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COORDINATED ATTITUDE CONTROL OF FLEXIBLE SPACECRAFT FORMATIONS VIA BEHAVIOR-BASED CONTROL APPROACH

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Abstract. *This paper is to investigate the coordinated attitude control problem for flexible spacecraft formations. Considering the attitude maneuver control problem of a formation composed of flexible spacecraft, two classes of decentralized attitude coordination control laws using behavior-based control approach are proposed. The first class of controllers can steer the attitude states of the flexible spacecraft formation to a desired reference state asymptotically in the absence of disturbances and communication delays. The second class of controllers is an improved one which simultaneously takes into account the effects of external disturbances and communication delays. The improved control strategies are effective to overcome these unexpected phenomena subject to arbitrary communication topologies. Finally, numerical simulations are provided to demonstrate the effectiveness of the designed control schemes.*

Keywords: Behavior-based control, Flexible spacecraft formation, Attitude maneuver, Robustness, Communication delays

1. **Introduction.** Spacecraft formation flying (SFF) has been studied extensively in recent decades due to numbers of advantages such as cost reduction and robustness improvement [1-12]. To perform a flying task cooperatively, each individual spacecraft within the formation must coordinate with others. Therefore, as one of the most important research topics in SFF, attitude coordinated control has attracted much research attention.

According to the place where the control decisions are made, attitude coordination control can be centralized or decentralized. Centralized control is fault-sensitive since a single spacecraft takes charge of making control decisions for the formation: the failure of the decision-making spacecraft leads to the failure of the global system. However, decentralized control scheme is fault-tolerant because the control action of each individual spacecraft is determined by its local information: the failure of a single spacecraft will be confined to the region of itself and will not lead to destabilization of the entire system.

In the field of decentralized control, some results have been reported [4-12]. The attitude synchronization problem of a fully autonomous and distributed formation system was solved in [4]. In [6], for depicting the information flow graph within a formation, the algebraic graph theory was employed for formation control of multiple agents modeled by linear dynamics. In particular, the attitude coordination control problems for rigid spacecraft formations were addressed in [8-10] by the use of behavior-based control approach. In [9], a class of behavior-based control laws was developed to guarantee global stability of a spacecraft formation, and the control performance with respect to different weights of the formation-keeping control action is investigated. By introducing sliding mode control method into control scheme design, a class of robust behavior-based control laws was proposed in [10]. The control strategies designed in [10] can drive the attitude states of a spacecraft formation to a desired dynamic reference state asymptotically, and are robust against both model uncertainties and external disturbances.

It is worth pointing out that the communication topologies considered in the aforementioned literature must be undirected graphs with no communication delay. This assumption may cause potential problems in practice because the information in SFF is always exchanged over a network and the communication is far from being perfect. In addition to these issues, spacecraft are always subject to unexpected environment disturbances which can jeopardize the mission. All these effects should be taken into account in the controller design for the spacecraft formation to pursue an ideal performance. On the other hand, modeling spacecraft as rigid bodies is an approximation whose validity needs to be checked all the time. Because of the existence of flexible attachments such as antennae and solar paddles, spacecraft within the formation are not rigid due to the flexibility inherent in these structures. The attitude control problem of flexible spacecraft has been addressed in [13-16]. In [13], an adaptive sliding mode control scheme was proposed to counterbalance the effects of uncertainties and disturbances. A control law with no measurement of angular velocity was developed in [14] to realize the attitude maneuver of a flexible spacecraft. The variable structure control method was introduced in [16]. Considering bounded disturbance torque, the controller proposed in [16] can guarantee asymptotical reachability of the desired trajectory. However, all these studies were done in the field of single spacecraft instead of formation systems. Therefore, the development of control strategies for flexible spacecraft formations is interesting and challenging. This scenario is important to some practical applications such as space-based interferometry and synthetic-aperture imaging: the satellite of the space-borne distributed Synthetic Aperture Radar (SAR) could be flexible because of the large radar antennae it takes for communication, and the solar paddles that the satellite takes also cause flexibility.

Inspired by these facts, it is desirable to design novel control schemes for a flexible spacecraft formation with external disturbances, communication delays and limited network. To the best of the authors' knowledge, such type of control problem has not been addressed in the existing literature.

In this paper, behavior-based approach is employed to solve the coordinated attitude control problem of a flexible spacecraft formation. The overall control action of behavior-based control is determined by a weighted sum of the control actions for each of the behaviors including station-keeping and formation-keeping. Station-keeping is the behavior that drives a spacecraft to its absolute desired attitude. Formation-keeping is the behavior that aligns a spacecraft with other spacecraft in the formation. The contributions of this study are 1) a novel class of control schemes is proposed to solve the attitude maneuver control problem of a flexible spacecraft formation, which has not been studied in the existing literature; 2) the designed control laws are robust against external disturbances, and can overcome the effects of communication delays which were not taken into account in [1-12]; 3) the developed control strategies are effective subject to arbitrary communication topologies rather than undirected graph used in the aforementioned literature.

This paper is organized as follows. Background and preliminaries are given in Section 2. In Section 3, a basic result is given, and the stability analysis for the proposed control schemes is provided. In Section 4, a class of coordinated behavior-based control laws which is proven to be robust against external disturbances and effective with communication delays is developed. In Section 5, numerical simulations are presented. The conclusions are given in Section 6.

2. Background and Preliminaries.

2.1. **Notations.** For a vector $\boldsymbol{\nu} = [(\boldsymbol{\nu})_1, \ldots, (\boldsymbol{\nu})_m]^T$, we use $(\boldsymbol{\nu})_k$ to represent the kth component of $\boldsymbol{\nu}$ and $|\boldsymbol{\nu}| = [|(\boldsymbol{\nu})_1|, \ldots, |(\boldsymbol{\nu})_m|]^T$. $\boldsymbol{\nu}_i$ denotes some parameters or variables $\boldsymbol{\nu}$ of the ith spacecraft. $\|\boldsymbol{\nu}\|$ represents the Euclidean norm of the vector $\boldsymbol{\nu}$. $\alpha_{\max}(*)$ and $\alpha_{\min}(*)$ are used to denote the maximum and minimum Eigenvalues of a matrix, respectively.

2.2. **Attitude kinematics and dynamics.** Unit quaternion parameters are adopted to describe the attitude of a flexible spacecraft. The unit quaternion parameters are defined by

$$\bar{q} = \begin{pmatrix} \boldsymbol{q} \\ q_0 \end{pmatrix} = \begin{pmatrix} \boldsymbol{n} \cdot \sin\left(\frac{\theta}{2}\right) \\ \cos\left(\frac{\theta}{2}\right) \end{pmatrix} \tag{1}$$

with the constraint $\boldsymbol{q}^T\boldsymbol{q} + q_0^2 = 1$, where $\boldsymbol{n}$ is the Euler axis; θ is the Euler angle.

The kinematic equation in terms of unit quaternion parameters is given by

$$\dot{\boldsymbol{q}} = \frac{1}{2}\left(q_0\boldsymbol{I} + \boldsymbol{q}^\times\right)\boldsymbol{\omega} \tag{2}$$

$$\dot{q}_0 = -\frac{1}{2}\boldsymbol{q}^T\boldsymbol{\omega} \tag{3}$$

where $\boldsymbol{q}^\times$ denotes the skew-symmetric matrix:

$$\boldsymbol{q}^\times = \begin{bmatrix} 0 & -(\boldsymbol{q})_3 & (\boldsymbol{q})_2 \\ (\boldsymbol{q})_3 & 0 & -(\boldsymbol{q})_1 \\ -(\boldsymbol{q})_2 & -(\boldsymbol{q})_1 & 0 \end{bmatrix} \tag{4}$$

and $\boldsymbol{I}$ denotes a 3×3 identity matrix.

