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IMPROVED ARITHMETIC OF TWO-POSITION FAST INITIAL ALIGNMENT FOR SINS USING UNSCENTED KALMAN FILTER

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Abstract. *An arithmetic of fast two-position initial alignment for Strapdown Inertial Navigation System (SINS) using Unscented Kalman Filter (UKF) is proposed in this paper to solve the initial alignment problems of SINS. Based on the analysis of initial alignment method of SINS, the nonlinear model for two-position attitude calculation is derived, and the two-position method is used to eliminate the constant error of inertial devices, while the errors of the inertial devices are not needed to be expanded to be states, and the amount of computation is reduced under the premise of ensuring the alignment accuracy of UKF. Furthermore, according to the characteristics of nonlinear model of two-position attitude algorithm, the UKF filter using hybrid model is designed to reduce the amount of initial alignment computation. Simulation results show that, within the nonlinear model of two-position attitude calculation, the heading angle is directly observable, and this system can improve the accuracy and speed of heading angle alignment, which satisfies the real-time requirements of UKF filter for the initial alignment of SINS.*
Keywords: Nonlinear filtering, Unscented Kalman filter, Strapdown inertial navigation system, Two-position initial alignment

1. **Introduction.** Strapdown Inertial Navigation System (SINS) could provide real-time position and attitude information of the carrier. The initial alignment technology is a hot issue in the application of SINS in recent years. And two-position initial alignment method is usually used for the merits of rapidity and convenience. As the process of initial alignment is to establish initialization operating conditions for SINS before it starts to work, the precision and rapidity of initial alignment will directly influence the performance of the SINS [1-3].

Before mid-1990s, the initial alignment research mainly concentrated on the study of the inertial navigation system error model, and its realization method. Then the research focuses on the advanced estimation theory and its engineering application in different carrier platforms [4,5]. The linear Kalman Filter (KF) is the representative and extensively used approach for initial alignment. It could establish the initial attitude transformation matrix for inertial navigation system by estimating the misalignment between the computing and navigating coordinate. However, in engineering application area, the process of initial alignment is nonlinear due to the large initial misalignment angles, which are not suitable for the linear Kalman Filter any more.

Consequently, some nonliner filters as Particle Filter (PF) and Unscented Kalman Filter (UKF), are used in the initial alignment to improve the precision. And several satisfied performances were produced [6-8]. However, PF could only be optimal in theory, which is hard to use in the real-time for its heavy computing burden. Furthermore, the initial value of the particle filter could be uncertain because of the large misalignment angle, which would lead to the filter divergence when the particle quantity is not good enough.

In order to improve the performance in precision, rapidity and robustness of initial alignment in real-time, the applications of another nonlinear filter method-UKF are discussed in this paper.

Though the real-time performance of UKF is better than PF [9-11], the calculation burden of UKF is still heavier than that of KF [12,13], especially in the process of two-position initial alignment, whose model order is usually 10 or more, which is also hard for the UKF to use in real-time.

To solve this problem, a nonlinear reduced-order initial alignment model is proposed, which could reduce the computational complexity of UKF, and the accuracy and rapidity of initial alignment are improved. To reduce the system's order, the two-position algorithm is adopted in initial alignment modeling by eliminating the errors of inertial instruments.

The paper is organized as follows: Section 2 studies the alignment method of SINS and the modeling of initial alignment in the case of conventional fixed-position and the characteristics of two-position alignment; Section 3 presents the basic theory of UT transformation, and the probable flow of UKF filter, the two-position fast alignment algorithm based on UKF and a nonlinear model for two-position attitude calculation are constructed in Section 4; the two-position alignment filter based on UKF is designed in Section 5; Section 6 presents the simulation results and Section 7 summarizes the conclusions.

The main contributions of this paper are in three aspects: 1) an initial alignment reduced-order model based on two-position method is constructed in which different nonlinear filter approaches can be applied in initial alignment; 2) a two-position initial alignment UKF filter based on linear/nonlinear hybrid model is designed, which reduces the computation complexity of UKF and improves the real-time performances of filter; and 3) a two-position fast alignment method using UKF is proposed, which can directly determine the initial attitude matrix by calculating the attitude angles of SINS to improve the accuracy, rapidity and real-time performance of initial alignment.

2. Initial Alignment Design of SINS.

2.1. Initial alignment at fixed-position.

The assumptions of the initial alignment are 1) the inertial navigation system is demarcated in initial alignment, 2) because the initial alignment time is shortened, the accelerator error model is approximated by the sum of a random constant value and a white noise, and the gyroscope error model is also approximated by the sum of a random constant value and a white noise. The random constant values of both the accelerator and gyroscope are expanded to the new system states. The navigation coordinate adopts East-North geographic coordinate. Thus, the stationary base initial alignment error model of the inertial navigation system is described as:

$$\dot{\boldsymbol{X}}(t) = \boldsymbol{A}(t)\boldsymbol{X}(t) + \boldsymbol{B}(t)\boldsymbol{W}(t) = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 \\ 0_{5\times5} & 0_{5\times5} \end{bmatrix} \boldsymbol{X}(t) + \begin{bmatrix} \boldsymbol{A}_2 \\ 0_{5\times5} \end{bmatrix} \boldsymbol{W}(t) \tag{1}$$

