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OBSERVER-BASED ADAPTIVE FUZZY BACKSTEPPING CONTROL FOR STRICT-FEEDBACK STOCHASTIC NONLINEAR SYSTEMS WITH TIME DELAYS

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Abstract. *In this paper, an adaptive fuzzy backstepping output feedback control approach is proposed for a class of strict-feedback stochastic nonlinear systems with time delays and immeasurable states. Fuzzy logic systems are firstly utilized to approximate the unknown nonlinear functions, and then a fuzzy state observer is designed to estimate the immeasurable states. By combining the fuzzy adaptive control theory with backstepping approach, an adaptive fuzzy output feedback control scheme is developed. It is proved that all the signals of the closed-loop of adaptive control system are semi-globally uniformly ultimately bounded (SUUB) in probability, and the observer errors and the output of the system can be made small by appropriate choice of the design parameters. Simulation results are provided to show the effectiveness of the proposed approach.*
Keywords: Nonlinear stochastic systems, Fuzzy logic systems, Fuzzy state observer, Adaptive backstepping control, Stability analysis

1. **Introduction.** In the past decades, many approximation-based adaptive backstepping control approaches have been developed to control the uncertain nonlinear strict-feedback systems via fuzzy logic systems (FLSs) approximators; see for example [1-9] and references herein. Adaptive fuzzy backstepping control approaches in [1-4] are for single-input and single-output (SISO) nonlinear systems, and [5,6] are for multiple-input and multiple-output (MIMO) nonlinear systems, while those in [7-9] are for SISO/MIMO nonlinear systems with immeasurable states. Two of the main features of the above adaptive fuzzy control approaches are as follows: (i) they can be used to deal with those nonlinear systems without satisfying the matching conditions, and (ii) they do not require that the unknown nonlinear functions are linearly parameterized. Therefore, the approximator-based adaptive fuzzy backstepping control becomes one of the most popular design approaches to deal with the uncertain nonlinear systems.

It is well known that stochastic disturbances often exist in many practical systems, such as chemical reactors, recycled storage tanks, wind tunnel, cold rolling mills and robotic systems. Their existence is a source of resulting in the instability of the control systems, thus, the investigations on stochastic systems have received considerable attention in the past decades, and many important results have been achieved [10-13]. Authors in [10] first proposed an adaptive backstepping control design approach for strict-feedback stochastic nonlinear systems by a risk-sensitive cost criterion. Authors in [11,12] studied the output feedback stabilization problem of strict-feedback stochastic nonlinear systems by using the quadratic Lyapunov function and a linear reduced-order state observer, and [13,14] investigated state feedback and output feedback adaptive control for a class of stochastic nonlinear systems with time delays. However, the above mentioned control schemes are only suitable for those nonlinear stochastic systems with nonlinear dynamics models known exactly or with the unknown parameters appearing linearly with respect to known nonlinear functions. In order to cope with the problems that the nonlinear dynamics models are unknown or the system uncertainties are not linearly parameterized, authors in [15-17] developed adaptive output feedback control approaches for a class of uncertain nonlinear stochastic systems by using neural networks and the stability proofs of the control systems are given. On the basis of [15-17], authors in [18,19] developed adaptive fuzzy backstepping output feedback controllers for SISO and MIMO strict-feedback stochastic nonlinear systems by deigning a fuzzy state observer. However, the control approaches in [15-19] did not consider the problem of the nonlinear systems with time delays. It is well known that the time delays frequently occur in real engineering systems, and they may destroy the stability or degrade the performance of the controlled systems. Therefore, the controller synthesis and stability analysis for the stochastic nonlinear systems with time delays are important both in theory and applications.

Motivated by the above observations, in this paper, an observer-based adaptive fuzzy backstepping output feedback is developed for a class of stochastic nonlinear strict-feedback systems with time delays and immeasurable states. In the design, the FLSs are first used to approximate the unknown functions, and a nonlinear fuzzy state observer is designed to estimate the unmeasured states. By combining the fuzzy adaptive control theory with backstepping approach, an adaptive fuzzy output feedback control is constructed recursively. It is proved that all the signals of the closed-loop system are SUUB in probability, and the observer errors and the output of the system can be made as small as the desired by appropriate choice of the design parameters. Compared with the existing results, the main advantages of the proposed control schemes are as follows: (i) by designing a new fuzzy nonlinear state observer, the proposed adaptive control method does not require that all the states of the system are measured directly; (ii) the considered nonlinear stochastic systems contain the time delays, and therefore, this paper has extended the research results of [15-19]; (iii) the proposed control schemes construct the state observer and controller simultaneously, instead of constructing the observer and controller separately, which is known as the separation principle in linear systems.

2. **Problem Formulation and Some Preliminaries.**

2.1. **Problem formulation.**

Consider the following a class of stochastic nonlinear strict-feedback system with time delays

$$\begin{aligned}
dx_1 &= (x_2 + f_1(x_1) + h_1(y, y(t-\tau_1(t))) + \Delta_1(\underline{x}_n))dt + g_1(y)^T dw \\
dx_2 &= (x_3 + f_2(\underline{x}_2) + h_2(y, y(t-\tau_2(t))) + \Delta_2(\underline{x}_n))dt + g_2(y)^T dw \\
&\vdots \\
dx_{n-1} &= (x_n + f_{n-1}(\underline{x}_{n-1}) + h_{n-1}(y, y(t-\tau_{n-1}(t))) + \Delta_{n-1}(\underline{x}_n))dt + g_{n-1}(y)^T dw \\
dx_n &= (u + f_n(\underline{x}_n) + h_n(y, y(t-\tau_n(t))) + \Delta_n(\underline{x}_n))dt + g_n(y)^T dw \\
y &= x_1
\end{aligned} \tag{1}$$

where $\underline{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i$ $(i = 1, 2, \ldots, n)$ is the system state vector; u and y are the control and output of the system, respectively. $f_i(\underline{x}_i)$ and $h_i(y, y(t-\tau_i(t)))$ are unknown smooth nonlinear functions. $\tau_i(t)$ is an unknown bounded time delays satisfying $|\tau_i(t)| \le \bar{\tau}_i$ and $\dot{\tau}_i(t) \le \tau^* \le 1$. $\Delta_i(\underline{x}_n)$ is a bounded disturbance. In this paper, it is assumed that only output y is available for measurement, and that $g_i(y) = y\psi_i(y)$, with $\psi_i(y)$ being a known smooth function satisfying local Lipschitz condition. w is an independent r-dimensional standard Wiener process.

