quad I \quad I \quad I \quad I \quad I \right\}$, and denote $X(i) = P^{-1}(i)$, $\hat{K}(i) = K(i)X(i)$, $\hat{F}(i) = F(i)X(i)$, then condition (15) can be obtained.

On the other hand, for given initial state ellipsoid set (17), if condition (16) is satisfied, then condition (7) is held; subsequently, condition (15) guarantees that the given initial state belongs to the invariant set, and then, the observer designed will make the system stochastically stable.

This completes the proof.

Next, we will design continuous gain-scheduled observer.

First, from Theorem 3.2, the gain matrices $K(i) \in R^{m\times n}$ and $F(i) \in R^{m\times n}$ can be obtained for the kth linear error dynamic system (5), and we will define $K(a,b,i)$ and $F(a,b,i)$ as each element of $K(i)$ and $F(i)$ where $a = 1,2,3,\ldots,m$, $b = 1,2,3,\ldots,n$ and

$$K(i) = \begin{bmatrix} K(1,1,i) & K(1,2,i) & \ldots & K(1,n,i) \\ K(2,1,i) & K(2,2,i) & \ldots & K(2,n,i) \\ \vdots & \vdots & \vdots & \vdots \\ K(m,1,i) & K_k(m,2,i) & \ldots & K(m,n,i) \end{bmatrix}$$

$$F(i) = \begin{bmatrix} F(1,1,i) & F(1,2,i) & \ldots & F(1,n,i) \\ F(2,1,i) & F(2,2,i) & \ldots & F(2,n,i) \\ \vdots & \vdots & \vdots & \vdots \\ F(m,1,i) & F(m,2,i) & \ldots & F(m,n,i) \end{bmatrix}$$

Second, we denote the matrices $\hat{K}(i)$ and $\hat{F}(i)$ as follows:

$$\hat{K}(i) = \begin{bmatrix} \hat{K}(1,1,i) & \hat{K}(1,2,i) & \ldots & \hat{K}(1,n,i) \\ \hat{K}(2,1,i) & \hat{K}(2,2,i) & \ldots & \hat{K}(2,n,i) \\ \vdots & \vdots & \vdots & \vdots \\ \hat{K}(m,1,i) & \hat{K}(m,2,i) & \ldots & \hat{K}(m,n,i) \end{bmatrix}$$

$$\hat{F}(i) = \begin{bmatrix} \hat{F}(1,1,i) & \hat{F}(1,2,i) & \ldots & \hat{F}(1,n,i) \\ \hat{F}(2,1,i) & \hat{F}(2,2,i) & \ldots & \hat{F}(2,n,i) \\ \vdots & \vdots & \vdots & \vdots \\ \hat{F}(m,1,i) & \hat{F}(m,2,i) & \ldots & \hat{F}(m,n,i) \end{bmatrix}$$

$$\tilde{x}_e(t) = \begin{bmatrix} x_e^{(1)}(t) & x_e^{(2)}(t) & \ldots & x_e^{(\mathrm{s})}(t) \end{bmatrix}, \quad e \in \{1,2,3\ldots n\}$$

where

$$\hat{K}(1,1,i) = \begin{bmatrix} K_1(1,1,i) & K_2(1,1,i) & \ldots & K_s(1,1,i) \end{bmatrix}$$
$$\hat{K}(1,2,i) = \begin{bmatrix} K_1(1,2,i) & K_2(1,2,i) & \ldots & K_s(1,2,i) \end{bmatrix}$$
$$\ldots$$
$$\hat{K}(m,n,i) = \begin{bmatrix} K_1(m,n,i) & K_2(m,n,i) & \ldots & K_s(m,n,i) \end{bmatrix}$$
$$\hat{F}(1,1,i) = \begin{bmatrix} F_1(1,1,i) & F_2(1,1,i) & \ldots & F_s(1,1,i) \end{bmatrix}$$
$$\hat{F}(1,2,i) = \begin{bmatrix} F_1(1,2,i) & F_2(1,2,i) & \ldots & F_s(1,2,i) \end{bmatrix}$$
$$\ldots$$
$$\hat{F}(m,n,i) = \begin{bmatrix} F_1(m,n,i) & F_2(m,n,i) & \ldots & F_s(m,n,i) \end{bmatrix}$$

Next, a fixed and appropriate value of e is selected and polynomial fitting approach is applied to matrices $\hat{K}(a,b,i)$, $\hat{F}(a,b,i)$ and $\tilde{x}_e(t)$, and then, each element of the gain matrices is described as a polynomial, and continuous observer is obtained for nonlinear system (1).