The states can be described as

$$\boldsymbol{X} = \begin{bmatrix} \sigma v_E & \sigma v_N & \phi_E & \phi_N & \phi_U & \nabla_x & \nabla_y & \varepsilon_x & \varepsilon_y & \varepsilon_z \end{bmatrix}^{\mathrm{T}} \tag{2}$$

where σv is the velocity error, ϕ is the error angle, ∇ is the random constant of accelerometer, and ε is the random constant of the gyroscope. Subscript x, y, z are three elements of the body frame, respectively. Subscript E, N, U are three elements to the geographical coordinate, respectively. $\boldsymbol{W}$ is the system noise, which subjects to $N(0, \boldsymbol{Q})$ distribution.

$$\boldsymbol{A}_1 = \begin{bmatrix} 0 & 2\Omega_U & 0 & -g & 0 \\ -2\Omega_U & 0 & g & 0 & 0 \\ 0 & 0 & 0 & \Omega_U & -\Omega_N \\ 0 & 0 & -\Omega_U & 0 & 0 \\ 0 & 0 & \Omega_N & 0 & 0 \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{21} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{11} & C_{12} & C_{13} \\ 0 & 0 & C_{21} & C_{22} & C_{23} \\ 0 & 0 & C_{31} & C_{32} & C_{33} \end{bmatrix} \tag{3}$$

where $\Omega_U = \omega_{ie} \sin L$, $\Omega_N = \omega_{ie} \cos L$, ω_{ie} is the rotation rate of earth, g is the local acceleration of gravity, L is the local latitude, and C_{ij} is the attitude matrix element.

Take the lateral velocity error of the inertial navigation system as the measurement objective, and then the measurement equation is:

$$\boldsymbol{Y}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{X}(t) + \boldsymbol{V}(t) = \boldsymbol{H}\boldsymbol{X}(t) + \boldsymbol{V}(t) \tag{4}$$

where $\boldsymbol{V}$ is the measurement noise, subjecting to $N(0, \boldsymbol{R})$ distribution; $\boldsymbol{R}$ is the covariance matrix of measurement noise.

The fixed-position initial alignment system is a linear model, which is derived from one-order linearization based on the assumption that the misalignment angle is small. Thus, it needs to be probably adjusted before the alignment, and then the misalignment angle can be limited to be small. Simultaneously, in the traditional alignment method, the measurements are only the two lateral-velocities, and the system states are not fully observable. Therefore, the accuracy and rapidity of the constringency of misalignment azimuth are lower than that of the horizontal misalignment angle.

2.2. **Multi-position alignment approach analysis.** To improve the observability of the initial alignment system and thereby estimate much more state variables, a method of changing the vehicle attitude or inertial turning measurement unit could be adopted [3]. The alignment method based on multi-position could change the system matrix of SINS error model, and therefore, improve the observability of the initial alignment system and estimate the random constant error of inertial devices. Thus, it could improve the constringency accuracy and rapidity of misalignment azimuth and obtain better alignment performances [14]. The conventional multi-position alignment method is that using two-position technique.

The alignment method using two-position technique divides the initial alignment processes into two steps. The first one is similar to the conventional alignment method. The second step is introduced when the misalignment azimuth tends to be stable, in which the inertial measurement devices and vehicle attitude are rotated with a certain angle to the second position, which is helpful to the estimation of the random constant value and the misalignment azimuth. When the azimuth is changed by 180°, the estimation error of the misalignment azimuth is minimized, and the position at this moment is the optimal

Table 1. The initial alignment observability comparison of the static base SINS

Difference alignment methods	The observability of states				
	ϕ_E, ϕ_N	ϕ_U	∇_x, ∇_y	$\varepsilon_x, \varepsilon_y$	ε_z
Fixed position alignment	observable	Low-observability	unobservable	observable	Low-observability
Two-position alignment	observable	observable	Low-observability	observable	High-observability

two-position for initial alignment. Thus this alignment method is considered to be the optimal two-position alignment method.

The initial alignment observability of SINS both in the fixed-position alignment and the two-position alignment are analyzed. It can be seen from Table 1 that the observability of gyroscope's random constant value ε_z and the random constant deflection of the accelerometer ∇_x, ∇_y are both improved using two-position method. Besides, the observability of the misalignment azimuth ϕ_U is obviously improved when the two-position alignment is introduced.

3. **Unscented Kalman Filter Theory.** In the practical applications, the system dynamics and measurement equations are usually nonlinear, and the traditional Extended Kalman Filter (EKF) technique is to get its linearization using Taylor series expansion to deal with the nonlinear filter problem. Due to that only the first term of the Taylor series expansion of the nonlinear function is used, the error is produced during the linearization process, and the filter divergence may occur sometimes. To improve the performances of the nonlinear filter, Julier et al. presented UKF to deal with the nonlinear filter problem [15,16].

