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VIBRATIONS IN PIPE SYSTEMS, TAKING INTO ACCOUNT SHEAR DEFORMATIONS

Arnulfo Luévanos Rojas[1], Nataliya I. Kalashnykova[2,3]
Ramón Luévanos Rojas[1], Facundo Cortés Martinez[1]
and Armando Cesar Uranga Sifuentes[1]

[1]Facultad de Ingeniería, Ciencias y Arquitectura
Universidad Juárez del Estado de Durango
Gómez Palacio, Durango, México
arnulfol_2007@hotmail.com

[2]Department of Physics and Mathematics
Universidad Autónoma de Nuevo León
San Nicolás de los Garza, Nuevo León, México
nkalash2009@gmail.com

[3]Department of Applied Mathematics
Sumy State University
2 Rimsky-Korsakov Str., Sumy 40007, Ukraine

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Abstract. *This paper proposes to take into account the deformations caused by shear, in the method of multiple response spectrum for dynamic analysis of piping systems with different movements at the supports and using the consistent mass or distributed. This methodology takes the response spectrum corresponding to each of the supports, including the flexure deformations and shear, the classical method of multiple response spectrum considering only the flexure deformations. Also it makes a comparison between the proposed method and the classical method; in the latter all values are not conservative, as you can see the problem considered. Then, the usual practice, neglecting shear deformations is not a recommendable solution. Therefore, it is proposed to consider the shear deformations and also more attached to the real conditions.*
Keywords: Form factor, Shear deformations, Matrix of pseudostatic influence, Modal analysis, Spectral analysis, Eigenvalues, Eigenvectors, Modal participation factor, Spectral acceleration

1. Introduction.

In the design of industrial facilities and nuclear, the study of its seismic-dynamic behavior constitutes a fundamental stage within its design, since, it has the probability that excitations appear by seismic effects during the useful life of these plants, and the damage caused by these effects can be predominant between the diverse requirements to consider for your design. This obviously, will be the main effect in facilities that are located in zones of median seismicity and high, as it happens in several parts of our planet.

Between the diverse industrial facilities or nuclear, it is frequent that we face with structural systems having multiple supports at different elevations and/or that are much extended in length. This situation implies that the seismic excitations in their supports are different, either because the excitations of equipment in high elevations are generally bigger than in low elevations, or because in structural systems very extended horizontally would present movements relative between their supports as a result of the propagation of the seismic waves through the ground [1-4].

The pipes systems of Industrial and Nuclear Plants constitute a typical example of those structural systems that present multiple supports, which interconnect diverse delicate equipment, which is supported directly on floor or in special structures [5-7].

It is frequent to find secondary structures or equipment (SS) attached to main structural systems (MS) [5,8,9]. These combined systems usually with different properties are found in buildings where there are antennas, or delicate equipment, piping in industrial buildings, security systems, etc. This type of systems has been studied by diverse researchers in the past using basically two methods to study the seismic response: analysis of history in the time and spectrum analysis of floor response. The first method, consists in obtaining the response spectrum of the alone MS; this response is used as excitation of the SS, in the connection points of the MS, in a later and independent analysis, which means that the interaction between the MS and SS is not considered; this is the case of the pioneering work of Kassawara and Peck [10]. The second method consists in knowing the response spectrum in the connection points between the MS and the SS by using only the response of the MS and without considering the SS response. The first studies, in which can be mentioned: Amin et al. [11], Shaw [12] and Vashi [5], among others, did not consider the interaction between the systems.

The work by Lee and Penzien [13], who used a model of stationary earthquake stochastic process to study the influence of the modal correlation in the structural response, Der Kiureghian and Igusa [14] studied the influence of the correlation between the modes in the response of systems MS+SS considering in addition the interaction between them. In this study, the authors show that depending on the reasons of mass and frequency there are many practical situations where the interaction between the MS and the SS can be highly significant in the structural response, and therefore must be included in the analysis. The concept of including the interaction is also adopted by Crandall and Mark [15], Amin et al. [11], Pickel [16] and Der Kiureghian et al. [17].

Asfura and Der Kiureghian [18] using stationary stochastic vibrations studied the behavior of systems MS and SS with generalized configurations of connection between the MS and the SS, determining a crossed floor spectrum for the connection points of the system considering the interaction between them. These spectrums are used for the design of the SS.

Authors like Suarez and Singh [1,19,20], Falsone et al. [2,21], have developed complete analysis, modeling MS along with SS.

To calculate the response in time of complex systems of several degrees of freedom that include systems MS+SS, Valladares [22], developed a numerical method to obtain the crossed correlation matrix of the structures subject to simulated earthquakes like stochastic processes. In this study, it is used a nonstationary earthquake model in amplitude and frequency, developed by Crempien and Aravena [23].

Indeed, stationary stochastic vibration approach has also been used to study the behavior of the SS structure, considering different excitations, i.e., taking the response spectrum corresponding to each of the supports and not considering the envelope the response spectrum of all the supports, as usually these analyses were developed. This method is more realistic, since it allows to consider different excitations in each support and it is attached to the requirements of the code ASME Section III that specifies that the efforts developed in a structure are decomposed in primary part and secondary, with different permissible limits in each one [3].

The approach adopted in this work is for secondary systems, as it is the case of pipes for industrial facilities and nuclear. Nevertheless, with this methodology it is possible to model the main structure and the secondary structure, for a complete analysis.

In this paper, the spectrum method of multiple response has been employed with consistent or distributed mass, and consider the shear deformations that is the important part of the present research, and addition is developing a comparison with the traditional method, when shear deformations are not considered.

2. Development.

2.1. Theoretical principles. Consider the system "pipe-equipment-structure" shown in Figure 1, in which the multiple excitations in the supports of the pipe are induced and furthermore they are non-uniform.

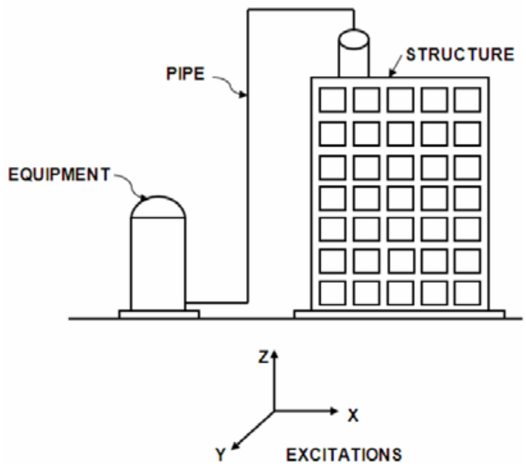

FIGURE 1. The system "pipe-equipment-structure"

The general equations of motion for the pipe system, without including conditions of border [24,25], can be written in matrix form [26]:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \tag{1}$$

where

U_1 = a vector of $n \times 1$ of absolute generalized displacements (not known), corresponding to the degrees of freedom not restricted "n".

U_2 = a vector of $m \times 1$ of absolute generalized displacements (null or prescribed), corresponding to the degrees of freedom of the support points "m".

M_{ij}, C_{ij}, K_{ij} = mass matrices, damping and stiffness, which are associated to the degrees of freedom "n" and/or "m" respectively.

P_1 = Vector of $n \times 1$ that represents associates dynamics requirements to the degrees of freedom "n".