Assumption 1: There exists a set of known constants Δ_{i0}, $i = 1, 2, \ldots, n$ satisfying $|\Delta_i(\underline{x}_n)| \leq \Delta_{i0}$.

Assumption 2 [8,9]: There exists a set of known constants m_i, $i = 1, 2, \ldots, n$, such that for $\forall X_1, X_2 \in R^i$ the following inequality holds

$$|f_i(X_1) - f_i(X_2)| \leq m_i \|X_1 - X_2\| \tag{2}$$

where $\|X\|$ denotes the 2-norm of a vector X.

Assumption 3 [14,15]: Nonlinear functions $h_i(X_1, X_2)$, $i = 1, 2, \ldots, n$, satisfy the following inequality for $i = 1, 2, \ldots, n$,

$$|h_i(X_1, X_2)| \leq |X_1| h_{i,1}(X_1) + |X_2| h_{i,2}(X_2) \tag{3}$$

where $h_{i,1}(\cdot)$ and $h_{i,2}(\cdot)$ are known functions.

2.2. **Stochastic system and stability definitions.** Consider the following stochastic nonlinear system

$$d\chi(t) = f(\chi(t))dt + g(\chi(t))d\omega(t) \tag{4}$$

where $\chi \in R^n$ is the state, ω is an r-dimensional independent standard Wiener process, and $f(\cdot) : R^n \to R^n$ and $g(\cdot) : R^n \to R^{n\times r}$ are locally Lipschitz and satisfy $f(0) = 0$, $g(0) = 0$. Define a differential operator ℓ for twice continuously differentiable function $V(\chi)$ as follows:

$$\ell V(\chi) = \frac{\partial V}{\partial \chi} f(\chi) + \frac{1}{2} Tr \left\{ g^T(\chi) \frac{\partial^2 V}{\partial \chi^2} g(\chi) \right\} \tag{5}$$

Definition 2.1. [10] *Consider system (4) with $f(0) = 0$ and $g(0) = 0$. The solution $\chi(t) = 0$ is said to be asymptotically stable in the large if for any $\varepsilon > 0$,*

$$\lim_{\chi(0)\to 0} P\left\{ \sup_{t\geq 0} \|\chi(t)\| \geq \varepsilon \right\} = 0$$

And for any initial condition $\chi(0)$,

$$P\left\{ \lim_{t\to\infty} \chi(t) = 0 \right\} = 1$$

Definition 2.2. [10] *The solution process $\{\chi(t), t \geq 0\}$ of stochastic differential system (4) is said to be bounded in probability, if*

$$\lim_{c\to\infty} \sup_{0\leq t\leq\infty} P\{\|\chi(t)\| \geq c\} = 0$$

Lemma 2.1. [10] *Consider the stochastic nonlinear system (4). If there exists a positive definite, radially unbounded, twice continuously differentiable Lyapunov $V : R^n \to R$, and constants $\rho > 0$ and $\mu \geq 0$, such that*

$$\ell V(\chi) \leq -\rho V(\chi) + \mu \tag{6}$$

then the following conclusions are true.
(i) The system (4) has a unique solution and almost surely is bounded in probability.
(ii) If $f(0) = 0$, $g(0) = 0$ and $\mu = 0$. Then the system (4) is asymptotically stable in the large.

Lemma 2.2. [10] *(Young's Inequality). For any vectors $x, y \in R^n$, the following inequality holds*

$$x^T y \leq \frac{a^p}{p} \|x\|^p + \frac{1}{qa^q} \|y\|^q \tag{7}$$

where $a > 0$, $p > 1$, $q > 1$ *and* $(p-1)(q-1) = 1$.

Write (1) in the state space form

$$\begin{aligned} dx &= \left(Ax + Ky + \sum_{i=1}^{n} B_i f_i(\underline{x}_i) + h + \Delta + Bu \right) dt + G(y)^T dw \\ y &= Cx \end{aligned} \tag{8}$$

where $x = [x_1, x_2, \ldots, x_n]^T$, $A = \begin{bmatrix} -k_1 & & \\ \vdots & & I \\ -k_n & 0 & \ldots & 0 \end{bmatrix}$, $K = \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}$, $\Delta = \begin{bmatrix} \Delta_1(\underline{x}_n) \\ \vdots \\ \Delta_n(\underline{x}_n) \end{bmatrix}$, $h = [h_1(y, y(t-\tau_1)), \cdots, h_n(y, y(t-\tau_n))]^T$, $B_i = [\ 0 \ \ldots \ 1 \ \ldots \ 0\ ]^T$, $C = [\ 1 \ \cdots \ 0 \ \cdots \ 0\ ]$, $G(y) = [\ g_1(y) \ \cdots \ g_n(y)\ ] = y[\psi_1(y) \cdots \psi_n(y)] = y\psi(y)$.

Choose vector K such that matrix A is a strict Hurwitz; therefore, given a positive definite matrix $Q = Q^T > 0$, there exists a positive definite matrix $P = P^T > 0$ such that

$$A^T P + PA = -Q \tag{9}$$

Control objective: Using the fuzzy logic systems to determine an output feedback controller and parameters adaptive laws such that all the signals involved in the closed-loop system are SUUB in probability and the observer errors and the outputs of the system are as small as the desired.

2.3. **Fuzzy logic systems.** Fuzzy logic systems are universal approximators and can approximate any smooth function on a compact space, i.e.,

Lemma 2.3. [20] *Let* $f(x)$ *be a continuous function defined on a compact set* Ω. *Then for any constant* $\varepsilon > 0$, *there exists a fuzzy logic system* $\theta^T \varphi(x)$ *such as*

$$\sup_{x \in \Omega} \left| f(x) - \theta^T \varphi(x) \right| \le \varepsilon \tag{10}$$

By Lemma 2.3, it can be assumed that the nonlinear function $f_i(\cdot)$ in (1) can be approximated by the following fuzzy logic systems

$$f_i(\underline{x}_i | \theta_i) = \theta_i^T \varphi(\underline{x}_i), \quad \hat{f}_i(\hat{\underline{x}}_i | \theta_i) = \theta_i^T \varphi(\hat{\underline{x}}_i), \quad 1 \le i \le n \tag{11}$$

where $\hat{\underline{x}}_i = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_i]^T$ is the estimate of $\underline{x}_i = [x_1, x_2, \ldots, x_i]^T$. Denote $\hat{\underline{x}}_n = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n]^T$ as $\hat{x} = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n]^T$.