An additive noise is considered in the nonlinear dynamics, and then the nonlinear system dynamics can be written:

$$x_{k+1} = f(x_k, u_k) + v_k \tag{5}$$

$$y_k = h(x_k) + n_k \tag{6}$$

where $x_k \in \Re^n$ is the state vector with n dimensions at time k, $y_k \in \Re^m$ is the measurement vector, u_k is the known input vector, v_k is the zero mean process noise, which is a gauss noise with the variance Q_v, n_k is the zero mean measurement noise, which is a gauss noise with the variance R_n.

The filter process is as follows:

1) Initialization:

$$\hat{x}_0 = E[x_0], \quad P_0 = E\left[(\hat{x}_0 - x_0)(\hat{x}_0 - x_0)^T\right] \tag{7}$$

2) Calculating the values in points Sigma, it obtains the following matrix with the elements of $2n+1$ Sigma points:

$$\chi_{k-1}^a = \left[\begin{array}{ccc} \hat{x}_0^a & \hat{x}_0^a + \sqrt{(n+\lambda)P_{k-1}^a} & \hat{x}_0^a - \sqrt{(n+\lambda)P_{k-1}^a} \end{array}\right] \tag{8}$$

where λ is the scale factor.

3) Time updating:

Nonlinear transformation of the system state equation is implemented at these Sigma points:

$$\chi_{k|k-1}^a = f(\chi_{k-1}^a, u_{k-1}) \tag{9}$$

By the way of Unscented Transformation (UT), we get the predictive value of the state and the predictive variance matrix as follows:

$$\hat{x}_k^- = \sum_{i=0}^{2n} W_i^m \chi_{i,k|k-1}^a, \quad P_k^- = \sum_{i=0}^{2n} W_i^c (\chi_{i,k|k-1}^a - \hat{x}_k^-)(\chi_{i,k|k-1}^a - \hat{x}_k^-)^T + Q_v \tag{10}$$

Nonlinear transformation of observation equation is implemented at the Sigma points $\chi_{i,k|k-1}$, and the predictive output of the system is:

$$y_{i,k/k-1} = h(\chi_{i,k/k-1}^x), \quad \hat{y}_k^- = \sum_{i=0}^{2n} W_i^m y_{i,k/k-1} \tag{11}$$

where W_i is the corresponding weight, satisfying $\sum W_i = 1$.

4) Measurement updating:
The output theoretical variance matrix of the computation system is:

$$P_{y_k y_k} = \sum_{i=0}^{2n} W_i^c [y_{i,k/k-1} - \hat{y}_k^-][y_{i,k/k-1} - \hat{y}_k^-]^T + R_n \tag{12}$$

The covariance and the filter gain matrix can be calculated as:

$$P_{x_k y_k} = \sum_{i=0}^{2n} W_i^c [\chi_{i,k/k-1}^x - \hat{x}_k^-][y_{i,k/k-1} - \hat{y}_k^-]^T \tag{13}$$

$$K = P_{x_k y_k} P_{y_k y_k}^{-1} \tag{14}$$

The updated filter value and the latter state variance matrix is:

$$\hat{x}_k = \hat{x}_k^- + K(y_k^- \hat{y}_k^-), \quad P_k = P_k^- - K P_{y_k y_k} K^T \tag{15}$$

4. Two-Position Fast Alignment Modeling Using UKF.

4.1. Two-position alignment approach using UKF. In the general alignment method, it always selects three misalignment angles, two horizontal velocity, three random constants of gyroscopes and two random constants of accelerometers as the state variables, and the dimension of the error dynamics of initial alignment system is 10. When the three misalignment angles are all large misalignment angles, it adopts additive quaternion as new state variables to replace the three misalignment angles, then the dimension of the initial alignment system error equation is 11.

As we know from Section 3, the calculation of UKF is mainly determined by the dimension of the model. So the two situations above cannot satisfy the real-time performance of the SINS while using UKF filter with a high order. In this case, SINS alignment process will have much longer time and a loss of accuracy.

According to the two-position alignment method and the north-finding technique of fiber gyroscope, the two-position method is capable of improving the observability of the misalignment, and can avoid the random constant error of the inertial components effectively. The key purpose of inertial alignment is to obtain the inertial attitude matrix of the fiber SINS. Therefore, it introduces the two-position method into the alignment modeling in order to improve the real-time performance, the rapidity and accuracy of the initial filter by reducing the dimension of the initial alignment model. The basic theory can be seen in Figure 1.

In the traditional alignment, the random noise of the inertial implement is generally supposed to be random constant and white noise, and the five-dimension random constants of the inertial device are expanded to be states of system. Within the two-position