P_2 = Vector of $m \times 1$ that represents the reactions (not known) associates to the degrees of freedom of the supports "m".

For the case of seismic excitations, "$P_1 = 0$" and the values of "$\ddot{U}_2$, $\dot{U}_2$, U_2", they are considered known. Therefore, the first expression of Equation (1) of the system is:

$$M_{11}\ddot{U}_1 + C_{11}\dot{U}_1 + K_{11}U_1 = -M_{12}\ddot{U}_2 - C_{12}\dot{U}_2 - K_{12}U_2 \tag{2}$$

The total displacement "U_1" can be expressed like the sum of the relative displacement "U_1^r" and the pseudostatic displacement "U_1^s" that would be from a static displacement of the support according to be seen in Figure 2, this is:

$$U = U^r + U^s \tag{3}$$

where

$$U_1 = U_1^r + U_1^s; \quad \dot{U}_1 = \dot{U}_1^r + \dot{U}_1^s; \quad \ddot{U}_1 = \ddot{U}_1^r + \ddot{U}_1^s \tag{4}$$

$$U_2 = U_2^s; \quad U_2^r = 0; \quad \dot{U}_2 = \dot{U}_2^s; \quad \ddot{U}_2 = \ddot{U}_2^s \tag{5}$$

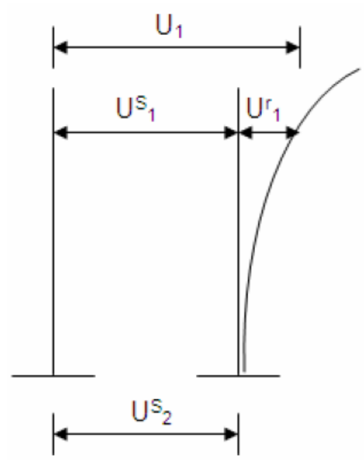

FIGURE 2. Total displacements

Substituting Equation (4) into Equation (2) gives:

$$M_{11}(\ddot{U}_1^r + \ddot{U}_1^s) + C_{11}(\dot{U}_1^r + \dot{U}_1^s) + K_{11}(U_1^r + U_1^s) = -M_{12}\ddot{U}_2 - C_{12}\dot{U}_2 - K_{12}U_2 \tag{6}$$

The pseudostatic displacements will be evaluated by the static equilibrium condition, which is obtained from Equation (6), that is:

$$K_{11}U_1^s = -K_{12}U_2 \tag{7}$$

$$U_1^s = \hat{r}U_2 \tag{8}$$

Being "$\hat{r}$" the pseudostatic influence matrix may be expressed as:

$$\hat{r} = -K_{11}^{-1}K_{12} \tag{9}$$

Substituting Equation (8) into Equation (4) gives:

$$U_1 = U_1^r + \hat{r}U_2 \tag{10}$$

The dynamic component of displacements will be expressed as:

$$U_1^r = U_1 - \hat{r}U_2 \tag{11}$$

Substituting Equation (10) into Equation (2), the equations of motion in terms of the component of dynamic displacements are obtained exclusively, with the result:

$$M_{11}(\ddot{U}_1^r + \hat{r}\ddot{U}_2) + C_{11}(\dot{U}_1^r + \hat{r}\dot{U}_2) + K_{11}(U_1^r + \hat{r}U_2) = -M_{12}\ddot{U}_2 - C_{12}\dot{U}_2 - K_{12}U_2 \tag{12}$$

Or:

$$M_{11}\ddot{U}_1^r + C_{11}\dot{U}_1^r + K_{11}U_1^r = -(M_{11}\hat{r} + M_{12})\ddot{U}_2 - (C_{11}\hat{r} + C_{12})\dot{U}_2 - (K_{11}\hat{r} + K_{12})U_2 \tag{13}$$

Then, substituting Equation (9) in the final member of Equation (13), the equations of motion in terms of the component of dynamic displacements may be rewritten:

$$M_{11}\ddot{U}_1^r + C_{11}\dot{U}_1^r + K_{11}U_1^r = -(M_{11}\hat{r} + M_{12})\ddot{U}_2 - (C_{11}\hat{r} + C_{12})\dot{U}_2 \tag{14}$$

It is important to indicate that when using a formulation of concentrated mass as is normal, implies that the term "$M_{12}\ddot{U}_2$" is null.

By other part, the damping in the excitation [26-28] is demonstrated that the second term of the right side in Equation (14) is very small in comparison with first, reason why usually it is not considered. In addition, when realizing a spectral analysis, the effect of the damping of the excitation, comes implicit in the spectrums.

Then, Equation (14) may be written:

$$\mathrm{M}_{11}\ddot{\mathrm{U}}_1^{\mathrm{r}} + \mathrm{C}_{11}\dot{\mathrm{U}}_1^{\mathrm{r}} + \mathrm{K}_{11}\mathrm{U}_1^{\mathrm{r}} = -(\mathrm{M}_{11}\hat{\mathrm{r}} + \mathrm{M}_{12})\ddot{\mathrm{U}}_2 \tag{15}$$

Substituting Equation (9) into Equation (15) gives:

$$\mathrm{M}_{11}\ddot{\mathrm{U}}_1^{\mathrm{r}} + \mathrm{C}_{11}\dot{\mathrm{U}}_1^{\mathrm{r}} + \mathrm{K}_{11}\mathrm{U}_1^{\mathrm{r}} = (\mathrm{M}_{11}\mathrm{K}_{11}^{-1}\mathrm{K}_{12} - \mathrm{M}_{12})\ddot{\mathrm{U}}_2 \tag{16}$$

Doing:

$$\mathrm{D} = \mathrm{M}_{11}\mathrm{K}_{11}^{-1}\mathrm{K}_{12} - \mathrm{M}_{12} \tag{17}$$

Substituting Equation (17) into Equation (16), with the result:

$$\mathrm{M}_{11}\ddot{\mathrm{U}}_1^{\mathrm{r}} + \mathrm{C}_{11}\dot{\mathrm{U}}_1^{\mathrm{r}} + \mathrm{K}_{11}\mathrm{U}_1^{\mathrm{r}} = \mathrm{D}\ddot{\mathrm{U}}_2 \tag{18}$$

Consequently, the vector of effective or equivalent forces "P_{eq}" acting in each one of the freedom degrees "n" of the structure or system will be expressed as:

$$[\mathrm{P}_{\mathrm{eq}}]_{n\times 1} = [\mathrm{D}]_{n\times m}[\ddot{\mathrm{U}}_2]_{m\times 1} \tag{19}$$

With this, the equations of motion may be written:

$$\mathrm{M}_{11}\ddot{\mathrm{U}}_1^{\mathrm{r}} + \mathrm{C}_{11}\dot{\mathrm{U}}_1^{\mathrm{r}} + \mathrm{K}_{11}\mathrm{U}_1^{\mathrm{r}} = \mathrm{P}_{\mathrm{eq}} \tag{20}$$

In case in that the movement of all the support is identical and is defined: the acceleration of the floor "$\ddot{\mathrm{U}}_g(t)$", the vector "$\ddot{\mathrm{U}}_2$", that is:

$$\ddot{\mathrm{U}}_2 = \begin{Bmatrix} 1 \\ \vdots \\ 1 \end{Bmatrix} \ddot{\mathrm{U}}_g(t) \tag{21}$$