The absolute attitude error $\bar{q}_e$ denotes the relative attitude from the desired reference frame to the body-fixed reference frame, which can be calculated as

$$\bar{q}_e = \bar{q}_d^* \otimes \bar{q} = \left(\boldsymbol{q}_e^T \; q_{0e}\right)^T \tag{5}$$

where $\bar{q}_d^* = \left(-\boldsymbol{q}_d^T \; q_{0d}\right)^T$ is the inverse of the desired attitude $\bar{q}_d$. The symbol $\otimes$ represents the quaternion multiplication:

$$\bar{q}_a \otimes \bar{q}_b = \begin{pmatrix} q_{0a}\boldsymbol{q}_b + q_{0b}\boldsymbol{q}_a + \boldsymbol{q}_a^\times\boldsymbol{q}_b \\ q_{0a}q_{0b} - \boldsymbol{q}_a^T\boldsymbol{q}_b \end{pmatrix} \tag{6}$$

The dynamic model of a flexible spacecraft is given by the following differential equation [13-16]:

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\delta}^T\ddot{\boldsymbol{\eta}} = -\boldsymbol{\omega}^\times\left(\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{\delta}^T\boldsymbol{\eta}\right) + \boldsymbol{u} \tag{7}$$

$$\ddot{\boldsymbol{\eta}} + \boldsymbol{C}\dot{\boldsymbol{\eta}} + \boldsymbol{K}\boldsymbol{\eta} = -\boldsymbol{\delta}^T\dot{\boldsymbol{\omega}} \tag{8}$$

where $\boldsymbol{J}$ represents the symmetric inertia matrix of the whole structure; $\boldsymbol{\delta}$ is the coupling matrix between the central rigid body and the flexible appendixes; $\boldsymbol{\eta}$ represents the modal displacement; $\boldsymbol{u}$ is the control torque; and $\boldsymbol{C}$ and $\boldsymbol{K}$ denote the damping matrix and stiffness matrix, respectively. We have that

$$\boldsymbol{C} = diag\left(2\vartheta_i\omega_{ni}, i = 1, \ldots, N\right) \tag{9}$$

$$\boldsymbol{K} = diag\left(\omega_{ni}^2, i = 1, \ldots, N\right) \tag{10}$$

with corresponding damping ϑ_i and natural frequency ω_{ni}, and N is the number of elastic modes considered.

Assuming $\boldsymbol{\zeta} = \begin{pmatrix} \boldsymbol{\eta}^T & \left(\dot{\boldsymbol{\eta}} + \boldsymbol{\delta}^T\boldsymbol{\omega}\right)^T \end{pmatrix}^T$ and substituting $\boldsymbol{\zeta}$ into (7) and (8), we have the following dynamics equation which will be involved in the stability analysis:

$$\boldsymbol{J}^*\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^\times \left(\boldsymbol{J}^*\boldsymbol{\omega} + \boldsymbol{H}\boldsymbol{\zeta}\right) + \boldsymbol{L}\boldsymbol{\zeta} - \boldsymbol{M}\boldsymbol{\omega} + \boldsymbol{u} \tag{11}$$

$$\dot{\boldsymbol{\zeta}} = \boldsymbol{A}\boldsymbol{\zeta} + \boldsymbol{B}\boldsymbol{\omega} \tag{12}$$

where $\boldsymbol{J}^* = \boldsymbol{J} - \boldsymbol{\delta}^T\boldsymbol{\delta}$; $\boldsymbol{H} = \begin{pmatrix} 0 & \boldsymbol{\delta}^T \end{pmatrix}$; $\boldsymbol{L} = \begin{pmatrix} \boldsymbol{\delta}^T\boldsymbol{K} & \boldsymbol{\delta}^T\boldsymbol{C} \end{pmatrix}$; $\boldsymbol{M} = \boldsymbol{\delta}^T\boldsymbol{C}\boldsymbol{\delta}$; $\boldsymbol{A} = \begin{pmatrix} 0 & \boldsymbol{I} \\ -\boldsymbol{K} & -\boldsymbol{C} \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} -\boldsymbol{\delta} \\ \boldsymbol{C}\boldsymbol{\delta} \end{pmatrix}$.

2.3. Lemmas.

Lemma 2.1. [17] *Suppose that a Hermitian matrix is partitioned as*

$$\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B}^* & \boldsymbol{C} \end{bmatrix} \tag{13}$$

where $\boldsymbol{A}$ and $\boldsymbol{C}$ are square. This matrix is positive definite if and only if $\boldsymbol{A}$ is positive definite and $\boldsymbol{C} > \boldsymbol{B}^*\boldsymbol{A}^{-1}\boldsymbol{B}$.

Lemma 2.2. [18] *Consider the autonomous system $\dot{x} = f(x)$ with $f(x)$ continuous, and let $V(x)$ be a scalar function with continuous first partial derivatives. Assume that $V(x) \to \infty$ as $\|x\| \to \infty$; $\dot{V}(x) \leq 0$ over the whole state space. Let $\boldsymbol{R}$ be the set of all points where $\dot{V}(x) = 0$, and $\boldsymbol{M}$ be the largest invariant set in $\boldsymbol{R}$. Then all solutions globally asymptotically converge to $\boldsymbol{M}$ as $t \to \infty$.*

Lemma 2.3. [19] *If $f(t)$, $\dot{f}(t) \in L_\infty$, and $f(t) \in L_p$ for some $p \in [1, \infty)$, then* $\lim_{t\to\infty} f(t) = 0$.

3. Basic Result.

3.1. Problem statement.

Consider an n-flexible-spacecraft formation. The desired angular velocity and desired attitude are $\mathbf{0}$ and $\boldsymbol{q}_d$, respectively. The attitude maneuver control problem in this section is to design the control torque $\boldsymbol{u}_i$ for each spacecraft within the formation, so that the attitude states of each flexible spacecraft can converge to the desired states asymptotically, namely, $\boldsymbol{\omega}_i \to 0$, $\boldsymbol{q}_{ei} \to 0$, $\boldsymbol{\zeta}_i \to 0$ as $t \to \infty$.

3.2. Behavior-based controllers design.

The behavior-based approach is used to solve the attitude maneuver control problem. Behavior-based control is to determine the control action through a weighted sum of control actions for each of the behaviors including station-keeping and formation-keeping. Based on the concept of behavior-based control, we propose the following class of behavior-based attitude control laws:

$$\boldsymbol{u}_i = \boldsymbol{u}_i^{sk} + \boldsymbol{u}_i^{fk}, \quad i = 1, 2, \cdots n \tag{14}$$

In (14),

$$\boldsymbol{u}_i^{sk} = -K_{pi}\boldsymbol{q}_{ei} - K_{di}\boldsymbol{\omega}_i \tag{15}$$

$$\boldsymbol{u}_i^{fk} = -\sum_{j=1}^{n} \left(K_{pij}\boldsymbol{q}_{ij} + K_{dij}\boldsymbol{\omega}_{ij}\right) \tag{16}$$

where $\boldsymbol{q}_{ei}$ calculated by (5) is the station-keeping attitude error and $\boldsymbol{\omega}_i$ is the station-keeping angular velocity; $K_{pi}, K_{di} > 0$ are station-keeping weight parameters, and $K_{pij} = K_{pji} > 0$, $K_{dij} = K_{dji} > 0$ are formation-keeping weight parameters which satisfy $K_{pij}/K_{dij} = \rho$ for $i, j = 1, \ldots, n$ where ρ is a positive constant that will be involved in the following proof procedure; $\boldsymbol{q}_{ij} = \boldsymbol{q}_{ei} - \boldsymbol{q}_{ej}$ and $\boldsymbol{\omega}_{ij} = \boldsymbol{\omega}_i - \boldsymbol{\omega}_j$ represent the formation-keeping attitude error and formation-keeping angular velocity error between the ith spacecraft and the jth spacecraft.