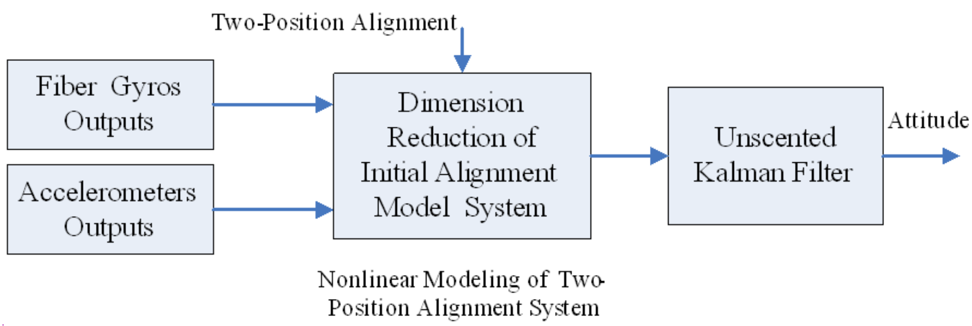

Figure 1. Nonlinear alignment theory using two-position

nonlinear alignment modeling, it could avoid the error of the inertial device in alignment modeling using two-position method. As the output of the inertial device is selected as the measurement in the new two-position nonlinear alignment modeling, the two-dimension velocity error of the states can also be avoided. Thus, the dimension of the state equation is reduced to 3, and then the real-time performance, the alignment rapidity and accuracy of UKF in practical alignment application are improved.

Compared with the traditional two-position alignment method, new method proposed in this paper make full consideration to the gyro's error characteristics in the process of alignment, which can effectively reduce the model dimension of the system and the calculation amount of UKF.

4.2. **Nonlinear modeling for the two-position attitude computation.**

4.2.1. *Computational model derivative of the horizontal attitude angle.* Firstly, it is supposed that the fiber SINS has been signed, and the random errors of the inertial components are random noise and white noise. Supposing the heading angle is ψ, pitching angle θ, roll angle γ, then the computational model for the accelerometer output is described as:

$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \boldsymbol{C}_n^b \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \begin{bmatrix} \Delta f_x \\ \Delta f_y \\ \Delta f_z \end{bmatrix} = \boldsymbol{C}_n^b \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \begin{bmatrix} \nabla_x \\ \nabla_y \\ \nabla_z \end{bmatrix} + \begin{bmatrix} w_{ax} \\ w_{ay} \\ w_{az} \end{bmatrix} \tag{16}$$

where f is the output of accelerometer to the body coordinate, Δf is the output error of the accelerometer, ∇ is the random constant error of the accelerometer in the body coordinate, w_a is the measurement gauss white noise of the accelerometer in the body coordinate, subscript x, y, z are the three directions of the body coordinate. C_n^b is the attitude transformation matrix from n-coordinate to b-coordinate:

$$\boldsymbol{C}_n^b = \begin{bmatrix} \sin\psi\sin\theta\sin\gamma + \cos\psi\cos\gamma & \cos\psi\sin\theta\sin\gamma - \sin\psi\cos\gamma & -\cos\theta\sin\gamma \\ \sin\psi\cos\theta & \cos\psi\cos\theta & \sin\theta \\ \cos\psi\sin\gamma - \sin\psi\sin\theta\cos\gamma & -\cos\psi\sin\theta\cos\gamma - \sin\psi\sin\gamma & \cos\theta\cos\gamma \end{bmatrix} \tag{17}$$

Substituting (17) to (16), the values at position-1 and position-2 are represented by superscripts 1 and 2, respectively, and then it gets:

$$\begin{bmatrix} -\sin\gamma\cos\theta\cdot g \\ \sin\theta\cdot g \\ \cos\gamma\cos\theta\cdot g \end{bmatrix} + \begin{bmatrix} \nabla_x \\ \nabla_y \\ \nabla_z \end{bmatrix} = \begin{bmatrix} f_x^1 \\ f_y^1 \\ f_z^1 \end{bmatrix} + \begin{bmatrix} w_{ax}^1 \\ w_{ay}^1 \\ w_{az}^1 \end{bmatrix} \tag{18}$$

Rotating 180 degrees around the vertical to the axis pedestal, the second position accelerometer output of the optical fiber SINS is:

$$\begin{bmatrix} -\sin(-\gamma)\cos(-\theta)\cdot g \\ \sin(-\theta)\cdot g \\ \cos(-\gamma)\cos(-\theta)\cdot g \end{bmatrix} + \begin{bmatrix} \nabla_x \\ \nabla_y \\ \nabla_z \end{bmatrix} = \begin{bmatrix} f_x^2 \\ f_y^2 \\ f_z^2 \end{bmatrix} + \begin{bmatrix} w_{ax}^2 \\ w_{ay}^2 \\ w_{az}^2 \end{bmatrix} \tag{19}$$

Subtracting (19) from (18), it obtains:

$$\begin{bmatrix} -2\sin\gamma\cos\theta\cdot g \\ 2\sin\theta\cdot g \\ 0 \end{bmatrix} = \begin{bmatrix} f_x^1 - f_x^2 \\ f_y^1 - f_y^2 \\ f_z^1 - f_z^2 \end{bmatrix} + \begin{bmatrix} w_{ax}^1 - w_{ax}^2 \\ w_{ay}^1 - w_{ay}^2 \\ w_{az}^1 - w_{az}^2 \end{bmatrix} \tag{20}$$

According to (20), it can obtain the computational model for the horizontal attitude:

$$\begin{cases} \theta = \arcsin \dfrac{(f_y^1 - f_y^2) + (w_{ay}^1 - w_{ay}^2)}{2g} \\ \gamma = \arcsin \dfrac{(f_x^1 - f_x^2) + (w_{ax}^1 - w_{ax}^2)}{-2\cos\theta \cdot g} \end{cases} \tag{21}$$

