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RECEDING HORIZON CONTROL FOR CONSTRAINED JUMP BILINEAR SYSTEMS

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Abstract. *In this paper, a receding horizon control strategy for a class of bilinear discrete-time systems with Markovian jumping parameters and constraints is investigated. Specifically, the stochastic jump system under consideration involves control and state multiplicative noise and partly unknown transition probabilities (TPs). The receding horizon formulation adopts an on-line optimization paradigm that utilizes open loop optimized control move plus linear feedback control and is solved as a semi-definite programming (SDP) problem. The mean square stability, control performance and constraint satisfaction properties are guaranteed, where the terminal-weighting matrix is determined off-line and the control move is calculated on-line. A numerical example is given to show the validity of the developed approach.*

Keywords: Markovian jump linear systems, Receding horizon control, Partly unknown transition probabilities, Multiplicative noise, Constraints

1. Introduction.

As a special class of stochastic hybrid systems, Markovian jump linear systems (MJLSs), which include both time-evolving and event-driven mechanisms, have received much attention in recent years. MJLSs work on different operation modes under a switching law governed by a Markov process and they are usually described by normal differential (or difference) equations and a Markov chain. A large variety of real systems, for example, solar thermal central receivers, economic models, manufacturing systems and networked control systems (NCS) with stochastic time delays, etc., can be modeled as MJLSs. Technical and economical reasons motivate the development of MJLSs with an ever-increasing complexity, for example, new stabilization method for MJLSs with time delays [1], state estimation and control via sliding mode approach [2]. As a significant fact, transition probabilities (TPs) of Markov process determine the system evolution to a large degree and many studies on MJLSs are based on the full access of them. Nowadays, a new research tendency is to study MJLSs with partly unknown TPs [3-5]. The underlying systems cover MJLSs with completely known TPs and switched systems under arbitrary switching law as two special cases.

On another research front line, receding horizon control (RHC), which is often known as model predictive control (MPC) [6-11] has become a popular strategy to handle hard/soft constraint and has guaranteed stability, feasibility, and optimality [7]. The RHC has been developed for classical discrete-time MJLSs [12], and it has been extended for MJLSs with polytopic uncertainties in both system parameters and TPs [13]. In this RHC approach, a linear state feedback control is calculated at each sampling time according to different operation modes and the resulting worst-case cost is smaller than guaranteed cost control. The robust one-step RHC [14] for MJLSs has been proposed to further reduce the minimum value of worst-case cost by introducing a one-step receding cost function. A nonlinear control sequence is obtained and a numerical example shows the robust one-step RHC can provide better performance in contrast with the guaranteed cost control and robust RHC. Following the research of robust RHC for MJLSs, an extension work has been developed for the uncertain MJLSs subject to actuator saturations [15]. As the sequel, the development of robust one-step RHC for MJLSs, constrained one-step RHC [16] and multiple-step RHC [17] for MJLSs have also been studied.

However, a literature review reveals that the issue of the RHC for jump bilinear stochastic system (JBSS) has not been fully investigated and remains important and challenging due to the difficulties of utilizing the existing results. Many real systems can be represented by stochastic bilinear model (see [18,19] and references therein) because the noise is dependent on the state and control move, for example, some of the cell biological or chemical reaction, the body temperature and water balance adjustment process, and blood circulation. Much attention in the last decade mainly focuses on the H_∞ control theory framework [20-23]. For a JBSS, every operation mode represents a stochastic dynamic rather than a deterministic one, and when the mode is fixed, the system evolves as a stochastic system due to the multiplicative noises appearing in the state and control input. Generally speaking, a JBSS can be regarded as a result of a series of stochastic systems switching from one to another accompanied with the governing of a Markov chain. Obviously, an MJLS is a special case of a JBSS, and consequently the JBSS with partly unknown TPs covers the MJLS with the same jump character.

In this paper, the motivation for the study of RHC problem of JBSSs with partly unknown TPs is from the robust point of view, because the existing RHC for MJLS cannot deal with multiplicative noise. The underlying systems are more general in comparison with the MJLSs with partly unknown TPs, which can be viewed as a special case of the ones tackled here. An open loop plus closed loop controller structure [11] applies to the constrained JBSS through a receding horizon method and the optimization problem is solved in terms of SDP on-line. Moreover, the relationship between the RHC strategy for the usual JBSS and switched bilinear system under arbitrary switching is revealed by the underlying system. Note that the proposed RHC strategy is quite different from [11]. First, the terminal-weighting matrix is obtained a priori under a cost monotonicity condition, which is presented in linear matrix inequality (LMI) form. Second, the terminal invariant ellipsoid is employed to relax the constraint condition, i.e., at time $k+N$ up to infinite horizon, we do not require constrained LMI condition while solving the controller on-line. Third, a fictitious feedback control law, which is theoretically employed out of the predictive horizon or equivalently, inside invariant ellipsoid, simplifies the feasibility and the mean square stability analysis.

2. **System Descriptions and Problem Formulation.** Consider the discrete-time MJLS

$$\boldsymbol{x}(k+1) = \boldsymbol{A}(r_k)\boldsymbol{x}(k) + \boldsymbol{B}(r_k)\boldsymbol{u}(k) + \sum_{q=1}^{m}\left[\boldsymbol{C}_q(r_k)\boldsymbol{x}(k) + \boldsymbol{D}_q(r_k)\boldsymbol{u}(k)\right] w_q(k), \tag{1}$$

where $\boldsymbol{x}(k) \in \mathrm{R}^{n_x}$ is the state vector, $\boldsymbol{u}(k) \in \mathrm{R}^{n_u}$ is the control input. $w_q(k) \in \mathrm{R}$ (where $q = 1, \ldots, m$) are independent and identically distributed random noises which satisfy

$$\mathrm{E}[w_q(k)] = 0, \quad \mathrm{E}[w_q(k)w_q(k)] = 1, \quad \mathrm{E}[w_q(k)w_p(k)] = 0 \text{ for } q \neq p.$$

$\boldsymbol{A}(r_k)$, $\boldsymbol{B}(r_k)$, $\boldsymbol{C}_q(r_k)$ and $\boldsymbol{D}_q(r_k)$ are matrices with appropriate dimensions which depend on the jump modes r_k. Suppose the initial state and initial mode are $\boldsymbol{x}_0$ and r_0. The mode process $\{r_k : \ k = 0, 1, \ldots\}$ is a discrete time Markov chain that takes values in a finite integer set $\mathbb{S} = \{1, 2, \ldots, s\}$ with the following TPs:

$$\Pr\{r_{k+1} = j \,|r_k = i\} = p_{ij}, \quad \sum_{j=1}^{s} p_{ij} = 1.$$

In addition, the TPs of the jump mode r_k are assumed to be partly unknown and partly accessed. For example, for the system (1) with four operation modes, the one-step TP matrix [3] can be denoted as

$$\boldsymbol{\Pi} = \begin{bmatrix} p_{11} & p_{12} & ? & ? \\ ? & ? & p_{23} & p_{24} \\ p_{31} & ? & ? & ? \\ p_{41} & ? & p_{43} & p_{44} \end{bmatrix}, \tag{2}$$

where "?" represents the unknown elements. For the notation clarity, $\forall i \in \mathbb{S}$, we denote that

$$\mathbb{S} = \mathbb{S}_k^i + \mathbb{S}_{uk}^i, \tag{3}$$

where

$$\mathbb{S}_k^i = \{j : p_{ij} \text{ is known}\}, \quad \mathbb{S}_{uk}^i = \{j : p_{ij} \text{ is unknown}\}. \tag{4}$$

If $\mathbb{S}_k^i \neq \varphi$, it can be further described as

$$\mathbb{S}_k^i = \{k_1^i, \ldots, k_a^i\}, \quad \forall\, 1 \leq a \leq s, \tag{5}$$

where k_a^i represents the jump mode j corresponding to the known element located in the ith row, ath column of matrix $\boldsymbol{\Pi}$. Also, we denote

$$\pi_k^i = \sum_{j \in \mathbb{S}_k^i} p_{ij} \tag{6}$$

throughout this paper. For the mode $r_{k+f} = g \in \mathbb{S}$, matrices $\boldsymbol{A}(r_{k+f})$, $\boldsymbol{B}(r_{k+f})$, $\boldsymbol{C}_q(r_{k+f})$ and $\boldsymbol{D}_q(r_{k+f})$ are noted as $\boldsymbol{A}(g)$, $\boldsymbol{B}(g)$, $\boldsymbol{C}_q(g)$ and $\boldsymbol{D}_q(g)$ for simplicity.