The optimal parameter vectors θ_i^* is defined as

$$\theta_i^* = \arg \min_{\theta_i \in \Omega_i} \left[\sup_{\hat{\underline{x}}_i \in U_i} \left| \hat{f}_i(\hat{\underline{x}}_i | \theta_i) - f_i(\hat{\underline{x}}_i) \right| \right], \quad 1 \le i \le n \tag{12}$$

where Ω_i and U_i are bounded sets for θ_i and $\hat{\underline{x}}_i$, respectively. The corresponding fuzzy minimum approximation error ε_i and approximation error δ_i are defined by

$$\varepsilon_i = f_i(\hat{\underline{x}}_i) - \hat{f}_i(\hat{\underline{x}}_i | \theta_i^*), \quad \delta_i = f_i(\hat{\underline{x}}_i) - \hat{f}_i(\hat{\underline{x}}_i | \theta_i) \tag{13}$$

Assumption 4 [8,9]: There are unknown positive constants ε_i^* and δ_i^* such as $|\varepsilon_i| \le \varepsilon_i^*$ and $|\delta_i| \le \delta_i^*$.

Denote $\omega_i = \varepsilon_i - \delta_i$, by Assumption 4, one has $|\omega_i| \le \varepsilon_i^* + \delta_i^* = \omega_i^*$, where ω_i^* is also an unknown constant.

3. **Fuzzy Adaptive Observer Design.** Note that the states $\underline{x}_i = (x_2, x_3, \cdots, x_i) \in R^i$ in system (1) are not available for measurement, thus, a state observer should be designed to estimate the unmeasured states. A fuzzy adaptive observer is designed for (1) as

$$\begin{aligned} \dot{\hat{X}} &= A\hat{X} + Ky + \sum_{i=1}^{n} B_i \hat{f}_i(\hat{\underline{x}}_i|\theta_i) + Bu \\ \hat{y} &= C\hat{x} \end{aligned} \tag{14}$$

Let $e^T = x - \hat{x} = [e_1, \cdots, e_n]$ be state estimation error vector. From (8) and (14), one can obtain the error dynamic equation

$$de = (Ae + F + \delta + h + \Delta)dt + G(y)^T dw \tag{15}$$

where $F = [F_1, \ldots, F_n]^T = [(f_1(x_1) - f_1(\hat{x}_1)), \ldots, (f_n(\underline{x}_n) - f_n(\hat{\underline{x}}_n))]^T$ and $\delta = [\delta_1, \ldots, \delta_n]^T$.

Combining (1), (14) and (15), one obtains the composite system

$$\begin{cases} de = (Ae + F + \Delta' + h)dt + G(y)^T dw \\ dy = (x_2 + f_1(x_1) + h_1 + \Delta_1(\underline{x}_n))dt + g_1(y)^T dw \\ d\hat{x}_i = \left(\hat{x}_{i+1} + \hat{f}_i(\hat{\underline{x}}_i\,|\theta_i) + k_i(y - \hat{x}_1)\right) dt, \quad i = 2, \cdots, n-1 \\ d\hat{x}_n = \left(u + \hat{f}_n(\hat{\underline{x}}_n\,|\theta_n) + k_n(y - \hat{x}_1)\right) dt \end{cases} \tag{16}$$

where $\Delta' = \delta + \Delta$.

Consider the following Lyapunov function candidate for the error system (18)

$$V_0 = \frac{1}{2}(e^T Pe)^2 + W_0 \tag{17}$$

where $W_0 = \frac{e^{r(\tau - t)}}{2(1-\tau^*)\eta^4} \sum_{i=1}^{n} \int_{t-\tau_i(t)}^{t} 2^{i+3} e^{rs} y^4(s) h_{i2}^4(y(s))ds$, with r being a positive design constant. $\tau = \max\{\bar{\tau}_1(t), \ldots, \bar{\tau}_n(t)\}$, and τ^* and τ are known constants. η is a positive design constant. Using (5), (9), (16) and (17), one has

$$\begin{aligned} \ell V_0 \leq & -\lambda\|e\|^4 + 2e^T Pee^T PF + 2e^T Pee^T P\Delta' + 2e^T Pee^T Ph \\ & + 2Tr\left\{G(y)^T(2Pee^T P + e^T PeP)G(y)\right\} + \dot{W}_0 \end{aligned} \tag{18}$$

Denote $\lambda = \lambda_{\min}(P) \cdot \lambda_{\min}(Q)$, where $\lambda_{\min}(P_j)$ and $\lambda_{\min}(Q_j)$ are the smallest eigenvalues of matrices P_j and Q_j, respectively. Choosing an appropriate constant $\eta > 0$ such that

$$p_0 = \lambda - \frac{9}{2}\eta^{\frac{4}{3}}\|P\|^{\frac{8}{3}} - \frac{1}{2\eta^4}\left(\sum_{i=1}^{n} m_i^2\right)^2 - \frac{3}{2}\eta^{\frac{4}{3}}\|e\|^4 - 3n\sqrt{n}\eta^2\|P\|^4 > 0$$