The vector of equivalent forces may be expressed as:

$$[\mathrm{P}_{\mathrm{eq}}]_{n\times 1} = [\mathrm{D}]_{n\times m}[1]_{m\times 1}\ddot{\mathrm{U}}_g(t) \tag{22}$$

In case that a supports receive one excitation "$a_1\ddot{\mathrm{U}}_g(t)$" and the other points receive an excitation "$a_2\ddot{\mathrm{U}}_g(t)$", where "$a_1$" and "$a_2$" are scalar, the vector of accelerations of the supports "$\ddot{\mathrm{U}}_2$" is presented as:

$$\ddot{\mathrm{U}}_2 = \begin{Bmatrix} a_1 \\ \vdots \\ a_1 \\ a_2 \\ \vdots \\ a_2 \end{Bmatrix} \ddot{\mathrm{U}}_g(t) \tag{23}$$

This procedure can be extended to the case of arbitrary excitations "m", "$\ddot{\mathrm{U}}_m(t)$" in each one of the supports "m". The result is

$$\ddot{\mathrm{U}}_2 = \begin{Bmatrix} \ddot{\mathrm{U}}_1(t) \\ \vdots \\ \ddot{\mathrm{U}}_m(t) \end{Bmatrix} \tag{24}$$

According to Equation (23) and Equation (24) it may be rewritten:

$$\ddot{\mathrm{U}}_2 = \sum_{j=1}^{m}\{\mathrm{V}^{(\mathrm{J})}\}\ddot{\mathrm{U}}_j(t) \tag{25}$$

where each vector "$\{V^{(j)}\}$" is defined, unit for line "j" and zero for the others.

For each vector "$\{V^{(j)}\}$" an equivalent force "$P_{eq}^{(j)}$" on the structure or system is defined as:

$$P_{eq}^{(j)} = D\{V^{(j)}\}\ddot{U}_j(t) \tag{26}$$

The total force as:

$$P_{eq}^{(T)} = \sum_{j=1}^{m}\{P_{eq}^{(j)}\} \tag{27}$$

For each one of the equivalent forces is originated a problem of response given as:

$$M_{11}\ddot{U}_1^{r(j)} + C_{11}\dot{U}_1^{r(j)} + K_{11}U_1^{r(j)} = P_{eq}^{(j)} \tag{28}$$

The relative displacement (dynamic) will be expressed as:

$$\ddot{U}_2 = \sum_{j=1}^{m} U_1^{r(j)} \tag{29}$$

According to the previously described, it is determined the dynamic response of a subject structure to different accelerations "m" in the supports "m" are decomposed in the solution of problems "m", each one corresponds to the excitation of a single support [26].

Considering linear systems, where conditions of orthogonality for stiffness matrices "K", Mass "M" and damping "C" exist; it turns out advisable to diagonalize the system of transformed equations of motion to a normal modal coordinate system. Considering the system of Equation (20), under the condition of free vibration without damping, which can exist in the absence of any excitation of the supports [29], may be expressed as:

$$M_{11}\ddot{U}_1^r + K_{11}U_1^r = 0 \tag{30}$$

Its solution is defined as:

$$U_1^r = \overrightarrow{\varnothing} e^{i\omega t} \tag{31}$$

where ω = natural frequency of vibration, $\overrightarrow{\varnothing}$ = modal vector (mode-shape vector) associated to "ω".

The values of "ω" and "$\overrightarrow{\varnothing}$" are determined by the solution of eigenproblems [20,30] as:

$$(K_{11} - \omega^2 M_{11})\overrightarrow{\varnothing} = 0 \tag{32}$$

With this, the equations of motion in the system are defined by Equation (20), it can be diagonalized if transformed to a normal modal coordinate system "$Y_n(t)$" is defined as:

$$U_1^r = \sum_{n=1}^{N} \overrightarrow{\varnothing}_n Y_n = \Phi\overrightarrow{Y} \tag{33}$$

where: Φ = modal matrix (mode-shape matrix), $\overrightarrow{Y}$ = vector of normal coordinates.

Substituting Equation (33) into Equation (20) and premultiplying by the transpose of the modal vector corresponding to mode "n" and applying conditions of orthogonality; it is obtained the undocking equations of motion [29]. Being the corresponding equation to mode "n" presented as:

$$M_n\ddot{Y}_n + 2M_n\eta_n\omega_n\dot{Y}_n + M_n\omega_n^2 Y_n = P_n(t) \tag{34}$$

where η_n = percent of damping to the mode "n".

Being:

$$\mathrm{M}_n = \overrightarrow{\varnothing}_n^t \mathrm{M}_{11} \overrightarrow{\varnothing}_n \tag{35}$$

$$\eta_n = \overrightarrow{\varnothing}_n^t \mathrm{C}_{11} \overrightarrow{\varnothing}_n / (2\mathrm{M}_n \omega_n) \tag{36}$$

$$\mathrm{P}_n(t) = \overrightarrow{\varnothing}_n^t \mathrm{P}_{\mathrm{eq}} \tag{37}$$

Now, when transforming to normal modal coordinates "Y_n", in the system of equations of motion of Equation (28) is obtained, for the system the degrees of freedom, "n", equations of motion uncoupled "n". In this case Equation (34) may be written:

$$\mathrm{M}_n \ddot{\mathrm{Y}}_n^{(j)} + 2\mathrm{M}_n \eta_n \omega_n \dot{\mathrm{Y}}_n^{(j)} + \mathrm{M}_n \omega_n^2 \mathrm{Y}_n^{(j)} = \overrightarrow{\varnothing}_n^t \mathrm{P}_{\mathrm{eq}}^{(j)} \tag{38}$$

Being "ω_n" and "$\overrightarrow{\varnothing}_n$" the eigenvalues and eigenvectors [30] corresponding to mode "n". Substituting Equation (26) into Equation (38) and realizing some transformations gives:

$$\ddot{\mathrm{Y}}_n^{(j)} + 2\eta_n \omega_n \dot{\mathrm{Y}}_n^{(j)} + \omega_n^2 \mathrm{Y}_n^{(j)} = \overrightarrow{\varnothing}_n^t \mathrm{D}\{\mathrm{V}^{(\mathrm{j})}\} \ddot{\mathrm{U}}_j(t)/\mathrm{M}_n \tag{39}$$

Or:

$$\ddot{\mathrm{Y}}_n^{(j)} + 2\eta_n \omega_n \dot{\mathrm{Y}}_n^{(j)} + \omega_n^2 \mathrm{Y}_n^{(j)} = \Gamma_n^{(j)} \ddot{\mathrm{U}}_j(t)/\mathrm{M}_n \tag{40}$$

$$\Gamma_n^{(j)} = \overrightarrow{\varnothing}_n^t \mathrm{D}\{\mathrm{V}^{(\mathrm{j})}\} \tag{41}$$

$$\Gamma_n^{(j)} = \left[\overrightarrow{\varnothing}_n^t \mathrm{M}_{11} \mathrm{K}_{11}^{-1} \mathrm{K}_{12} - \overrightarrow{\varnothing}_n^t \mathrm{M}_{12}\right] \{\mathrm{V}^{(j)}\} \tag{42}$$

where "$\mathrm{Y}_n^{(j)}$" represents the modal response for mode "n" due to the excitation of support "j", "$\Gamma_n^{(j)}$" represents the participation factor for the mode "n" and the seismic excitation in support "j".