We are interested in regulating system (1) to the origin in the mean square sense when the quadratic and linear constraints [11] should be satisfied. That is

$$\varpi : \quad \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{u}(k) \end{bmatrix}^{\mathrm{T}} \boldsymbol{G}_l \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{u}(k) \end{bmatrix} + \boldsymbol{f}_l^{\mathrm{T}} \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{u}(k) \end{bmatrix} \leq \eta_l, \quad l = 1, \ldots, L. \tag{7}$$

with $\boldsymbol{G}_l = \boldsymbol{G}_l^{\mathrm{T}} \geq 0$. If $\boldsymbol{G}_l$ equals zero, (7) is viewed as a linear constraint. If $\boldsymbol{f}_l^{\mathrm{T}}$ equals zero, (7) degenerates to a quadratic constraint.

Then we seek to minimize the performance objective given by the quadratic cost function

$$J(k) = J_1(k, N-1) + J_2(k, N), \tag{8}$$

where

$$J_1(k, N-1) = \mathrm{E}_k \left\{ \sum_{f=0}^{N-1} \begin{bmatrix} \boldsymbol{x}(k+f\,|k) \\ \boldsymbol{u}(k+f\,|k) \end{bmatrix}^{\mathrm{T}} \boldsymbol{H} \begin{bmatrix} \boldsymbol{x}(k+f\,|k) \\ \boldsymbol{u}(k+f\,|k) \end{bmatrix} \right\}, \tag{9}$$

$$J_2(k, N) = \mathrm{E}_k\{\boldsymbol{x}^{\mathrm{T}}(k+N\,|k)\ \boldsymbol{P}(r_{k+N})\boldsymbol{x}(k+N\,|k)\,\}, \tag{10}$$

where $\mathrm{E}_k\{\cdot\}$ denotes the conditional mathematical expectation $\mathrm{E}\{\cdot\,|x_0\,, r_0\}$, $\boldsymbol{H} = \mathrm{diag}\{\boldsymbol{Q}, \boldsymbol{R}\} \geq 0$.

An open loop optimized control move plus linear feedback control law will be obtained through optimizing a finite horizon quadratic performance index (9) with the terminal cost (10) when constraints (7) are also satisfied. It can be denoted as

$$\begin{aligned} &\text{Op1:} && \min_{\boldsymbol{u}(k+f|k)} J(k) \\ &\text{subject to} && \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{u}(k) \end{bmatrix}^{\mathrm{T}} \boldsymbol{G}_l \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{u}(k) \end{bmatrix} + \boldsymbol{f}_l^{\mathrm{T}} \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{u}(k) \end{bmatrix} \leq \eta_l, \quad l = 1, \ldots, L. \end{aligned}$$

We try to design controller

$$\boldsymbol{u}(k+f\,|k) = \boldsymbol{u}_{op}(k+f\,|k) + \boldsymbol{F}(r_{k+f})\left[\boldsymbol{x}(k+f\,|k) - \bar{\boldsymbol{x}}(k+f\,|k)\right], \tag{11}$$

$$\boldsymbol{u}(k+N\,|k) = \boldsymbol{F}(r_{k+N})\bar{\boldsymbol{x}}(k+N\,|k), \tag{12}$$

to minimize performance index $J(k)$, where, $\bar{\boldsymbol{x}}(k+f\,|k) = \mathrm{E}_k\{\boldsymbol{x}(k+f\,|k)\}$, $(f = 0, 1, \ldots, N-1)$. The controller $\boldsymbol{u}_{op}(k+f\,|k)$ is the open loop optimized variable and $\boldsymbol{F}(r_{k+f})$ is the linear feedback gain matrix. $\boldsymbol{u}(k+N\,|k)$ is a fictitious feedback control employed to guarantee the mean square stability of the JBSS (1) over an infinite horizon [8].

To obtain the control law, which guarantees the stability of JBSS (1), we also define the mean square stability, which is selected as the stability concept in this paper.

Definition 2.1. [14]. *The JBSS (1) is mean square stable, if for any initial state $\boldsymbol{x}_0$ and initial mode r_0,*

$$E_k\{\boldsymbol{x}^T(k)\boldsymbol{x}(k)\} \to 0 \text{ as } k \to \infty. \tag{13}$$

3. **Optimization Problem.** In this section, we present a theorem to solve the constrained on-line optimization Op1, which is constructed by a linear optimized objective subject to a series of sufficient LMI conditions.

Theorem 3.1. *Optimization problem Op1 can be transferred into the following SDP problem:*

$$\min_{\substack{u_{op}(0), \Sigma(g,f), \Sigma(g,N), \Sigma(r_k,1), \Sigma(f), \gamma, \\ \Omega(f), \Omega_1(0), \Omega_2(0), \Omega_3(0), \Omega(N), \bar{\boldsymbol{x}}(N)}} \left\{ \sum_{f=0}^{N-1} tr[\boldsymbol{H}\Omega(f)] \right\} + tr[\boldsymbol{P}(i)\Omega(N)] \tag{14}$$

subject to

$$\begin{bmatrix} \begin{bmatrix} \boldsymbol{I} & 0 \end{bmatrix} \Sigma(g, f+1) & \boldsymbol{U}_1(g,f) & \boldsymbol{U}_2(g,f) & \boldsymbol{U}_3(g,f) \\ * & \begin{bmatrix} \boldsymbol{I} & 0 \end{bmatrix} \Sigma(g,f) & 0 & 0 \\ * & * & \boldsymbol{Z} & 0 \\ * & * & * & \boldsymbol{I} \end{bmatrix} \geq 0,\ f = 1, 2, \ldots, N-1, \tag{15}$$

$$\begin{bmatrix} \begin{bmatrix} \boldsymbol{I} & 0 \end{bmatrix} \Sigma(r_k, 1) & \boldsymbol{U}_3(r_k, 0) \\ * & \boldsymbol{I} \end{bmatrix} \geq 0, \tag{16}$$

$$\begin{bmatrix} \Omega(f) & \Sigma(g,f) & \Sigma(f) \\ * & \begin{bmatrix} \boldsymbol{I} & 0 \end{bmatrix} \Sigma(g,f) & 0 \\ * & * & \boldsymbol{I} \end{bmatrix} \geq 0, \quad f = 1, 2, \ldots, N-1, \tag{17}$$

$$\begin{bmatrix} \Omega_1(0) & \Omega_2(0) & \bar{\boldsymbol{x}}(0) \\ * & \Omega_3(0) & \boldsymbol{u}_{op}(0) \\ * & * & \boldsymbol{I} \end{bmatrix} \geq 0, \tag{18}$$

$$\begin{bmatrix} \Omega(N) - \begin{bmatrix} \boldsymbol{I} & 0 \end{bmatrix} \sum(g,N) & \bar{\boldsymbol{x}}(N) \\ * & \boldsymbol{I} \end{bmatrix} \geq 0, \tag{19}$$