And by Assumption 2, 3 and Lemma 2.2, one can obtain the following inequalities

$$2e^T Pee^T PF \leq \frac{3}{2}\eta^{\frac{4}{3}}\|P\|^{\frac{8}{3}}\|e\|^4 + \frac{1}{2\eta^4}\left(\sum_{i=1}^{n} m_i^2\right)^2\|e\|^4 \tag{19}$$

$$2e^T Pee^T Ph \leq \frac{3}{2}\eta^{\frac{4}{3}}\|P\|^{\frac{8}{3}}\|e\|^4 + \frac{1}{2\eta^4}\sum_{i=1}^{n} 2^{i+3}(y^4 h_{i1}^4(y) + y^4(t-\tau_i)h_{i2}^4(y(t-\tau_i))) \tag{20}$$

$$2e^T Pee^T P\Delta' \leq \frac{3}{2}\eta^{\frac{4}{3}}\|P\|^{\frac{8}{3}}\|e\|^4 + \frac{1}{2\eta^4}\|\Delta_0'\|^4 \tag{21}$$

$$2Tr\left\{G^T(y)(2Pee^T P + e^T PeP)G(y)\right\} \leq \frac{3n\sqrt{n}}{\eta^2}y^4\|\psi(y)\|^4 + 3n\sqrt{n}\eta^2\|P\|^4\|e\|^4 \tag{22}$$

$$\dot{W}_0 \le \frac{-r}{2(1-\tau^*)b} e^{-rt} \sum_{i=1}^{n} \int_{t-\tau_i(t)}^{t} e^{rs} 2^{i+3} y^4(s) h_{i2}^4(y(s)) ds + \frac{1}{2(1-\tau^*)b} \sum_{i=1}^{n} 2^{i+3} y^4 h_{i2}^4(y)$$
$$-\frac{1}{2b} \sum_{i=1}^{n} 2^{i+3} e^{-r\tau_i(t)} y^4(t-\tau_i(t)) h_{i2}^4(y(t-\tau_i)) \tag{23}$$

where $\Delta_0' = [\delta_1^* + \Delta_{10}, \ldots, \delta_n^* + \Delta_{n0}]^T$ and $b = \eta^4 e^{-r\tau}$. Substituting (19)-(23) into (18) results in

$$\begin{aligned} \ell V_0 \le & -p_0\|e\|^4 + \Xi_1 + \frac{3n\sqrt{n}}{\eta^2} y^4 \|\psi(y)\|^4 + \frac{1}{2\eta^4} \sum_{i=1}^{n} 2^{i+3} y^4 h_{i1}^4(y) \\ & + \frac{1}{2(1-\tau^*)b} \sum_{i=1}^{n} 2^{i+3} y^4 h_{i2}^4(y) - rW_0 \end{aligned} \tag{24}$$

where $\Xi_1 = \frac{1}{2\eta^4} \|\Delta'{}_0\|^4$.

4. **Controller Design and Stability Analysis.**

The n-step adaptive fuzzy output feedback backstepping design is based on the change of coordinates

$$\begin{aligned} \chi_1 &= y \\ \chi_i &= \hat{x}_i - \alpha_{i-1} \end{aligned} \tag{25}$$

where $\alpha_{i-1}(\cdot)$ $(i = 2, \cdots, n)$ is an intermediate control, and u will be designed in the last step.

Step 1: From (1), (16) and (25), one has

$$d\chi_1 = \left(\chi_2 + \alpha_1 + e_2 + F_1 + \theta_1^{*T} \varphi_1(\hat{x}_1) + \varepsilon_1 + \Delta_1(\underline{x}_n) + h_1\right) dt + g_1(y)^T dw \tag{26}$$

where $\tilde{\theta}_1 = \theta_1^* - \theta_1$ is the parameter error vector, and θ_1 is the estimate of θ_1^*.

Choose the Lyapunov function candidate

$$V_1 = V_0 + \frac{1}{4}\chi_1^4 + \frac{1}{2\gamma_1} \tilde{\theta}_1^T \tilde{\theta}_1 + \frac{1}{2\bar{\gamma}_1} \tilde{\pi}_1^2 + W_1 \tag{27}$$

where $\gamma_1 > 0$ and $\bar{\gamma}_1 > 0$ are design parameters. $\pi_1^* = \Delta_{10} + \varepsilon_1^*$, $\tilde{\pi}_1 = \pi_1^* - \hat{\pi}_1$, $\hat{\pi}_1$ is the estimate of π_1^*, $W_1 = \frac{2}{(1-\tau^*)\eta^4} e^{r(\tau-t)} \int_{t-\tau_1(t)}^{t} e^{rs} y^4(s) h_{12}^4(y(s)) ds$.

From (24), (26) and (27), one has

$$\begin{aligned} \ell V_1 \le & -p_0\|e\|^4 - rW_0 + \chi_1^3(\chi_2 + \alpha_1 + e_2 + F_1 + \theta_1^T \varphi_1(\hat{x}_1) + h_1) + \left|\chi_1^3\right| \pi_1^* \\ & + \frac{3}{2}\chi_1^2 g_1(y)^T g_1(y) + \tilde{\theta}_1^T \left(\varphi_1(\hat{x}_1)\chi_1^3 - \frac{1}{\gamma_1}\dot{\theta}_1\right) + \frac{1}{\bar{\gamma}_1}\tilde{\pi}_1 \dot{\hat{\pi}}_1 + \dot{W}_1 + \Xi_1 \\ & + \frac{3n\sqrt{n}}{\eta^2} y^4 \|\psi(y)\|^4 + \frac{1}{2\eta^4} \sum_{i=1}^{n} 2^{i+3} y^4 h_{i1}^4(y) + \frac{1}{2(1-\tau^*)b} \sum_{i=1}^{n} 2^{i+3} y^4 h_{i2}^4(y) \end{aligned} \tag{28}$$

By Assumption 2, 3 and Lemma 2.2, the following inequalities can be obtained

$$\chi_1^3 e_2 \le \frac{3}{4}\eta^{\frac{4}{3}}\chi_1^4 + \frac{1}{4\eta^4}\|e\|^4 \tag{29}$$

$$\chi_1^3 F \le |\chi_1|^3 |f_1(x_1) - f_1(\hat{x}_1)| \le m_1 |\chi_1|^3 \|e\| \le \frac{3}{4}\eta^{\frac{4}{3}}\chi_1^4 + \frac{1}{4\eta^4} m_1^4 \|e\|^4 \tag{30}$$

$$\chi_1^3 h_1 \le \frac{3}{4}\eta^{\frac{4}{3}}\chi_1^4 + \frac{1}{4\eta^4}\|h_1\|^4 \le \frac{3}{4}\eta^{\frac{4}{3}}\chi_1^4 + \frac{2}{\eta^4}\chi_1^4 h_{11}^4(y) + \frac{2}{\eta^4} y^4(t-\tau_1) h_{12}^4(y(t-\tau_1)) \tag{31}$$