The solution of Equation (40) can be obtained considering the first integral of Duhamel [29], as follows:

$$\mathrm{Y}_n^{(j)} = \left(\frac{\Gamma_n^{(j)}}{\mathrm{M}_n}\right)\left(\frac{1}{\omega_n}\right)\int_0^t \ddot{\mathrm{U}}_j(T)[e^{-\eta_n \omega_n (t-T)}] \sin \omega_n (t-T) dT \tag{43}$$

It is denoted:

$$S_{vn}^{(j)}(t) = \int_0^t \ddot{\mathrm{U}}_j(\tau)[e^{-\eta_n \omega_n (t-\tau)}] \sin \omega_n (t-\tau) d\tau \tag{44}$$

Thus, in general:

$$\mathrm{Y}_n^{(j)} = \left(\frac{\Gamma_n^{(j)}}{\mathrm{M}_n}\right)\left(\frac{S_{vn}^{(j)}}{\omega_n}\right) \tag{45}$$

where $S_{vn}^{(j)}$ = spectral velocity.

Now, according to the procedure, the spectrum of seismic response [29], will be sufficient to determine solely the maximum values of response, and not all the complete history. Of the expressions, Equation (43) and Equation (44), it is observed that the maximum responses are defined when considering the maximum value of the response function.

That in terms of the spectral acceleration "$S_{an}^{(j)}$", is obtained for mode "n" from the response spectrum corresponding to the excitation of the support "j" may be written:

$$S_{vn}^{(j)} = \frac{S_{an}^{(j)}}{\omega_n} \tag{46}$$

Substituting Equation (46) into Equation (45), it is obtained the maximum modal responses due to each excitation of the supports "$\left(\mathrm{Y}_n^{(j)}\right)_{\max}$", [29] is presented as:

$$\left(\mathrm{Y}_n^{(j)}\right)_{\max} = \left[\frac{\Gamma_n^{(j)}}{\mathrm{M}_n}\right]\left[\frac{S_{an}^{(j)}}{\omega_n^2}\right] \tag{47}$$

The maximum modal responses due to each excitation of the supports could not happen simultaneously, and total maximum modal response obtained by modal simple superposition, would give a quite preservative prediction. Reason why a maximum modal response, based in a probabilistic consideration, can be obtained considering the "Square Root of the Sum of the Squares"; procedure known as method "SRSS" [29], is defined as:

$$(\mathrm{Y}_n)_{\max} = \left\{\sum_{j=1}^{n}\left(\mathrm{Y}_n^{(j)}\right)_{\max}^2\right\}^{1/2} \tag{48}$$

The vectors corresponding to the components of the maximum relative displacement vector for each mode "$\{\mathrm{U}_{1n}^{\mathrm{r}}\}_{\max}$" [29], is denoted as:

$$\{\mathrm{U}_{1n}^{\mathrm{r}}\}_{\max} = \left\{\overrightarrow{\varnothing}_n\right\}(\mathrm{Y}_n)_{\max} \tag{49}$$

Once that was obtained the maximum responses for each one of the modes, the maximum value of the vector of relative displacements of the pipe system "$\{\mathrm{U}_1^{\mathrm{r}}\}_{\max}$"[29], is obtained as:

$$\{\mathrm{U}_1^{\mathrm{r}}\}_{\max} = \left\{\sum_{j=1}^{n}(\mathrm{U}_{1j}^{\mathrm{r}})_{\max}^2\right\}^{1/2} \tag{50}$$

The value of the equivalent mechanical elements that act in free joints "P" [29] may be expressed as:

$$\mathrm{P} = \mathrm{K}_{11}\{\mathrm{U}_1^{\mathrm{r}}\}_{\max} \tag{51}$$

Finally the mechanical elements that act on members "F" [29], is defined as:

$$\mathrm{F} = \mathrm{K}_i'\mathrm{U}_i \tag{52}$$

where K_i' = stiffness matrix of member "i", in the global or general system, U_i = displacements vector of member "i", in global system.

3. **Application.** The illustration of the methods of analyses previously considered to study the behavior of structural systems subjected to multiple excitation in its supports during seismic effects. In addition, it is presented the analysis of a typical pipe system, that is interconnecting equipment to two different elevations that induce different excitations in the ends of the pipe, according to that is observed in Figure 3 and in Figure 4 it is showed the excitations in each one of the supports, also the surrounding of these movements in the supports, whereas in Table 1 the nomenclature used for these excitations of the considered problem.

Data of the pipe:
d (diameter) = 76.2cm
t (thickness) = 0.635cm
I (the inertia moment) = 107637.4467cm^4
A (total area) = 150.9674cm^2
A_C (shear area) = 0.53A = 0.53(150.9674) = 80.0127cm^2
E (elasticity modulus) = 17258.277kN/cm^2
W (weight) = 17.5344N/cm = 1753.44N/m

m (mass) $= 18.22$kg/m
ν (Poisson's ratio) $= 0.25$
g (acceleration of the gravity) $= 9.80665$m/seg^2