$$\begin{cases} \gamma + \frac{1}{2}(\boldsymbol{f}_l^{\mathrm{T}}\Sigma(f) + \Sigma^{\mathrm{T}}(f)\boldsymbol{f}_l) \leq \eta_l \\ \boldsymbol{G}_l\Omega(f) + \Omega(f)\boldsymbol{G}_l^{\mathrm{T}} \leq \frac{2\gamma}{m+n}\boldsymbol{I} \end{cases}, \quad f = 0, 1, \ldots, N-1, \tag{20}$$

for $i = 1, 2, \ldots, s$, $g = 1, 2, \ldots, s$, $l = 1, 2, \ldots, L$, *where*

$$\boldsymbol{U}_1(g,f) = \bar{\boldsymbol{A}}(g)\Sigma(g,f),\ \boldsymbol{U}_2(g,f) = \begin{bmatrix} \phi_1 & \cdots & \phi_m \end{bmatrix},\ \boldsymbol{U}_3(g,f) = \begin{bmatrix} \vartheta_1 & \cdots & \vartheta_m \end{bmatrix},$$
$$\bar{\boldsymbol{A}}(g) = \begin{bmatrix} \boldsymbol{A}(g) & \boldsymbol{B}(g) \end{bmatrix}, \quad \phi_1 = \bar{\boldsymbol{C}}_1(g)\Sigma(g,f), \quad \phi_m = \bar{\boldsymbol{C}}_m(g)\Sigma(g,f),$$
$$\bar{\boldsymbol{C}}_1(g) = \begin{bmatrix} \boldsymbol{C}_1(g) & \boldsymbol{D}_1(g) \end{bmatrix}, \quad \bar{\boldsymbol{C}}_m(g) = \begin{bmatrix} \boldsymbol{C}_m(g) & \boldsymbol{D}_m(g) \end{bmatrix},$$
$$\vartheta_1 = \bar{\boldsymbol{C}}_1(g)\Sigma(f), \quad \vartheta_m = \bar{\boldsymbol{C}}_m(g)\Sigma(f), \quad \boldsymbol{U}_3(r_k,0) = \begin{bmatrix} \vartheta_{01} & \cdots & \vartheta_{0m} \end{bmatrix},$$
$$\vartheta_{01} = \boldsymbol{C}_1(r_k)\boldsymbol{x}(0) + \boldsymbol{D}_1(r_k)\boldsymbol{u}_{op}(0), \quad \vartheta_{0m} = \boldsymbol{C}_m(r_k)\boldsymbol{x}(0) + \boldsymbol{D}_m(r_k)\boldsymbol{u}_{op}(0),$$

$$\boldsymbol{Z} = \mathrm{diag}\overbrace{\begin{bmatrix} \begin{bmatrix} \boldsymbol{I} & 0 \end{bmatrix}\sum(g,f) & \begin{bmatrix} \boldsymbol{I} & 0 \end{bmatrix}\sum(g,f) & \cdots & \begin{bmatrix} \boldsymbol{I} & 0 \end{bmatrix}\sum(g,f) \end{bmatrix}}^{m}.$$

Matrices $\boldsymbol{P}(i)$ *can be calculated from*

$$\begin{bmatrix} \boldsymbol{\pi}_k^i\boldsymbol{X}(i) & \boldsymbol{U}^{\mathrm{T}}(i) & \boldsymbol{X}(i)\boldsymbol{Q}^{1/2} & \boldsymbol{Y}^{\mathrm{T}}(i)\boldsymbol{R}^{1/2} \\ * & \boldsymbol{W}(j) & 0 & 0 \\ * & * & \boldsymbol{I} & 0 \\ * & * & * & \boldsymbol{I} \end{bmatrix} \geq 0, \tag{21}$$

$$\begin{bmatrix} \boldsymbol{X}(i) & \boldsymbol{\Theta}^{\mathrm{T}}(i) \\ * & \boldsymbol{X}(j) \end{bmatrix} \geq 0, \quad j \in \mathbb{S}_{uk}^i, \tag{22}$$

where

$$\boldsymbol{\Theta}(i) = \boldsymbol{A}(i)\boldsymbol{X}(i) + \boldsymbol{B}(i)\boldsymbol{Y}(i), \quad \boldsymbol{W}(j) = \mathrm{diag}\{\boldsymbol{X}(k_1^i), \boldsymbol{X}(k_2^i), \cdots, \boldsymbol{X}(k_a^i)\},$$
$$\boldsymbol{U}^{\mathrm{T}}(i) = \begin{bmatrix} \sqrt{p_{ik_1^i}}\boldsymbol{\Theta}^{\mathrm{T}}(i) & \sqrt{p_{ik_2^i}}\boldsymbol{\Theta}^{\mathrm{T}}(i) & \cdots & \sqrt{p_{ik_a^i}}\ \boldsymbol{\Theta}^{\mathrm{T}}(i) \end{bmatrix},$$
$$\boldsymbol{\Theta}(i) = \boldsymbol{A}(i)\boldsymbol{X}(i) + \boldsymbol{B}(i)\boldsymbol{Y}(i), \quad i = 1, 2, \cdots, s.$$

The problem can be solved in two steps. First, The terminal-weighting matrix $\boldsymbol{P}(i)$*, which appears in the objective function of SDP (14), can be determined off-line by solving* $\boldsymbol{P}(i) = \boldsymbol{X}^{-1}(i)$*, if there exist matrices* $\boldsymbol{X}(i) = \boldsymbol{X}^{\mathrm{T}}(i) > 0$*,* $\boldsymbol{X}(j) = \boldsymbol{X}^{\mathrm{T}}(j) > 0$*, and* $\boldsymbol{Y}(i)\,(i \in \mathbb{S})$ *satisfying LMIs (21) and (22). Then, the control move* $\boldsymbol{u}(k) = \boldsymbol{u}_{op}(0)$ *can be directly obtained by solving SDP (14) on-line at every sampling time, if there exist scalar* $\gamma > 0$*, matrices* $\Sigma(g,f)$*,* $\Sigma(g,N)$*,* $\Sigma(r_k,1)$*,* $\Sigma(f)$*,* $\Omega(f)$*,* $\Omega_1(0)$*,* $\Omega_2(0)$*,* $\Omega_3(0)$*,* $\Omega(N)$ *and* $\bar{\boldsymbol{x}}(N)$ *satisfying LMI constraints (15)-(20).*

Proof: For notational ease in the proof, we use $\boldsymbol{x}(f)$ to represent state $\boldsymbol{x}(k+f\,|k)$ which is denoted as the predicted state from current sampling time k and similar notations will be used for all predicted quantities (for example $\Xi(f) = \Xi(k+f\,|k)$, $\Omega(N) = \Omega(k+N\,|k)$). Current time state is denoted as $\boldsymbol{x}(k) = \boldsymbol{x}(k\,|k)$.