$$\frac{3}{2}\chi_1^2 g_1(y)^T g_1(y) = \frac{3}{2}\chi_1^4 \psi_1(y)^T \psi_1(y) \tag{32}$$

$$\chi_1^3\chi_2 \le \frac{3}{4}\nu_1^{\frac{4}{3}}\chi_1^4 + \frac{1}{4\nu_1^4}\chi_2^4 \tag{33}$$

where $v_1 > 0$ is design parameter. Substituting (29)-(33) into (31) results in

$$\begin{aligned}\ell V_1 \le & -\left(p_0 - \frac{1}{4\eta_1^4} - \frac{1}{4\eta_1^4}m_1^4\right)\|e\|^4 + \Xi_1 + \chi_1^3\left(\frac{3}{4}\nu_1^{\frac{4}{3}}\chi_1 + \alpha_1 + \frac{9}{4}\eta^{\frac{4}{3}}\chi_1 + \theta_1^T\varphi_1(\hat{x}_1)\right.\\ & \left. + \frac{2}{\eta^4}h_{11}^4(y)\chi_1 + \frac{3n\sqrt{n}}{\eta^2}\chi_1\|\psi(y)\|^4 + \frac{2}{(1-\tau^*)b}h_{12}^4(y)\chi_1 + \frac{3}{2}\chi_1\psi_1(y)^T\psi_1(y) + H_1\right)\\ & + \left|\chi_1^3\right|\pi_1^* + \frac{1}{4\nu_1^4}\chi_2^4 - \tilde{\theta}_1^T\left(\frac{\dot{\theta}_1}{\gamma_1} - \varphi_1(\hat{x}_1)\chi_1^3\right) + \frac{1}{\bar{\gamma}_1}\tilde{\pi}_1\dot{\hat{\pi}}_1 - rW_0 - 2rW_1\end{aligned} \tag{34}$$

where $H_1 = \frac{1}{2\eta^4}\sum\limits_{i=1}^{n} 2^{i+3}h_{i1}^4(y)\chi_1 + \frac{1}{2(1-\tau^*)b}\sum\limits_{i=1}^{n} 2^{i+3}h_{i2}^4(y)\chi_1$.

Design the intermediate control function θ_1 and the adaptation functions $\hat{\pi}_1$ as

$$\begin{aligned}\alpha_1 = & -c_1\chi_1 - \frac{3}{4}\nu_1^{\frac{4}{3}}\chi_1 - \frac{9}{4}\eta_1^{\frac{4}{3}}\chi_1 - \theta_1^T\varphi_1(\hat{x}_1) - \hat{\pi}_1\tanh\left(\chi_1^3/k\right) - \frac{3n\sqrt{n}}{\eta^2}\chi_1\|\psi(y)\|^4\\ & + \frac{3}{2}\chi_1\psi_1(y)^T\psi_1(y) - \frac{2n}{\eta^4}h_{11}^4(y)\chi_1 - \frac{2n\chi_1}{(1-\tau^*)b}h_{12}^4(y) - H_1\\ & - \frac{3(n-1)}{4}\eta^2\chi_1(\psi_1(y)^T\psi_1(y))^2\end{aligned} \tag{35}$$

$$\dot{\theta}_1 = \gamma_1\varphi_1(\hat{x}_1)\chi_1^3 - \sigma_1\theta_1 \tag{36}$$

$$\dot{\hat{\pi}}_1 = \bar{\gamma}_1\chi_1^3\tanh\left(\frac{\chi_1^3}{k}\right) - \bar{\sigma}_1\hat{\pi}_1 \tag{37}$$

where $\sigma_1 > 0$ and $\bar{\sigma}_1 > 0$ are design parameters.

Substituting (35)-(37) into (34) and utilizing the inequalities

$$\left|\chi_1^3\right| - \chi_1^3\tanh(\chi_1^3/k) \le 0.2785k = k',\ \text{for } \forall k > 0 \tag{38}$$

(34) becomes

$$\begin{aligned}\ell V_1 \le & -p_1\|e\|^4 - c_1\chi_1^4 + \frac{1}{4\nu_1^4}\chi_2^4 + \pi_1^*k' + \Xi_1 + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_1^T\theta_1 + \frac{\bar{\sigma}_1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 - rW_0 - 2rW_1\\ & - \frac{3(n-1)}{4}\eta^2\chi_1^4(\psi_1(y)^T\psi_1(y))^2 - \frac{2(n-1)}{\eta^4}h_{11}^4(y)\chi_1^4 - \frac{2(n-1)}{(1-\tau^*)b}h_{12}^4(y)\chi_1^4\end{aligned} \tag{39}$$

where $p_1 = p_0 - \frac{1}{4\eta^4} - \frac{1}{4\eta^4}m_1^4$.

Step i ($2 \le i \le n-1$): From (14) and (25), one has

$$\begin{aligned}d\chi_i = & \left(\chi_{i+1} + \alpha_i + H_i + \tilde{\theta}_i^T\varphi_i(\underline{\hat{x}}_i) + \omega_i - \frac{\partial\alpha_{i-1}}{\partial y}(e_2 + F_1 + \delta_1 + \Delta_1 + h_1)\right.\\ & \left. -\frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial y^2}g_1(y)^Tg_1(y)\right)dt - \frac{\partial\alpha_{i-1}}{\partial y}g_1(y)^Tdw\end{aligned} \tag{40}$$

where

$$\begin{aligned}H_i = & k_ie_1 + \theta_i^T\varphi_i(\underline{\hat{x}}_i) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_j}\dot{\hat{x}}_j - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\theta_j}\dot{\theta}_j - \frac{\partial\alpha_{i-1}}{\partial\hat{\pi}_1}\dot{\hat{\pi}}_1\\ & - \sum_{j=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\omega}_2}\dot{\hat{\omega}}_2 - \frac{\partial\alpha_{i-1}}{\partial y}(\hat{x}_2 + \theta_1^T\varphi_1(\hat{x}_1))\end{aligned}$$

Consider the following Lyapunov function candidate

$$V_i = V_{i-1} + \frac{1}{4}\chi_i^4 + \frac{1}{2\gamma_i}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2\bar{\gamma}_i}\tilde{\omega}_i^2 + W_i \tag{41}$$

where $\gamma_i > 0$ and $\bar{\gamma}_i > 0$ are design parameters, $\tilde{\theta}_i = \theta_i^* - \theta_i$ and $\tilde{\omega}_i = \omega_i^* - \hat{\omega}_i$. θ_i and $\hat{\omega}_i$ are the estimates of θ_i^* and ω_i^*. $W_i = \frac{2e^{r(\tau-t)}}{(1-\tau^*)\eta^4}\int_{t-\tau_1(t)}^t e^{rs}y^4(s)h_{12}^4(y(s))ds$.