Remark 3.1. *The station-keeping errors $\boldsymbol{q}_{ei}$ and $\boldsymbol{\omega}_i$ denote the attitude states of an individual spacecraft with respect to its desired attitude states. The formation-keeping errors $\boldsymbol{q}_{ij}$ and $\boldsymbol{\omega}_{ij}$ denote the attitude states of a spacecraft with respect to other spacecraft. In (14), $\boldsymbol{u}_i^{sk}$ is the station-keeping control action to drive the attitude of the spacecraft to the desired state and $\boldsymbol{u}_i^{fk}$ is the formation-keeping control action to maintain certain relative attitude within the formation. $\boldsymbol{u}_i$ is determined by a weighted sum of the station-keeping control action and the formation-keeping control action.*

In order to facilitate the stability analysis, we restate the control laws (14) as

$$\boldsymbol{u}_i = \boldsymbol{u}_i^{sk} + \boldsymbol{u}_i^{fk} = -K_{pi}\boldsymbol{q}_{ei} - K_{di}\boldsymbol{\omega}_i - \sum_{j=1}^{n} K_{dij}\left(\rho \boldsymbol{q}_{ij} + \boldsymbol{\omega}_{ij}\right) \tag{17}$$

Theorem 3.1. *The controller (14) can solve the attitude maneuver problem of the flexible spacecraft formation stated before.*

Proof: We consider the following candidate Lyapunov function:

$$V = \sum_{i=1}^{n} V_i \tag{18}$$

with

$$V_i = 2\left(K_{pi} + \rho K_{di}\right)\left(1 - q_{0ei}\right) + \frac{1}{2}\boldsymbol{\omega}_i^T \boldsymbol{J}_i^* \boldsymbol{\omega}_i + \rho \boldsymbol{q}_{ei}^T \boldsymbol{J}_i^* \boldsymbol{\omega}_i + \frac{1}{2}\boldsymbol{\zeta}_i^T \boldsymbol{P}_i \boldsymbol{\zeta}_i \tag{19}$$

where V_i is the component Lyapunov function; $\boldsymbol{P}_i$ is a positive definite matrix which is a solution of the Lyapunov equation $\boldsymbol{A}_i^T\boldsymbol{P}_i + \boldsymbol{P}_i\boldsymbol{A}_i = -\boldsymbol{Q}_i$ with a positive definite matrix $\boldsymbol{Q}_i$.

V_i can be bounded by

$$V_i \geq \frac{1}{2}\boldsymbol{\xi}_i^T \boldsymbol{\Theta}_i \boldsymbol{\xi}_i \tag{20}$$

where

$$\boldsymbol{\xi}_i = \left(\|\boldsymbol{q}_{ei}\| \ \|\boldsymbol{\omega}_i\| \ \|\boldsymbol{\zeta}_i\|\right)^T \tag{21}$$

$$\boldsymbol{\Theta}_i = \begin{pmatrix} 4\left(K_{pi} + \rho K_{di}\right) & -\rho\sigma_{\max}\left(\boldsymbol{J}_i^*\right) & 0 \\ -\rho\sigma_{\max}\left(\boldsymbol{J}_i^*\right) & \sigma_{\min}\left(\boldsymbol{J}_i^*\right) & 0 \\ 0 & 0 & \sigma_{\min}\left(\boldsymbol{P}_i\right) \end{pmatrix} \tag{22}$$

According to Lemma 2.1, $\boldsymbol{\Theta}_i$ is positive definite if the following inequality holds:

$$4\left(K_{pi} + \rho K_{di}\right)\sigma_{\min}\left(\boldsymbol{J}_i^*\right) > \rho^2\sigma_{\max}^2\left(\boldsymbol{J}_i^*\right) \tag{23}$$

Therefore, the positive definiteness of $\boldsymbol{\Theta}_i$ is obtained provided that ρ is small enough.

Calculating the derivative of V, we have that

$$\begin{aligned} \dot{V} = \sum_{i=1}^{n} \dot{V}_i = \sum_{i=1}^{n} \Big\{ &\left(K_{pi} + \rho K_{di}\right)\boldsymbol{q}_{ei}^T \boldsymbol{\omega}_i + \boldsymbol{\omega}_i^T \boldsymbol{J}_i^* \dot{\boldsymbol{\omega}}_i + \rho \boldsymbol{q}_{ei}^T \boldsymbol{J}_i^* \dot{\boldsymbol{\omega}}_i \\ &+\rho\left(q_{0ei}\boldsymbol{I} + \boldsymbol{q}_{ei}^{\times}\right)\boldsymbol{J}_i^* \boldsymbol{\omega}_i + \boldsymbol{\zeta}_i^T \boldsymbol{P}_i \dot{\boldsymbol{\zeta}}_i \Big\} \end{aligned} \tag{24}$$

Substituting (11) and (12) into (24) yields

$$\begin{aligned}\dot{V} = \sum_{i=1}^{n} \Big\{ &(K_{pi} + \rho K_{di})\, \boldsymbol{q}_{ei}^T \boldsymbol{\omega}_i + \boldsymbol{\omega}_i^T \left(-\boldsymbol{\omega}_i^{\times} (\boldsymbol{J}_i^* \boldsymbol{\omega}_i + \boldsymbol{H}\boldsymbol{\zeta}) + \boldsymbol{L}\boldsymbol{\zeta} - \boldsymbol{M}\boldsymbol{\omega}_i + \boldsymbol{u}_i\right) \\ &+ \rho \boldsymbol{q}_{ei}^T \left(-\boldsymbol{\omega}_i^{\times} (\boldsymbol{J}_i^* \boldsymbol{\omega}_i + \boldsymbol{H}\boldsymbol{\zeta}) + \boldsymbol{L}\boldsymbol{\zeta} - \boldsymbol{M}\boldsymbol{\omega}_i + \boldsymbol{u}_i\right) \\ &+\rho \left(q_{0ei}\boldsymbol{I} + \boldsymbol{q}_{ei}^{\times}\right) \boldsymbol{\omega}_i^T \boldsymbol{J}_i^* \boldsymbol{\omega}_i + \boldsymbol{\zeta}_i^T \boldsymbol{P}_i \left(\boldsymbol{A}_i \boldsymbol{\zeta}_i + \boldsymbol{B}_i \boldsymbol{\omega}_i\right) \Big\}\end{aligned} \tag{25}$$