Noticing that the state vector $\boldsymbol{x}(f)$ can be written as $\boldsymbol{x}(f) = \bar{\boldsymbol{x}}(f) + \hat{\boldsymbol{x}}(f)$ and with the aim to find the upper bound of index (8), we first try to find the upper bound of the covariance matrix of $\hat{\boldsymbol{x}}(f)$, i.e., $\Xi(f) = \mathrm{E}_k\{\hat{\boldsymbol{x}}(f)\hat{\boldsymbol{x}}^{\mathrm{T}}(f)\}$. Obviously, $\hat{\boldsymbol{x}}(f)$ represents the difference between the state $\boldsymbol{x}(f)$ and the mean of the state $\bar{\boldsymbol{x}}(f)$. Control inputs (11) and (12) can be rewritten as

$$\boldsymbol{u}(f) = \boldsymbol{u}_{op}(f) + \boldsymbol{F}(g)[\boldsymbol{x}(f) - \bar{\boldsymbol{x}}(f))], \quad \boldsymbol{u}(N) = \boldsymbol{F}(i)\bar{\boldsymbol{x}}(i) \tag{23}$$

with $\bar{\boldsymbol{x}}(f) = \mathrm{E}_k\{\boldsymbol{x}(f)\}$, $r_{k+N} = i$ for notational ease. Then the mean of the system dynamics have the form

$$\bar{\boldsymbol{x}}(f+1) = \mathrm{E}_k[\boldsymbol{x}(f+1)] = \boldsymbol{A}(g)\bar{\boldsymbol{x}}(f) + \boldsymbol{B}(g)\boldsymbol{u}_{op}(f). \tag{24}$$

It can be concluded that

$$\begin{aligned}\hat{\boldsymbol{x}}(f+1) =& \boldsymbol{x}(f+1) - \bar{\boldsymbol{x}}(f+1)\\ =& [\boldsymbol{A}(g) + \boldsymbol{B}(g)\boldsymbol{F}(g)]\hat{\boldsymbol{x}}(f) + \sum_{q=1}^{m}\{[\boldsymbol{C}_q(g) + \boldsymbol{D}_q(g)\boldsymbol{F}(g)]\hat{\boldsymbol{x}}(f)\}\, w_q(f)\\ &+ \sum_{q=1}^{m}[\boldsymbol{C}_q(g)\bar{\boldsymbol{x}}(f) + \boldsymbol{D}_q(g)\boldsymbol{u}_{op}(f)]w_q(f).\end{aligned}$$

From the above equation, we have

$$\begin{aligned}\Xi(f+1) =& \mathrm{E}_k\left\{\hat{\boldsymbol{x}}(f+1)\hat{\boldsymbol{x}}^{\mathrm{T}}(f+1)\right\}\\ =& [\boldsymbol{A}(g) + \boldsymbol{B}(g)\boldsymbol{F}(g)]\Xi(f)\,[\boldsymbol{A}(g) + \boldsymbol{B}(g)\boldsymbol{F}(g)]^{\mathrm{T}}\\ &+ \sum_{q=1}^{m}\left\{[\boldsymbol{C}_q(g) + \boldsymbol{D}_q(g)\boldsymbol{F}(g)] \times \Xi(f)[\boldsymbol{C}_q(g) + \boldsymbol{D}_q(g)\boldsymbol{F}(g)]^{\mathrm{T}}\right\}\\ &+ \sum_{q=1}^{m}\left\{[\boldsymbol{C}_q(g)\boldsymbol{x}(f) + \boldsymbol{D}_q(g)\boldsymbol{u}_{op}(f)][\boldsymbol{C}_q(g)\boldsymbol{x}(f) + \boldsymbol{D}_q(g)\boldsymbol{u}_{op}(f)]^{\mathrm{T}}\right\}.\end{aligned} \tag{25}$$

Equation (25) can be viewed as a constraint of optimization problem Op1. To obtain a feasible optimized solution, it can be transformed into a matrix inequality form. Actually, the solution of the modified "inequality" (25) gives an upper bound on $\Xi(f)$. Under the assumption of $\Xi(f) > 0$, and the notation of

$$\boldsymbol{\Psi}(g,f) = \boldsymbol{F}(g)\Xi(f), \quad \Sigma(g,f) = \begin{bmatrix}\Xi(f)\\ \boldsymbol{\Psi}(g,f)\end{bmatrix}, \quad \Xi(f) = \begin{bmatrix}\boldsymbol{I} & 0\end{bmatrix}\Sigma(g,f),$$

and replacing the equality sign "=" in (25) with a matrix inequality sign "≥", we obtain LMI (15) for $f = 1, 2, \ldots, N-1$.

For $f = 0$, we have $\boldsymbol{x}(0) - \bar{\boldsymbol{x}}(0) = 0$ and $\boldsymbol{u}(0) = \bar{\boldsymbol{u}}(0) = \boldsymbol{u}_{op}(0)$

$$\hat{\boldsymbol{x}}(1) = \sum_{q=1}^{m}[\boldsymbol{C}_q(r_k)\bar{\boldsymbol{x}}(0) + \boldsymbol{D}_q(r_k)\boldsymbol{u}_{op}(0)]w_q(0),$$

$$\Xi(1) = \sum_{q=1}^{m}[\boldsymbol{C}_q(r_k)\bar{\boldsymbol{x}}(0) + \boldsymbol{D}_q(r_k)\boldsymbol{u}_{op}(0)][\boldsymbol{C}_q(r_k)\bar{\boldsymbol{x}}(0) + \boldsymbol{D}_q(r_k)\boldsymbol{u}_{op}(0)]^{\mathrm{T}}.$$

Similarly as the acquisition of LMI (15), we obtain LMI (16).

Second, we try to find upper bound of the performance index $J(k)$, and prove that Op1 can be transferred into an SDP. We put an upper bound $\Omega(f)$ to $\mathrm{E}_k\left\{\begin{bmatrix}\boldsymbol{x}(f)\\ \boldsymbol{u}(f)\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(f)\\ \boldsymbol{u}(f)\end{bmatrix}^{\mathrm{T}}\right\}$, and then we have

$$\begin{aligned}\Omega(f) \geq& \mathrm{E}_k\left\{\begin{bmatrix}\boldsymbol{x}(f)\\ \boldsymbol{u}(f)\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(f)\\ \boldsymbol{u}(f)\end{bmatrix}^{\mathrm{T}}\right\}\\ =& \begin{bmatrix}\bar{\boldsymbol{x}}(f)\\ \boldsymbol{u}_{op}(f)\end{bmatrix}\begin{bmatrix}\bar{\boldsymbol{x}}(f)\\ \boldsymbol{u}_{op}(f)\end{bmatrix}^{\mathrm{T}} + \begin{bmatrix}\Xi(f)\\ \boldsymbol{\Psi}(g,f)\end{bmatrix}\Xi^{-1}(f)\begin{bmatrix}\Xi(f)\\ \boldsymbol{\Psi}(g,f)\end{bmatrix}^{\mathrm{T}}.\end{aligned} \tag{26}$$

Denoting $\Sigma(f) = \begin{bmatrix} \bar{\boldsymbol{x}}(f) \\ \boldsymbol{u}_{op}(g,f) \end{bmatrix}$ and applying Schur complement, this relationship can be written as LMI (17) for $f = 1, 2, \dots, N-1$.

For $f = 0$, we have $\Omega(0) \geq \begin{bmatrix} \bar{\boldsymbol{x}}(0) \\ \boldsymbol{u}_{op}(0) \end{bmatrix}$. Because $\bar{\boldsymbol{x}}(0) = \boldsymbol{x}(0)$ and $\boldsymbol{u}_{op}(0)$ must be obtained and calculated on-line, respectively, we cannot deal $\sum(0)$ as an LMI variable and then we decompose matrix $\Omega(0)$ into $\begin{bmatrix} \Omega_1(0) & \Omega_2(0) \\ * & \Omega_3(0) \end{bmatrix}$ to construct LMI (18) in a component form.

For $f = N$, we have

$$\Omega(N) \geq \mathrm{E}_k[\boldsymbol{x}(N)\boldsymbol{x}^{\mathrm{T}}(N)] = \bar{\boldsymbol{x}}(N)\bar{\boldsymbol{x}}^{\mathrm{T}}(N) + \Xi(N).$$

The above inequality can be transferred into LMI (19) by using Schur complement. According to the inequality (26), we have

$$J_1(k, N-1) \leq \sum_{f=0}^{N-1} tr[\boldsymbol{H}\Omega(f)], \quad J_2(k,N) \leq tr[\boldsymbol{P}(i)\Omega(f)], \quad \text{for } i = 1, 2, \dots, s.$$

Then the upper bound of index $J(k)$ can be obtained as the term established in the minimization objective (14).