Using Assumption 2, 3, Lemma 2.2 and the similar derivations to Step 1, one has

$$\begin{aligned}\ell V_i \leq & -p_i\|e\|^4 - \sum_{j=1}^{i-1} c_j\chi_j^4 + \pi_1^* k' + \sum_{j=2}^{i-1}\omega_j^* k' + \Xi_i + \sum_{j=1}^{i-1}\frac{\sigma_j}{\gamma_j}\tilde{\theta}_j^T\theta_j + \frac{\bar{\sigma}_1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 \\ & + \sum_{j=2}^{i-1}\frac{\bar{\sigma}_j}{\bar{\gamma}_j}\tilde{\omega}_j\hat{\omega}_j - \frac{3(n-i)}{4}\eta^2\chi_1^4(\psi_1(y)^T\psi_1(y))^2 - \frac{2(n-i)}{\eta^4}h_{11}^4(y)\chi_1^4 \\ & - \frac{2(n-i)}{(1-\tau^*)b}h_{12}^4(y)\chi_1^4 - rW_0 - 2r\sum_{j=1}^{i}W_j + \chi_i^3\left(\frac{1}{4\nu_{i-1}^4}\chi_i + \frac{3}{4}\nu_i^{\frac{4}{3}}\chi_i + \alpha_i\right. \\ & \left. + H_i + \frac{3}{4\eta^2}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^4\chi_i + \frac{15}{4}\eta^{\frac{4}{3}}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^{\frac{4}{3}}\chi_i - \frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial y^2}g_1(y)^Tg_1(y)\right) \\ & + \left|\chi_i^3\right|\omega_i^* + \frac{1}{4\nu_i^4}\chi_{i+1}^4 - \tilde{\theta}_i^T\left(\frac{\dot{\theta}_i}{\gamma_i} - \varphi_i(\underline{\hat{x}}_i)\chi_i^3\right) - \frac{1}{\bar{\gamma}_i}\tilde{\omega}_i\dot{\hat{\omega}}_i\end{aligned} \tag{42}$$

where $v_i > 0$ is a design parameter. $p_i = p_{i-1} - \frac{1}{4\eta^4} - \frac{m_1^4}{4\eta^4}$, $\Xi_i = \Xi_{i-1} + \frac{1}{4\eta^4}\delta_1^{*4} + \frac{1}{4\eta^4}\Delta_{10}^4$.

Design the intermediate control function α_i and the adaptation functions θ_i and $\hat{\omega}_i$ as

$$\begin{aligned}\alpha_i = & -c_i\chi_i - H_i - \hat{\omega}_i\tanh(\chi_i^3/k) - \frac{1}{4\nu_{i-1}^4}\chi_i - \frac{3}{4}\nu_i^{\frac{4}{3}}\chi_i - \frac{3}{4\eta^2}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^4\chi_i \\ & - \frac{15}{4}\eta^{\frac{4}{3}}\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^{\frac{4}{3}}\chi_i + \frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial y^2}g_1(y)^Tg_1(y)\end{aligned} \tag{43}$$

$$\dot{\theta}_i = \gamma_i\varphi_i(\underline{\hat{x}}_i)\chi_i^3 - \sigma_i\theta_i \tag{44}$$

$$\dot{\hat{\omega}}_i = \bar{\gamma}_i\chi_i^3\tanh\left(\frac{\chi_i^3}{k}\right) - \bar{\sigma}_i\hat{\omega}_i \tag{45}$$

where $\sigma_i > 0$ and $\bar{\sigma}_i > 0$ are design parameters. Substituting (43)-(45) into (42), (42) becomes

$$\begin{aligned}\ell V_i \leq & -p_i\|e\|^4 - \sum_{j=1}^{i-1}c_j\chi_j^4 + \frac{1}{4\nu_i^4}\chi_{i+1}^4 + \pi_1^*k' + \sum_{j=2}^{i-1}\omega_j^*k' + \Xi_i + \sum_{j=1}^{i}\frac{\sigma_j}{\gamma_j}\tilde{\theta}_j^T\theta_j + \frac{\bar{\sigma}_1}{\bar{\gamma}_1}\tilde{\varepsilon}_1\hat{\varepsilon}_1 \\ & + \sum_{j=2}^{i}\frac{\bar{\sigma}_j}{\bar{\gamma}_j}\tilde{\omega}_j\hat{\omega}_j - \frac{3(n-i)}{4}\eta^2\chi_1^4(\psi_1(y)^T\psi_1(y))^2 - \frac{2(n-i)}{\eta^4}h_{11}^4(y)\chi_1^4 \\ & - \frac{2(n-i)}{(1-\tau^*)b}h_{12}^4(y)\chi_1^4 - rW_0 - 2r\sum_{j=1}^{i}W_j\end{aligned} \tag{46}$$

Step *n*: In the final step, the actual control input u appears. From (16) and (25), one has

$$\begin{aligned} d\chi_n = &\left(u + H_n + \tilde{\theta}_n^T \varphi_n(\hat{\underline{x}}_n) + \omega_n - \frac{\partial \alpha_{n-1}}{\partial y}(e_2 + F_1 + \delta_1 + \Delta_1 + h_1)\right. \\ &\left. -\frac{1}{2}\frac{\partial^2 \alpha_{n-1}}{\partial y^2} g_1(y)^T g_1(y)\right) dt - \frac{\partial \alpha_{n-1}}{\partial y} g_1(y)^T dw \end{aligned} \tag{47}$$

where

$$\begin{aligned} H_n = &k_n e_1 + \theta_n^T \varphi_n(\hat{\underline{x}}_n) - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{x}_i}\dot{\hat{x}}_i - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \theta_i}\dot{\theta}_i \\ &- \frac{\partial \alpha_{i-1}}{\partial \hat{\pi}_1}\dot{\hat{\pi}}_1 - \sum_{i=2}^{n-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\omega}_i}\dot{\hat{\omega}}_i - \frac{\partial \alpha_{n-1}}{\partial y}(\hat{x}_2 + \theta_1^T \varphi_1(\hat{x}_1)) \end{aligned}$$