Substituting the control law (14) into (25), it is derived that

$$\begin{aligned}\dot{V} = &\sum_{i=1}^{n} \Bigg\{ \Bigg(K_{pi} + \rho K_{di}\Bigg) \boldsymbol{q}_{ei}^T \boldsymbol{\omega}_i + \boldsymbol{\omega}_i^T \Bigg(\boldsymbol{L}_i \boldsymbol{\zeta}_i - \boldsymbol{M}_i \boldsymbol{\omega}_i - K_{pi} \boldsymbol{q}_{ei} - K_{di} \boldsymbol{\omega}_i \\ &- \sum_{j=1}^{n} K_{dij} \left(\boldsymbol{\omega}_{ij} + \rho \boldsymbol{q}_{ij}\right) \Bigg) + \rho \boldsymbol{q}_{ei}^T \Bigg(- \boldsymbol{\omega}_i^{\times} \left(\boldsymbol{J}_i^* \boldsymbol{\omega}_i + \boldsymbol{H}_i \boldsymbol{\zeta}_i\right) + \boldsymbol{L}_i \boldsymbol{\zeta}_i - \boldsymbol{M}_i \boldsymbol{\omega}_i \\ &- K_{pi} \boldsymbol{q}_{ei} - K_{di} \boldsymbol{\omega}_i - \sum_{j=1}^{n} K_{dij} \left(\boldsymbol{\omega}_{ij} + \rho \boldsymbol{q}_{ij}\right) \Bigg) \\ &+ \frac{1}{2} \rho \left(q_{0ei} \boldsymbol{I} + \boldsymbol{q}_{ei}^{\times}\right) \boldsymbol{\omega}_i^T \boldsymbol{J}_i^* \boldsymbol{\omega}_i - \frac{1}{2} \boldsymbol{\zeta}_i^T \boldsymbol{Q}_i \boldsymbol{\zeta}_i + \boldsymbol{\zeta}_i^T \boldsymbol{P}_i \boldsymbol{B}_i \boldsymbol{\omega}_i \Bigg\} \\ = &\sum_{i=1}^{n} \Bigg\{ - \rho K_{pi} \boldsymbol{q}_{ei}^T \boldsymbol{q}_{ei} - \boldsymbol{\omega}_i^T \boldsymbol{M}_i \boldsymbol{\omega}_i - K_{di} \boldsymbol{\omega}_i^T \boldsymbol{\omega}_i + \frac{1}{2} \rho \left(q_{0ei} \boldsymbol{I} + \boldsymbol{q}_{ei}^{\times}\right) \boldsymbol{\omega}_i^T \boldsymbol{J}_i^* \boldsymbol{\omega}_i \\ &- \frac{1}{2} \boldsymbol{\zeta}_i^T \boldsymbol{Q}_i \boldsymbol{\zeta}_i - \rho \boldsymbol{q}_{ei}^T \boldsymbol{\omega}_i^{\times} \boldsymbol{J}_i^* \boldsymbol{\omega}_i - \rho \boldsymbol{q}_{ei}^T \boldsymbol{M}_i \boldsymbol{\omega}_i + \rho \boldsymbol{q}_{ei}^T \boldsymbol{L}_i \boldsymbol{\zeta}_i - \rho \boldsymbol{q}_{ei}^T \boldsymbol{\omega}_i^{\times} \boldsymbol{H}_i \boldsymbol{\zeta}_i \\ &+ \boldsymbol{\omega}_i^T \boldsymbol{L}_i \boldsymbol{\zeta}_i + \boldsymbol{\zeta}_i^T \boldsymbol{P}_i \boldsymbol{B}_i \boldsymbol{\omega}_i - \left(\boldsymbol{\omega}_i + \rho \boldsymbol{q}_{ei}\right) \sum_{j=1}^{n} K_{dij} \left(\boldsymbol{\omega}_{ij} + \rho \boldsymbol{q}_{ij}\right) \Bigg\}\end{aligned} \tag{26}$$

In light of (26), it can be derived that

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{n} \left\{ -\boldsymbol{\xi}_i^T \Xi_i \boldsymbol{\xi}_i - \left(\boldsymbol{\omega}_i + \rho \boldsymbol{q}_{ei}\right) \sum_{j=1}^{n} K_{dij} \left(\boldsymbol{\omega}_{ij} + \rho \boldsymbol{q}_{ij}\right) \right\} \\ &= - \sum_{i=1}^{n} \boldsymbol{\xi}_i^T \Xi_i \boldsymbol{\xi}_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} K_{dij} \left(\boldsymbol{\omega}_{ij} + \rho \boldsymbol{q}_{ij}\right)^T \left(\boldsymbol{\omega}_{ij} + \rho \boldsymbol{q}_{ij}\right) \\ &\leq - \sum_{i=1}^{n} \boldsymbol{\xi}_i^T \Xi_i \boldsymbol{\xi}_i\end{aligned} \tag{27}$$

with

$$\Xi_i = \begin{pmatrix} \rho K_{pi} \boldsymbol{I} & \frac{1}{2} \rho \boldsymbol{M}_i & -\frac{1}{2} \rho \boldsymbol{L}_i \\ \frac{1}{2} \rho \boldsymbol{M}_i & K_{di} \boldsymbol{I} + \boldsymbol{M}_i - \frac{1}{2} \rho \boldsymbol{J}_i^* & -\frac{1}{2} \left(\boldsymbol{L}_i + \boldsymbol{B}_i^T \boldsymbol{P}_i\right) - \frac{1}{2} \rho \boldsymbol{H}_i \\ -\frac{1}{2} \rho \boldsymbol{L}_i & -\frac{1}{2} \left(\boldsymbol{L}_i + \boldsymbol{B}_i^T \boldsymbol{P}_i\right) - \frac{1}{2} \rho \boldsymbol{H}_i & \frac{1}{2} \boldsymbol{Q}_i \end{pmatrix} = \begin{pmatrix} \boldsymbol{D}_i & \boldsymbol{E}_i \\ \boldsymbol{E}_i^T & \boldsymbol{F}_i \end{pmatrix} \tag{28}$$

where $\boldsymbol{D}_i = \begin{pmatrix} \rho K_{pi} \boldsymbol{I} & \frac{1}{2} \rho \boldsymbol{M}_i \\ \frac{1}{2} \rho \boldsymbol{M}_i & K_{di} \boldsymbol{I} + \boldsymbol{M}_i - \frac{1}{2} \rho \boldsymbol{J}_i^* \end{pmatrix}$, $\boldsymbol{E}_i = \begin{pmatrix} -\frac{1}{2} \rho \boldsymbol{L}_i \\ -\frac{1}{2} \left(\boldsymbol{L}_i + \boldsymbol{B}_i^T \boldsymbol{P}_i\right) - \frac{1}{2} \rho \boldsymbol{H}_i \end{pmatrix}$, and $\boldsymbol{F}_i = \frac{1}{2} \boldsymbol{Q}_i$.

According to Lemma 2.1, if the following inequality holds,

$$\rho K_{pi} \boldsymbol{I} \left(K_{di} \boldsymbol{I} + \boldsymbol{M}_i - \frac{1}{2} \rho \boldsymbol{J}_i^*\right) - \frac{1}{4} \rho^2 \boldsymbol{M}_i^T \boldsymbol{M}_i > 0 \tag{29}$$

and $\boldsymbol{D}_i - \boldsymbol{E}_i\boldsymbol{F}_i^{-1}\boldsymbol{E}_i^T$ is positive definite, then Ξ_i is positive definite. As for $\boldsymbol{D}_i - \boldsymbol{E}_i\boldsymbol{F}_i^{-1}\boldsymbol{E}_i^T$, we have the following inequality

$$\boldsymbol{\delta}^T\left(\boldsymbol{D}_i - \boldsymbol{E}_i\boldsymbol{F}_i^{-1}\boldsymbol{E}_i^T\right)\boldsymbol{\delta} \geq \|\boldsymbol{\delta}\|^2\left(\sigma_{\min}\left(\boldsymbol{D}_i\right) - \frac{\sigma_{\max}^2\left(\boldsymbol{E}_i\right)}{\sigma_{\min}\left(\boldsymbol{F}_i\right)}\right), \quad \forall\boldsymbol{\delta} \in R^n \tag{30}$$