As the constraint (7) is considered, we have

$$\begin{bmatrix} \boldsymbol{x}(f) \\ \boldsymbol{u}(f) \end{bmatrix}^{\mathrm{T}} \boldsymbol{G}_l \begin{bmatrix} \boldsymbol{x}(f) \\ \boldsymbol{u}(f) \end{bmatrix} = tr\left\{ \begin{bmatrix} \boldsymbol{x}(f) \\ \boldsymbol{u}(f) \end{bmatrix}^{\mathrm{T}} \boldsymbol{G}_l \begin{bmatrix} \boldsymbol{x}(f) \\ \boldsymbol{u}(f) \end{bmatrix} \right\} \leq tr\left\{\boldsymbol{G}_l \Omega(f)\right\},$$
$$\text{for } f = 0, 1, \dots, N-1.$$

Put an upper bound γ on $tr\{\boldsymbol{G}_l\Omega(f)\}$, we obtain $tr\{\boldsymbol{G}_l\Omega(f)\} \leq \gamma$ which immediately implies the second item of (20) holds. After that, the constraint (7) will be satisfied if the first item of (20) holds. Then, the constraint is established in LMI form.

It should be noted that at time $k+N$, we do not require the constraint (20) because it will be satisfied naturally by using the terminal invariant ellipsoid. It will be fully discussed in the following section.

Next, we try to obtain cost monotonicity condition, which leads to mean square stability of jump bilinear system (1) under control (23). The mean square stabilizability will be proved in the sequel section. Assume jump mode r_{k+N} and r_{k+N+1} can be represented by i and j respectively, variables $\boldsymbol{u}_{op}^*(f)$ and $\boldsymbol{x}^*(f)$ are optimal solutions for the performance index $J(k+f+1)$ and variables $\boldsymbol{u}_{op}^\circ(f)$ and $\boldsymbol{x}^\circ(f)$ are optimal for $J(k+f)$. It is noted that if $\boldsymbol{u}_{op}^\circ(f)$ and $\boldsymbol{x}^\circ(f)$ are optimal solutions at time $k+f$ and they are also feasible ones at time $k+f+1$. At time $k+f+1$, the optimal solutions $\boldsymbol{u}_{op}^*(f)$ and $\boldsymbol{x}^*(f)$ are always smaller than the feasible ones $\boldsymbol{u}_{op}^\circ(f)$ and $\boldsymbol{x}^\circ(f)$. If we replace $\boldsymbol{u}_{op}^*(f)$ and $\boldsymbol{x}^*(f)$ by $\boldsymbol{u}_{op}^\circ(f)$ and $\boldsymbol{x}^\circ(f)$ on the receding horizon $[k, k+f-1]$, we have

$$\begin{aligned} J^*(k+f+1) &= \sum_{f=0}^{N} \left[(\bar{\boldsymbol{x}}^*(f))^{\mathrm{T}}\boldsymbol{Q}\bar{\boldsymbol{x}}^*(f) + (\boldsymbol{u}_{op}^*(f))^{\mathrm{T}}\boldsymbol{R}\boldsymbol{u}_{op}^*(f)\right] \\ &\quad + (\bar{\boldsymbol{x}}^*(N+1))^{\mathrm{T}}\boldsymbol{P}(j)\bar{\boldsymbol{x}}^*(N+1) \\ &\leq \sum_{f=0}^{N-1}[(\bar{\boldsymbol{x}}^\circ(f))^{\mathrm{T}}\boldsymbol{Q}\bar{\boldsymbol{x}}^\circ(f) + (\boldsymbol{u}_{op}^\circ(f))^{\mathrm{T}}\boldsymbol{R}\boldsymbol{u}_{op}^\circ(f)] + (\bar{\boldsymbol{x}}^\circ(N))^{\mathrm{T}}\boldsymbol{Q}\bar{\boldsymbol{x}}^\circ(N) \\ &\quad + (\bar{\boldsymbol{x}}^\circ(N))^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}(i)\boldsymbol{R}\boldsymbol{F}(i)\bar{\boldsymbol{x}}^\circ(N) \end{aligned}$$

$$+ (\bar{\boldsymbol{x}}^\circ(N))^{\mathrm{T}} \phi^{\mathrm{T}}(i) \left(\sum_{j \in \mathbb{S}} p_{ij} \boldsymbol{P}(j) \right) \phi(i) \bar{\boldsymbol{x}}^\circ(N), \tag{27}$$

where $\phi(i) = \boldsymbol{A}(i) + \boldsymbol{B}(i)\boldsymbol{F}(i)$.

$$J^*(k+f) = \sum_{f=0}^{N-1} [(\bar{\boldsymbol{x}}^\circ(f))^{\mathrm{T}} \boldsymbol{Q} \bar{\boldsymbol{x}}^\circ(f) + (\boldsymbol{u}_{op}^\circ(f))^{\mathrm{T}} \boldsymbol{R} \boldsymbol{u}_{op}^\circ(f)] + (\bar{\boldsymbol{x}}^\circ(N))^{\mathrm{T}} \boldsymbol{P}(i) \bar{\boldsymbol{x}}^\circ(N). \tag{28}$$

Because

$$\begin{aligned}
&(\bar{\boldsymbol{x}}^\circ(N))^{\mathrm{T}} \phi^{\mathrm{T}}(i) \left(\sum_{j \in \mathbb{S}} p_{ij} \boldsymbol{P}(j) \right) \phi(i) \bar{\boldsymbol{x}}^\circ(N) \\
&= (\bar{\boldsymbol{x}}^\circ(N))^{\mathrm{T}} \phi^{\mathrm{T}}(i) \pi_k^i \boldsymbol{P}(j) \phi(i) \bar{\boldsymbol{x}}^\circ(N) + \sum_{j \in \mathbb{S}_{uk}^i} p_{ij} \left[(\bar{\boldsymbol{x}}^\circ(N))^{\mathrm{T}} \phi^{\mathrm{T}}(i) \boldsymbol{P}(j) \phi(i) \bar{\boldsymbol{x}}^\circ(N) \right]
\end{aligned}$$

and

$$(\bar{\boldsymbol{x}}^\circ(N))^{\mathrm{T}} \boldsymbol{P}(i) \bar{\boldsymbol{x}}^\circ(N) = \pi_k^i \left[(\bar{\boldsymbol{x}}^\circ(N))^{\mathrm{T}} \boldsymbol{P}(i) \bar{\boldsymbol{x}}^\circ(N) \right] + \sum_{j \in \mathbb{S}_{uk}^i} p_{ij} \left[(\bar{\boldsymbol{x}}^\circ(N))^{\mathrm{T}} \boldsymbol{P}(i) \bar{\boldsymbol{x}}^\circ(N) \right],$$

it follows from (27) and (28) that the cost monotonicity condition should satisfy

$$\begin{aligned}
\delta J^* &= J^*(k+f+1) - J^*(k+f) \\
&= (\bar{\boldsymbol{x}}^\circ(N))^{\mathrm{T}} \Big\{ \boldsymbol{Q} + \boldsymbol{F}^{\mathrm{T}}(i) \boldsymbol{R} \boldsymbol{F}(i) + \phi^{\mathrm{T}}(i) \pi_k^i \boldsymbol{P}(j) \phi(i) \\
&\quad - \pi_k^i \boldsymbol{P}(i) + \sum_{j \in \mathbb{S}_{uk}^i} p_{ij} [\phi^{\mathrm{T}}(i) \boldsymbol{P}(j) \phi(i) - \boldsymbol{P}(i)] \Big\} \bar{\boldsymbol{x}}^\circ(N) \leq 0
\end{aligned} \tag{29}$$