TABLE 1. Direction and joint where the excitation spectrum is applied

Spectrum	Direction	Joint	Type
1	X	A	··········
2	X	E	– – –
3	X	envelope	——

2% damping

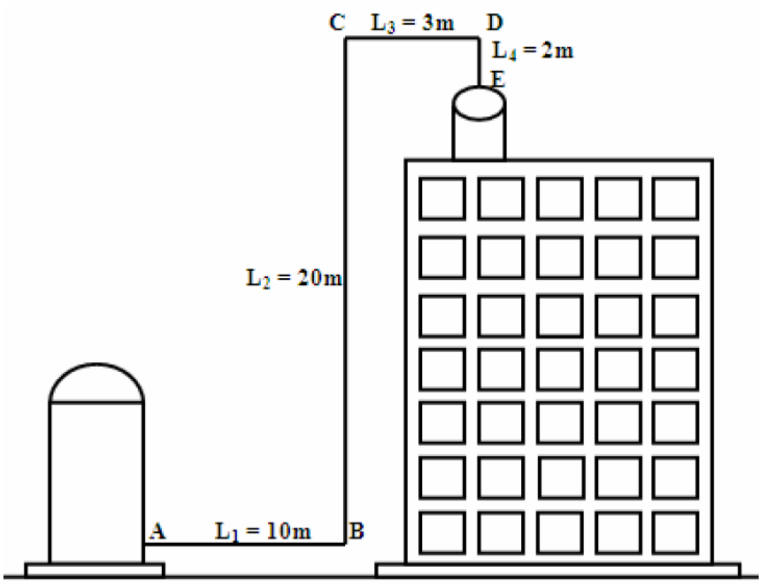

FIGURE 3. Typical system of pipe

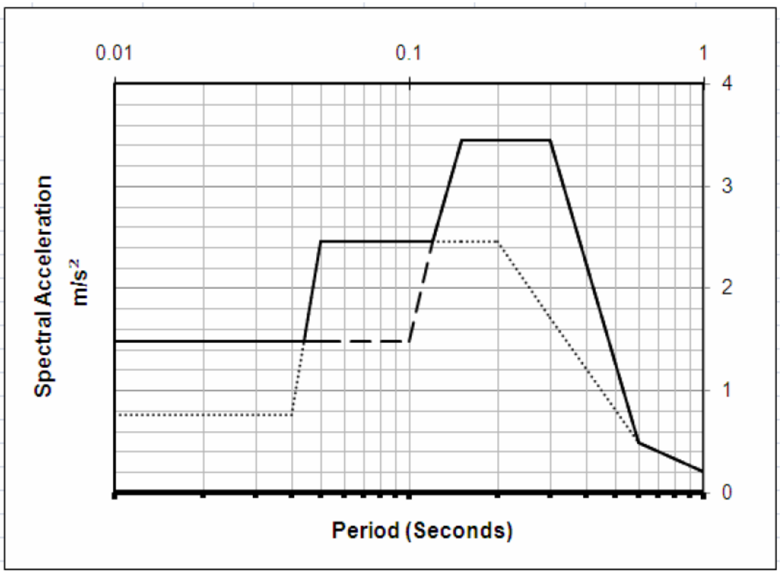

FIGURE 4. The response spectrum

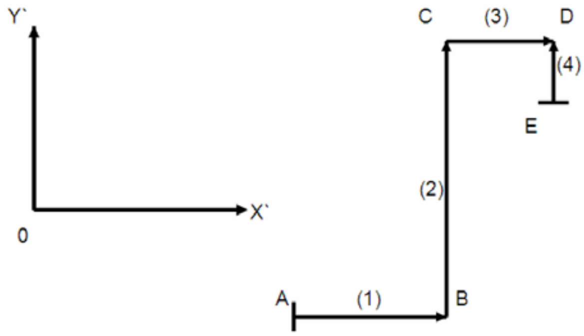

Figure 5. Model vector of the pipes system

In Figure 5 is shown the vector model of the pipes system of the problem considered for the obtaining of the mass matrix and stiffness.

Matrix of consistent mass of a member [31] can be written:

$$\mathrm{M}_i = \frac{m}{420}L\begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22L & 0 & 54 & -13L \\ 0 & 22L & 4L^2 & 0 & 13L & -3L^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13L & 0 & 156 & -22L \\ 0 & -13L & -3L^2 & 0 & -22L & 4L^2 \end{bmatrix}$$

Stiffness matrix for a member without considering the shear deformations [31] is:

$$\mathrm{K}_i = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

Stiffness matrix for a member considering the shear deformations [31], that is,

$$\mathrm{K}_i = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3(1+\lambda)} & \frac{6EI}{L^2(1+\lambda)} & 0 & -\frac{12EI}{L^3(1+\lambda)} & \frac{6EI}{L^2(1+\lambda)} \\ 0 & \frac{6EI}{L^2(1+\lambda)} & \left(\frac{4+\lambda}{1+\lambda}\right)\frac{EI}{L} & 0 & -\frac{6EI}{L^2(1+\lambda)} & \left(\frac{2-\lambda}{1+\lambda}\right)\frac{EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3(1+\lambda)} & -\frac{6EI}{L^2(1+\lambda)} & 0 & \frac{12EI}{L^3(1+\lambda)} & -\frac{6EI}{L^2(1+\lambda)} \\ 0 & \frac{6EI}{L^2(1+\lambda)} & \left(\frac{2-\lambda}{1+\lambda}\right)\frac{EI}{L} & 0 & -\frac{6EI}{L^2(1+\lambda)} & \left(\frac{4+\lambda}{1+\lambda}\right)\frac{EI}{L} \end{bmatrix}$$

$$\lambda = \frac{12EI}{GA_cL^2}; \quad G = \frac{E}{2(1+\nu)}$$

where λ = form factor, G = shear modulus.

For the considered problem is:

$$\lambda_1 = 0.04035761393; \quad \lambda_2 = 0.01008940348; \quad \lambda_3 = 0.4484179326; \quad \lambda_4 = 1.008940348$$

The mass matrix of the resulting global system, defined as

$$M = \begin{bmatrix} M_{11}^{(1)} & M_{12}^{(1)} & 0 & 0 & 0 \\ M_{21}^{(1)} & M_{22}^{(1)} + M_{11}^{(2)} & M_{12}^{(2)} & 0 & 0 \\ 0 & M_{21}^{(2)} & M_{22}^{(2)} + M_{11}^{(3)} & M_{12}^{(3)} & 0 \\ 0 & 0 & M_{21}^{(3)} & M_{22}^{(3)} + M_{22}^{(4)} & M_{21}^{(4)} \\ 0 & 0 & 0 & M_{12}^{(4)} & M_{11}^{(4)} \end{bmatrix}$$

The stiffness matrix of the resulting global system as

$$K = \begin{bmatrix} K_{11}^{(1)} & K_{12}^{(1)} & 0 & 0 & 0 \\ K_{21}^{(1)} & K_{22}^{(1)} + K_{11}^{(2)} & K_{12}^{(2)} & 0 & 0 \\ 0 & K_{21}^{(2)} & K_{22}^{(2)} + K_{11}^{(3)} & K_{12}^{(3)} & 0 \\ 0 & 0 & K_{21}^{(3)} & K_{22}^{(3)} + K_{22}^{(4)} & K_{21}^{(4)} \\ 0 & 0 & 0 & K_{12}^{(4)} & K_{11}^{(4)} \end{bmatrix}$$

Calculate mass matrix and stiffness of each member. Later making the change of the local system to the general system and then to realize the coupler of each mass matrix and stiffness, in order to obtain the general matrix system. Immediately a "similar transformation" would be applied, which is the matrix of interchange of lines and columns, to separate the degrees of freedom of the structure "M_{11} and K_{11}" and degrees of freedom of the supports "M_{22} and K_{22}".

Without considering the effect of the damping, under the condition of free vibration this dice by Equation (30). Being "U_1^r" a vector of "9×1" of relative displacements corresponding to the nine degrees of freedom of the pipe system. Later solving the determinant of Equation (32), it gives the eigenvalues and eigenvectors [30].

Using Software: MATLAB solve the determinant obtains the polynomial and the roots. The results appear in Table 2.

In Table 3, the values are obtained of the response spectrum of Figure 4, for each support in mode "n".