Consider the following Lyapunov function candidate

$$V_n = V_{n-1} + \frac{1}{4}\chi_n^4 + \frac{1}{2\gamma_n}\tilde{\theta}_n^T \tilde{\theta}_n + \frac{1}{2\bar{\gamma}_n}\tilde{\omega}_n^2 + W_n \tag{48}$$

where $\gamma_n > 0$ and $\bar{\gamma}_n > 0$ are design parameters, $\tilde{\theta}_n = \theta_n^* - \theta_n$ and $\tilde{\omega}_n = \omega_n^* - \hat{\omega}_n$, θ_n and $\hat{\omega}_n$ are the estimates of θ_n^* and ω_n^*. $W_n = \frac{2e^{r(\tau-t)}}{(1-\tau^*)\eta^4}\int_{t-\tau_1(t)}^t e^{rs} y^4(s) h_{12}^4(y(s)) ds$.

Design controller u and adaptation functions θ_n and $\hat{\omega}_n$ as

$$\begin{aligned} u = &-c_n\chi_i - H_n - \hat{\omega}_n \tanh(\chi_n^3/k) - \frac{1}{4\nu_{n-1}^4}\chi_i^4 - \frac{3}{4\eta^2}\left(\frac{\partial \alpha_{n-1}}{\partial y}\right)^4 \chi_n \\ &- \frac{15}{4}\eta^{\frac{4}{3}}\left(\frac{\partial \alpha_{n-1}}{\partial y}\right)^{\frac{4}{3}} \chi_n + \frac{1}{2}\frac{\partial^2 \alpha_{n-1}}{\partial y^2} g_1(y)^T g_1(y) \end{aligned} \tag{49}$$

$$\dot{\theta}_n = \gamma_n \varphi_n(\hat{\underline{x}}_n)\chi_n^3 - \sigma_n \theta_n \tag{50}$$

$$\dot{\hat{\omega}}_n = \bar{\gamma}_n \chi_n^3 \tanh\left(\frac{\chi_n^3}{k}\right) - \bar{\sigma}_n \hat{\omega}_n \tag{51}$$

where $\sigma_n > 0$ and $\bar{\sigma}_n > 0$ are design parameters. Similar to the derivations in Step i, one has

$$\begin{aligned} \ell V_n \le &- p_n\|e\|^4 - \sum_{i=1}^n c_i\chi_i^4 - \sum_{i=1}^n \frac{\sigma_i \left\|\tilde{\theta}_i\right\|^2}{2\gamma_i} - \frac{\bar{\sigma}_1 \tilde{\pi}_1^2}{2\bar{\gamma}_1} - \sum_{i=2}^n \frac{\bar{\sigma}_i \tilde{\omega}_i^2}{2\bar{\gamma}_i} - rW_0 - 2r\sum_{i=1}^n W_i \\ &+ \Xi_n + \left(\pi_1^* + \sum_{i=2}^n \omega_i^*\right)k' + \sum_{i=1}^n \frac{\sigma_i\|\theta_i^*\|^2}{2\gamma_i} + \frac{\bar{\sigma}_1 \pi_1^{*2}}{2\bar{\gamma}_1} + \sum_{i=2}^n \frac{\bar{\sigma}_i \omega_i^{*2}}{2\bar{\gamma}_i} \end{aligned} \tag{52}$$

Choose the constant η which satisfies that $p_i = p_{i-1} - \frac{1}{4\eta^4} - \frac{m_1^4}{4\eta^4} > 0$, $(i = 1, 2, \cdots, n)$. Let

$$\rho = \min(2p_n/\lambda_{\max}^2(P), 4c_1, 4c_2, \cdots, 4c_n, \sigma_1, \cdots, \sigma_n, \bar{\sigma}_1, \cdots, \bar{\sigma}_n, r) \tag{53}$$

$$\mu = \Xi_n + \left(\pi_1^* + \sum_{i=2}^n \omega_i^*\right)k' + \sum_{i=1}^n \frac{\sigma_i\|\theta_i^*\|^2}{2\gamma_i} + \frac{\bar{\sigma}_1 \pi_1^{*2}}{2\bar{\gamma}_1} + \sum_{i=2}^n \frac{\bar{\sigma}_i \omega_i^{*2}}{2\bar{\gamma}_i} \tag{54}$$

where $\lambda_{\max}(P)$ is the largest eigenvalue of P, and (52) becomes

$$\ell V_n \le -\rho V_n + \mu \tag{55}$$

By Lemma 2.1 and inequality (55), and using the same arguments as [15-17], one can obtain that all the signals of the closed-loop system are bounded by μ/ρ, that is, e and

χ_i are SUUB in probability. $\tilde{\theta}_i$, $\tilde{\pi}_1$ and $\tilde{\omega}_i$ are also SUUB in probability ($i = 1, 2, \cdots, n$). Moreover, choosing appropriate design parameters, the states observer errors and the outputs of the control system can be made as small as the desired.

5. **Simulation Studies.** Let us apply the proposed adaptive control scheme to a pendulum system with stochastic disturbances.

$$ml\ddot{q} = -mg\sin q - kl\dot{q} = \frac{1}{l}u \tag{56}$$

where $u \in IR$ is the torque applied to the pendulum, $q \in IR$ is the anticlockwise angle between the vertical axis through the pivot point and the rod, g is the gravity acceleration, and the constants k, l and m denote a coefficient of friction, the length of the rod, and the mass of the bob, respectively. It is assumed that the constants k, l and m are unknown. Let $x_1 = ml^2(q - \pi)$, $x_2 = ml^2\left(\dot{q} + \frac{k}{m}(q - \pi)\right)$. $m = g^{-2}$, $k = g^{-2}$ and $l = g$.