If (29) is expected to be held, then we only need to have

$$\boldsymbol{Q} + \boldsymbol{F}^{\mathrm{T}}(i) \boldsymbol{R} \boldsymbol{F}(i) + \phi^{\mathrm{T}}(i) \pi_k^i \boldsymbol{P}(j) \phi(i) - \pi_k^i \boldsymbol{P}(i) \leq 0 \tag{30}$$

and

$$\phi^{\mathrm{T}}(i) \boldsymbol{P}(j) \phi(i) - \boldsymbol{P}(i) \leq 0, \quad \forall j \in \mathbb{S}_k^i \tag{31}$$

hold. Pre- and post multiplying $\boldsymbol{X}(i) = \boldsymbol{P}^{-1}(i)$ on both sides of (30) and (31), denoting $\boldsymbol{\Theta}(i) = \boldsymbol{A}(i)\boldsymbol{X}(i) + \boldsymbol{B}(i)\boldsymbol{Y}(i)$, and using Schur complement, we obtain sufficient LMI conditions (21) and (22) from (30) and (31). Given a receding horizon length N and an initial state $\boldsymbol{x}_0$, assume state $\boldsymbol{x}(k)$ can be obtained at every sampling time, the on-line optimizations can be computed in a tractable manner by solving SDP (14) subject to LMIs (15)-(20). This completes the proof.

Remark 3.1. *In the diagonal of (15) and (17), there are some matrices, such as* $\begin{bmatrix} \boldsymbol{I} & 0 \end{bmatrix}$ $\Sigma(g,f)$*, and they are not symmetric ones. However, due to* $\begin{bmatrix} \boldsymbol{I} & 0 \end{bmatrix} \Sigma(g,f) = \boldsymbol{\Xi}(f)$ *and* $\boldsymbol{\Xi}(f)$ *are symmetric matrices, then we have*

$$\begin{bmatrix} \boldsymbol{I} & 0 \end{bmatrix} \Sigma(g,f) = \frac{1}{2} \left(\begin{bmatrix} \boldsymbol{I} & 0 \end{bmatrix} \Sigma(g,f) + \Sigma^{\mathrm{T}}(g,f) \begin{bmatrix} \boldsymbol{I} \\ 0 \end{bmatrix} \right). \tag{32}$$

This means inequalities (15)-(17) can be solved as LMI constraints.

Remark 3.2. *In contrast with [11], the terminal-weighting matrix is directly solved via cost monotonicity condition, which implies the mean square stability. The stability analysis will be further discussed in the following part based on the feasibility analysis.*

Remark 3.3. *It should be pointed out that our RHC strategy is not completely the same as the one proposed by [11]. The predictive controller (11) is not the real one that applied to the real plant but used to obtain a better performance because the optimized quadratic performance index approaches to a smaller level by increasing the predictive horizon length [7]. Controller (12) is not applied to the real plant and it is a theoretically fictitious feedback control move to guarantee the mean square stability of JBSS (1) over infinite horizon. That is to say in the predictive horizon* $[k, k+N-1]$*, optimized control sequence* $\begin{bmatrix} \boldsymbol{u}^{\mathrm{T}}(k) & \cdots & \boldsymbol{u}^{\mathrm{T}}(k+N-1) \end{bmatrix}^{\mathrm{T}}$ *is used to drive the state into the terminal invariant set* $\varepsilon(\boldsymbol{P}(r_{k+N}))$*; out of the predictive horizon, feed back control* $\boldsymbol{F}(r_{k+N})\bar{\boldsymbol{x}}(k+N\,|k)$ *is theoretically used to guarantee the mean square stability. The first-step control move* $\boldsymbol{u}(k)$ *is applied to the real plant. The feasibility and stabilizability problem will be discussed in the following sections.*

Remark 3.4. *The SDP (14) gives a general framework when the RHC for stochastic switched linear systems is considered. If* $\mathbb{S}^i_{uk} = \phi$*,* $\mathbb{S}^i_k = \mathbb{S}$*, JBSS (1) degenerates to a usual JBSS with completely known TPs. If* $\mathbb{S}^i_k = \phi$*,* $\mathbb{S}^i_{uk} = \mathbb{S}$*, JBSS (1) degenerates to a switched bilinear systems under arbitrary switching.*

4. **Feasibility and Mean Square Stabilizability.** Since the controllers (11) and (12) are calculated over different time horizon, there is no direct connection between cost $J(k)$ and $J(k+1)$. In order to prove the mean square stabilizability of RHC scheme proposed in Section 3, the feasibility of SDP (14) at every sampling time needs to be established first by observing the cost monotonicity condition (21) and (22).

From (21) and (22), we have (29) and it implies that

$$\begin{aligned} \mathrm{E}_k\{\boldsymbol{x}^{\mathrm{T}}(N)\boldsymbol{P}(i)\boldsymbol{x}(N)\} \geq{}& \mathrm{E}_k\{\boldsymbol{x}^{\mathrm{T}}(N+1)\boldsymbol{P}(j)\boldsymbol{x}(N+1) + \boldsymbol{x}^{\mathrm{T}}(N)\boldsymbol{Q}\boldsymbol{x}(N) \\ &+ \boldsymbol{x}^{\mathrm{T}}(N)\boldsymbol{F}^{\mathrm{T}}(i)\boldsymbol{R}\boldsymbol{F}(i)\boldsymbol{x}(N)\}. \end{aligned} \tag{33}$$

Follows from (33), we conclude that

$$\begin{aligned} &\mathrm{E}_k\left\{\boldsymbol{x}^{\mathrm{T}}(k+N\,|k)\boldsymbol{P}(r_{k+N})\boldsymbol{x}(k+N\,|k)\right\} \\ \geq{}& \mathrm{E}_k\left\{\boldsymbol{x}^{\mathrm{T}}(k+N+1\,|k)\boldsymbol{P}(r_{k+N+1})\boldsymbol{x}(k+N+1\,|k)\right\} \end{aligned} \tag{34}$$

Inequality (34) implies that if we have an ellipsoid $\varepsilon(\boldsymbol{P}(r_{k+N})) : \boldsymbol{x}^{\mathrm{T}}(k+N|k)\boldsymbol{P}(r_{k+N})\boldsymbol{x}(k+N|k) \leq \beta$ (where β is a given scalar), then this ellipsoid is invariant. It can be noted as $\varepsilon(\boldsymbol{P}(r_{k+N}))$ which depends on the jump mode and it is a stochastic invariant ellipsoid. Obviously, the terminal invariant ellipsoid $\varepsilon(\boldsymbol{P}(r_{k+N}))$ should be located inside the constrained region ϖ. Therefore, we have answered why constraint (20) is not considered at time $k+N$.

Assume that the receding horizon length is fixed as N and the control pair $(\boldsymbol{u}_{op}(f-1), \boldsymbol{F}(r_{k+f-1}))$ (where $f = 1, 2, \ldots, N-1$) is found at time $k-1$. At time k, we compute another optimal control pair $(\boldsymbol{u}_{op}(f), \boldsymbol{F}(g))$ which is better than the proceeding control pair in the sense that

$$J(\boldsymbol{x}(f), k, \boldsymbol{u}(f), N) \leq J(\boldsymbol{x}(f-1), k-1, \boldsymbol{u}(f-1), N), \tag{35}$$

where

$$J(\boldsymbol{x}(f), k, \boldsymbol{u}(f), N) = \mathrm{E}_k\left\{\sum_{f=1}^{N-1}\begin{bmatrix} \boldsymbol{x}(k+f|k) \\ \boldsymbol{u}(k+f|k) \end{bmatrix}^{\mathrm{T}}\boldsymbol{H}\begin{bmatrix} \boldsymbol{x}(k+f\,|k) \\ \boldsymbol{u}(k+f\,|k) \end{bmatrix}\right\}. \tag{36}$$

$\boldsymbol{u}(f-1)$ is the optimal control move at time $k-1$ and it can be viewed as an admissible control move for cost $J(\boldsymbol{x}(f), k, \boldsymbol{u}(f), N)$ on the interval $[k, k+N-1]$.