4.2.2. *The computation model derivation of heading angle.* The method of derivation for heading angle computational model is similar to that of horizontal attitude angle, and when the fiber SINS possesses a certain attitude angle, the outputs of the gyroscopes can be described as:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \boldsymbol{C}_n^b \begin{bmatrix} 0 \\ \omega_{ie}\cos L \\ \omega_{ie}\sin L \end{bmatrix} + \begin{bmatrix} \Delta\omega_x \\ \Delta\omega_y \\ \Delta\omega_z \end{bmatrix} = \boldsymbol{C}_n^b \begin{bmatrix} 0 \\ \omega_{ie}\cos L \\ \omega_{ie}\sin L \end{bmatrix} + \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} + \begin{bmatrix} w_{gx} \\ w_{gy} \\ w_{gz} \end{bmatrix} \tag{22}$$

where ω is the output of the gyroscope in the body axis coordinate, ε is the random constant error of the gyroscope, $\Delta\omega$ is the output error of the gyroscope, w_g is the measurement gauss white noise of the gyroscope in the body axis coordinate.

Substituting (17) to (22), it has:

$$\begin{aligned} &\begin{bmatrix} (-\cos\gamma\sin\psi + \sin\gamma\sin\theta\cos\psi)\omega_{ie}\cos L - \sin\gamma\cos\theta\cdot\omega_{ie}\sin L \\ \cos\theta\cos\psi\cdot\omega_{ie}\cos L + \sin\theta\cdot\omega_{ie}\sin L \\ (-\sin\gamma\sin\psi - \cos\gamma\sin\theta\cos\psi)\omega_{ie}\cos L + \cos\gamma\cos\theta\cdot\omega_{ie}\sin L \end{bmatrix} + \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} \\ &= \begin{bmatrix} \omega_x^1 \\ \omega_y^1 \\ \omega_z^1 \end{bmatrix} + \begin{bmatrix} w_{gx}^1 \\ w_{gy}^1 \\ w_{gz}^1 \end{bmatrix} \end{aligned} \tag{23}$$

Rotating 180 degrees around the vertical to the axis pedestal, the second position gyroscope output of the optical fiber SINS is:

$$\begin{aligned} &\begin{bmatrix} (\cos\gamma\sin\psi - \sin\gamma\sin\theta\cos\psi)\omega_{ie}\cos L + \sin\gamma\cos\theta\cdot\omega_{ie}\sin L \\ -\cos\theta\cos\psi\cdot\omega_{ie}\cos L - \sin\theta\cdot\omega_{ie}\sin L \\ (-\sin\gamma\sin\psi - \cos\gamma\sin\theta\cos\psi)\omega_{ie}\cos L + \cos\gamma\cos\theta\cdot\omega_{ie}\sin L \end{bmatrix} + \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} \\ &= \begin{bmatrix} \omega_x^2 \\ \omega_y^2 \\ \omega_z^2 \end{bmatrix} + \begin{bmatrix} w_{gx}^2 \\ w_{gy}^2 \\ w_{gz}^2 \end{bmatrix} \end{aligned} \tag{24}$$

Subtracting (24) from (23), it gets:

$$\begin{aligned} &\begin{bmatrix} (-2\cos\gamma\sin\psi + 2\sin\gamma\sin\theta\cos\psi)\omega_{ie}\cos L - 2\sin\gamma\cos\theta\cdot\omega_{ie}\sin L \\ 2\cos\theta\cos\psi\cdot\omega_{ie}\cos L + 2\sin\theta\cdot\omega_{ie}\sin L \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \omega_x^1 - \omega_x^2 \\ \omega_y^1 - \omega_y^2 \\ \omega_z^1 - \omega_z^2 \end{bmatrix} + \begin{bmatrix} w_{gx}^1 - w_{gx}^2 \\ w_{gy}^1 - w_{gy}^2 \\ w_{gz}^1 - w_{gz}^2 \end{bmatrix} \end{aligned} \tag{25}$$

The analytical model of the heading angles can be obtained from (25):

$$\psi = \arccos \frac{(\omega_y^1 - \omega_y^2) + (w_{gy}^1 - w_{gy}^2) - 2\sin\theta\cdot\omega_{ie}\sin L}{2\cos\theta\cdot\omega_{ie}\cos L} \tag{26}$$

5. Two-Position Alignment Filter Design Using UKF.

5.1. Nonlinear filter model for two-position alignment. For the two-position attitude nonlinear computational model of the SINS is established in the above work, the nonlinear filter model of two-position alignment will be established here. During the alignment process of the SINS, the attitude angle can be kept invariant. Then the alignment state equation could be modeled as the sum of a constant and a small disturbance, which is set as gauss white noise:

$$\begin{bmatrix} \theta_k \\ \gamma_k \\ \psi_k \end{bmatrix} = \begin{bmatrix} \theta_{k-1} \\ \gamma_{k-1} \\ \psi_{k-1} \end{bmatrix} + \begin{bmatrix} w_\theta \\ w_\gamma \\ w_\psi \end{bmatrix} \tag{27}$$