TABLE 2. Eigenvalues

Mode	Circular frequency (rad/sec)		Frequency (hz)		Period (sec)		ω_n^2 (rad/sec)^2	
	M 1	M 2	M 1	M 2	M 1	M 2	M 1	M 2
1	38.5867	38.1678	6.1413	6.0746	0.1628	0.1646	1489	1457
2	83.8672	80.9134	13.3479	12.8778	0.0749	0.0777	7034	6547
3	184.4477	174.4133	29.3558	27.7587	0.0341	0.0360	34021	30420
4	453.1971	391.2515	72.1286	62.2696	0.0139	0.0161	205388	153078
5	538.3848	498.3707	85.6866	79.3182	0.0117	0.0126	289858	248373
6	587.1852	580.7751	93.4534	92.4332	0.0107	0.0108	344786	337300
7	2023.3322	1930.3692	322.0233	307.2278	0.0031	0.0033	4093873	3726325
8	2661.7476	2580.0766	423.6303	410.6319	0.0024	0.0024	7084900	6656795
9	4411.0047	3884.1340	702.0332	618.1791	0.0014	0.0016	19456962	15086497

where M 1 = Model 1, without considering the shear deformations
M 2 = Model 2, considering the shear deformations

Table 3. Spectral accelerations

Mode	Period (sec)		S_{an}^{A} (m/sec^2)		S_{an}^{E} (m/sec^2)		S_{an}^{T} (m/sec^2)	
	M 1	M 2	M 1	M 2	M 1	M 2	M 1	M 2
1	0.1628	0.1646	2.46	2.46	3.45	3.45	3.45	3.45
2	0.0749	0.0777	2.46	2.46	1.47	1.47	2.46	2.46
3	0.0341	0.0360	0.76	0.76	1.47	1.47	1.47	1.47
4	0.0139	0.0161	0.76	0.76	1.47	1.47	1.47	1.47
5	0.0117	0.0126	0.76	0.76	1.47	1.47	1.47	1.47
6	0.0107	0.0108	0.76	0.76	1.47	1.47	1.47	1.47
7	0.0031	0.0033	0.76	0.76	1.47	1.47	1.47	1.47
8	0.0024	0.0024	0.76	0.76	1.47	1.47	1.47	1.47
9	0.0014	0.0016	0.76	0.76	1.47	1.47	1.47	1.47

Being S_{an}^{A} = Spectral acceleration for the support A in mode "n"
S_{an}^{E} = Spectral acceleration for the support E in mode "n"
S_{an}^{T} = Spectral acceleration of envelope in mode "n"

Table 4. The participation factors "$\Gamma_n^{(j)}$" for the support A and E, mass "M_n" corresponding to mode "n"

Mode	Γ_n^{A} (kg-s^2/cm^2)		Γ_n^{E} (kg-s^2/cm^2)		M_n (kg-s^2/cm^2)	
	M 1	M 2	M 1	M 2	M 1	M 2
1	−0.5435	−0.5283	−0.3874	−0.8320	1.5909	1.5536
2	−0.5574	+0.5573	−0.5859	+0.3876	1.9737	1.9722
3	+0.3143	−0.2767	−0.7413	+0.5566	1.7251	1.3196
4	−0.2433	+0.1072	−0.1628	+0.1676	0.2791	0.2168
5	−0.2710	+0.3827	+0.1615	−0.0708	0.2753	0.4257
6	+0.2318	−0.2838	+0.0795	−0.0176	0.3270	0.3225
7	−0.0011	+0.0011	+0.0121	−0.0186	0.1635	0.2016
8	+0.0024	−0.0025	+0.0261	−0.0252	0.1046	0.1135
9	+0.0007	+0.0009	−0.0373	−0.0257	0.1232	0.0997

Vectors "$V^{(j)}$" for the excitations in the nozzles (A) and (E) are:

$$V^{A} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad V^{E} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Equation (35) can be evaluated for the mass "M_n" corresponding in mode "n", the participation factors "$\Gamma_n^{(j)}$" for the support A and E is denoted from Equation (42). They are observed in Table 4.

The maximum modal responses "$(Y_n^{(j)})_{\max}$" due to each excitation of the supports is presented from Equation (47), the maximum normal coordinates "$(Y_n)_{\text{máx}}$" of the system for each mode can be obtained from Equation (48). They appear in Table 5.

Equation (49) can be obtained for the vectors corresponding to the components of the vector of maximum relative displacements "$\{U_{1n}^r\}_{\text{máx}}$" for each mode and finally the maximum value of the vector of relative displacements of the pipe system "$\{U_1^r\}_{\text{máx}}$" is defined from Equation (50). These values are in Table 6.

Once obtained the deformations, is used Equation (51) and are obtained the values of the forces in "X" and "Y" and moments, which are applied in the free joints, these effects are equivalent to that if a movement in the ends of the pipe system are presented, the results appear in Table 7.

Since the mechanical elements in the joints were obtained, the forces in the members by means of Equation (52) are determined and presented in Table 8.

4. **Results and Discussions.** In the following figures are presented the differences of both models.

TABLE 5. Maximum modal response "$(Y_n^{(j)})_{\max}$" due to each excitation of the supports and maximum normal coordinates "$(Y_n)_{\text{máx}}$" of the system for each mode

Mode	Y_n^A (cm)		Y_n^E (cm)		$(Y_n)_{\text{máx}}$ (cm)	
	M 1	M 2	M 1	M 2	M 1	M 2
1	−0.056523	−0.056523	−0.056490	−0.126967	0.079911	0.139380
2	−0.009893	+0.009893	−0.006218	+0.004422	0.011684	0.011519
3	+0.000409	−0.000526	−0.001862	+0.002042	0.001905	0.002108
4	−0.000323	+0.000246	−0.000419	+0.000884	0.000528	0.000917
5	−0.000513	+0.000277	+0.000297	−0.000099	0.000592	0.000295
6	+0.000157	−0.000198	+0.000104	−0.000023	0.000188	0.000201
7	−0.000000	+0.000000	+0.000003	−0.000003	0.000003	0.000003
8	+0.000000	−0.000000	+0.000005	−0.000005	0.000005	0.000005
9	+0.000000	+0.000000	−0.000003	−0.000003	0.000003	0.000003

TABLE 6. Vector of deformations

Relative deformations	Joint	Concept	Unit	Spectral method of multiple response		
				M 1	M 2	M 1/M 2
$U_1^r 1$	B	Displacement "X"	cm	0.00086	0.00114	0.7544
$U_1^r 2$		Displacement "Y"	cm	0.05677	0.09855	0.5761
$U_1^r 3$		Rotation	rad	0.00014	0.00024	0.5833
$U_1^r 4$	C	Displacement "X"	cm	0.01158	0.01829	0.6331
$U_1^r 5$		Displacement "Y"	cm	0.05507	0.09576	0.5751
$U_1^r 6$		Rotation	rad	0.00022	0.00036	0.6111
$U_1^r 7$	D	Displacement "X"	cm	0.01166	0.01847	0.6313
$U_1^r 8$		Displacement "Y"	cm	0.00036	0.00051	0.7059
$U_1^r 9$		Rotation	rad	0.00012	0.00019	0.6316

TABLE 7. Mechanical elements that act in the equivalent joints

Joint	Unit	Concept	Spectral method of multiple response		
			M 1	M 2	M 1/M 2
B	N	Force "X"	+1217.0637	+1275.5453	0.9771
	N	Force "Y"	+1922.0262	+3180.2323	0.6044
	N-m	Moment	+13662.2546	+22666.2679	0.6028
C	N	Force "X"	+371.2104	+158.3138	2.3448
	N	Force "Y"	+85060.2616	+97679.9329	0.8708
	N-m	Moment	+148180.9349	+180645.4857	0.8203
D	N	Force "X"	+66585.4123	+53510.2480	1.2443
	N	Force "Y"	−82644.7609	−94701.9170	0.8727
	N-m	Moment	+201793.1735	+211066.6253	0.9561