The nonlinear stochastic system with time delays can be expressed as follows:

$$\begin{aligned} dx_1 &= (x_2 - sx_1 - (1-s)x_1(t-\tau_1))dt + g_1(y)^T dw \\ dx_2 &= (u + s\sin(x_1) + (1-s)\sin(x_1(t-\tau_2)))dt + g_2(y)^T dw \\ y &= x_1 \end{aligned} \tag{57}$$

where $s \in [0, 1]$ is time-delay coefficient, which is chosen as $s = 0.9$. $\tau_1 = \tau_2 = 0.5(1 + \sin(t))$ are the time delays and $g_1(y) = g_2(y) = 0.5y^2$.

Fuzzy membership functions ($l = 1, \cdots, 5$) are chosen as

$$\begin{aligned} \mu_{F_1^l}(\hat{x}_1) &= \exp\left[-\frac{(\hat{x}_1 - 3 + l)^2}{16}\right], \\ \mu_{F_2^l}(\hat{x}_1, \hat{x}_2) &= \exp\left[-\frac{(\hat{x}_1 - 3 + l)^2}{4}\right] \times \exp\left[-\frac{(\hat{x}_2 - 3 + l)^2}{16}\right]. \end{aligned}$$

The design parameters are chosen as $k_1 = 9$, $k_2 = 11$, $\gamma_1 = 1$, $\gamma_2 = 2$, $\bar{\gamma}_1 = 5$, $\bar{\gamma}_2 = 6$, $\sigma_1 = 0.1$, $\sigma_2 = 0.12$, $\bar{\sigma}_1 = 0.01$, $\bar{\sigma}_2 = 0.02$, $c_1 = 6$, $c_2 = 7$, $\nu_1 = 0.2$, $\nu_2 = 0.3$, $k = 0.05$, $\eta = 0.5$, $b = 1$, $n = 2$.

In the simulation, $x_1(0) = 0.1$, $\hat{x}_1(0) = 0.05$, the other initial conditions are all chosen as zeros. The simulation results are shown by Figures 1-3.

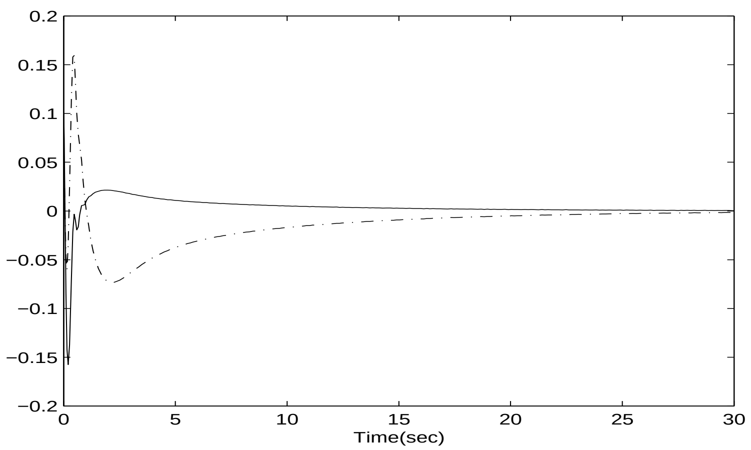

FIGURE 1. State $\hat{x}_1$ (dotted) follows state x_1 (solid)

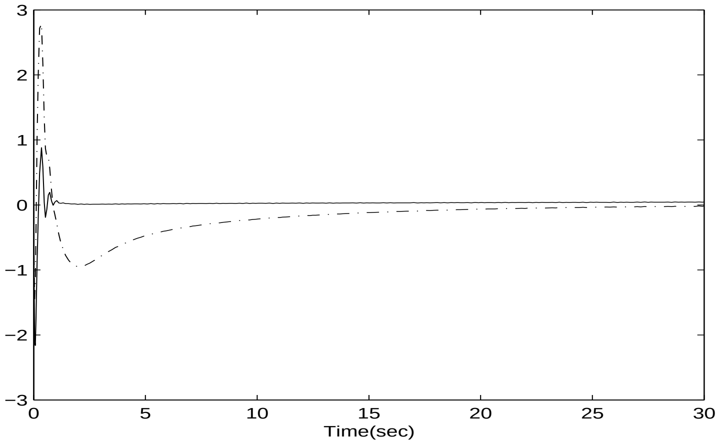

FIGURE 2. State $\hat{x}_2$ (dotted) follows state x_2 (solid)

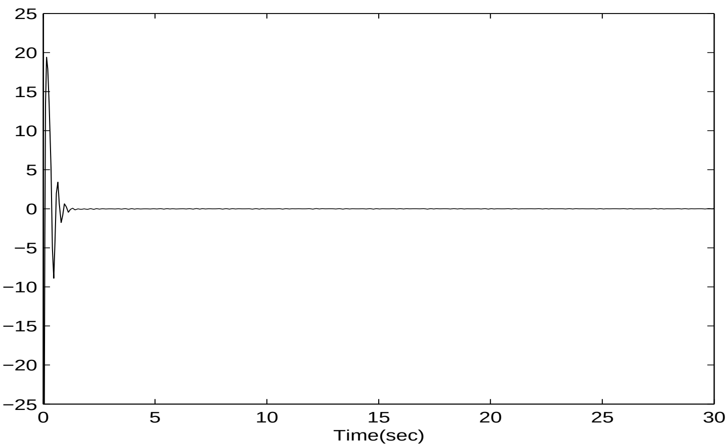

FIGURE 3. The trajectory of u

6. **Conclusions.** In this paper, an observer-based adaptive fuzzy output feedback control approach has been proposed for a class of stochastic nonlinear the time-delay systems with unmeasured states. Fuzzy logic systems are used to approximate the unknown nonlinear functions and a fuzzy state observer is designed to estimate those immeasurable states. By applying the backstepping design technique and combining with fuzzy control theory, an adaptive fuzzy output feedback backstepping control approach have been developed. It has proved that all the signals of the system are SUUB in probability, and the observer errors and the outputs of the control system can be made as small as the desired by appropriate choice of the design parameters. Simulation results are provided to show the effectiveness of the proposed approach.

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