Selecting the first two equations of (20) and the first one of (25), the corresponding initial alignment nonlinear measurement dynamic is established as follows:

$$\begin{bmatrix} f_{xk}^1 - f_{xk}^2 \\ f_{yk}^1 - f_{yk}^2 \\ \omega_{xk}^1 - \omega_{xk}^2 \end{bmatrix} = \begin{bmatrix} -2\sin\gamma_k \cos\theta_k \cdot g \\ 2\sin\theta_k \cdot g \\ (-2\cos\gamma_k \sin\psi_k + 2\sin\gamma_k \sin\theta_k \cos\psi_k)\omega_{ie}\cos L \\ -2\sin\gamma_k \cos\theta_k \cdot \omega_{ie}\sin L \end{bmatrix} - \begin{bmatrix} w_{axk}^1 - w_{axk}^2 \\ w_{ayk}^1 - w_{ayk}^2 \\ w_{gxk}^1 - w_{gxk}^2 \end{bmatrix} \tag{28}$$

Equations (27) and (28) are nonlinear dynamics of two-position alignment, where the state equation is linear while the measurement equation is nonlinear. The dimension of each equation is 3, which is suitable for nonlinear filter to be applied in alignment. Secondly, the system equation is not established under any linearization assumption or approximation in the modeling process, so that the two-position nonlinear model describes the physical process of initial alignment accurately. Finally, the heading angle of the new model has corresponding direct observation information, which is different from the indirect observation of the general method and could improve the alignment rapidity.

5.2. UKF design using hybrid model. In Subsection 5.1, the state equation of the two-position alignment nonlinear filter model is linear while the measurement equation is nonlinear, and the noise is gauss white noise. For this class of linear/nonlinear hybrid system, we can deal with UKF to reduce the calculating when UKF is applied in initial alignment. The specific progress is presented as follows.

It is supposed that the posterior probability density subjects to gauss distribution. Obviously the prior probability density of hybrid system subjects to gauss distribution, thus one-step state prediction can be obtained by linear updating. Then the time updating process of UKF can be simplified to the following steps:

The one-step predictive value and variance matrix of the state are:

$$\hat{x}_k^- = f(x_{k-1}, u_{k-1}), \quad P_k^- = f_k P_{k-1} f_k^T + P_v \tag{29}$$

The one-step predictive Sigma points of the state can be constructed as:

$$\chi_{k|k-1}^a = \left[\begin{array}{ccc} \hat{x}_k^- & \hat{x}_k^- + \sqrt{(L+\lambda)P_{k-1}^-} & \hat{x}_k^- - \sqrt{(L+\lambda)P_{k-1}^-} \end{array}\right] \tag{30}$$

Nonlinear transformation is implemented at each Sigma points $\chi_{i,k|k-1}$, and the predictive output of the system is obtained by weighted summation:

$$y_{i,k/k-1} = h(\chi_{i,k/k-1}^x), \quad \hat{y}_k^- = \sum_{i=0}^{2L} W_i^m y_{i,k/k-1} \tag{31}$$

Obviously, the one-step prediction of the states adopts Kalman linear recurrence form for the UKF of hybrid model, while the other nonlinear portions adopt UKF recurrence technique still. When this approach constituted by linear Kalman filter and UKF filter applied in initial alignment filter, it would reduce the calculation of UKF in some degree, and improve the real-time performance of initial alignment filter when UKF is applied.

6. **Simulation Results.** The simulation condition is set as follows:

(1) Gyroscopes: the random constant value is 0.1 (°)/h, and the white noise is 0.1 (°)/h;
(2) Accelerometers: the random constant value is $1 \times 10^{-4}g$, and the white noise is $1 \times 10^{-4}g$;
(3) Initial attitude angles are 0°, 0° and 60° (for pitching, rolling, heading, respectively);
(4) Initial attitude angle errors are 1°, 1° and 1° (for pitching, rolling, heading, respectively);
(5) The simulation time is set as 900s.

In order to compare the effect of the proposed method in this paper, simulations are separately carried out by using these three methods:

(1) Fixed-position alignment method;
(2) Traditional two-position alignment method;
(3) Proposed two-position fast alignment method.

The heading angle error of alignment using the first method is shown as Figure 2, and the heading angle error of alignment using the second method is shown as Figure 3.