TABLE 8. Mechanical elements that act on the members

Member	Joint	Unit	Concept	Spectral method of multiple response		
				M 1	M 2	M 1/M 2
1	A	N	Force "X"	−2250.0559	−2978.0158	0.7556
		N	Force "Y"	+294.9406	+459.5642	0.6418
		N-m	Moment	−1125.9879	−2160.5073	0.5212
	B	N	Force "X"	+2250.0559	+2978.0158	0.7556
		N	Force "Y"	−294.9406	−459.5642	0.6418
		N-m	Moment	+4075.3948	+6756.1499	0.6032
2	B	N	Force "X"	−1032.9910	−1702.4696	0.6068
		N	Force "Y"	+2216.9668	+3639.7965	0.6091
		N-m	Moment	+9586.8588	+15910.1180	0.6026
	C	N	Force "X"	+1032.9910	+1702.4696	0.6068
		N	Force "Y"	−2216.9668	−3639.7965	0.6091
		N-m	Moment	+11072.9688	+18139.2825	0.6104
3	C	N	Force "X"	−661.7816	−1544.1558	0.4286
		N	Force "Y"	+87277.2293	+101319.7293	0.8614
		N-m	Moment	+137107.9670	+162506.2032	0.8437
	D	N	Force "X"	+661.7816	+1544.1558	0.4286
		N	Force "Y"	−87277.2293	−101319.7293	0.8614
		N-m	Moment	+124723.7210	+141452.9847	0.8817
4	D	N	Force "X"	+65923.6316	+51966.0912	1.2686
		N	Force "Y"	+4632.4684	+6617.8123	0.7000
		N-m	Moment	+77069.4525	+69613.6416	1.1071
	E	N	Force "X"	−65923.6316	−51966.0912	1.2686
		N	Force "Y"	−4632.4684	−6617.8123	0.7000
		N-m	Moment	+54777.8108	+34318.5408	1.5962

In Figure 6 is observed that all the deformations are smaller when the shear deformations are not considered, with respect to when are considering the shear deformations and they are very bigger the differences.

In Figure 7 is presented the mechanical elements in the free joints, in all the case are smaller in absolute value when the shear deformations are not considered, with exception in the forces in "X" of the joints, C and D that happens the opposite.

The axial forces that act in the members, are smaller in absolute value of Model 1 (without considering the shear deformations), according to Figure 8.

Finally, it is analyzed in Figure 9, which provides the results of the shear forces and Figure 10 presents the moments that act in the members; in all the members are smaller in absolute value of Model 1 (without considering the shear deformations), except in the member DE, that are F_x, (shear force) and M (Moment), that are minor in Model 2 (considering the shear deformations).

5. **Conclusions.** According to the analysis previously done, the differences between the spectral method of multiple response in the Model 1 (without considering the shear deformations) and Model 2 (considering the shear deformations), they are presented bigger