$$K(i)=\begin{bmatrix} K(1,1,i) & K(1,2,i) & \dots & K(1,n,i) \\ K(2,1,i) & K(2,2,i) & \dots & K(2,n,i) \\ \vdots & \vdots & \vdots & \vdots \\ K(m,1,i) & K(m,2,i) & \dots & K(m,n,i) \end{bmatrix}$$

$$F(i)=\begin{bmatrix} F(1,1,i) & F(1,2,i) & \dots & F(1,n,i) \\ F(2,1,i) & K(2,2,i) & \dots & F(2,n,i) \\ \vdots & \vdots & \vdots & \vdots \\ F(m,1,i) & F(m,2,i) & \dots & F(m,n,i) \end{bmatrix}$$

where

$$\begin{aligned} K(1,1,i) &= q_0(1,1)+q_1(1,1)x_e(t)+q_2(1,1)x_e^2(t) \\ &\quad + q_3(1,1)x_e^3(t)+\dots+q_{g(11)}(1,1)x_e^{g(11)}(t) \\ K(1,2,i) &= q_0(1,2)+q_1(1,2)x_e(t)+q_2(1,2)x_e^2(t) \\ &\quad + q_3(1,2)x_e^3(t)+\dots+q_{g(12)}(1,2)x_e^{g(12)}(t) \\ &\cdots \\ K(m,n,i) &= q_0(m,n)+q_1(m,n)x_e(t)+q_2(m,n)x_e^2(t) \\ &\quad + q_3(m,n)x_e^3(t)+\dots+q_{g(mn)}(m,n)x_e^{g(mn)}(t) \\ F(1,1,i) &= l_0(1,1)+l_1(1,1)x_e(t)+l_2(1,1)x_e^2(t) \\ &\quad + l_3(1,1)x_e^3(t)+\dots+l_{g(11)}(1,1)x_e^{g(11)}(t) \\ F(1,2,i) &= l_0(1,2)+l_1(1,2)x_e(t)+l_2(1,2)x_e^2(t) \\ &\quad + l_3(1,2)x_e^3(t)+\dots+l_{g(12)}(1,2)x_e^{g(12)}(t) \\ &\cdots \\ F(m,n,i) &= l_0(m,n)+l_1(m,n)x_e(t)+l_2(m,n)x_e^2(t) \\ &\quad + l_3(m,n)x_e^3(t)+\dots+l_{g(mn)}(m,n)x_e^{g(mn)}(t) \end{aligned}$$

$g(11), g(12), \dots, g(mn)$ are selected and fitted values subject to the fitted error; $q_0(1,1)$, $\dots, q_{g(mn)}(m,n)$, $l_0(1,1), \dots, l_{g(mn)}(m,n)$ are fitted coefficients which are found in polynomial fitting approach.

Remark 3.2. *We proposed a sufficient condition for the existence of nonlinear observer via linear matrix inequality approach. It is obvious that more working points we select, less conservativeness we obtain.*

4. **Numerical Example.** The initial condition for uncertain CSTR system is given as $x_0 = [\ 0.5 \quad 0.5\ ]^{\mathrm{T}}$, disturbance is given as $w(t) = [\ 0.5\sin 0.25\pi t \quad 0.5 + 0.5\sin 0.25\pi t\ ]^{\mathrm{T}}$, and the system state trajectories under the observer are obtained as shown in Figures 1 and 2.

Remark 4.1. *It is worth mentioning that the continuous gain-scheduled observer approach designed in this paper can be applied to many practical nonlinear systems, the SCTR system mentioned above is only a successful application example. The continuous H-infinity observer designed in this paper varies its parameters with the variation of states over the time interval one concerned; this observer is applicable, in practice, such as*