Therefore, if

$$\sigma_{\min}\left(\boldsymbol{D}_i\right)\sigma_{\min}\left(\boldsymbol{F}_i\right) > \sigma_{\max}^2\left(\boldsymbol{E}_i\right) \tag{31}$$

then, $\boldsymbol{D}_i - \boldsymbol{E}_i\boldsymbol{F}_i^{-1}\boldsymbol{E}_i^T$ is positive definite. From (29) and the expressions of $\boldsymbol{D}_i$, $\boldsymbol{E}_i$ and $\boldsymbol{F}_i$, it can be seen that there always exist appropriate parameters ρ, K_{pi} and K_{di} such that (29) and (31) hold. Then, (27) can be restated as

$$\dot{V} \leq -\sum_{i=1}^{n}\sigma_{\min}\left(\Xi_i\right)\boldsymbol{\xi}_i^T\boldsymbol{\xi}_i \tag{32}$$

According to Lemma 2.2, $\{\boldsymbol{\omega}_i \to 0, \boldsymbol{q}_{ei} \to 0, \boldsymbol{\zeta}_i \to 0\}$ as $t \to \infty$ can be achieved. Hence, the proof of Theorem 3.1 is completed.

Remark 3.2. *It should be noted that in the previous analysis we have used*

$$\sum_{i=1}^{n}\sum_{j=1}^{n}K_{dij}\left(\boldsymbol{\omega}_i + \rho\boldsymbol{q}_{ei}\right)\left(\boldsymbol{\omega}_{ij} + \rho\boldsymbol{q}_{ij}\right) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}K_{dij}\left(\boldsymbol{\omega}_{ij} + \rho\boldsymbol{q}_{ij}\right)^T\left(\boldsymbol{\omega}_{ij} + \rho\boldsymbol{q}_{ij}\right) \tag{33}$$

for $K_{dij} = K_{dji}$, which is a similar analysis with that in [10].

Remark 3.3. *The communication topology of the formation is an undirected graph, because we assume that the formation-keeping weight parameters satisfy $K_{pij} = K_{pji} > 0$, $K_{dij} = K_{dji} > 0$ in the controller design. That means if the ith spacecraft within the formation is able to get the knowledge of the jth spacecraft, then the converse is also true.*

4. Main Result.

In SFF, the information is transmitted over a network, so the communication delays are ineluctable and the communication topologies are not fixed as undirected graphs. In addition, there are always unexpected disturbances in practice, which should be considered in the control laws design. In this section, we propose a class of improved control schemes by taking into account external disturbances and communication delays simultaneously. The improved control strategies can overcome the effects of both disturbances and communication delays subject to arbitrary communication topologies.

4.1. Problem statement.

We assume that the external disturbances are bounded as $|(\boldsymbol{d})_k| \leq \upsilon$, $k = 1, 2, 3$, where $\upsilon > 0$; the communication delay τ_{ij} from jth spacecraft to ith spacecraft is slowly time-varying and satisfies $\dot{\tau}_{ij} < 1$. The attitude maneuver problem in this section is to design the control torque $\boldsymbol{u}_i$ for each spacecraft within the formation, so that the attitude states of each spacecraft can converge to the desired state in the presence of external disturbances and communication delays.

4.2. Improved control laws.

We proposed the improved behavior-based attitude control laws as follows:

$$\boldsymbol{u}_i = \boldsymbol{u}_i^{sk} + \boldsymbol{u}_i^{fk} = -K_{pi}\boldsymbol{q}_{ei} - K_{di}\boldsymbol{\omega}_i - K_{si}\text{sgn}\left(\boldsymbol{s}_i\right) - \sum_{j=1}^{n}\left(K_{ij}^i\boldsymbol{s}_i - K_{ij}^j\boldsymbol{s}_j\left(t - \tau_{ij}\right)\right) \tag{34}$$

where $\boldsymbol{s}_i = \boldsymbol{\omega}_i + \rho\boldsymbol{q}_{ei}$; $K_{si} > 0$, $K_{ij}^i > 0$ and $K_{ij}^j > 0$ are weight parameters.

Theorem 4.1. *The controller (34) can solve the attitude maneuver problem of a flexible spacecraft formation in the presence of external disturbances and communication delays with arbitrary communication topologies, if 1) the parameters K_{pi}, K_{di}, ρ are chosen properly; 2) $K_{si} > \upsilon$; 3) $K_{ij}^i > \beta_{ij}$; 4) $4\beta_{ij}\left(K_{ij}^i - \beta_{ij}\right)(1-\dot{\tau}_{ij}) > \left(K_{ij}^j\right)^2$, where β_{ij} is a positive constant that will be involved in the following stability analysis.*

Proof: Consider the following Lyapunov function:

$$V = V_1 + V_2 \tag{35}$$

where

$$V_1 = \sum_{i=1}^{n} V_i \tag{36}$$

$$V_2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\tau_{ij}}^{t} \beta_{ij}\boldsymbol{s}_i^T\boldsymbol{s}_i dx \tag{37}$$

In (36),

$$V_i = 2\left(K_{pi} + \rho_i K_{di}\right)(1 - q_{0ei}) + \frac{1}{2}\boldsymbol{\omega}_i^T\boldsymbol{J}_i^*\boldsymbol{\omega}_i + \rho_i\boldsymbol{q}_{ei}^T\boldsymbol{J}_i^*\boldsymbol{\omega}_{ei} + \frac{1}{2}\boldsymbol{\zeta}_i^T\boldsymbol{P}_i\boldsymbol{\zeta}_i \tag{38}$$

Similar with the proof of Theorem 3.1, we have

$$\dot{V}_i \leq -\sum_{i=1}^{n}\boldsymbol{\xi}_i^T\Xi_i\boldsymbol{\xi}_i - \sum_{i=1}^{n}\boldsymbol{s}_i^T\left(K_{si}\mathrm{sgn}\left(\boldsymbol{s}_i\right) + \sum_{j=1}^{n}\left(K_{ij}^i\boldsymbol{s}_i - K_{ij}^j\boldsymbol{s}_j\left(t-\tau_{ij}\right)\right) - \boldsymbol{d}_i\right) \tag{39}$$

Because $K_{si} > \upsilon$, (39) can be rewritten as

$$\dot{V}_i \leq -\sum_{i=1}^{n}\boldsymbol{\xi}_i^T\Xi_i\boldsymbol{\xi}_i - \sum_{i=1}^{n}\left(K_{si}-\upsilon\right)\|\boldsymbol{s}_i\| - \sum_{i=1}^{n}\boldsymbol{s}_i^T\sum_{j=1}^{n}\left(K_{ij}^i\boldsymbol{s}_i - K_{ij}^j\boldsymbol{s}_j\left(t-\tau_{ij}\right)\right) \tag{40}$$