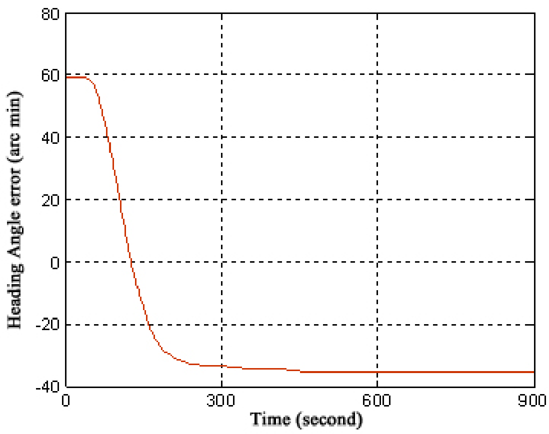

FIGURE 2. Heading angle error using fixed-position alignment method, $(1°, 1°, 1°)$

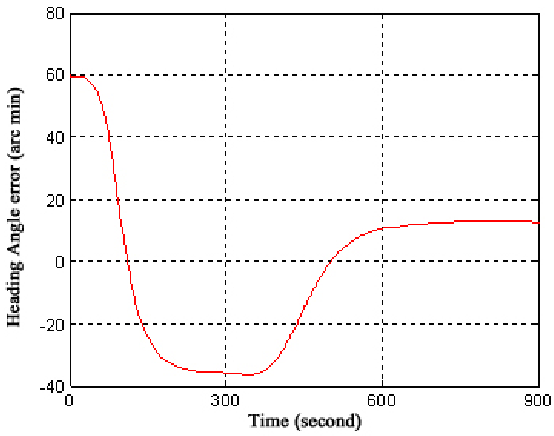

FIGURE 3. Heading angle error using two-position alignment method, $(1°, 1°, 1°)$

Using the fixed-position initial alignment method, the alignment accuracy and time are: the horizontal misalignment angle error is $21''$, the misalignment azimuth is $35'$, the alignment time is about 300s. Using the traditional two-position initial alignment method, the alignment accuracy and time are: the horizontal misalignment angle error is $6''$, the misalignment azimuth is $12'$, the alignment time is about 600s.

For the method proposed in this paper, simulations are done with the initial attitude errors as $(1°, 1°, 1°)$, $(5°, 5°, 5°)$, $(10°, 10°, 10°)$, $(15°, 15°, 15°)$, respectively. The results of the first two groups are shown in Figures 4-9, and the other results are shown in Table 2.

Figures 4-6 show that when the attitude error is $(1°, 1°, 1°)$, the horizontal attitude angles converge rapidly, with the horizontal attitude error converging to $6''$ after several two-position measurements. This accuracy is similar to that of the theoretical accuracy of general two-position method. Figure 6 illustrates that the convergence rapidity of heading

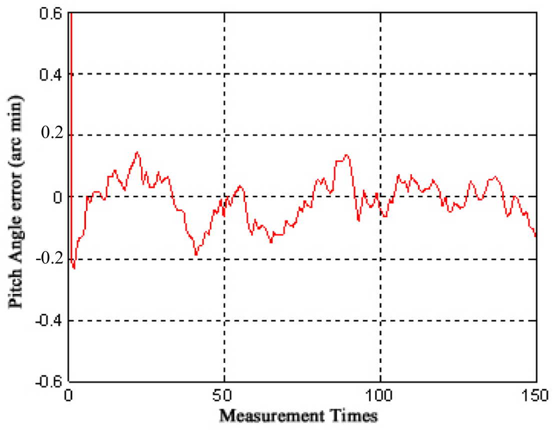

FIGURE 4. Pitching angle error using two-position fast alignment method, $(1^\circ, 1^\circ, 1^\circ)$

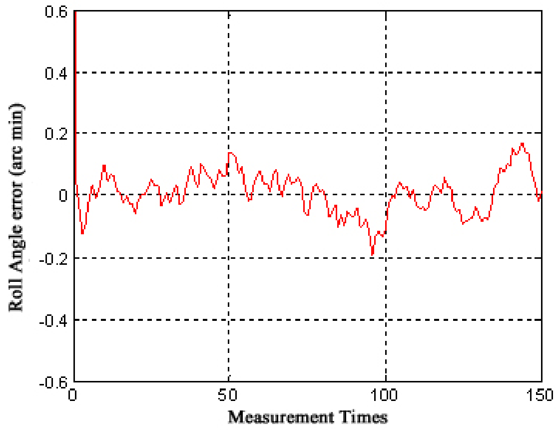

FIGURE 5. Rolling angle error using two-position fast alignment method, $(1^\circ, 1^\circ, 1^\circ)$

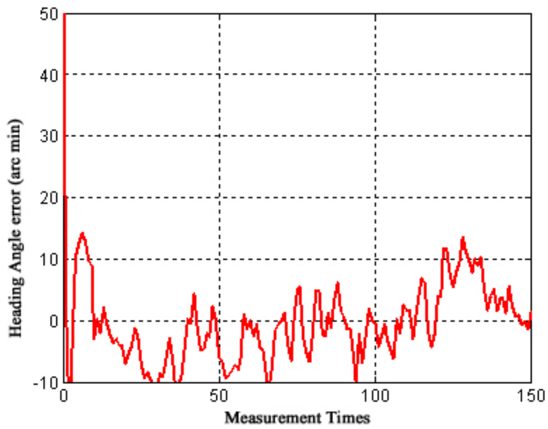

FIGURE 6. Heading angle error using two-position fast alignment method, $(1^\circ, 1^\circ, 1^\circ)$