If we denote predictive step $f = 0$ in (9), then we obtain real time cost

$$J(\boldsymbol{x}(k), k, \boldsymbol{u}(k), N) = \boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{Q}\boldsymbol{x}(k) + \boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{R}\boldsymbol{u}(k). \tag{37}$$

The control $\boldsymbol{u}(k)$ is applied to the real system and it always exists since there is at least one feasible solution if N is fixed. It will be proved in the next theorem.

Theorem 4.1. *For a given predictive horizon length $N > 1$, there exists a control $\boldsymbol{u}(k)$ such that the following relationship is satisfied:*

$$J(\boldsymbol{x}(k), k, \boldsymbol{u}(k), N) \leq J(\boldsymbol{x}(k-1), k-1, \boldsymbol{u}(k-1), N) \tag{38}$$

and there exists a limited constant α satisfying

$$J(\boldsymbol{x}(k), k, \boldsymbol{u}(k), N) - J(\boldsymbol{x}(k+1), k+1, \boldsymbol{u}(k+1), N) \geq \alpha \tag{39}$$

for all k such that once current time state $\boldsymbol{x}(k)$ is inside ellipsoid $\varepsilon(\boldsymbol{P}(r_{k+N}))$ then the state $\boldsymbol{x}(k+1)$ is also inside $\varepsilon(\boldsymbol{P}(r_{k+N}))$.

Proof: The statement of (38) can be immediately obtained since (35) is established. If we take control $\boldsymbol{u}(k)$ and state $\boldsymbol{x}(k)$ as feasible solutions at time $k+1$ on the horizon $[k+1, k+N]$, we have

$$\begin{aligned} & J(\boldsymbol{x}(k), k, \boldsymbol{u}(k), N) - J(\boldsymbol{x}(k+1), k+1, \boldsymbol{u}(k+1), N) \\ \geq & \, \boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{Q}\boldsymbol{x}(k) + \boldsymbol{u}^{\mathrm{T}}(k)\boldsymbol{R}\boldsymbol{u}(k) \geq \inf\left\{\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{Q}\boldsymbol{x}(k)\,|\boldsymbol{x}(k) \in \varepsilon(\boldsymbol{P}(r_{k+N}))\right\} \\ \geq & \, \frac{\lambda_{\min}(\boldsymbol{Q})}{\lambda_{\max}(\boldsymbol{P}(r_{k+N}))}\beta = \alpha. \end{aligned}$$

The above inequality implies that the real time cost (37) decreased, so we can conclude that once current time state $\boldsymbol{x}(k)$ is inside ellipsoid $\varepsilon(\boldsymbol{P}(r_{k+N}))$ then the state $\boldsymbol{x}(k+1)$ is also inside $\varepsilon(\boldsymbol{P}(r_{k+N}))$. This completes the proof.

According to Theorem 4.1, the existence of control move $\boldsymbol{u}(k)$ for a given finite predictive horizon length N guarantees the feasibility of SDP (14) at every sampling time and next we direct our attention to prove the mean square stabilizability of JBSS (1) under the cost monotonicity condition (20) and (21).

Theorem 4.2. *If the optimization problem (14) has a solution at the initial time, then the RHC law $\boldsymbol{u}(k)$, which stems from the SDP (14) at every sampling time k, stabilizes the discrete time JBSS (1) for all the time that satisfies the constraints on the input and state.*

Proof: As mentioned in Section 4, from (21) and (22), we have (33) and it follows that

$$\mathrm{E}_k\left\{\boldsymbol{x}^{\mathrm{T}}(k+N\,|k)\boldsymbol{Q}\boldsymbol{x}(k+N\,|k) + \boldsymbol{x}^{\mathrm{T}}(k+N\,|k)\boldsymbol{F}^{\mathrm{T}}(i)\boldsymbol{R}\boldsymbol{F}(i)\boldsymbol{x}(k+N\,|k)\right\} \to 0,$$

as $k \to \infty$. It is easy to obtain that

$$\mathrm{E}_k\{\boldsymbol{x}^{\mathrm{T}}(k+N\,|k)\boldsymbol{\Gamma}(i)\boldsymbol{x}(k+N\,|k)\} \to 0, \text{ as } k \to \infty, \tag{40}$$

where $\boldsymbol{\Gamma}(i) = \boldsymbol{Q} + \boldsymbol{F}^{\mathrm{T}}(i)\boldsymbol{R}\boldsymbol{F}(i)$. From [14], we get

$$\mathrm{E}_k\{\boldsymbol{x}^{\mathrm{T}}(k+N\,|k)\boldsymbol{\Gamma}(i)\boldsymbol{x}(k+N\,|k)\} \geq \boldsymbol{\Gamma}\mathrm{E}_k\{\boldsymbol{x}^{\mathrm{T}}(k+N\,|k)\boldsymbol{x}(k+N\,|k)\}, \tag{41}$$

with $\boldsymbol{\Gamma} = \min\limits_{r_k=1,2,\ldots,s} \lambda_{\min}(\boldsymbol{\Gamma}(i))$.

Since $\boldsymbol{\Gamma} > 0$, we have $\mathrm{E}_k\{\boldsymbol{x}^{\mathrm{T}}(k+N\,|k)\boldsymbol{x}(k+N\,|k)\} \to 0$ as $k \to \infty$ hold, which directly implies $\mathrm{E}_k\{\boldsymbol{x}^{\mathrm{T}}(k)\boldsymbol{x}(k)\} \to 0$ as $k \to \infty$. Combined with Definition 2.1, JBSS (1) is mean square stable under the control law (23), which are equivalent to (11) and (12). This completes the proof.

Remark 4.1. *The terminal invariant ellipsoid $\varepsilon(\boldsymbol{P}(r_{k+N}))$ is employed to relax the constraint condition, i.e., at time $k+N$ and the time followed, the state is limited inside the ellipsoid, which always locates inside the constraint region. The feasibility can be guaranteed without extra state-only constraint. A fictitious feedback control law $\boldsymbol{F}(i)$, which is theoretically employed inside the invariant ellipsoid, simplifies the feasibility and the mean square stability analysis.*

5. **Numerical Example.** In this section, a numerical example is given to show the validity of the RHC strategy. Consider the MJLS (1) with 4 operation modes and the following data [3]:

$$\boldsymbol{A}_1 = \begin{bmatrix} 0.32 & -0.40 \\ 0.80 & -0.80 \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} 0.08 & -0.26 \\ 0.80 & -1.12 \end{bmatrix},$$

$$\boldsymbol{A}_3 = \begin{bmatrix} 0.16 & -0.08 \\ 0.80 & -0.96 \end{bmatrix}, \quad \boldsymbol{A}_4 = \begin{bmatrix} 0.48 & -0.18 \\ 0.80 & -0.88 \end{bmatrix},$$

$$\boldsymbol{B}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \boldsymbol{B}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \boldsymbol{B}_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$\boldsymbol{B}_4 = \begin{bmatrix} 0.8 \\ -1 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