In light of (40), the first-order derivative of (35) can be calculated as

$$\begin{aligned}
\dot{V} &= \dot{V}_1 + \dot{V}_2 \\
&\leq -\sum_{i=1}^{n}\left(\boldsymbol{\xi}_i^T\Xi_i\boldsymbol{\xi}_i + \left(K_{si}-\upsilon\right)\|\boldsymbol{s}_i\|\right) - \sum_{i=1}^{n}\sum_{j=1}^{n}\boldsymbol{s}_i^T\left(K_{ij}^i\boldsymbol{s}_i - K_{ij}^j\boldsymbol{s}_j\left(t-\tau_{ij}\right)\right) \\
&\quad + \sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\left(\boldsymbol{s}_i^T\boldsymbol{s}_i - \left(1-\dot{\tau}_{ij}\right)\boldsymbol{s}_j^T\left(t-\tau_{ij}\right)\boldsymbol{s}_j\left(t-\tau_{ij}\right)\right) \\
&= -\sum_{i=1}^{n}\left(\boldsymbol{\xi}_i^T\Xi_i\boldsymbol{\xi}_i + \left(K_{si}-\upsilon\right)\|\boldsymbol{s}_i\|\right) \\
&\quad - \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\boldsymbol{\varsigma}^T\boldsymbol{\varsigma} + \left(\beta_{ij}\left(1-\dot{\tau}_{ij}\right) - \frac{\left(k_j^i\right)^2}{4\left(k_i^i - \beta_{ij}\right)}\right)\boldsymbol{s}_j^T\left(t-\tau_{ij}\right)\boldsymbol{s}_j\left(t-\tau_{ij}\right)\right) \\
&\leq -\sum_{i=1}^{n}\left(\boldsymbol{\xi}_i^T\Xi_i\boldsymbol{\xi}_i + \left(K_{si}-\upsilon\right)\|\boldsymbol{s}_i\|\right) - \sum_{i=1}^{n}\sum_{j=1}^{n}\boldsymbol{\varsigma}^T\boldsymbol{\varsigma}
\end{aligned} \tag{41}$$

where $\boldsymbol{\varsigma} = \sqrt{K_{ij}^i - \beta_{ij}}\boldsymbol{s}_i - \frac{K_{ij}^j}{2\sqrt{K_{ij}^i - \beta_{ij}}}\boldsymbol{s}_j\left(t-\tau_{ij}\right)$.

If ρ, K_{pi} and K_{di} are chosen properly such that the matrices Ξ_i for $i = 1, \ldots n$ are positive definite, then Theorem 4.1 holds obviously. If the parameters are chosen such that Ξ_i are positive semi-definite, then $\dot{V}$ is negative semi-definite and V is bounded which means $(\boldsymbol{q}_{ei}, \boldsymbol{\omega}_i) \in L_\infty^6$ and $\boldsymbol{s}_i \in L_\infty^3$. According to (2), (3), (11) and (12), $(\dot{\boldsymbol{q}}_{ei}, \dot{\boldsymbol{\omega}}_i) \in L_\infty^6$ and $\dot{\boldsymbol{s}}_i \in L_\infty^3$ can be obtained. Integrating both sides of (41) from 0 to ∞ yields $\boldsymbol{s}_i \in L_1^3$. Then according to Lemma 2.3, $\lim\limits_{t\to\infty} \boldsymbol{s}_i \to 0$ and it follows that $\lim\limits_{t\to\infty} (\boldsymbol{q}_{ei}, \boldsymbol{\omega}_i) \to 0$ [20]. From (12), $\lim\limits_{t\to\infty} \boldsymbol{\zeta}_i \to 0$ with $\lim\limits_{t\to\infty} \boldsymbol{\omega}_i \to 0$ can be obtained. This completes the proof.

Remark 4.1. *Note that the control schemes presented in [1-12] are not effective when flexible spacecraft are taken into account. In this study, attitude coordination problem of a flexible spacecraft formation with disturbances and communication delays are solved. Furthermore, both the basic controllers proposed in Section 3 and the improved controllers developed in Section 4 have simple and intuitive structures. The basic control laws are in PD-control form, and the improved control laws are designed by adding a variable structure term to the basic ones. In this setting, the controllers are simple to be implemented to pursue a low cost in practice. In addition, although the control problem of a single flexible spacecraft is solved in [13-16], the system investigated herein is a formation composed of multiple flexible spacecraft in the presence of external disturbances, communication delays and time-varying topologies, which has not been addressed before.*

Remark 4.2. *The control objective $\lim\limits_{t\to\infty} (\boldsymbol{q}_{ei}, \boldsymbol{\omega}_i, \boldsymbol{\zeta}_i) \to 0$ is asymptotically stable provided that the parameters K_{pi}, K_{di}, ρ are chosen properly, $K_{si} > \upsilon$, $K_{ij}^i > \beta_{ij}$, and $4\beta_{ij}\left(K_{ij}^i - \beta_{ij}\right)(1 - \dot{\tau}_{ij}) > \left(K_{ij}^j\right)^2$. In the stability analysis, the positive definiteness of the matrix Ξ_i can be guaranteed by choosing the parameters K_{pi}, K_{di}, ρ properly according to Lemma 2.1 and the expression of the matrix Ξ_i. Furthermore, by virtue of the parameter ρ, one can not only select relatively large K_{pi} and K_{di} to guarantee the positive definiteness of this matrix, but also choose small enough ρ with moderate K_{pi} and K_{di}. This is easier for practical implementation. $K_{si} > \upsilon$ where υ is the upper bound of external disturbances can guarantee the robustness of the control schemes. In this setting, the attitude of the formation can be stabilized and synchronized in the presence of bounded disturbance torques. The effectiveness of the proposed control laws with non-uniform communication delays is guaranteed by the conditions $K_{ij}^i > \beta_{ij}$ and $4\beta_{ij}\left(K_{ij}^i - \beta_{ij}\right)(1 - \dot{\tau}_{ij}) > \left(K_{ij}^j\right)^2$. It should be noted that there always exist parameters K_{ij}^i and K_{ij}^j such that the inequalities hold because $(1 - \dot{\tau}_{ij})$ is positive. In addition, the attitude states can always converge to the desired attitude if the time interval of non-positive $(1 - \dot{\tau}_{ij})$ is finite. Because these inequalities will hold after a finite time interval, $\dot{V}$ will be negative semi-definite again, and then the control objective $\lim\limits_{t\to\infty} (\boldsymbol{q}_{ei}, \boldsymbol{\omega}_i, \boldsymbol{\zeta}_i) \to 0$ can be achieved.*

Remark 4.3. *The proposed behavior-based control schemes are effective subject to arbitrary communication topologies. When $K_{ij}^j = 0$, it means no information transmission from the jth spacecraft to the ith spacecraft. In this sense, the weight parameter K_{ij}^j is able to describe the communication topology. Furthermore, if the parameter K_{ij}^j switches between zero and a positive constant during the maneuver process, then the communication topology will be a time-varying type. Hence, the weight parameter K_{ij}^j can achieve the dual objectives of representing the weight of the formation-keeping control action and describing the communication topology simultaneously.*

Remark 4.4. *Note that the control schemes (34) are discontinuous due to the sign functions. We propose a class of modified ones as follows*

$$\boldsymbol{u}_i = -K_{pi}\boldsymbol{q}_{ei} - K_{di}\boldsymbol{\omega}_i - K_{si}\mathrm{cont}\left(\boldsymbol{s}_i\right) - \sum_{j=1}^{n}\left(K_{ij}^i\boldsymbol{s}_i - K_{ij}^j\boldsymbol{s}_j\left(t - \tau_{ij}\right)\right) \tag{42}$$

with $\operatorname{cont}(\boldsymbol{s}_i) = \frac{\boldsymbol{s}_i}{|\boldsymbol{s}_i| + \boldsymbol{\psi}_i}$ *where* $(\boldsymbol{\psi}_i)_k > 0$, $k = 1, 2, 3$ *satisfies* $\int_0^\infty \left(\sum_{k=1}^{3} K_{si} (\boldsymbol{\psi}_i)_k \right) dt = \mathrm{M} < +\infty$ *where* M *is a constant. Using the same Lyapunov function (35), it can be proven that*

$$\begin{aligned} \dot{V} &\le -\sum_{i=1}^{n} \left(\sigma_{\min}(\Xi_i) \boldsymbol{\xi}_i^T \boldsymbol{\xi}_i + \boldsymbol{s}_i^T K_{si} \frac{\boldsymbol{s}_i}{|\boldsymbol{s}_i| + \boldsymbol{\psi}_i} - \boldsymbol{s}_i^T \boldsymbol{d}_i \right) \\ &\le -\sum_{i=1}^{n} \left(\sigma_{\min}(\Xi_i) \boldsymbol{\xi}_i^T \boldsymbol{\xi}_i + (K_{si} - \upsilon) |\boldsymbol{s}_i| - K_{si} \boldsymbol{\psi}_i \right) \end{aligned} \tag{43}$$