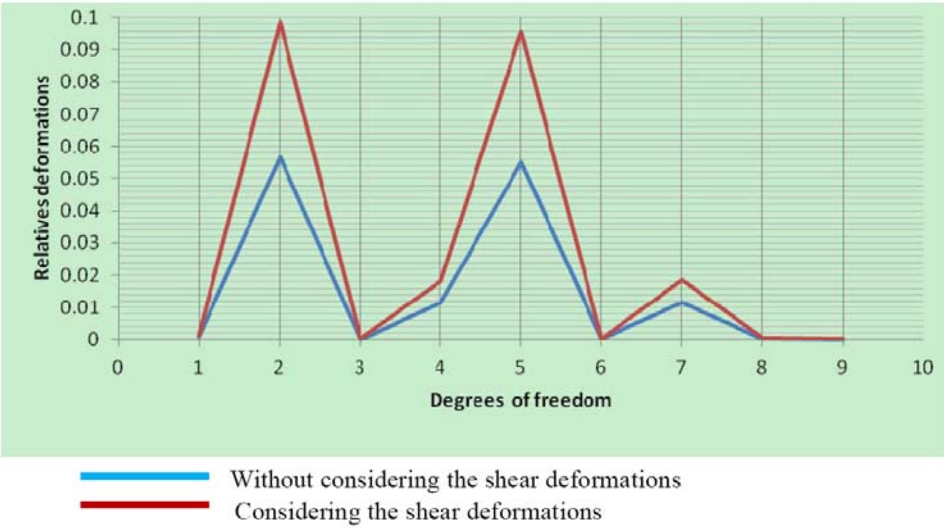

Figure 6. Relative deformations

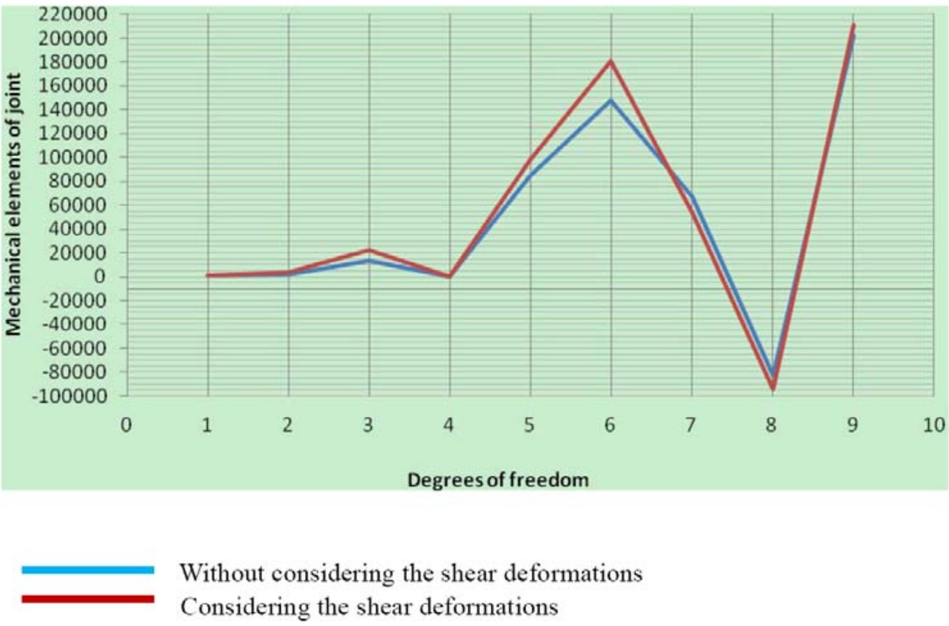

Figure 7. Mechanical elements that act in the joint

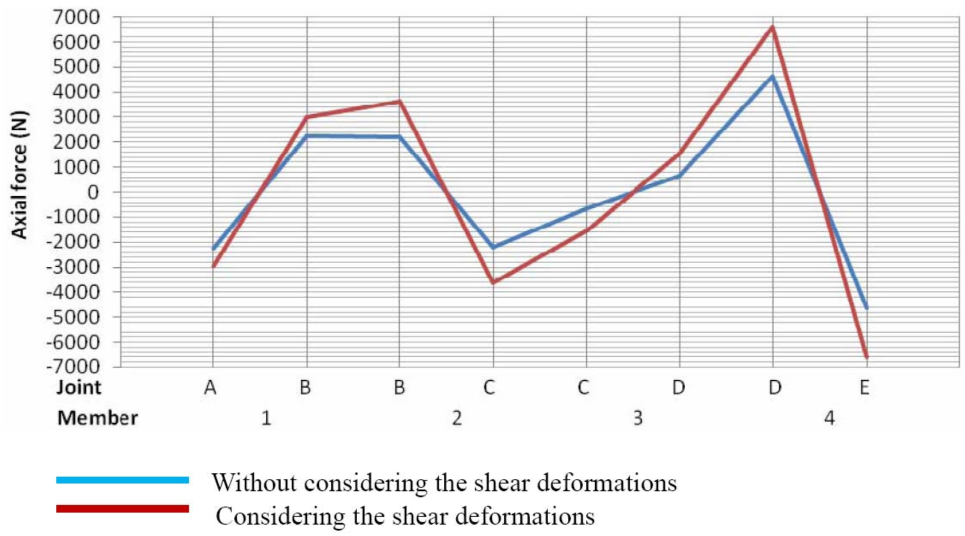

Figure 8. Axial force

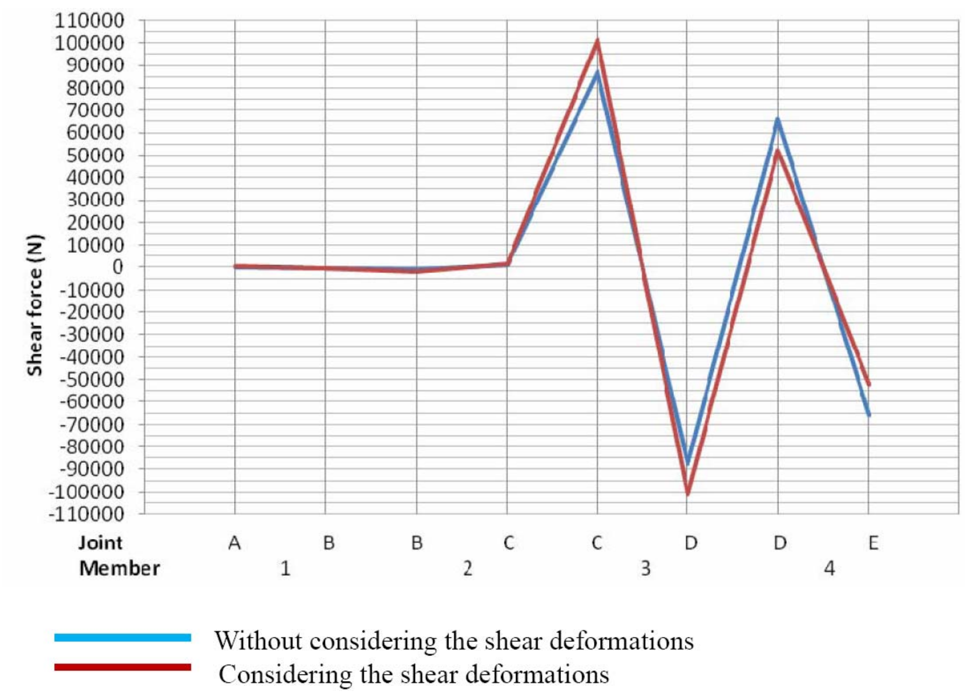

Figure 9. Shear force

differences as far as the deformations in all the joints, being the Model 1, in minor magnitude, see Figure 6. This situation is logical, because considering the shear deformations, the cross-sectional section is minor and therefore the stiffness is also minor.

Nevertheless, in regard to the mechanical elements which act in the joints, that is the equivalent force due to an excitation in the supports of pipes system, it is observed that the Model 1 that is the classic, when it is compared with Model 2, in this last all the values are major, with exception joints, C and D in regard to the forces in "X", as it is noticed in Figure 7 of results of the considered problem. Finally, in Figures 8-10, where are presented axial forces, shear forces and moments that normally govern the design of the system of pipes, in most of the points are not the side of the security in Model 1.

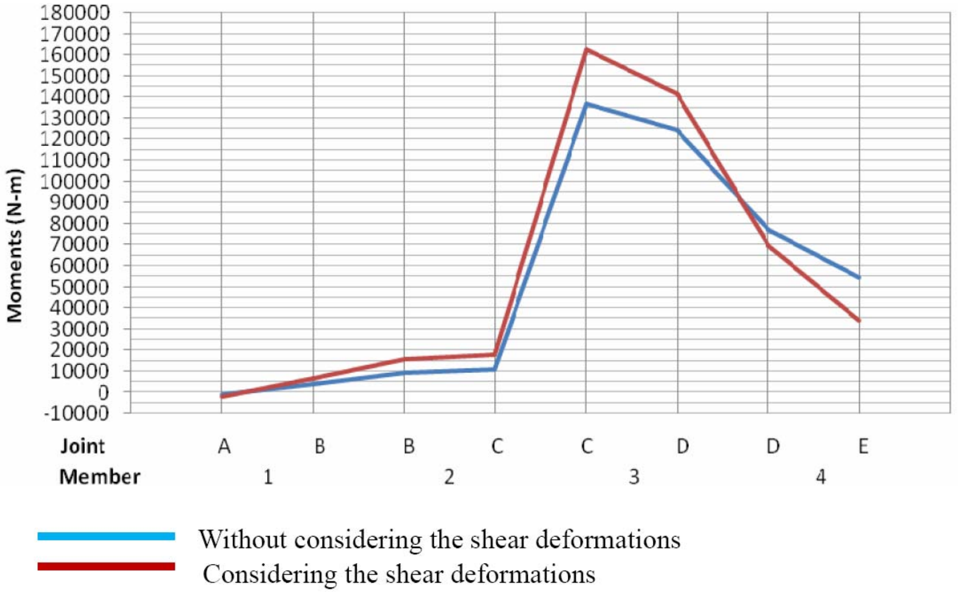

FIGURE 10. Moments

Therefore, the general practice of considering the spectral method of multiple response of Model 1 (without considering the shear deformations), will not be a recommendable solution. Now, considering the numeric approximation, the spectral method of multiple response in Model 2 (including the shear deformations), it turns out to be the most suitable method for the seismic-dynamic analysis of systems of pipes subjected to different excitations between its supports.

With respect to the formulation considering consistent mass or discreet mass, in the second case, it is not consider the effect of the mass in the excitation forces, which definitively is reflected in the response of the system and not of the preservative side. On the other hand, when realizing the frequency analysis, demonstrates that considering discreet mass beforehand implies are not consider certain modal forms symmetrical and/or anti-symmetrical modes, of the system, which in the case of different excitations in the supports are present and must be considered, since in some cases correspond to relatively low frequencies.

With regard to the state-of-art in the analysis of response seismic-dynamic of structural systems with multiple supports subjected to nonuniform excitations in its supports; is treated like the classic method that is the Model 1, already mentioned. When it is studied the seismic-dynamic behavior in facilities of nuclear plants, where due to risk that presents its operation, is demanded a greater refinement in the analysis techniques to be more realistic and to consider all effects that are acting in the structural systems for the design.

Finally, since it is typical to find, in the oil industry and nuclear plant, piping systems subjected to multiple excitations different between its supports by the seismic effects, the analysis technique through spectral method of multiple response Model 2 (considering the shear deformations), turns out to be a simple practice that must be considered as one of the stages within the analysis and design of the systems of pipes.

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