Assume the one-step partly unknown transition probability matrix is

$$\boldsymbol{\Pi} = \begin{bmatrix} 0.3 & ? & 0.1 & ? \\ ? & ? & 0.3 & 0.2 \\ ? & 0.1 & ? & 0.3 \\ 0.2 & ? & ? & ? \end{bmatrix}.$$

The weighting matrix is chosen as $\boldsymbol{Q}_1 = \boldsymbol{Q}_2 = \boldsymbol{Q}_3 = \boldsymbol{Q}_4 = \boldsymbol{I} \in \mathrm{R}^{2\times 2}$, $\boldsymbol{R}_1 = \boldsymbol{R}_2 = \boldsymbol{R}_3 = \boldsymbol{R}_4 = 1$. We consider linear constraint $\begin{bmatrix} -1 & 1 \end{bmatrix} \boldsymbol{x} \leq -1.5$ and quadratic constraint $\boldsymbol{x}^{\mathrm{T}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{x} \leq 1.5$, respectively. The multiplicative stochastic noise $w(k)$ follows $0 \sim 1$ distribution and is independent identically distributed. The initial state is set to be $\boldsymbol{x}_0 = \begin{bmatrix} -2 & 1 \end{bmatrix}^{\mathrm{T}}$ and the initial mode is $r_0 = 1$. The receding horizon length is taken as $N = 5$. Simulation time is chosen as 20 time units and each unit is taken as $\mathrm{Ts} = 1$. The mode path from time step 0 to the time step 20 is generated randomly, 20 times. The quadratic performance index is taken as

$$\sum_{f=0}^{4} \left[\|\boldsymbol{x}(k+f\,|k)\|_{\boldsymbol{Q}}^2 + \|\boldsymbol{u}(k+f\,|k)\|_{\boldsymbol{R}}^2 \right] + \|\boldsymbol{x}(k+5\,|k)\|_{\boldsymbol{P}(i)}^2 .$$

The terminal-weighting matrix $\boldsymbol{P}(i)$ can first be obtained by solving LMIs (21) and (22) off-line and we have

$$\boldsymbol{P}(1) = \begin{bmatrix} 5.5998 & -2.7073 \\ -2.7073 & 5.1607 \end{bmatrix}, \quad \boldsymbol{P}(2) = \begin{bmatrix} 4.2424 & -3.0560 \\ -3.0560 & 6.8165 \end{bmatrix},$$

$$\boldsymbol{P}(3) = \begin{bmatrix} 5.4362 & -3.5441 \\ -3.5441 & 7.3135 \end{bmatrix}, \quad \boldsymbol{P}(4) = \begin{bmatrix} 7.7675 & -2.1396 \\ -2.1396 & 7.2321 \end{bmatrix}.$$

Then we solve SDP (14) subject to LMIs (15)-(20) at every sampling time to regulate the system into the mean square stable sense while optimizing quadratic performance index and satisfying state/input constraint under a given mode evolution. Only the first element of the control law (11), i.e., $\boldsymbol{u}_{op}(k)$, or equivalently $\boldsymbol{u}_{op}(0)$ in (14), is applied to the real plant. The simulation results are shown in Figures 1 and 2. The real line represents state trajectory and the dash-dot line represents the constraint borderline.

Figure 1 shows the state trajectory of the system when the RHC strategy is adopted in the case of linear constraint and Figure 2 shows the case under quadratic constraint. In both cases, the controlled system is mean square stable. In Figure 2, it is quite clear to see, the initial state is out of the constraint region and the RHC strategy drives the state to meet the constraint and finally converges to zero in the mean square sense.

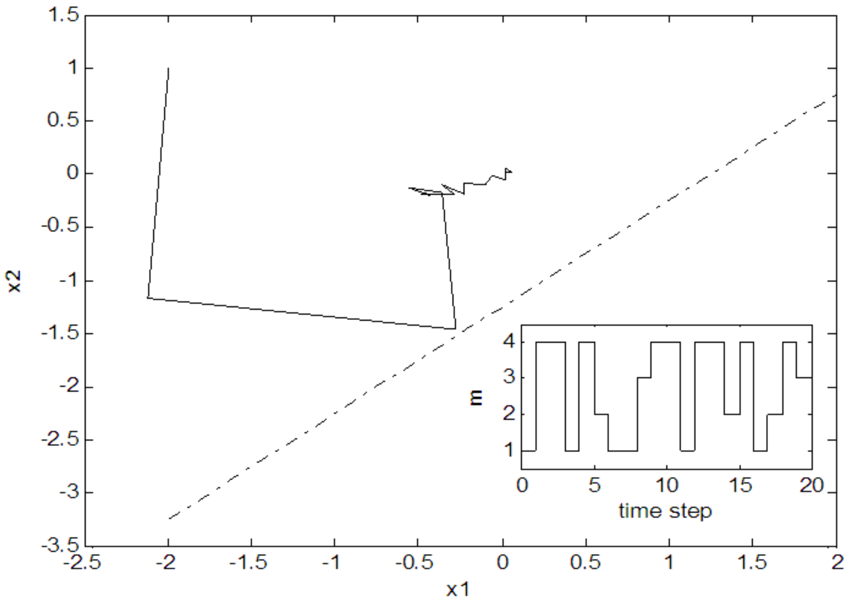

FIGURE 1. State trajectory with linear constraint

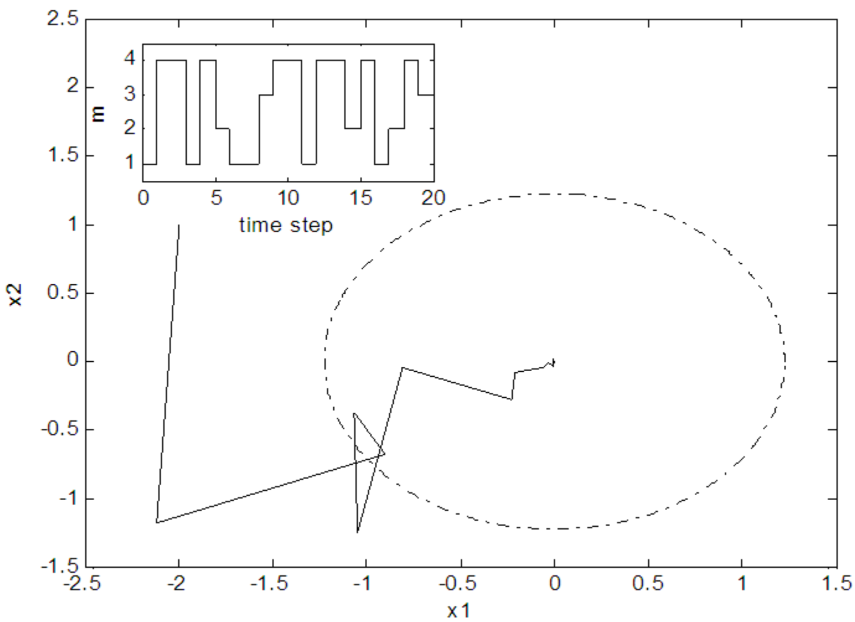

FIGURE 2. State trajectory with quadratic constraint

6. **Conclusions.** In this work, we have developed a tractable RHC strategy for JBSS with partly unknown TPs. The system under consideration covers usual JBSS and switched bilinear systems under arbitrary switching. We have focused on the design of the controller constructed by open loop optimized control move plus linear feedback control, which allows the optimization problem to be solved as an SDP problem. Furthermore, the terminal-weighting matrix is obtained from off-line LMI computation. The terminal invariant set together with a fictitious feedback control is introduced to relax the constraint condition and guarantees the mean square stabilizability. A numerical example has illustrated the applicability of the proposed scheme.

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