Integrating both sides of (43), we can obtain that V *is bounded, which leads to* $(\boldsymbol{q}_{ei}, \boldsymbol{\omega}_i) \in L_\infty^6$ *and* $\boldsymbol{s}_i \in L_\infty^3$. *With the similar analysis of Theorem 4.1, the control objective* $\lim_{t\to\infty} (\boldsymbol{q}_{ei}, \boldsymbol{\omega}_i, \boldsymbol{\zeta}_i) \to 0$ *can be achieved asymptotically.*

Remark 4.5. *The sign function in control laws (31) will cause chattering in control signals. Chattering is a harmful phenomenon for spacecraft formations, which should be eliminated for the controllers to perform properly. To alleviate the chattering, we can replace the sign function by the saturation function or the hyperbolic tangent function below.*

$$\operatorname{sgn}(x) \to \operatorname{sat}(x) = \begin{cases} 1 & x > \mu \\ x/\mu & |x| \le \mu \\ -1 & x < \mu \end{cases} \quad \text{or } \operatorname{sgn}(x) \to \tanh\left(\frac{x}{\mu}\right) \tag{44}$$

where $\operatorname{sat}(*)$ *denotes the saturation function and* $\tanh(*)$ *denotes the hyperbolic tangent function;* μ *is a small positive constant.*

5. **Numerical Simulations.** In this section, numerical simulations of a five-spacecraft formation are provided to investigate the effectiveness of the proposed control schemes (34). The simulations will test the validity of the controller with large disturbances, time-varying communication delays, time-varying communication topologies and control input saturation.

The model parameters of each flexible spacecraft within the formation are chosen as [14]

$$\boldsymbol{J}_i^* = \begin{pmatrix} 350 & 3 & 4 \\ 23 & 280 & 10 \\ 4 & 10 & 190 \end{pmatrix} kg \cdot m^2, \text{ for } i = 1, 2, 3, 4, 5$$

with the coupling matrices, the natural frequencies in rad/s and the dampings associated to the first four natural modes:

$$\begin{aligned} \boldsymbol{\delta}_i &= \begin{pmatrix} 6.45637 & 1.27814 & 2.15629 \\ -1.25619 & 0.91756 & -1.67264 \\ 1.11687 & 2.48901 & -0.83674 \\ 1.23637 & -2.6581 & -1.12503 \end{pmatrix} kg^{\frac{1}{2}} m, \\ &\omega_{1i} = 0.7681,\ \omega_{2i} = 1.1038,\ \omega_{3i} = 1.8733,\ \omega_{4i} = 2.5496, \\ &\vartheta_{1i} = 0.005607,\ \vartheta_{2i} = 0.00862,\ \vartheta_{3i} = 0.01283,\ \vartheta_{4i} = 0.02516. \end{aligned}$$

The desired attitude states are given by $\bar{q}_d = (0\ 0\ 0\ 1)^T$, $\boldsymbol{\omega}_d = 0$, $\boldsymbol{\zeta}_d = 0$.

In order to highlight the performance of the control laws, the initial attitude states of the spacecraft are chosen to be the following large values,

$$\begin{array}{ll}
\boldsymbol{\omega}_1(0) = (0.02 \quad 0.04 \quad -0.03)^T \, rad/s, & \bar{q}_1(0) = (0.7 \quad 0.5 \quad 0.5 \quad 0.1)^T \\
\boldsymbol{\omega}_2(0) = (0.01 \quad -0.03 \quad 0.02)^T \, rad/s, & \bar{q}_2(0) = (0.5 \quad -0.7 \quad 0.1 \quad 0.5)^T \\
\boldsymbol{\omega}_3(0) = (-0.01 \quad 0.02 \quad -0.01)^T \, rad/s, & \bar{q}_3(0) = (-0.5 \quad 0.7 \quad -0.1 \quad -0.5)^T \\
\boldsymbol{\omega}_4(0) = (0.04 \quad -0.01 \quad -0.03)^T \, rad/s, & \bar{q}_4(0) = (-0.7 \quad 0.5 \quad -0.5 \quad -0.1)^T \\
\boldsymbol{\omega}_5(0) = (-0.02 \quad 0.03 \quad 0.01)^T \, rad/s, & \bar{q}_5(0) = (-0.1 \quad -0.5 \quad -0.5 \quad 0.7)^T
\end{array}$$

and the external disturbances are chosen as $\boldsymbol{d}_i = \begin{pmatrix} 0.2 + 0.08\sin(t/5)\cos(t/i) \\ -0.15 - 0.02\sin(t/5)\cos(t/i) \\ 0.1 + 0.06\sin(t/5)\cos(t/i) \end{pmatrix} N\cdot m$ which is larger in magnitude than that in the practical situations. We also bound the magnitude of the control torque as $|(\boldsymbol{u}_i)_k| \le 2N\cdot m$ with the consideration that the control torque provided by the actuator is limited in practice. The actuators can be implemented by thrusters.

We suppose the delays in communication links take the time-varying form that

$$\begin{array}{ll}
T_{12} + 0.2 = T_{21} = 0.9 + 0.3\sin(t/10), & T_{13} - 0.1 = T_{31} = 1.5 + 0.7\sin(t/14) \\
T_{14} + 0.2 = T_{41} = 1.2 + 0.4\sin(t/14), & T_{15} - 0.1 = T_{51} = 1.1 + 0.5\sin(t/20) \\
T_{23} + 0.2 = T_{32} = 0.6 + 0.6\sin(t/8), & T_{24} - 0.1 = T_{42} = 0.8 + 0.3\sin(t/12) \\
T_{25} + 0.2 = T_{52} = 0.3 + 0.5\sin(t/16), & T_{34} - 0.1 = T_{43} = 1.2 + 0.4\sin(t/7) \\
T_{35} + 0.2 = T_{53} = 0.4 + 0.7\sin(t/6), & T_{45} - 0.1 = T_{54} = 0.5 + 0.6\sin(t/15)
\end{array}$$

In the simulations, the control parameters are chosen through numerical trial-and-error. $K_{pi} = 100$, $K_{di} = 800$, $K_{si} = 0.5$, $\rho = 0.2$, $K_{ij}^i = 6$ and K_{ij}^j is chosen as Table 1 to describe a time-varying communication topology.