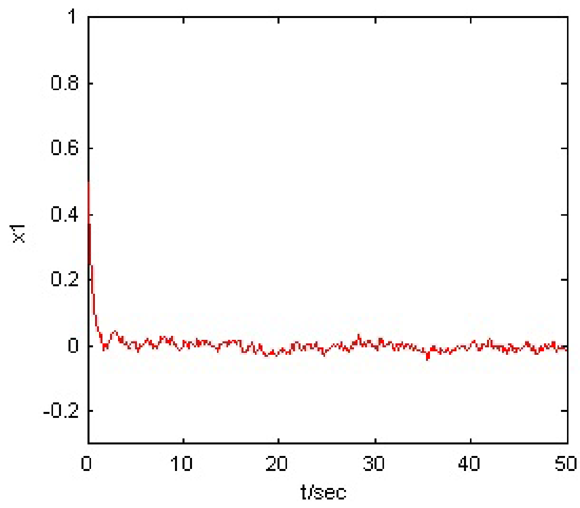

FIGURE 1. Trajectory of state x_1

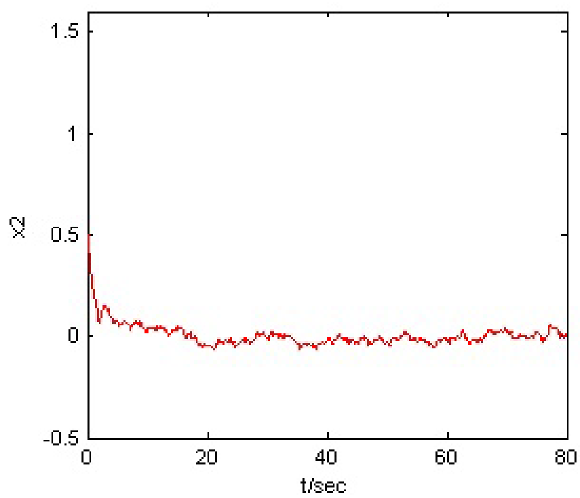

FIGURE 2. Trajectory of state x_2

manufacturing systems, bioreactor systems and networked systems, and there is no sudden switch of the observer parameters.

5. **Conclusions.** In this paper, the issue on gain-scheduled H-infinity observer for a class of uncertain nonlinear systems is addressed. Gradient linearization approach is applied to such systems and linear error dynamic systems are obtained. Actuator saturation is expressed in terms of linear matrix inequalities. Taylor fitting approach is investigated and continuous gain-scheduled observer is designed. The simulation shows the potential of the proposed techniques.

Acknowledgment. This work was partially supported by the National Natural Science Foundation of China (61134007), the Program for Excellent Innovative Team of Jiangsu Higher Education Institutions, the Fundamental Research Funds for the Central Universities (JUDCF10032). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

[1] H. Liu, E. K. Boukas, F. Sun and W. C. Daniel, Controller design for Markov jumping systems subject to actuator saturation, *Automatica*, vol.42, no.3, pp.459-465, 2006.

[2] H. Liu, F. Sun and E. K. Boukas, Robust control of uncertain discrete-time Markovian jump systems with actuator saturation, *International Journal of Control*, vol.79, no.7, pp.805-812, 2006.

[3] Y. Cao, Z. Lin and Y. Shamash, Set invariance analysis and gain-scheduling control for LPV systems subject to actuator saturation, *Systems and Control Letters*, vol.46, no.2, pp.137-151, 2002.

[4] T. Hu, Z. Lin and M. Chen, An analysis and design method for linear systems subject to actuator saturation and disturbance, *Automatica*, vol.38, no.2, pp.351-359, 2002.

[5] T. Hu and Z. Lin, Output regulation of linear systems with bounded continuous feedback, *IEEE Trans. on Automatic Control*, vol.49, no.11, pp.1941-1953, 2004.

[6] T. Hu, Z. Lin and B. Chen, Analysis and design for discrete-time linear systems subject to actuator saturation, *Systems and Control Letters*, vol.45, no.2, pp.97-112, 2002.

[7] Y. Xia, Z. Zhu, C. Li, H. Yang and Q. Zhu, Robust adaptive sliding mode control for uncertain discrete-time systems with time delay, *Journal of the Franklin Institute*, vol.347, no.1, pp.339-357, 2010.

[8] L. V. Hien and V. N. Phat, Exponential stability and stabilization of a class of uncertain linear time-delay systems, *Journal of the Franklin Institute*, vol.346, no.6, pp.611-625, 2009.

[9] J. Qiu and K. Lu, New robust passive stability criteria for uncertain singularly Markov jump systems with time delays, *ICIC Express Letters*, vol.3, no.3(B), pp.651-656, 2009.

[10] Y. Xia, Z. Zhu and M. S. Mahmoud, H2 control for networked control systems with Markovian data losses and delays, *ICIC Express Letters*, vol.3, no.3(A), pp.271-276, 2009.

[11] Y. Yin, F. Liu and P. Shi, Finite-time continuous gain-scheduled control on stochastic hyperchaotic systems, *J. Systems and Control Engineering*, vol.224, no.I6, pp.679-688, 2010.

[12] Y. Yin, P. Shi and F. Liu, Gain-scheduled robust fault detection on time-delay stochastic nonlinear systems, *IEEE Trans. on Industrial Electronics*, vol.58, no.10, pp.4908-4916, 2011.

[13] Y. Yin, F. Liu and P. Shi, Finite-time gain-scheduled control on stochastic bioreactor systems with partially known transition jump rates, *Circuits, Systems, and Signal Processing*, vol.30, no.3, pp.609-627, 2011.

[14] H. Sira-Ramirez and M. I. Angulo-Nunez, Passivity-based control of nonlinear chemical processes, *International Journal of Control*, vol.68, no.5, pp.971-996, 1997.

[15] C. Marcelo and H. Stanislaw, Stabilizing controller design for uncertain nonlinear systems using fuzzy models, *IEEE Trans. on Fuzzy Systems*, vol.7, no.2, pp.133-142, 1999.

[16] Y. Wang, L. Xie and C. E. de Souza, Robust control of a class of uncertain nonlinear systems, *Systems and Control Letters*, vol.19, no.2, pp.139-149, 1992.