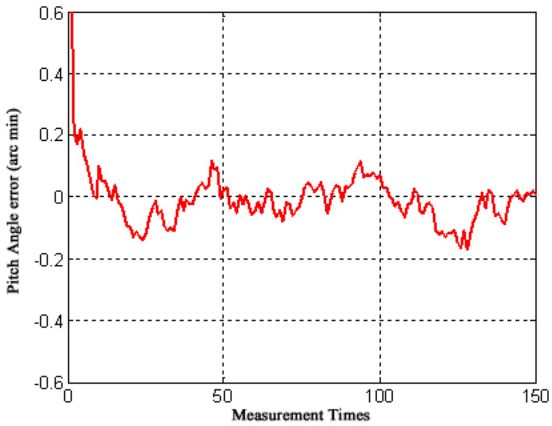

FIGURE 7. Pitching angle error using two-position fast alignment method, $(5^\circ, 5^\circ, 5^\circ)$

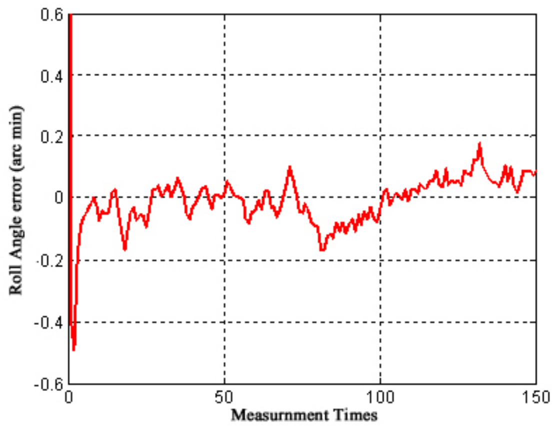

FIGURE 8. Rolling angle error using two-position fast alignment method, $(5^\circ, 5^\circ, 5^\circ)$

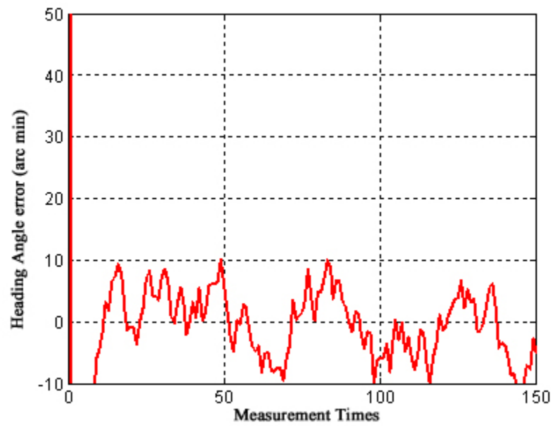

FIGURE 9. Heading angle error using two-position fast alignment method, $(5^\circ, 5^\circ, 5^\circ)$

angle is slightly less than that of horizontal attitude angle. However, it can also converge to $10'$ rapidly.

Both the fixed-position alignment method and the traditional two-position alignment method are only suitable for small misalignment angle. However, the proposed approach in this paper can also deal with the problem when the misalignment angles are large.

As shown in Figures 7-9 with large initial attitude errors $(5°, 5°, 5°)$, the alignment precision of heading angle with new method has improved to $6'$, compared with the $35'$ in Figure 2 and $12'$ in Figure 3.

Table 2 shows the simulation result of initial alignment accuracy with new method in the case of different large misalignment angles. It can draw the conclusion that the proposed two-position fast alignment method in this paper is adaptive to the initial alignment of different misalignment angles (especially large misalignment angles) using UKF.

Table 2. The comparison of initial alignment accuracy in the case of different misalignment angles with new method

Misalignment angle	First group $(1°, 1°, 1°)$	Second group $(5°, 5°, 5°)$	Third group $(10°, 10°, 10°)$	Fourth group $(15°, 15°, 15°)$
Pitching angle error ($''$)	2.6348	3.1162	2.5386	2.4877
rolling angle error ($''$)	3.3316	2.9303	3.2835	2.9906
heading angle error ($'$)	5.7493	5.8980	5.3009	5.7007

On the other side, alignment time is another important factor which also determine the method whether can be used in the engineering. As shown in Figure 9, using the proposed approach, the time of heading angle error getting into the convergence state is no more than 10 seconds, which is much less than the fixed-position alignment method (Figure 2) and the traditional two-position alignment method (Figure 3).

For the new reduce-order nonlinear model of two-position initial alignment method, the random constants of both accelerometers and gyroscopes are eliminated, and the dimension of the system is reduced from 10 to 3. The amount of calculation of UKF is reduced and the convergence time is shorter when it is applied to initial alignment of inertial navigation system.

7. Conclusions.

To solve the initial alignment problems of SINS, two-position fast initial alignment method using UKF is proposed in this paper. The initial alignment nonlinear model is established based on two-position method, and this two-position measurement method is used in the new model to eliminate the influence on the attitude angle estimation of random constants of accelerometers and gyroscopes, which can reduce the dimension of the initial alignment system and the amount of the computation of UKF. Furthermore, an UKF using a hybrid model is designed to improve the real-time performance in the initial alignment applications. The accuracy of two-position initial alignment nonlinear model is higher than that of the conventional initial alignment model, and the heading angle of the nonlinear model is directly observable, so that it can improve the accuracy and the rapidity of the initial alignment.

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