TABLE 1. The parameter K_{ij}^j

	Agent 1	Agent 2	Agent 3	Agent 4	Agent 5
Agent 1		$K_{12}^2(t)$	$K_{12}^2(t+1)$	$K_{12}^2(t+2.8)$	$K_{12}^2(t+3.3)$
Agent 2	$K_{12}^2(t+1.5)$		$K_{12}^2(t+0.5)$	$K_{12}^2(t+2.5)$	$K_{12}^2(t+3.2)$
Agent 3	$K_{12}^2(t+0.9)$	$K_{12}^2(t+4.9)$		$K_{12}^2(t+5.6)$	$K_{12}^2(t+2.1)$
Agent 4	$K_{12}^2(t+3.7)$	$K_{12}^2(t+6)$	$K_{12}^2(t+5)$		$K_{12}^2(t+4.1)$
Agent 5	$K_{12}^2(t+2.4)$	$K_{12}^2(t+1.7)$	$K_{12}^2(t+0.9)$	$K_{12}^2(t+1.3)$	

with $K_{12}^2(t) = \begin{cases} 4 & \text{for } \bmod(t,8) \le 4 \\ 0 & \text{for } \bmod(t,8) > 4 \end{cases}$ where $\bmod(x,y)$ denotes the remainder of dividing x by y.

The performance of the flexible spacecraft formation is measured by station-keeping error metrics and formation-keeping error metrics. The station-keeping error metrics are defined as $\mathrm{SK}_{qe} = \frac{1}{n}\sum_{i=1}^{n}\sqrt{\sum_{k=1}^{3}(\boldsymbol{q}_{ei})_k^2}$, $\mathrm{SK}_{\omega e} = \frac{1}{n}\sum_{i=1}^{n}\sqrt{\sum_{k=1}^{3}(\boldsymbol{\omega}_i)_k^2}$, and $\mathrm{SK}_{\eta e} = \frac{1}{n}\sum_{i=1}^{n}\sqrt{\sum_{k=1}^{3}(\boldsymbol{\eta}_i)_k^2}$ with $n = 5$ for a five-flexible-spacecraft formation. The formation-keeping error metrics are calculated using $\mathrm{FK}_{qe} = \frac{1}{10}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\sqrt{\sum_{k=1}^{3}(\boldsymbol{q}_{ij})_k^2}\right)$ and $\mathrm{FK}_{\omega e} = \frac{1}{10}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\sqrt{\sum_{k=1}^{3}(\boldsymbol{\omega}_{ij})_k^2}\right)$.

Simulations are presented in Figures 1-5. Figures 1-3 show the station-keeping error metrics SK_{qe}, $SK_{\omega e}$, $SK_{\eta e}$ of the spacecraft formation. The formation-keeping error metrics FK_{qe}, $FK_{\omega e}$ of the spacecraft formation are shown in Figures 4 and 5, respectively.

The simulation results validate the stability and convergence analysis. In the presence of external disturbances, communication delays and time-varying communication topologies, Figures 1 and 2 show rapid transient response and high steady accuracy of the station-keeping errors. Fine control performance of the formation-keeping attitude errors and the formation-keeping angular velocity errors are illustrated in Figures 4 and 5. Both the station-keeping errors and the formation-keeping errors fall to the tolerance in approximately 100s. In Figure 3, the response of the modal displacements is presented. It can be seen that the vibration magnitudes of the flexible attachments keep on a low level.

6. **Conclusions.** In this paper, we have solved the attitude maneuver control problem of a flexible spacecraft formation via behavior-based control approach. The presented robust behavior-based control schemes could guarantee asymptotical convergence of the attitude states of the formation in the presence of external disturbances and communication delays. Besides, by virtue of a weight parameter, the communication network could be switched arbitrarily: full-connected or not, directed or undirected, fixed or time-varying. Finally,

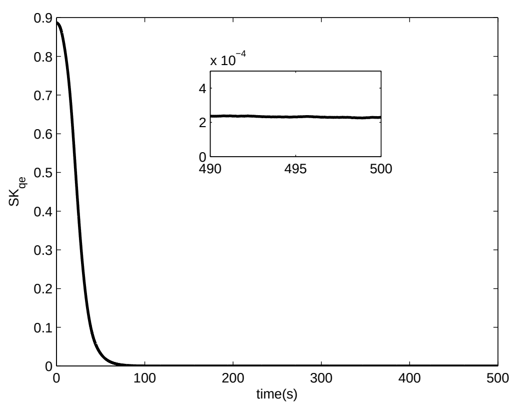

FIGURE 1. The station-keeping attitude error metrics

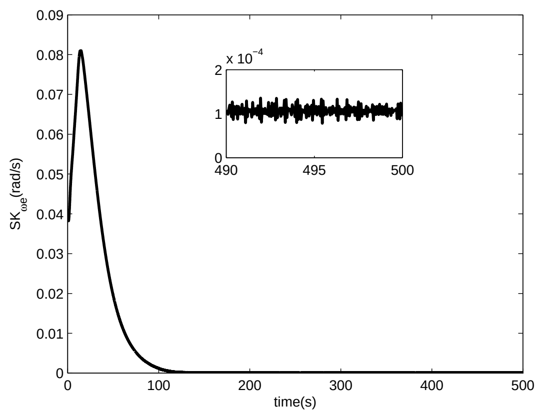

FIGURE 2. Station-keeping angular velocity metrics

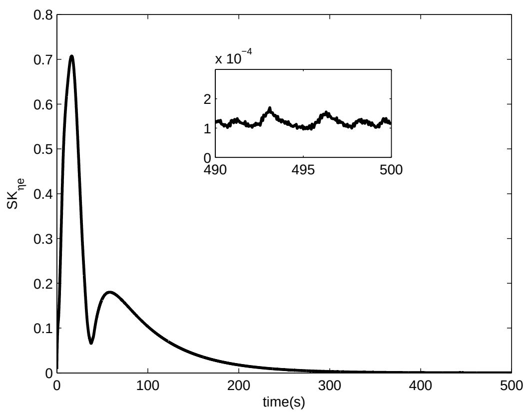

FIGURE 3. Station-keeping modal displacement metrics

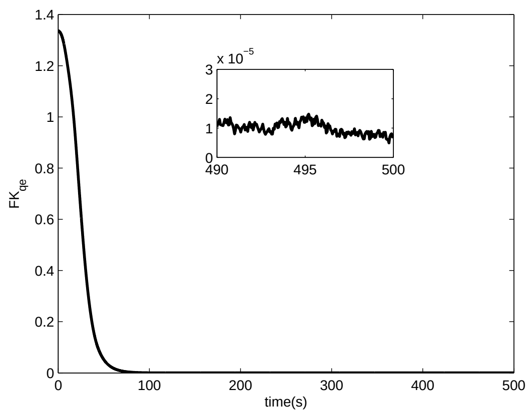

FIGURE 4. Formation-keeping attitude error metrics

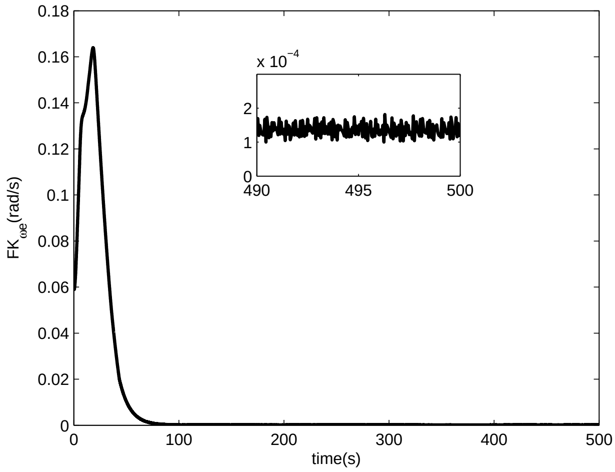

FIGURE 5. Formation-keeping angular velocity error metrics

numerical simulations were performed to support the theoretical analysis. The numerical simulation of a five-spacecraft formation demonstrated ideal control performance and superior robustness, and validated the effectiveness of the proposed control strategies.

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